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Explanation of Kapitza's Linear Law of Magnetoresistance

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Abstract

A theoretical interpretation of the linear dependence of magnetoresistance of metallic samples $R(B) \propto B$ on the strong external magnetic field B is presented. This behavior was discovered by Kapitza [1, 2] and, to the best of our knowledge, has not previously been explained. It is shown that Kapitza's law can be ascribed to the current redistribution in the sample produced by the Hall effect. As a result, the main Ohmic dissipation takes place in two types thin layer. The first type reside in the vicinity of electrodes. The second type, which is due to three-dimensional geometry of the sample, lies on a surface in the interior of the sample. It is concluded that the linear behavior of $R(B)$ observed in [1, 2] can be directly related to the (inappropriate) two-terminal scheme of Kapitza's magnetoresistance measurements. In this case the voltage drop is proportional to the Hall component ρ_{xy} of the microscopic resistivity tensor. The slowly varying linear magnetoresistance of simple metals is also briefly discussed. This may be attributed to the distortion introduced by potential leads.

According to contemporary microscopic theory [3], the dependence of electron orbits on the topology of the Fermi surface implies two alternative descriptions of the magnetoresistance behavior in a strong external magnetic field $\mathbf{B} = B\hat{z}$. The dissipative components ρ_{xx}, ρ_{yy} of the resistivity tensor $\hat{\rho}$ either saturate,

$$\rho_{xx}(B) \approx \rho_0 = \text{Const}, \ \beta \equiv \Omega\tau \gg 1, \tag{1}$$

or increase quadratically with the magnetic field,

$$\rho_{xx}(B) \approx \rho_0 \beta^2 \propto B^2. \tag{2}$$

Here $\Omega = eB/(m_* c)$ is cyclotron frequency, τ is collision time, and m_* the effective mass. The saturation (1) takes place for the case of a closed Fermi surface, while the quadratic dependence (2) is characteristic of an open Fermi surface, but in the latter case the saturation of ρ_{xx} is also possible at special crystal orientations.

This behavior is in striking contrast to the experimental results by Kapitza [1, 2], who was the first to perform systematic measurements of magnetoresistance of metals in strong magnetic fields. In these pioneering experiments pulsed magnetic fields up to 320 kG were created, and at nitrogen temperatures the increase in transverse resistance R was observed to vary from several percent (Hg, Pd, Pt) to several hundred (Sb, As) and thousand (Bi) times the resistance at $B = 0$. In the case of a strong effect, viz. $R(B) \gg R(0)$, Kapitza found, and emphasized, the linear scaling of the transverse (with respect to $\mathbf{B}$) magnetoresistance as a function of magnetic field strength:

$$R(B) \propto B. \tag{3}$$

It has been long believed (cf. Ref. [4]) that the polycrystalline structure of samples can lead to the Kapitza's law (3) or a similar dependence. Dreizin and Dykhne [5] have calculated the average transverse resistivity of a polycrystal and found it to scale as $\rho_0 \beta^{2/3}$

for a noncompensated metal with a "goffered-cylinder" type Fermi surface, and as $\rho_0\beta^{4/3}$ for a compensated metal (i.e. with equal densities of electrons and holes: $n_e = n_h$) with the same Fermi surface topology. For "space-mesh" type open, and closed Fermi surfaces, the authors found polycrystalline magnetoresistance scalings as a combination of Eqs. (1) and (2).

On the other hand, the first work by Kapitza [1] dealt with monocrystals of bismuth, which is a compensated semimetal with closed Fermi surface. Antimony and arsenic, whose polycrystals were studied in [2], possess a similar electronic structure. Hence the contradiction between the theory and the experimental scaling (3) has not been eliminated but rather ignored, especially since later experiments did not reproduce the Kapitza's law [7].

In this letter we propose an explanation of the Kapitza's result. Our suggestion is that in a two-terminal scheme of resistance measurements – the one used by Kapitza – the sample resistance R is not proportional to any of the transverse dissipative components of the resistivity tensor $\widehat{\rho}$, because of a strong current contraction in narrow layers caused by the Hall effect and the boundary conditions. We find that this geometric redistribution of current flow depends on B in such a way that the linear scaling (3) emerges, yielding an excess resistance R, which exceeds, by orders of magnitude, the "naive" microscopic estimate given by Eq. (1). Later experiments used more accurate multiple contact schemes of measurements that avoided these complexities, thus making Kapitza's results "irreproducible".

In a wider sense, "Kapitza's law" refers to any linear dependence of small to moderate magnetoresistance of simple metals like potassium, $\Delta R/R \equiv (R(B) - R(0))/R(0) \leq 1$ [6], which also remains somewhat mysterious [7]. As discussed below, the Hall effect coupled to electrode geometry can give rise to such a linear component of the magnetoresistance.

We begin our analysis with the boundary problem of current flow in the presence of Hall effect. The Ohm's law

$$\mathbf{E} = \widehat{\rho}\mathbf{j}, \quad \nabla \times \mathbf{E} = 0, \quad \nabla \cdot \mathbf{j} = 0, \tag{4}$$

where $\mathbf{E}$ is the electric field and $\mathbf{j}$ is the current density, will be considered with the plasma-type resistivity tensor

$$\widehat{\rho} = \rho_0 \begin{pmatrix} 1 & \beta & 0 \\ -\beta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

which models a noncompensated metal with closed Fermi surface. In the limit of a strong magnetic field, $\beta = \Omega\tau \gg 1$, the Hall component of the tensor (5) is

$$\rho_{xy} = \rho_0 \beta = B/[ec(n_h - n_e)]. \tag{6}$$

First, consider the two-dimensional (x, y) geometry shown in Fig. 1. Let AB and CD be ideally conducting electrodes. Then the tangential electric field E_t should be zero on these lines. According to Eq. (5), this implies that the current lines encounter electrodes at a nearly tangential angle, so that $j_t/j_n = \beta \gg 1$. This means a strongly nonuniform current distribution (see Fig. 1) and, consequently, anomalously high resistance between AB and CD. To calculate the resistance, let us introduce the streamfunction ψ of the plane incompressible flow $\mathbf{j} = \nabla\psi(x, y) \times \widehat{z}$, and the electric potential ϕ: $\mathbf{E} = -\nabla\phi(x, y)$. If we neglect small diagonal components of $\widehat{\rho}$ in comparison with the Hall component $\rho_{xy} = \rho_0\beta$, then the approximate Ohm's law

$$\mathbf{E} = \rho_{xy} \mathbf{j} \times \widehat{z} \tag{7}$$

yields

$$\phi = \rho_{xy} \psi. \tag{8}$$

Eq. (8) should be supplemented with the boundary conditions

$$\phi|_{AB} = 0, \quad \phi|_{CD} = V, \quad \psi|_{AD} = 0, \quad \psi|_{BC} = I, \tag{9}$$

where V and I are the net voltage and current between the electrodes. The current I is referred to the unit length of the sample in z direction, hence the 2-D resistance R has the dimension of $\widehat{\rho}$. One can see that Eqs. (8) and (9) contradict to each other. In order to remove the contradiction, note that the current density at the points B and D diverges, thus making the small diagonal components of (5) also important [8]. So Eq. (8) is valid everywhere except in the vicinity of the points B and D. Writing Eqs. (8)–(9) for A and C we find the universal 2-D result

$$R = V/I = \rho_{xy} = \rho_0\beta, \ \beta \gg 1, \tag{10}$$

which is well known in the (intrinsically two-dimensional) quantized Hall effect where the ratio ρ_{xy}/ρ_{xx} is especially large [8]. The resistance (10) does not depend on the shape of the 2-D sample and is directly proportional to the external magnetic field.

The universality of (10) is due to the strong localization of dissipation in narrow current layers of width $\delta \approx |\mathrm{AB}|/\beta$ near electrodes. For a sufficiently long sample, the volume resistance $\rho_0|\mathrm{BC}|/|\mathrm{AB}|$ may exceed the surface resistance (10). Generally, Eq. (10) holds if the sample aspect ratio $\xi = |\mathrm{BC}|/|\mathrm{AB}|$ lies in the range

$$\beta^{-1} \ll \xi \ll \beta. \tag{11}$$

Notice that to obtain the resistance we need not calculate the detailed current distribution, which can in principle be done with the conformal mapping technique [9, 8, 10].

The universal result (10) can be readily extended to three-dimensional geometry. Let a 3-D sample be bounded in z-direction by arbitrary surfaces: $z_1(x,y) \le z \le z_2(x,y)$. Again, we write the approximate Ohm's law (7). Then the requirement $\nabla \times \mathbf{E} = 0$ yields

$$\partial \mathbf{j}/\partial z = 0, \ \nabla \cdot \mathbf{j}_\perp = 0. \tag{12}$$

Hence the current distribution in the limit $\beta \to \infty$ becomes two-dimensional, as pointed out by Herring [11]. This implies that 3-D effects are introduced solely by boundary conditions. On the free surfaces $z = z_{1,2}(x,y)$ of the sample, the normal component of the current density $j_n = 0$, so that

$$j_z = \mathbf{j}_\perp \nabla z_1(x,y) = \mathbf{j}_\perp \nabla z_2(x,y). \tag{13}$$

Since $\mathbf{j}$ does not depend on z, Eqs. (13) are valid everywhere in the sample except for the regions, which are projected along $\hat{z}$ onto electrodes or external current layers (see below). From Eqs. (13) it follows that the family of streamlines is given by the system

$$Z(x,y) = C_1, \; z - z_0(x,y) = C_2, \tag{14}$$

where

$$Z(x,y) = z_2(x,y) - z_1(x,y), \; z_0(x,y) = [z_1(x,y) + z_2(x,y)]/2, \tag{15}$$

and the constants C_1 and C_2 enumerate the lines of the electric current. (Note that one can equivalently choose $z_0(x,y) = \mu z_1(x,y) + (1-\mu)z_2(x,y)$, at any constant μ.) The current lines (14) are defined solely by the shape of free surfaces of the sample. If the current electrodes do not match this shape, viz. are not completely projected to each other along the lines (14), then additional external current layers must form near free surfaces [10], as well as those near electrodes. In the particular case of a cylindrical sample with its axis along $\hat{x}$ and the electrodes on its ends, this is not the case. Here we obtain $z_0(x,y) = 0$, $j_z = 0$, and the current flows strictly parallel to the cylinder's axis, because $Z = Z(y)$. Furthermore, it can be seen that in the 3-D case there is an even stronger constraint for current flow than those given by Eqs. (14). Let us imagine the sample to be cut by the family of surfaces $z = z_0(x,y) + C_2$ into equally thin slices (for the cylindrical sample the slices are parallel to the (x,y) plane). According to Eq. (10), every slice has the same 2-D resistance. Since the voltage is also the same, the net currents in the slices should be equal. This complies with Eq. (12) and with a generic shape of the sample only if all the current flows in the region of zero volume (for $\beta \to \infty$), corresponding to the isoline of maximum $Z(x,y)$. This is the second – internal – type of current layer. In the special case of a cylinder, the current flows in the maximum cross-section parallel to the (x,z) plane and the resistance between the bases of the cylindrical sample equals

$$R_{cyl} = \rho_0 \beta / d, \; \beta^{-1} \ll \xi \ll \beta, \tag{16}$$

where the aspect ratio $\xi = L/d$, L is the length, and d the diameter of the cylinder. A more detailed analysis [10] predicts the internal current layer thickness $\delta \approx (Ld^2/\beta)^{1/3}$. Thus the internal current contraction in a generic 3-D sample produces much weaker dissipation (at $\xi \ll \beta$) than the external one (where the layer thickness $\delta \approx d/\beta$). In any event, both types of current layer imply a highly nonuniform current distribution in the limit of a strong magnetic field $\beta \gg 1$. This behavior can result in a strong dependence of measured voltage on the position of potential leads with respect to the volume current layer in a matchstick-like sample. Another consequence of the current contraction is that local sample heating can be much more important than one would expect from a quasi-uniform current distribution.

Let us now return back to the Kapitza experiments. It is interesting to note that he rejected a multi-terminal measurement scheme in favour of a two-pole one. In paper [1] on page 395 he states

> ...we took advantage of the fact that the specific resistance of bismuth in normal conditions is about 75 times that of silver, and, as in our experiments we limit ourselves to an accuracy of 1 per cent., it is not necessary to make proper potential leads. It is sufficient to solder on the end of the bismuth rod two silver discs, 1/4 mm. thick, and to bring the power and potential leads from these discs.

The work in Ref. [2] used the same experimental apparatus. If the Bi crystals used in [1] were exactly compensated, viz. $n_e = n_h$, then their resistance would grow as the square of magnetic field. The geometric effects described above clearly cannot diminish the resistance. The only way to explain the linear behavior $R(B) \propto B$ is to assume the presence of impurities, especially since the number of intrinsic charge carriers in Bi is very small – of order of 10^{-5} per atom [12], and small impurities can make bismuth strongly noncompensated. This would give rise to a linear regime and, due to the smallness of n_e and n_h, the Hall component (6) becomes large at moderate B, making it possible to observe a strong magnetoresistance $R(B) \gg R(0)$ in Bi [1], Sb and As [2] at the temperature of liquid nitrogen. Kapitza used cylindrical samples with the aspect ratio $\xi = L/d = 3 \div 5$, and the Hall parameter β reached thousands for Bi and hundreds for Sb and As. This implies that formula (16) should be applicable, in accordance with the linear magnetoresistance observed in [1, 2]. We thus conclude that the original "linear Kapitza's law" is an artifact due to an inappropriate measurement of the voltage on power leads.

For potential leads separate from power contacts, the geometrical effects are not so essential, especially for samples of sufficiently long wire ($\xi \gg \beta$). Yet, even in this case the potential contacts provide a similar boundary condition and distort current flow in their vicinity. This effect is proportional to the current in the voltage measuring circuit. Hence one can expect a small linear component of magnetoresistance with the slope of order of the ratio of the "saturated" sample resistance and the inner resistance of the voltage measuring circuit. Since this parameter changes from experiment to experiment, the irreprodicibility of observed Kohler's slopes [6] looks natural. However, the possibility of explaining the linear magnetoresistance with the help of voids and other samples' inhomogeneities still exists [7].

The effect of anomalous resistance in the presence of strong Hall effect is not quite new. It has been discussed both for a uniform bounded medium [8, 13] and for a randomly inhomogeneous case [11, 5, 14], in respect of semiconductors, inversion layers, and plasmas. In the present letter we have linked these effects to the magnetoresistance of metals, and demonstrated how the Hall effect played a joke on the Kapitza law.

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Figure Caption

Figure 1. Scheme of a two-dimensional sample. Arrows indicate current lines.

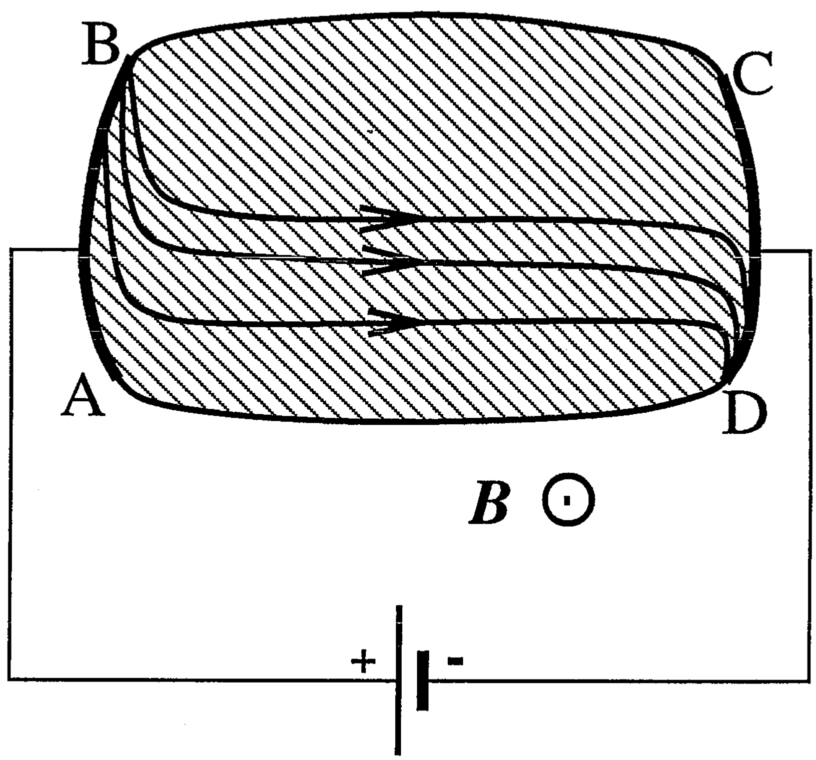

Figure 1.