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Statistical Geometry of Multiscale Isolines. Part I. Fractal Dimension of Coastlines and Number-Area Rule for Islands

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Abstract

The fractal and percolation properties of isolines of a random function of two variables $\psi(x,y)$ (coastlines on a random relief) are studied. Previously [1,2] such a problem has been discussed for a function possessing a single characteristic spatial scale λ, with the help of percolation theory. In the present paper the previous approach is extended to the case of a multiscale random function with a power spectrum of scales, $\psi_\lambda \propto \lambda^H$ in a wide range of wavelengths $\lambda_0 < \lambda < \lambda_m$. It is found that the coastlines pattern differs significantly from a monoscale landscape provided that $-3/4 < H < 1$. The expression for the fractal dimension of an individual coastline is derived: $d_h = (10-3H)/7$, which varies in the interval from $d_h = 1$, at $H = 1$, to $d_h = 7/4$, the monoscale value, at $H = -3/4$. The self-similar behavior of the contours $\psi(x,y) = h$ on scales less than the

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correlation length ξ_h is used to derive the number-area distribution for islands, which can be expressed through fractal dimensionalities. A comparison of the theoretical results with geographical data is presented.

I. Introduction

The patterns of coastlines of continents and islands are surprising in their whimsicality and diversity. Behind the apparent chaos in the spread of the border between the sea and the land, however, an observer still can identify underlying regularities. One of them is that the length of a coastline L depends on the map resolution (or, say, measuring dividers' opening) λ as a power, $L \propto \lambda^{1-d_h}$, with a well defined exponent d_h ("the fractal dimension") for most seaboards. This observation, that there is no "actual" coast length [3], became one of stimuli for introducing the concept of fractals [4] which is now widely used in various branches of knowledge [5].

Coastlines represent isolines of a function of two variables describing the Earth's relief: $z = \psi(x, y)$. In the present paper we undertake a consistent study of *statistical topography*, meaning the theory of level lines of a random function. The model proposed here differs from the "Poisson relief" [4] or "Mandelbrot's mountains" [6] in that our starting point is not a graph of a (fractional) Brownian function but rather a smooth monoscale relief related to the 2D continuous percolation problem. Being a generalization of the models used in Refs. [4,6], our model enables us to advance further the study of topographic properties, particularly to determine the fractal dimension of a separate isoline d_h and the correlation length (maximum island size for the given sea level h) ξ_h.

Generally, the study of the properties of iso-sets of a random function arises in a variety of physical problems. Among them are turbulent diffusion in two dimensions [1,2] (where stream lines coincide with the isolines of a stream function), a metal-dielectric transition in doped semiconductors [7] (where surfaces of constant potential embrace classically permitted domains for charge carriers), the quantum Hall effect [8], and many others.

In two-dimensional case, the "Flood-In-The-Hills" argument is well known (cf. [7]). This consideration implies that at the water level $z = h < 0$ (here and further we put the plane

average $\langle\psi(x,y)\rangle = 0$ which makes no restriction on generality) one can observe only lakes in an infinite land (see Fig. 1a) , while at $h > 0$ there are solely islands in an infinite ocean (see Fig. 1c). Hence, only for the critical level $h = 0$, does there exist an infinitely long coastline (Fig. 1b). Such a behavior is due to the fact that for a generic 2D profile the simultaneous existence of both infinite marine and land paths is impossible, while the substitution $\psi(x,y) \to -\psi(x,y)$ results in the same problem.

The full set of isolines $\psi(x,y) = h > 0$ at a specified h includes both shores of ocean islands (free isolines) and those of lakes in islands, islands in the lakes, etc. (enclosed isolines). See Fig. 1b). The diameters of both are bounded from above by the correlation length ξ_h tending to infinity at $h \to 0$. One of the most fruitful ideas introduced in the theory of fractals is that of the self-similarity [4]. In the case of isolines it can be formulated as follows. Let λ_0 be some characteristic (internal) length of $\psi(x,y)$, $\lambda_0 \ll \xi_h$. Then at the scales λ, such as $\lambda_0 \ll \lambda \ll \xi_h$, the pattern of isolines $\psi(\mathbf{r}) = h$ is statistically invariant with respect to the scale transformation $\mathbf{r} \to c\mathbf{r}$, this same being also true for the subset of the free isolines. In physical terms the self-similarity hypothesis is a consequence of the absence of any intermediate characteristic scale; its application goes back, for example, to the Kolmogorov spectrum in the inertial range.

Below, in this article, we shall argue both in terms of a function $\psi(x,y)$ and of a 2D incompressible flow $\mathbf{v}(x,y) = \nabla\psi(x,y) \times \hat{z}$, whose stream lines are the isolines of $\psi(x,y)$. The latter presentation of the problem is equivalent to the former one, while making in some cases our arguments more clear. Besides, in a companion paper [9], this ansatz is given a more literal sense by considering a passive transport in the multiscale flow $\mathbf{v}(x,y)$ in the presence of a small background diffusion.

Previously [1,2] we were concerned with the statistical geometry of isolines of a random function $\psi(x,y)$ with the only characteristic spatial scale λ. We concluded that the system of the isolines $\psi(x,y) = h > 0$ is qualitatively analogous to the system of hulls (external and

internal perimeters) of 2D percolation clusters on a regular lattice with the period λ, where the bond (or site) occupation probability equals[1] $p = p_c - h/\psi$, and p_c is the percolation threshold. (In more detail about the percolation problem see review [10].)

More exactly, for the random function $\psi(x,y)$ in [1,2] the following was assumed. The function $\psi(x,y)$, which is characterized by a single space-scale λ and by a single characteristic amplitude $\psi = \lambda v$, is bounded, statistically homogeneous, and isotropic; any kinds of degeneracy (like periodicity, or singularity in the saddle point height distribution) are not allowed. For brevity, let us refer to the function $\psi(x,y)$, obeying these conditions, as "the λ-function", simultaneously specifying its scale. In what follows, we will denote similarly the flow, whose stream function is a λ-function.

The results of Refs. [1,2], which are relevant for the further discussion, may be shortly presented as follows:

(i) Every isoline $\psi(x,y) = h$ is closed with the probability of one.

(ii) There exists only one infinite (unclosed) isoline; it has the horizontal cross-section altitude $h = 0$.

(iii) The distribution function of the isolines $\psi(x,y) = h \neq 0$ over their diameter a is algebraic[2], $n_h(a) \approx a^{-2}$, up to the cut-off at the correlation length

$$\xi_h = \lambda(\psi/|h|)^{\nu}, \quad \nu = 4/3 ;$$

for the values of $a > \xi_h$ $n_h(a)$ decays exponentially.

(iv) Long isolines (length $L \gg \lambda$) are fractal curves with the fractal dimension $d_{h\lambda} = 1 + 1/\nu = 7/4$. In particular, the dependence of the curve length L on its displacement a is

$$L(a) \approx \lambda(a/\lambda)^{d_{h\lambda}} . \quad (2)$$

The robustness of these properties, as well as the wideness and importance of the λ-flow class are connected with the looseness of the restrictions imposed on the function $\psi(x,y)$.

[1] Here and further $\psi(x,y)$ means the actual function while ψ its characteristic (positive) amplitude.
[2] $n_h(a)$ is the number of isolines with the diameter of order of a (e.g. $[a, 2a]$) per unit area.

and thus by its structural stability (the latter means that (i)–(iv) remain valid also after any small smooth perturbation of $\psi(x, y)$ without long correlations). The boundary of the universality class of λ-function, which is closely connected to the universality class of random percolation [10], are determined below in terms of a permissible spectrum of $\psi(x, y)$.

Properties (i) and (ii) may be simply derived from the "Flood-In-The-Hills" argument: the others demand the knowledge of percolation exponents. Fortunately, in two dimensions, all the exponents we need (the correlation exponent ν and the fractal dimension of cluster perimeter $d_{h\lambda}$) are known exactly [11,12].

Previously [8] Trugman used an approach to monoscale isolines close to ours. However obscure as it was at that time, the perimeter exponent $d_{h\lambda}$ in Ref. [8] was flippantly ta... as equaling 2. (The results of Ref. [8] seem to be widely accepted [13].)

The objective of the present article is to extend our earlier approach [1,2] and to estab... the statistical properties of isolines of a multiscale random function. For simplicity suppose this function to possess a power spectrum of scales in a wide range of wavelen... $\lambda_0 < \lambda < \lambda_m$, $\lambda_0 \ll \lambda_m$, and a sufficiently abrupt cut-off outside it. Mathematically, ... express the appropriate function in the form

$$\psi(x, y) = \sum_{\lambda=\lambda_0, 2\lambda_0, 4\lambda_0, \ldots, \lambda_m} \psi_\lambda(x, y)$$

with the components $\psi_\lambda(x, y)$ being λ-functions and having the amplitudes

$$\psi_\lambda \approx \psi_0 (\lambda/\lambda_0)^H \ .$$

In other words, the component $\psi_\lambda(x, y)$ represents the λ-scale pulsation of $\psi(x, y)$ and ma... be expressed through the Fourier component $\psi(\mathbf{k})$ as

$$\psi_\lambda(\mathbf{r}) = \int_{1/2<|\mathbf{k}|\lambda<1} \psi(\mathbf{k}) e^{i\mathbf{k}\mathbf{r}} d^2\mathbf{k} \ .$$

The λ-spectrum (4) then corresponds to the Fourier spectrum $|\psi(\mathbf{k})| \propto k^{-(1+H)}$.

II. Random 2D Flow in a Weak Homogeneous Stream

In this section we study the geometry of stream lines of the flow, being the superposition of a λ-flow and a very weak uniform stream, or, which is just the same, the isolines of a monoscale function with a small mean gradient:

$$\mathbf{v}(x,y) = \mathbf{v}_0(x,y) + \mathbf{u}\ , \quad \mathbf{v}_0(x,y) = \boldsymbol{\nabla}\psi_0(x,y) \times \hat{\mathbf{z}}\ , \tag{5}$$

$$\mathbf{u} = u\hat{\mathbf{x}} = \text{Const}\ , \quad \varepsilon \equiv u/v_0 \ll 1\ .$$

Here $\mathbf{v}_0(x,y)$ is a λ_0-flow, and $\mathbf{u}$ is the small homogeneous component.

We have seen that non-perturbed streamlines of $\mathbf{v}_0(x,y)$ are closed: the average flux is absent. The perturbation $\mathbf{u}$ keeps most of the stream lines still closed, according to its weakness, but inevitably opens some of them, to carry the arising average flux. Thus the mean flux is localized in a set of channels averagely oriented along the x-axis. Since the small perturbation cannot appreciably affect the absolute value of $\mathbf{v}(x,y)$, the velocity in the channels is clearly of the order of v_0. The smallness of $\mathbf{u}$ results only in that the channels are narrow (the characteristic width $\delta_\varepsilon \ll \lambda_0$) and lie well apart from each other (the characteristic distance between them $\Delta_\varepsilon \gg \lambda_0$).

In order to visualize the picture, one can imagine the stream lines as the coastlines of a hilly continental slope at successive stages of "the Flood", in other words, the isolines of a function

$$\psi(x,y) = \psi_0(x,y) + uy\ . \tag{6}$$

For the evaluation of δ_ε and Δ_ε, let us set the origin on some infinite isoline of (6). Near $y = 0$ (or, to be more exact, at $|y| \ll \Delta_\varepsilon$) the qualitative behavior of the curve passing through the origin is qualitatively the same as for the non-perturbed function $\psi_0(x,y)$. According to (ii), in the vicinity of the origin the isoline altitude corresponds to the local

mean level of $\psi(x,y)$ $(h=0)$. Drifting over y, the curve will get into the region, where its level appears to acquire a mismatch $(h=-uy)$ with respect to the local mean value of $\psi(x,y)$. According to (iii), this forces the curve to behave like as tending to close, that is, to return to the region $y=0$. The process described will continue, and our isoline will wander around the x-axis, being constrained in a stripe with the width Δ_ε. The amplitude of the wandering Δ_ε may be assessed as the self-consistent correlation length (1),

$$\Delta_\varepsilon \approx \lambda_0 (u\Delta_\varepsilon/\psi_0)^{-\nu} ,$$

hence,

$$\Delta_\varepsilon \approx \lambda_0 \varepsilon^{-\nu/(\nu+1)} \gg \lambda_0 . \tag{7}$$

Apparently, the distance between the channels is approximately the same. The requirement that the total flux **u** be concentrated in the channels should be written as $v\delta_\varepsilon = u\Delta_\varepsilon$; so, we obtain the width of the channels

$$\delta_\varepsilon \approx \lambda_0 \varepsilon^{1/(\nu+1)} \ll \lambda_0 . \tag{8}$$

The formulas (7), (8) agree with those of Trugman [8], who derived them from similar arguments of "graded percolation".

Completing this section let us note that the picture drawn above is somewhat simplified. Actually, the channels of various widths compound a web-like fractal net with multiple holes of different size. The quantities δ_ε and Δ_ε are the typical "web thread" thickness and the maximum hole diameter, respectively. The simplification had evidently no effect on the previous derivation. An attempt to present the flow is given in Fig. 2.

III. Fractal Dimension of Multiscale Isolines

Now, let us proceed to the multiscale function, given by the power spectrum (4). Let us rewrite the spectrum in terms of the flow $\mathbf{v}(\mathbf{r}) = \nabla\psi(\mathbf{r}) \times \hat{\mathbf{z}}$,

$$v_\lambda \approx v_0(\lambda/\lambda_0)^{H-1} \ , \tag{9}$$

and recall that outside the wide inertial range $\lambda_0 \leq \lambda \leq \lambda_m$, $\lambda_0 \ll \lambda_m$, a sufficiently sharp cut-off is supposed.

In order to study the geometric properties of the flow with the spectrum (9), we shall make use of the results obtained in the previous section. Namely, at first we consider a flow with strongly separated spectral components:

$$\begin{array}{l} \mathbf{v}(\mathbf{r}) = \mathbf{v}_{\lambda_0}(\mathbf{r}) + \ \mathbf{v}_{\lambda_1}(\mathbf{r}) + \ \cdots + \ \mathbf{v}_{\lambda_m}(\mathbf{r}) \ , \\ \qquad\quad \lambda_0 \qquad \ll \lambda_1 \qquad \ll \cdots \ll \lambda_m \ , \end{array} \tag{10}$$

the components $\mathbf{v}_{\lambda_i}(\mathbf{r})$ being λ_i-flows and their intensities given by the formula (9). Running ahead, we announce our approach to the continuous spectrum: we will use the results obtained for the flow (10) on the limit of their applicability, $\mu \equiv \lambda_{i+1}/\lambda_i = 2$.

Note that the case $H > 1$ presents nothing new: the appearance of the relief is essentially the same as for the monoscale function $\psi_{\lambda_m}(x,y)$ (see Fig. 3a). Indeed, the long-scale perturbations being much more intensive than the small-scale ones ($v_{\lambda_{i+1}} \gg v_{\lambda_i}$), the latter will cause only a slight "trembling" of the stream-lines of the largest scale λ_m.

For $H < 1$ (Fig. 3b,c), let us consider the flow $\mathbf{v}_{\lambda_i}(\mathbf{r}) + \mathbf{v}_{\lambda_{i+1}}(\mathbf{r})$. According to the ratio $v_{\lambda_{i+1}}/v_{\lambda_i} \approx (\lambda_{i+1}/\lambda_i)^{H-1}$ being much less than 1, the flow $\mathbf{v}_{\lambda_{i+1}}(\mathbf{r})$ may be considered as a locally homogeneous small perturbation of $\mathbf{v}_{\lambda_i}(\mathbf{r})$. Applying the results of the previous secton we find that the interaction of scales λ_i and λ_{i+1} produces channels, which, on the average, follow the stream lines of $\mathbf{v}_{\lambda_{i+1}}(\mathbf{r})$, and wander, according to (7), in stripes of the characteristic width

$$\Delta_{i,i+1} = \lambda_i \left[\frac{v_{\lambda_i}}{v_{\lambda_{i+1}}}\right]^{\frac{\nu}{\nu+1}} = \lambda_i \left[\frac{\lambda_{i+1}}{\lambda_i}\right]^{\frac{(1-H)\nu}{(\nu+1)}} \ , \tag{11}$$

while their curvature radius is of order of λ_i.

Of course, the result $\Delta_{i,i+1} > \lambda_{i+1}$ makes no sense – this would be the case for $H < -1/\nu$. That large value of $\Delta_{i,i+1}$ means that the assumption of the local homogeneity is wrong: during the course of the stream line on the distance $\Delta_{i,i+1}$ the perturbing field $\mathbf{v}_{\lambda_{i+1}}(\mathbf{r})$ changes strongly. This can also be interpreted in another way: the large-scale flow $\mathbf{v}_{\lambda_{i+1}}(\mathbf{r})$ is too weak to have any feasible influence on stream-lines (i.e. to swing them around on the length λ_{i+1}). Returning now to the flow (10), one can infer that under the condition $H < -1/\nu$ the behavior of stream lines is determined solely by $\mathbf{v}_{\lambda_0}(x,y)$ and correspond- analogously to the case $H > 1$, to the monoscale flow.

Thus we come to the conclusion that a nontrivial interaction of scales takes plac $-1/\nu < H < 1$, which will be the subject of our further interest.

The width of a channel, produced by the interaction between scales λ_i and λ_{i+1} (le call it "$(i, i+1)$-channel"), according to (8), is

$$\delta_{i,i+1} = \lambda_i(\lambda_i/\lambda_{i+1})^{(1-H)/(\nu+1)} \ .$$

Now suppose $0 < H < 1$. Then, as it can be easily seen, $\delta_{i,i+1} \gg \Delta_{i-1,i}$, particu $\delta_{m-1,m} \gg \Delta_{m-2,m-1}$. Thus, we infer that the velocity field $\mathbf{v}_{\lambda_{m-1}}(\mathbf{r}) + \mathbf{v}_{\lambda_m}(\mathbf{r})$ ma regarded as a weak long-wave perturbation of $\mathbf{v}_{\lambda_{m-2}}(\mathbf{r})$: the stream lines of the superpo of these three components are typically closed on the scale λ_{m-2}, however, some line compound channels $\delta_{m-2,m-1}$ wide, which follow the stream-lines of $\mathbf{v}_{\lambda_{m-1}}(\mathbf{r}) + \mathbf{v}_{\lambda_m}(\mathbf{r})$. latters are typically closed in the corresponding holes (see Fig. 2b) but can also be to $(m-1,m)$-channels to wander in the stripes of the width $\Delta_{m-1,m}$. The velocity $\mathbf{v}_{\lambda_{m-3}}(\mathbf{r}) + \mathbf{v}_{\lambda_{m-2}}(\mathbf{r}) + \mathbf{v}_{\lambda_{m-1}}(\mathbf{r}) + \mathbf{v}_{\lambda_m}(\mathbf{r})$ may be studied in this same manner. The proce should be repeated down to the lowest scale.

Thus we have the following behavior of percolating (that is, with the diameter n greater than λ_0) isolines of $\psi(x,y)$. For relatively short displacements $a < \Delta_{0,1}$ the isoli

walk is indistinguishable from that of the monoscale function $\psi_{\lambda_0}(\mathbf{r})$. Further the isoline turns out to be constrained into a stripe of the width $\Delta_{0,1}$, whose curvature radius is of the order of λ_1. Presented on a map with the resolution scale $\Delta_{0,1}$ (let us call it as "the first map"), the isoline looks like a smooth curve, however, according to (2), the genuine length of the line is $(\Delta_{0,1}/\lambda_0)^{d_{h\lambda}-1}$ times greater than it shows on the map. The curve on the first map behaves as a stream line of the flow $\mathbf{v}_{\lambda_1}(\mathbf{r})$ — up to the displacement $a = \Delta_{1,2}$. Let us now look at the second map — with the resolution $\Delta_{1,2}$: there the curve will be smooth again and $[(\Delta_{0,1}/\lambda_0)\cdot(\Delta_{1,2}/\lambda_1)]^{d_{h\lambda}-1}$ times shorter than actually. This procedure shall be continued; for the contour diameter exceeding λ_m, the last map will be the m-th.

Summarizing the above discussion, it is convenient to plot the length $L(a)$ of a percola... isoline versus its displacement a on the diagram with logarithmic axes (Fig. 4). We see

$$L(\lambda_i) = \lambda_i \prod_{j=0}^{i} (\Delta_{j,j+1}/\lambda_j)^{d_{h\lambda}-1} = \lambda_i(\lambda_i/\lambda_0)^{(1-H)/(\nu+1)} ,$$

and in the limit $\lambda_{i+1}/\lambda_i = 2$ the subscripts may be omitted and λ replaced by a: L ... $a^{1+(1-H)/(\nu+1)}$. In Fig. 2 this limit corresponds to the smoothing of the fractures ... graph. So the expression for the fractal dimensionality will be

$$d_h = 1 + (1-H)/(\nu+1) = (10-3H)/7 .$$

We have derived it for $\Delta_{i-1,i} < \delta_{i,i+1}$, that is, for $0 < H < 1$. However, this restricti... be weakened, and the condition $\Delta_{i-1,i} < \lambda_i$ is sufficient. Indeed, let $\delta_{i,i+1} \ll \Delta_{i-1,i}$. Then percolating stream lines of $\mathbf{v}_{\lambda_{i-1}}(\mathbf{r}) + \mathbf{v}_{\lambda_i}(\mathbf{r}) + \mathbf{v}_{\lambda_{i+1}}(\mathbf{r})$ wander on the average a... $(i,i+1)$-channel, coming out of its width $\delta_{i,i+1}$ essentially. However, due to the ineq... $\Delta_{i-1,i} \ll \lambda_i$ the direction of stream lines of $\mathbf{v}_{\lambda_i}(\mathbf{r}) + \mathbf{v}_{\lambda_{i+1}}(\mathbf{r})$ in the region of this wande... is almost the same as in the very $(i,i+1)$-channel. Furthermore, a line of $\mathbf{v}_{\lambda_i}(\mathbf{r}) + \mathbf{v}_{\lambda_{i-1}}(\mathbf{r})$ lying the distance $\Delta_{i-1,i}$ apart from the channel, can leave it encountering a nearest sa... point, that is, passing a distance much longer than $\Delta_{i-1,i}$. Still, the percolating lines

$\mathbf{v}_{\lambda_{i-1}}(\mathbf{r}) + \mathbf{v}_{\lambda_i}(\mathbf{r}) + \mathbf{v}_{\lambda_{i+1}}(\mathbf{r})$ have to return back to the $(i, i+1)$-channel, thus producing seldom excursions aside it. This means that in the case $H < 0$ the correlation length ξ_h can behave otherwise than for $H > 0$ (see the next section). However, the expression for the fractal dimensionality (14) remains the same. So, the whole nontrivial region is covered, and (14) is valid for $-1/\nu < H < 1$.

On the upper limit of this interval (and for $H > 1$) the dimensionality turns out to be unity (unless the displacement exceeds the maximum diameter of the hills λ_m; at $a \gg \lambda_m$ one again obtains $d_h = d_{h\lambda} = 7/4$). For $H \leq -3/4$ the picture of the coastlines is essentially the same as for the landscape with the single scale λ_0, $d_h = d_{h\lambda}$.

If one assumes a power spectrum and the randomness of the Earth's relief $z = \psi(x, y)$ then fractal dimensions of natural coastlines should fall into the same interval $1 < d_h < 1.75$ According to available data [3], d_h takes values 1.25 (Western England), 1.13 (Australia), 1.02 (South Africa, one of the smoothest shores), which correspond to the exponents $H =$ 0.42, 0.70, and 0.95, respectively. Some more elaborate analysis of this issue is given in the Sec. VI.

In addition to the fractal dimension of a separate coastline d_h, let us conclude this section by the discussion of the dimensionality of the full system of isolines (d_{hf}) and its subset ocean islands coastlines (d_{hi}). Apparently, $d_h \leq d_{hi} \leq d_{hf} \leq 2$. Let us proceed with the main case of positive H: $0 < H < 1$. As one can guess from Fig. 3b, both islands and lakes[3] lie in the vicinity of a continental coast, the number of islands and lakes being approximately the same, which means $d_{hi} = d_{hf}$. This statement can be given a more precise sense, introducing again the separated scales (10). Firstly, consider a relief formed only by the two scales λ_{m-1} and λ_m. Then islands with sizes of order of λ_m lie in a bank with the width $B_{m-1,m}$ near the coastline of $\psi_{\lambda_m}(\mathbf{r})$ where the variation of $\psi_{\lambda_m}(\mathbf{r})$ does not exceed the small amplitude $\psi_{\lambda_{m-1}}$ (see Fig. 5): $B_{m-1,m}|\nabla\psi_{\lambda_m}(\mathbf{r})| \approx \psi_{\lambda_{m-1}}$. Hence, $B_{m-1,m} \approx \lambda_0(\lambda_{m-1}/\lambda_m)^{1-H} \gg \Delta_{m-1,m}$. In

[3]Here for simplicity we assume that the level of lakes is the same as the sea level.

the bank with the length l there are approximately $B_{m-1,m}l/\lambda_{m-1}^2$ islands with diameters of order of λ_{m-1}, consequently, their full perimeter is[4] $\mathcal{L} \approx B_{m-1,m}l/\lambda_{m-1}$. On a map with the resolution $B_{m-1,m}$, all these islands fade into a single coastline of $\psi_{\lambda_m}(\mathbf{r})$ with the length l, hence, the apparent shore length is $B_{m-1,m}/\lambda_{m-1} = (\lambda_m/\lambda_{m-1})^{1-H}$ times less than actually. Adding shorter scales, we recover a similar fine structure of every coastline with the curvature radius λ_{m-1}, which is split into a bank with multiple islands. Continuing this argument down to the shortest scale we conclude that on a map with the resolution $\lambda(\lambda_0 \ll \lambda \ll \lambda_m)$ the apparent perimeter of the system $\mathcal{L}$ is $(\lambda/\lambda_0)^{1-H}$ times less than real islands' perimeter, that is, $d_{hf} - 1 = 1 - H$. These same arguments apply to islands (free isolines), which make up of the order of one half of all the isolines. Thus we obtain

$$d_{hi} = d_{hf} = 2 - H \ , \quad 0 < H < 1 \ . \tag{15}$$

This expression for d_{hf} coincides with Mandelbrot's formula [4], which has been derived in another way, based on a simplest generalization of the Brownian graph — the fractional Brownian surface $z = B_H(x,y)$ defined by its delta-variance $\langle[B_H(\mathbf{r}+\Delta\mathbf{r}) - B_H(\mathbf{r})]^2\rangle \propto |\Delta\mathbf{r}|^{2H}$. This function presents a fractal relief for $0 < H < 1$ (a standard Brownian surface corresponds to the exponent $H = 1/2$). The fractal dimension of this relief equals $d_r = 3 - H$. The spectrum of the function $B_H(x,y)$ is a power, $(B_H)_\lambda \propto \lambda^H$, with $\lambda_0 = 0$ and $\lambda_m = \infty$. As the fractal dimension of a non-degenerate plane cross-section of a 3D fractal is the dimension of the whole fractal minus one, one readily obtains the dimension of the full system of isolines [4]: $d_{hf} = d_r - 1 = 2 - H$, in accordance with (15).

At $H = 0$ the expression (15) yields $d_{hf} = 2$; i.e. the dense packing of the isolines with the minimum scale λ_0. At $H < 0$ the role of shorter wavelengths is still more significant (compare Figs. 3b and 3c), hence, as before,

$$d_{hf} = 2 \ , \quad H < 0 \ . \tag{16}$$

[4]This estimate will not change if one takes into account actual distribution of islands over diameters which, in its turn, is different in different regions inside the bank.

As regards the dimensionality d_{hi} at $H < 0$, we will return to its calculation in Sec. V.

IV. Correlation Length

Given the horizontal cross-section altitude h, the diameters of islands exhibit a power-law distribution (see the next section), which is cut off by the correlation length ξ_h – the diameter of the largest isoline[5]. As mentioned above, $\xi_h \to \infty$ for $h \to 0$.

For a monoscale landscape, the correlation length expression is given by Eq. (1), according to the direct percolation analogy. For a multiscale random function $\psi(x,y)$ with the power spectrum (4), the mechanism of isoline diameter limitation is different for positive and negative H. Suppose $H > 0$. According to the explanations given above, isolines of $\psi(x,y)$ wander around the isoline of $\psi_{\lambda_m}(x,y)$, as the longer is the space-scale, the greater is the ψ-function component. So the maximum isoline diameter ξ_h will be determined by that of $\psi_{\lambda_m}(x,y)$ (3):

$$\xi_h = \lambda_m(|h|/\psi_{\lambda_m})^{-\nu} , \quad H > 0 , \quad |h| < \psi_{\lambda_m} .$$

The situation with $-1/\nu < H < 0$ is more complicated. At first, let us suppose $\psi_{\lambda_m} < |h| < \psi_{\lambda_0}$. Denote λ_h the solution of the equation $\psi_{\lambda_h} = |h|$. It is evident that the scales $\lambda >$ λ_h have too small amplitudes and cannot influence the diameter of the isolines $\psi(x,y) = h$. The maximum diameter ξ_h is determined by such a scale λ, $\lambda < \lambda_h$, that gives the maximum monoscale correlation length $\xi_h^{(\lambda)} = \lambda(|h|/\psi_\lambda)^{-\nu}$. As $\xi_h^{(\lambda)} \propto \lambda^{\nu H+1}$, we come to the conclusion that at $H > -1/\nu$ this takes place for the maximum allowed $\lambda = \lambda_h$. Thus we find

$$\xi_h = \lambda_h = \lambda_0(|h|/\psi_{\lambda_0})^{1/H} , \quad -1/\nu < H < 0 , \quad \psi_{\lambda_m} < |h| < \psi_{\lambda_0} .$$

For the completeness of the picture we must consider the case $|h| < \psi_{\lambda_m}$. Now, as for $H > 0$, the closure of the isoline will be again determined by the λ_m-flow, so that

[5] For $\xi_h > \lambda_m$, there will be different contour distributions in scaling ranges $[\lambda_0, \lambda_m]$ and $[\lambda_m, \xi_h]$.

$\xi_h = \lambda_m(|h|/\psi_{\lambda_m})^{-\nu}$. Bringing these results together, we obtain the following expression for the correlation length:

$$\xi_h = \begin{cases} \lambda_0(|h|/\psi_{\lambda_0})^{1/H} , & \psi_{\lambda_m} < |h| < \psi_{\lambda_0} , \quad -1/\nu < H < 0 , \\ \lambda_m(|h|/\psi_{\lambda_m})^{-\nu} , & |h| < \psi_{\lambda_m} , \quad -1/\nu < H < 1 . \end{cases} \tag{17}$$

V. Area Distribution of Islands

In this section we discuss self-similar properties of coastlines and establish connection between the size-distribution of islands with the fractal dimension d_{hi} and other exponents.

Empirically known is the Korčak number-area rule according to which the number of islands with area exceeding S is proportional to the quantity

$$\Phi(S) = S^{-k} ,$$

with k being some exponent characterizing the archipelago [3]. For example, $k = 0.5$ (Afr... 0.75 (Indonesia and Canada), 0.65 (Earth averaged).

Let us begin with the distribution of all the isolines $\psi(x, y) = h$ over their sizes in main inertial range. Under the inertial range we imply the self-similar wave-length r... $[\lambda_0, \lambda_{\max}]$, defined depending on the relief spectrum. For a monoscale λ_0-function, $\lambda_{\max}$ = In the multiscale case for $H > 0$ and $|h| < \psi_{\lambda_m}$, we have $\lambda_{\max} = \lambda_m$. (For $|h| > \psi_{\lambda_m}$ t... would be no contours at all.) At $H < 0$, $\lambda_{\max} = \min(\lambda_m, \xi_h)$.

Denote $n_h(a) = N(a)/S$ the number of isolines $\psi(x, y) = h$ with diameters a to $2a$ ... unit square. At $H \leq 0$ these lines fill the plane (x, y) densely: $d_{hf} = 2$. Due to this prop... the quantity $n_h(a)$ is scale-invariant in the inertial range; i.e. does not change under ... scaling transformation $\mathbf{r} \to c\mathbf{r}$. Accounting for the transformation of area $S \to c^2 S$ and ... of isolines number $N(a) \to N(a/c)$, we conclude $n_h(a) = c^{-2} n_h(a/c)$, hence

$$n_h(a) = \zeta a^{-2} , \quad \lambda_0 \ll a \ll \lambda_{\max} , \quad H \leq 0 ,$$

with ζ being a numerical constant. This number-size distribution is the same as for contours of a monoscale function.

Before proceeding with a more subtle question about the distribution of free, i.e. not surrounded by others, isolines, let us dwell upon a simpler example with the Sierpinski carpet (Fig. 6). It represents a regular fractal which is constructed by dividing an initial set in q (say, equal) parts and subsequent removal of $\tilde{q}$ of them, the procedure being repeated with every one of $q - \tilde{q}$ remaining parts, etc. ad infinitum. On the n-th step one removes pieces with the area $S_n = a_n^2 = q^{-n}$, the remainder can be covered by $N_n = (q - \tilde{q})^n$ squares of the same size a_n. It follows that the fractal (Hausdorff) dimension of the carpet equals

$$d = -\frac{\log N_n}{\log a_n} = 2\log(q - \tilde{q})/\log q \; .$$

If we treat the Sierpinski carpet as "an ocean," and its holes are identified with "is...
then the index k in (18) turns out to be related rather simply with the fractal dim...
(20). Indeed, the number of holes with sizes greater than a_n is of the same order ...
hence,

$$k = -\frac{\log N_n}{\log S_n} = d/2 \; .$$

It can be seen that relation (21) [4] holds not only for the Sierpinski carpet and i...
but also for any other gappy fractal whose fractal dimension exceeds the dimension
perimeter of its hole.

Now we turn to the contours of a random function. Let us begin with the mon...
λ-function $\psi(x, y)$. As mentioned above, its lines of the constant level h (we set $0 < h$ ...
behave like the hulls of subcritical percolation clusters with the bond (or site) prob...
$p = p_c - h/\psi$. Let us call these clusters "dry," and their complementary supercritical cl...
($p = p_c + h/\psi$) "wet." Among the wet clusters there is one infinite, which corresponds ...
ocean. Then free isolines, i.e. coasts of ocean islands, correspond to the hulls of the h...
the infinite wet cluster. At first sight, the picture is quite similar to holes in the Sierp...

carpet whose parameters ($q, \tilde{q}$, etc.) could be adjusted to model the infinite wet cluster in the inertial range. However, the whole wet cluster ($d = d_c = 91/48 \approx 1.90$ [10]) is redundant with respect to islands because it contains almost all its mass in dead-end branches. These branches correspond to the bays of the islands, which do not affect the islands' size. On the other hand, if one cuts all the dead branches off, then the remaining backbone (skeleton) of the wet cluster ($d = d_b \approx 1.67$ [14]) is insufficient, because it does not account for smaller islands lying in the bays; these correspond to the dead-end branches terminating with loops, as shown in Fig. 7. Let us call the backbone of a cluster together with these loop-like dead braches "the tailless cluster", and denote its fractal dimension d_{tc}. Apparently, $d_b < d_{tc} \leq d_c$. Thus, the fractal dimension of the set of free monoscale isolines (near the sea level $h \approx 0$) equals d_{tc}, and from Eq. (21) we find

$$k = d_{tc}/2 \ , \quad \lambda - \text{function} \ . \qquad (22)$$

According to the said above, Eq. (22) is valid provided that $d_{tc} > d_{h\lambda} = 1.75$. As the probability for every four consequent bonds of a dead-end branch to form a loop is finite, the loop-like dead-end branches appear to consist the finite fraction of the net cluster mass. Hence, it is plausible to suppose that $d_{tc} = d_c = 91/48 \approx 1.90$.

The number-area distribution of islands (18) may be presented in the form of the probability $P_f(a)$ for a given contour $\psi(x,y) = h$ with the diameter a to be free, viz. not surrounded by another isoline with the same h:

$$P_f(a) \approx (a/\xi_h)^f \ . \qquad (23)$$

Then the distribution of free isolines (18), $\Phi(a) \propto a^{-2k}$, can be presented as the product of the distribution function of all the isolines $n_h(a)$ and the freedom probability (23). In particular, for a monoscale relief we have:

$$f = f_\lambda = 2(1-k) = 2 - d_{tc} \ , \quad \lambda - \text{function} \ . \qquad (24)$$

Similar arguments can be applied to a multiscale landscape. Firstly, consider the case $H > 0$. As mentioned in Sec. III, the islands of the size a can lie only in the vicinity of, i.e. on the distance $\approx a$ from, continents or larger islands, for more remote perturbations ψ_a produce only little knobs on a deep bottom. This observation leads immediately to the result (21) with $d = d_{hi}$ — the fractal dimension of all the islands' coasts. Now, according to (15), we obtain

$$k = d_{hi}/2 = 1 - H/2 \ , \quad 0 < H < 1 \ . \tag{25}$$

The case $H < 0$ is somewhat more complicated. Like the monoscale case, here there are also very many islands in the sense that at $0 < h \ll \psi_{\lambda_0}$ the area fraction occupied by open ocean is much less than one. For the determination of the exponent f in (23) at $-1/\nu < H < 0$ we shall restrict ourselves by the condition $\psi_{\lambda_m} \ll h \ll \psi_{\lambda_0}$ under which the correlation length, according to (17), is $\xi_h = \lambda_h$. Distinguishing shorter scales, let us denote them for brevity $\lambda_{h-1}, \lambda_{h-2}, \ldots, \lambda_0$. The probability for an isoline of diameter λ_i to be free on the scales up to λ_{i+1} can be determined as the monoscale probability given by Eqs. (23) and (24), where instead of ξ_h one should use the monoscale correlation length of $\psi_{\lambda_i}(\mathrm{r})$ for the cross-section level $h' = \psi_{\lambda_{i+1}}$:

$$\xi_h^{(i)} = \lambda_i (\psi_{\lambda_i}/h')^\nu = \lambda_i (\lambda_i/\lambda_{i+1})^{H\nu} \ . \tag{26}$$

The probability for a contour of diameter λ_i to be absolutely free is equal to the product of the freedom probabilities on subsequent scales, up to the correlation length λ_h:

$$P_f(\lambda_i) = \left[\frac{\lambda_i}{\xi_h^{(i)}}\right]^{f_\lambda} \cdots \left[\frac{\lambda_{h-1}}{\xi_h^{(h-1)}}\right]^{f_\lambda} \ .$$

Using expression (24) for f_λ we finally find

$$P_f(a) = (a/\xi_h)^f \ , \ f = -H\nu f_\lambda = -H\nu(2 - d_{tc}) \ , \ -1/\nu < H < 0 \ . \tag{27}$$

As expected, in the monoscale limit $H = -1/\nu$ the exponent f becomes equal to its monoscale value (24).

Knowing the probability (27) and the integral distribution (19), one easily finds the exponent k of the island's distribution:

$$k = 1 - f/2 = 1 + H\nu(1 - d_{tc}/2) \ , \ -1/\nu < H < 0 \ . \tag{28}$$

Now the formula (28) makes it possible to determine the fractal dimensionality of the set of the ocean coasts:

$$d_{hi} = 2k = 2 + H\nu(2 - d_{tc}) \ , \quad -1/\nu < H < 0 \ , \tag{29}$$

which is a counterpart of the previous result (15).

VI. Comparison with Geographical Data

To compare the theoretical results with the geographical reality, we have measured the fractal dimension of the Baltic shore of Estonia [15] and analyzed the distribution of adjacent islands [16] (Fig. 8).

For the calculation of d_h we have measured the length of the continental coastline l as a function of the spread of the pair of measuring compasses λ, the latter having been varied in the range from 0.8 km up to 25.6 km. From the dependence $L(\lambda) \propto \lambda^{1-d_h}$ we have calculated average $d_h = 1.17 \pm 0.02$, although on separate parcels d_h differs from the mean value strongly: for instance, the fractal dimension of the North-East parcel of the Estonian coast (between Aseri and Narva) $d_h = 1.02 \pm 0.01$, while for the South-East coast of the Saaremaa island $d_h = 1.35 \pm 0.03$.

Then the length of the coastlines of islands with diameters greater than λ was also taken into account, to obtain the net coasts' fractal dimension $d_{hi} = 1.25 \pm 0.02$.

Having analyzed in sum about 1350 Estonian islands with areas $10^{-3}\,\mathrm{km}^2$ to $10^2\,\mathrm{km}^2$, we have found the distribution (18) with the exponent $k = 0.56 \pm 0.02$.

According to the model of the random, power-spectrum relief, the spectral exponent H should be calculated from Eqs. (14), (15) , and (24), as shown in the following table:

	measured	calculated	H
d_h	1.17 ± 0.02	$H = (10 - 7d_h)/3$	0.60 ± 0.05
d_{hi}	1.25 ± 0.02	$H = 2 - d_{hi}$	0.75 ± 0.02
k	0.56 ± 0.02	$H = 2(1 - k)$	0.88 ± 0.04

Thus, the results show not much of a quantitative agreement between the real relief and the power-spectrum model. The resulting discrepancies can be explained by the geological peculiarities. Due to small sizes of the most of Estonian islands (diameter less than 1 k... and their low heights above the sea level (of order of several meters), the sea and the float... ice destroy the islands intensively. It should also be mentioned that the land in Est... raises dozens of centimeters per century, hence in the past these islands were still lower ... were destroyed more effectively. This same (but to a lower degree) applies to peninsula... means that ancient values of d_{hi} and especially k were higher (and corresponding H ... than now, which would fit better the model of random, power-spectrum relief.

This kind of processes are not likely to be significant everywhere. Mandelbrot [4] st... that on the average over the Earth the relation $k = d_{hi}/2$ holds well, although locally pat... behave quite diversely. For example, the coasts of Finland look much more self-similar ... Fig. 8).

VII. Summary and Discussion

The main result of the present paper lies in working out a new analytical approach to stud... contours of a multiscale random function. The proposed method of virtual separation ... scales reduces the problem of the multiscale statistical topography to a simpler monos... one, which belongs to the universality class of the random percolation problem, and to ... notion of the interaction of scales.

Such an approach makes it possible to advance further in the description of isoli...

than the fractional Brownian relief method [4], and to determine all the basic exponents for any power spectrum. These exponents are summarized below once more for the reference purpose:

- d_h - fractal dimension of an individual (but sufficiently long) isoline $\psi(x,y) = h$;
- d_{hi} - fractal dimension of the aggregate of free isolines $\psi(x,y) = h$ (islands);
- d_{hf} - fractal dimension of the aggregate of all the isolines $\psi(x,y) = h$;
- d_r - fractal dimension of the relief $z = \psi(x,y)$;
- k - exponent in the number-area rule for islands (18).

	$H < -3/4$	$-3/4 < H < 0$	$0 < H < 1$	$H > 1$
d_h	$7/4$	$(10-3H)/7$	$(10-3H)/7$	1
$d_{hi} = 2k$	d_{tc}	$2 + 4H(2-d_{tc})/3$	$2-H$	1
$d_{hf} = d_r - 1$	2	2	$2-H$	1

The scaling range for d_h is $[\lambda_0, a]$, where a is the diameter of the given isoline. The ot... exponents are valid for the scales $[\lambda_0, \min(\xi_h, \lambda_m)]$, with the correlation length ξ_h determ... by Eq. (17). The fractal dimension of the tailless percolation cluster d_{tc} is supposingly eq... to (but in no event greater than) the fractal dimension of the whole cluster $d_c = 91/48$.

The method of the separation of scales may be not universally applicable and seem... require an extra verification of the results obtained with its help. In particular, while ca... lating the fractal dimension (14), the isoline length $L(\lambda_m)$, according to (13), is determ... as a product of $N_\mu = \log(\lambda_m/\lambda_0)/\log\mu$ multipliers (the number of intermediate sca... with $\mu = \lambda_{i+1}/\lambda_i \gg 1$. For the transition to the continuous spectrum, $\mu = 2$, the nu... N_μ becomes large and an error can be accumulated. If we accepted that a systematic ... $E(\mu)$ took place in every factor of RHS of Eq. (13) ($E(\mu) \to$ Const at $\mu \to \infty$), then in... of (14) we would find

$$d_h(H) = 1 + (1-H)/(\nu+1) + \log_\mu E(\mu)/\log\mu \ ,$$

which would yield (14) for $\mu \gg 1$, but could differ from (14) at $\mu = 2$. However, for d_h ...

valid are the independent on μ requirements $d_h(1) = 1$, $d_h(-1/\nu) = 1 + 1/\nu$, pointing out the absence of such a systematic error and the validity of the result (14). Perhaps, generally, our doubts are in vain, too. A good sign is that, for the case $0 < H < 1$, the method of the virtual separation of scales predicts the same fractal dimension $d_{hf} = 2 - H$ as the fractional Brownian approach [4].

It should be emphasized that both fractal dimension and correlation length of random function isolines for spectra decaying or growing too rapidly ($H < -3/4$ or $H > 1$). are identical to those for the case of a single space scale. For $H > 1$, the upper cut-off scale λ_m plays the role of this single scale (particularly, the spectrum may be extended to $\lambda_0 = 0$). For $H < -3/4$ the lower scale λ_0 plays the same role (while λ_m can be extended to infinity).

Also, the results obtained allow us to determine the applicability of the monoscale approximation used in [1,2]. Now we have a ground to affirm that a function with the spectrum ψ_λ decaying faster than $\lambda^{-3/4}$, for $\lambda > \lambda_*$, and faster than λ, for $\lambda < \lambda_*$, belongs to the universality class of, and hence can be referred to as, a monoscale λ_*-function.

The percolation and fractal properties of isolines of random functions, being of a definite fundamental interest, have a number of first-hand application in several scopes of nonlinear physics. In this way the results obtained could be employed in the problem of anomalous transport in turbulent and chaotic media — fluids, plasmas, etc. — where power (e.g. Kolmogorov) spectra arise in a rather natural way. Other important implications of the statistical topography concern the electronic properties of disordered materials, in particular, the intrinsically two dimension quantized Hall effect [13]. One can also suggest a possible importance of the continuous universality classes, viz. those depending on a continuous parmeter H, for cooperative critical phenomena in phase tansitions.

The analysis of geographical data shows that the model of random, power-spectrum function describes the average Earth's landscape qualitatively right, with the relief spectrum exponent H lying in the interval $0 < H < 1$. We have shown that, for an "ideally random"

relief, the observable exponents d_h, d_{hi}, and k are not independent, being functions of the only parameter H. A comparison of "map-experimental" and theoretical data can cast a light on geological processes, which occurred in different places on the Earth's surface. The fact draws one's attention that the amplitude of variation of various geometric exponents (d_h, k etc.) is appreciably higher than the deviations of local distribution from their power laws. Such a behavior corresponds to the spectral exponent H, depending on coordinates relatively slowly. This suggests the idea that the very function $H(x, y)$ (and also $d_h(x, y)$ etc.) is self-similar: compare for instance, on the one hand, Africa and Indonesia, and, on the other hand, North-East sea-side of Estonia and the Saaremaa island. It would be interesting to investigate the question of such "double scaling" in more detail.

Acknowledgments

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Figure Captions

1 Coastlines arising during "the Flood." Hatched region is water. The water levels: undercritical $h < 0$ (a); critical $h = 0$; supercritical $g > 0$ (c).

2 Isolines of a monoscale random function with a small mean gradient along y-axes. a) typical percolating line; b) hatched region shows the aggregate of percolating isolines.

3 Vertical cross-section of multiscale 2D profiles for different spectral exponent H: a) $H > 1$; b) $0 < H < 1$; c) $H < 0$.

4 On the calculation of the fractal dimension of multiscale isoline. Double-logar... dependence of the length of the curve L on its displacement a. Solid line descri... limit of strongly separated scales. Dashed line corresponds to the smooth sp... ($\mu = 2$). Near the lines the corresponding slopes are shown. It is evident th... $d_h \leq 7/4$.

5 The isolines on the two-scale relief $\psi_{\lambda_{m-1}} + \psi_{\lambda_m}$. The islands of smaller size ... stripe ("bank") in the vicinity of large-scale coast.

6 Sierpinski carpet for $q = 9, \tilde{q} = 1$.

7 The tailless cluster (solid lines). "The tails" are shown in dashed lines.

8 The map of Estonia.

a

b

c

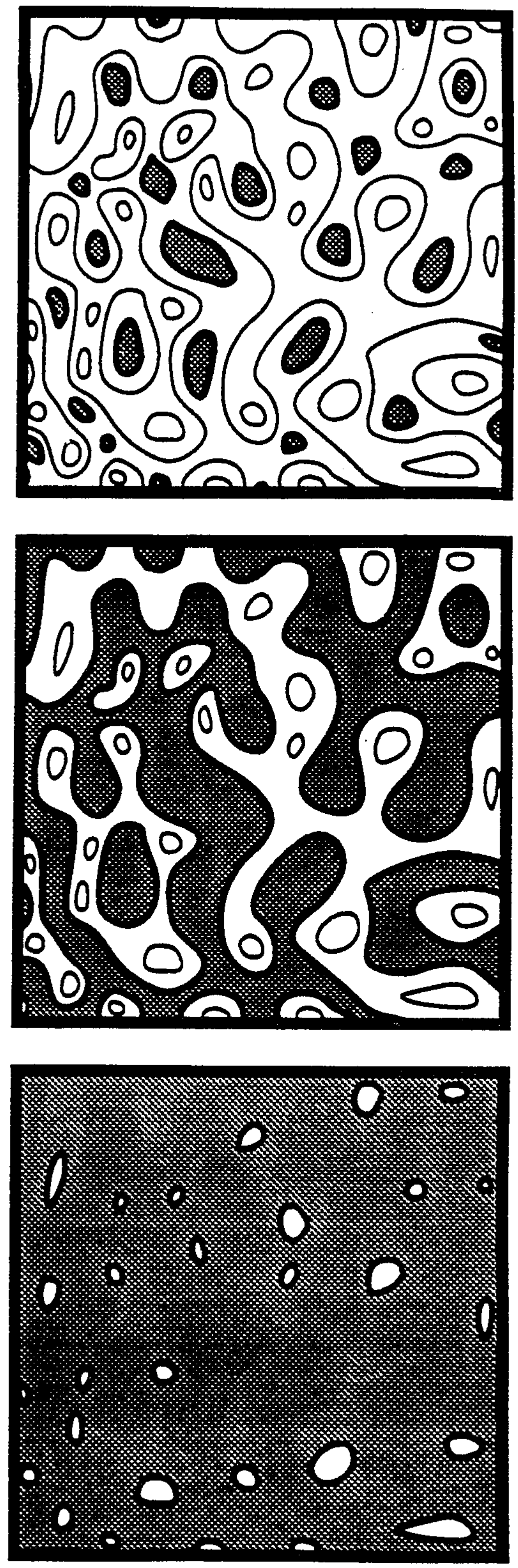


Figure 1.

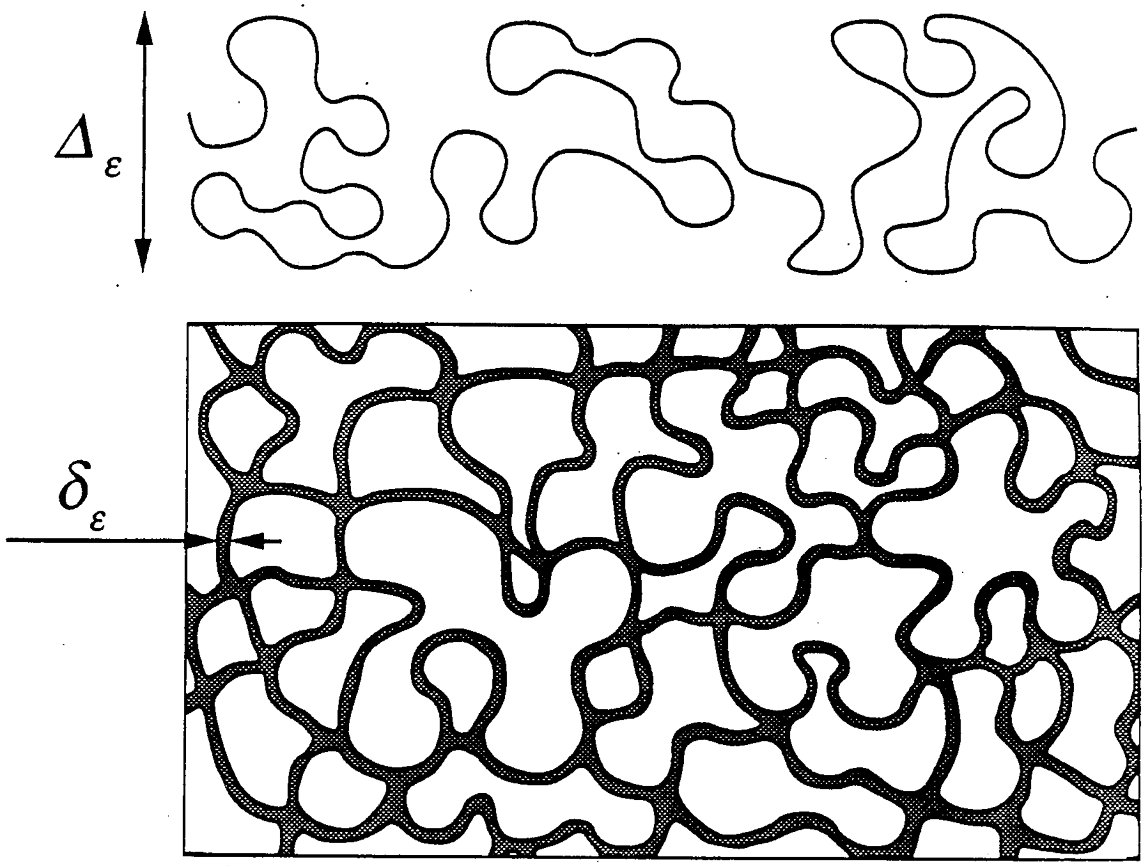


Figure 2.

a

b

c

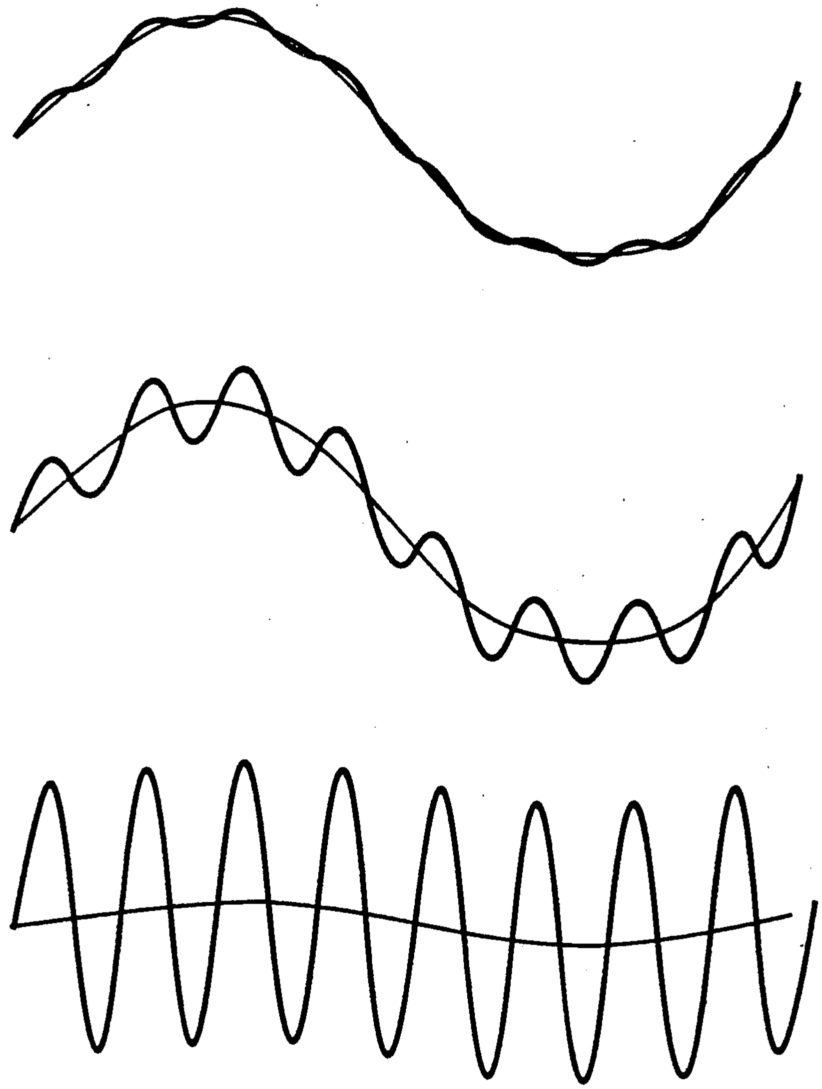
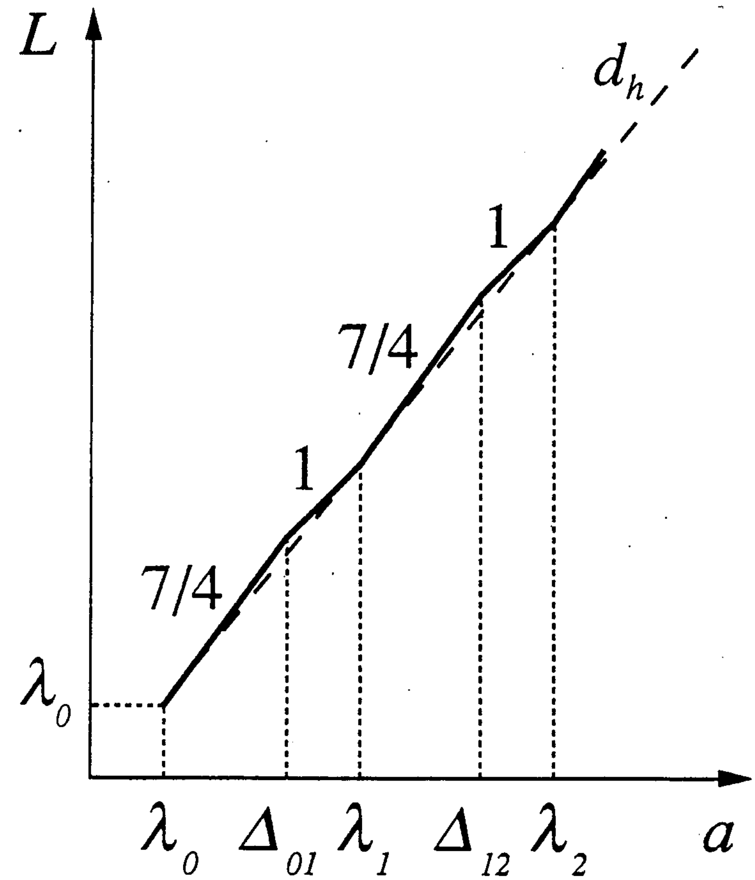


Figure 3.


L
d_h
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1
7/4
λ_0
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a

Figure 4.

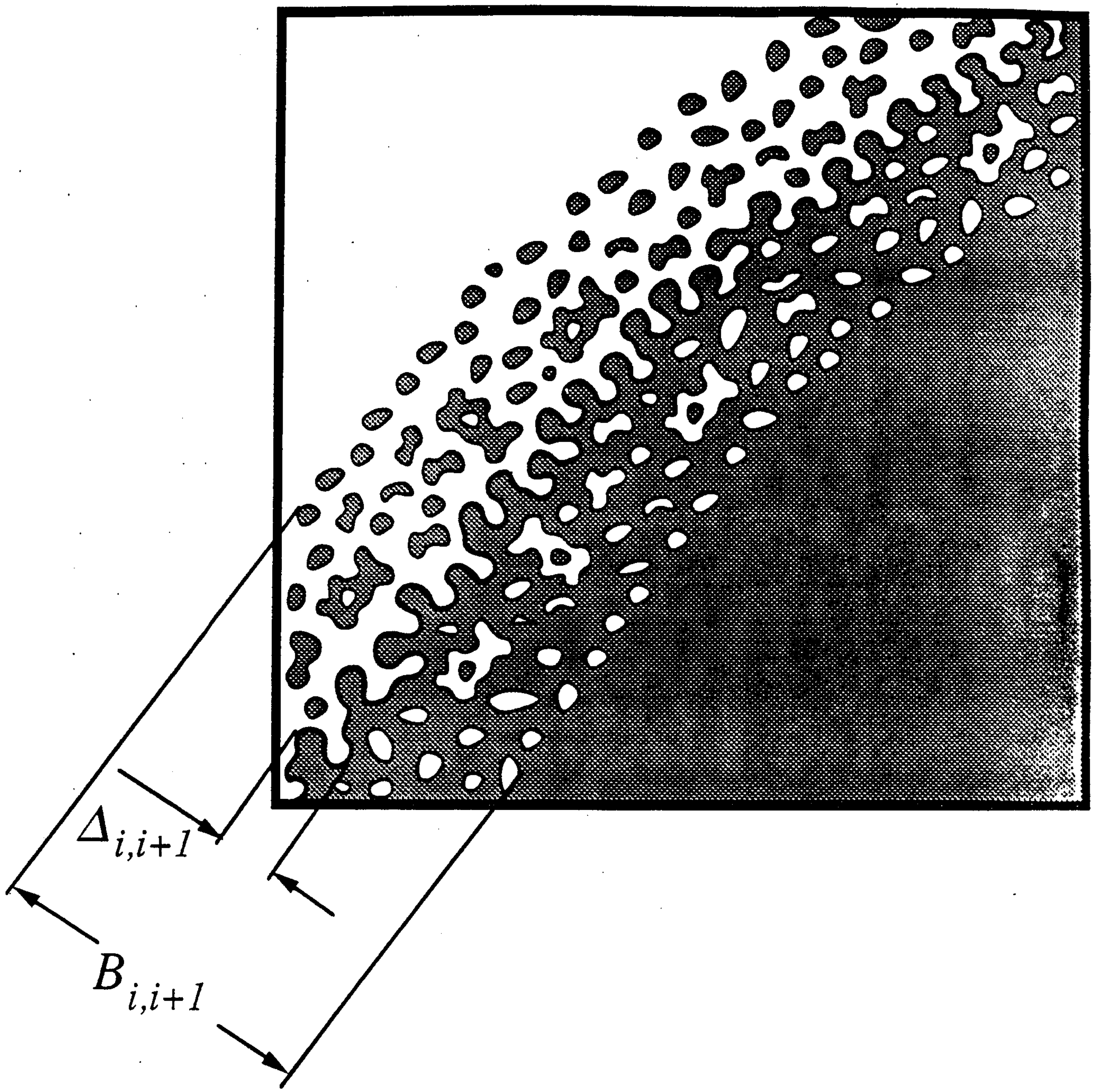


Figure 5.

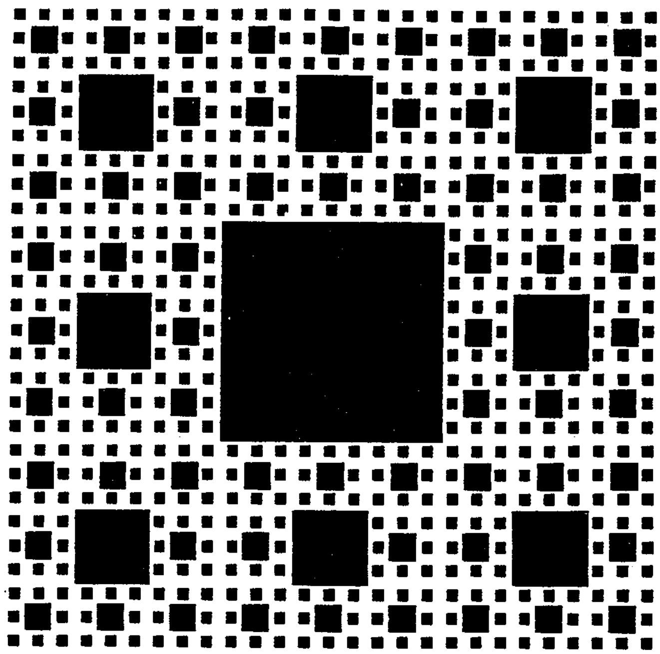


Figure 6.

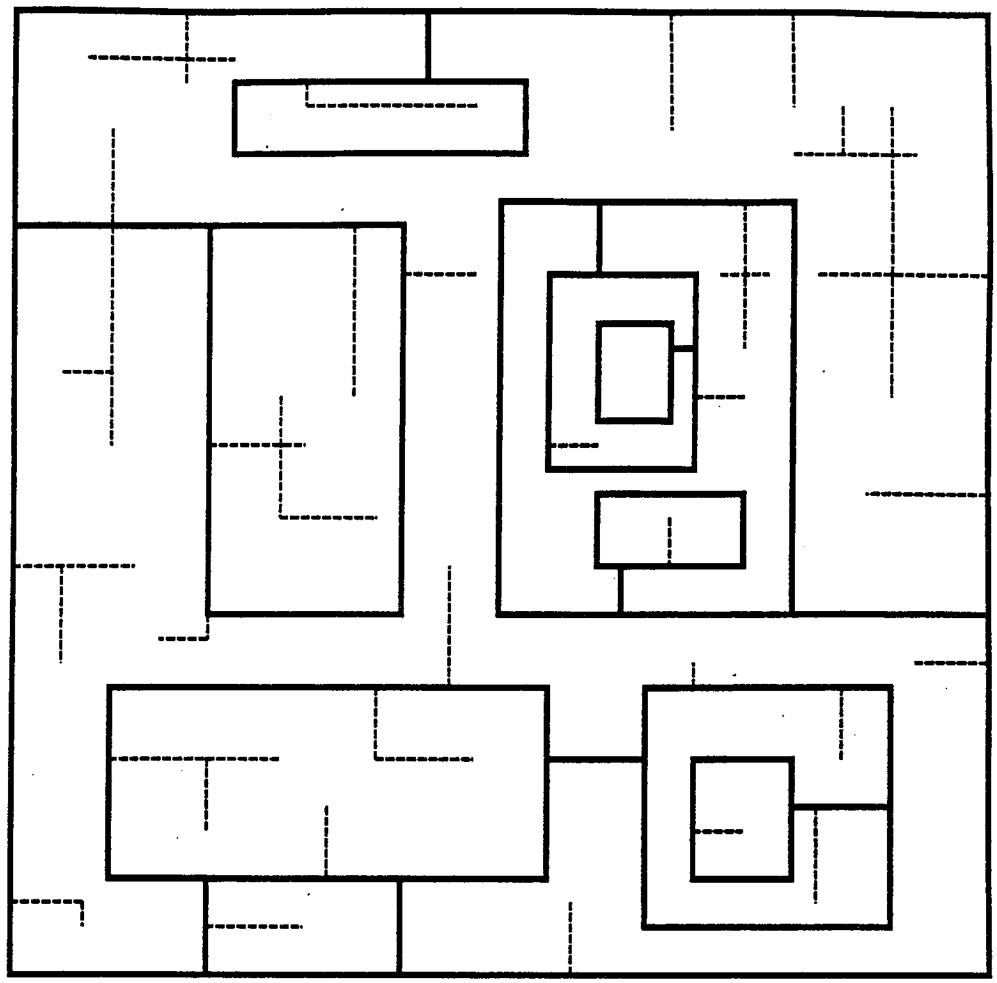
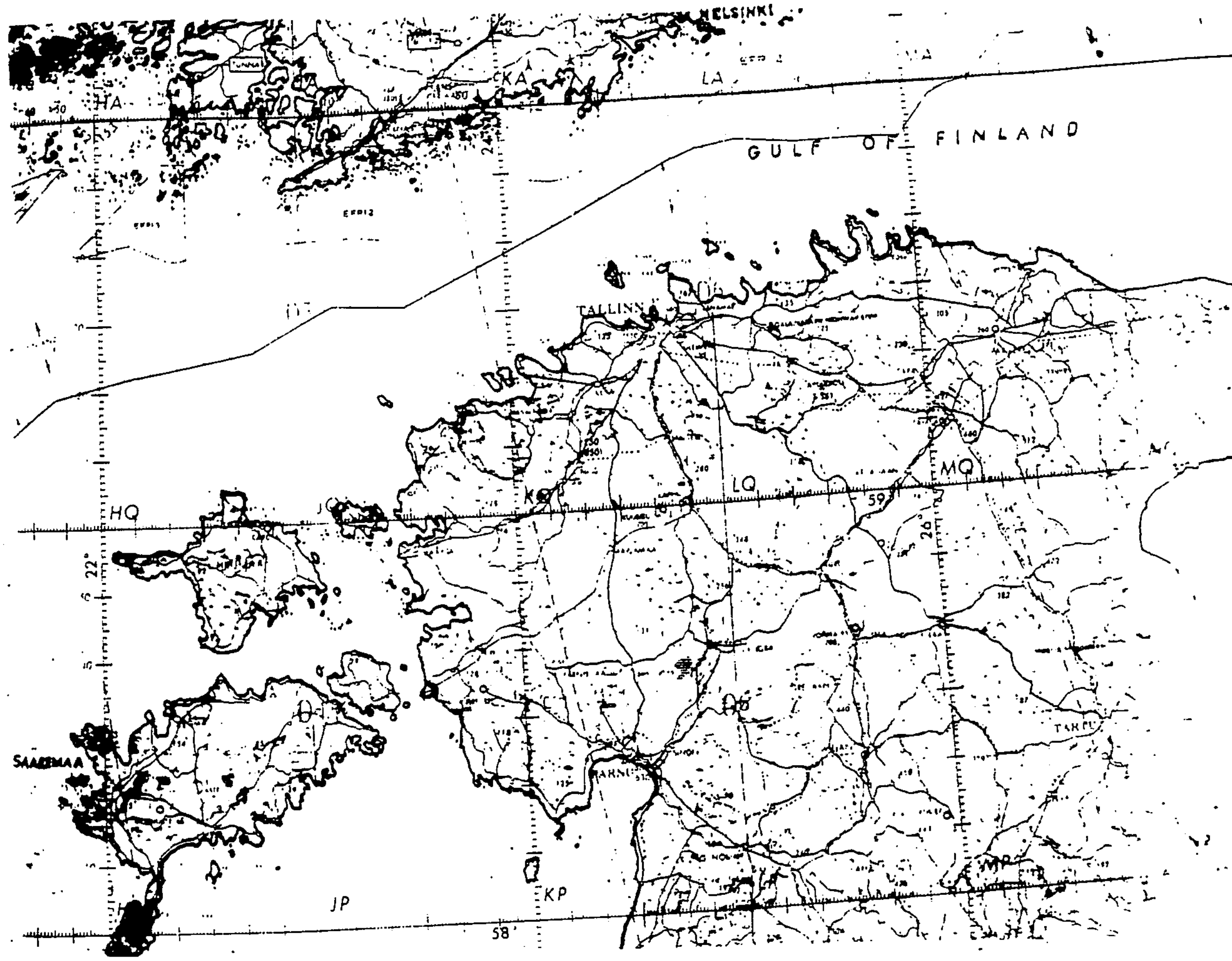


Figure 7.


HELSINKI
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59°
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Figure 8.