INSTITUTE FOR FUSION STUDIES

DOE/ET-53088-463 IFSR #463

Effective Plasma Heat Conductivity in "Braided" Magnetic Field. Part I. Quasi-Linear Limit

M.B. ISICHENKO[a)]
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

November 1990

[a)] *permanent address: Kurchatov Institute of Atomic Energy, 123182 Moscow, U.S.S.R.*

THE UNIVERSITY OF TEXAS

AUSTIN

Errata "Effective Plasma Heat Conductivity in "Braided" Magnetic Field. Part I. Quasi-Linear Limit".

M.B.Isichenko

1. [p. 5, line 14] The text

   ", due to the KAM theorem"

should be omitted.

2. [p. 18, Eq. (26b)] "$\nu_e t_d > 1$" should read "$\nu_e t_d < 1$".

3. [p. 19, Eq. (30b)] "$l/v_e \log(\delta/r)$" should read "$l/v_e \log[v_e\delta/(\omega l(D_m l)^{1/2})]$".

4. [p. 22] The reference

   Krommes, J.A. (1978) Prog. Theor. Phys. Suppl. **64**, 137.

should be inserted between Kadomtsev & Pogutse (1978) and Krommes, Oberman & Kleva (1983).

Effective Plasma Heat Conductivity in "Braided" Magnetic Field. Part I. Quasi-Linear Limit

M.B. Isichenko [a)]
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

Abstract

Anomalous cross-field electron transport in a specified magnetic field with weakly destroyed flux surfaces is discussed. Following the approach developed by Rechester and Rosenbluth (1978), and by Krommes, Oberman and Kleva (1983), in the present paper the regimes of effective transverse plasma transport are studied systematically, both in collisional and collisionless limits. The present analysis incorporates non-stationarity of magnetic perturbations, which was not included in the previous works. Part I of this study deals with quasi-linear approximation, which may be written as $b_0 L_0/\delta \ll 1$, where $b_0 = \delta B_\perp / B_0$ represents the relative magnitude of the transverse magnetic perturbation, while L_0 and δ represent the longitudinal and transverse correlation lengths, respectively. It is found that some of previously described transport regimes cannot be considered as anomalous (in the sense that effective heat conduction $\chi_{\text{eff}} \gg \chi_\perp$) for time-independent magnetic perturbations. However, these regimes can exist in the presence a finite-frequency magnetic flutter. A unified classification of quasi-linear regimes of anomalous transport is introduced in order to further extend the analysis

[a)]Permanent address: I.V. Kurchatov Institute of Atomic Energy, 123182 Moscow, U.S.S.R.

to the strong magnetic turbulence limit $b_0 L_0/\delta \gg 1$ (see Part II), which is considered as a companion paper (Isichenko 1990).

I. Introduction

Anomalous transport in magnetic confinement systems is a central issue in fusion studies (cf. Rechester & Rosenbluth 1978, Stix 1978, Kadomtsev & Pogutse 1978, Krommes 1978, Krommes, Oberman & Kleva 1983, Galeev and Zelenyi 1981, Horton 1983, Haas & Thyagaraja 1986). Up to the present time, no widely adopted self-consistent anomalous transport theory exists. Such a theory would require the knowledge of plasma turbulence parameters whose analytical evaluation in terms of density and temperature gradients or other plasma parameters is scaresly reliable, if possible at all. In most theoretical papers devoted to this problem average transport characteristics, such as heat conductivity, diffusion coefficients, etc. are expressed in terms of electromagnetic fluctuations that are treated as given, or taken from an experiment (cf. review Liewer 1985). This approach is an integral part of a self-consistent transport theory, which still remains to be developed.

In the present paper a similar approach is taken. We consider transport in a specified "braided" magnetic field of the form

$$\mathbf{B} = B_0(\widehat{z} + \alpha(x)\widehat{y}) + \delta\mathbf{B}(x, y, z, t) \ , \tag{1}$$

where x, y, z represent Cartesian coordinates with the corresponding unit vectors $\widehat{x}, \widehat{y}, \widehat{z}$, $d\alpha(x)/dx$ is the background shear, and $\delta\mathbf{B}$ represents a short-scale random magnetic perturbation.

Since in the majority of applicable cases electrons are strongly magnetized, i.e. $\chi_{\|}/\chi_{\perp} = (\omega_{Be}\tau_e)^2 \gg 1$ ($\chi_{\|}$ and $\chi_{\perp}$ are the longitudinal and transverse electron heat conductivities, respectively, ω_{Be} is gyrofrequency and τ_e Coulomb collision time), a weak destruction of magnetic flux surfaces with $\delta B \ll B_0$ can lead to a significant increase in transverse heat transport.

Magnetic stochasticity is characterized by a random, diffusion-like walk of a magnetic line with respect to the unperturbed magnetic surface (Rosenbluth *et al.*1966, Stix 1973), and the exponential scattering of close magnetic lines (Krommes 1978, Rechester, Rosenbluth & White 1979, Krommes, Oberman & Kleva 1983). The magnetic line diffusivity D_m and the length of exponentiation l, which influence the effective heat conductivity χ_{eff}, are defined in the most simple way in the so-called quasi-linear approximation with the small parameter introduced by Kadomtsev and Pogutse (1978) (see also Krommes 1978):

$$R = (\delta B_\perp / B_0) L_0 / \delta \equiv b_0 L_0 / \delta \ll 1 \ , \tag{2}$$

where L_0 and δ are the characteristic inhomogeneity scales of the perturbation $\delta\mathbf{B}(x, y, z)$ in longitudinal and transverse directions, respectively. The present article deals with this limit only. The opposite case is discussed in a companion paper Isichenko (1990).

In this article the problem of stochastic magnetic transport is treated systematically, proceeding from the most simple and clear arguments of test particle motion. Results of this approach are presented in a convenient form, that includes additional effects, such as the non-stationarity of magnetic perturbations.

Our consideration of this problem has been motivated by the desire to examine the limit of strong magnetic perturbations $R \gg 1$ (Part II), which is the opposite of the quasi-linear limit. However, the present paper reports some new results concerning the quasi-linear limit $R \ll 1$. We follow the smooth transition between the week and strong shear regimes, described in Kadomtsev & Pogutse (1978) and Krommes, Oberman & Kleva (1983), respectively, and estimate the related critical shear length l_s. It is found that some of previously introduced magnetic transport regimes, namely, the "fluid limit", "double streaming" and "Kadomtsev-Pogutse" regimes (with nomenclature adopted in Krommes, Oberman & Kleva 1983) are not literally anomalous (understood as $\chi_{\text{eff}} \gg \chi_\perp$) in the limit of a stationary magnetic flutter. A consideration of non-stationary magnetic perturbations reveals that those regimes can be recovered. Also, it shown that one can follow the transition between all the transport regimes, as the characterictic frequency of magnetic turbulence ω is changed.

When $\delta\mathbf{B}$ is sufficiently small, it gives rise to the formation of a set of magnetic islands separated by regions of "good" flux surfaces (Rosenbluth *et al.* 1966, Stix 1978). As $\delta\mathbf{B}$ increases, the islands coalesce, resulting in the formation of stochastic regions. One should emphasize that according to the KAM theory (cf. Sagdeev, Usikov & Zaslavsky 1988) at sufficiently small perturbation, $\delta B \ll B_0$, the regions of globally stochastic magnetic lines must occupy a very small fraction of the plasma volume (Cook, Thyagaraja & Haas 1982). On the other hand, the Kadomtsev & Pogutse (1978) "braided" model, which ignores toroidal effects, as well as shear, predicts a stochastic behavior of every field line, at arbitrarily small $\delta\mathbf{B}$. These limiting cases of KAM theory and the Kadomtsev-Pogutse model, although different, do not contradict each other. Indeed, a microturbulent magnetic flutter with correlation scales much less than both minor and major radii of tokamak make magnetic lines behave locally as in the "braided" model. At the same time, according to KAM theory, most of the short-range disordered flux surfaces are long-range topologically ordered, i.e. exactly closed, due to the KAM theorem. To more clearly understand this point, one should bear in mind that *locally*, any solenoidal field possesses invariant flux surfaces whose choice is quite arbitrary (see, for example, Fig.1). Also, the global nested topology of magnetic surfaces does not prevent a local stochastic behavior of field lines. If we consider a finite decorrelation length z_d, over which a test particle can be considered to move along a given magnetic line, and if this length is much smaller than a global topology scale (say, the minor radius), then we come to the conclusion that in this case the KAM-backed long-range magnetic order is irrelevant and the local stochastic features of the magnetic field govern anomalous transport.

Another remark concerns the assumed arbitrariness of the magnetic perturbation. In the limiting case of $\delta\mathbf{B}$ being produced by a short scale ideal MHD activity, $\delta\mathbf{B} = \nabla \times (\boldsymbol{\xi} \times \mathbf{B}_0)$, not only every flux surface is preserved, but also local topological restraints are imposed on field lines, confining their transverse walk within the range of the displacement $\boldsymbol{\xi}(\mathrm{r}, t)$. This limit requires a separate examination. Let us only note that at sufficiently small scales (for instance, c/ω_{pe} and less) the magnetic field is no longer "frozen" to plasma and thus can change its topology (Kadomtsev & Pogutse 1978).

The paper is organized in the following way. In Sec. II we briefly discuss the geometric properties of the "braided" magnetic field. The exponentiation rate of close magnetic lines is evaluated, with an emphasis on shear dependence. In Sec. III we discuss the test particle approach to the heat transport and express the effective diffusivity χ_{eff} through the test particle decorrelation time t_d. Section IV is devoted to the evaluation of t_d in the case of stationary magnetic perturbations. Here, we also analyze stationary anomalous transport regimes both for collisional and collisionless limits. Results of our analysis are compared to those of others. In Sec. V the effect of non-stationary magnetic fluctuations is studied. In Sec. VI we summarize and discuss the results and demonstrate the transition between various regimes of anomalous transport.

II. Geometry of Stochastic Magnetic Field

In this section, we discuss the behavior of a separate magnetic line, of two close lines and of a magnetic flux tube in the "braided" magnetic field (1).

Let us take the shear parameter $\alpha(x)$ in the formula (1) to be less than or of order of unity and the amplitude of the fluctuations to be small: $b_0 \equiv \delta B_\perp / B_0 \ll 1$. The mean value of magnetic perturbation is taken to be zero, $\langle \delta \mathbf{B}(x, y, z) \rangle = 0$. Under these constraints, the equation of a magnetic line takes the form

$$d\mathbf{r}_\perp / dz = \mathbf{b}(\mathbf{r}_\perp, z) + \alpha(x)\widehat{y} \ , \qquad (3)$$

$$\mathbf{b} \equiv \delta \mathbf{B}_\perp / B_0 \quad , \quad \mathbf{r}_\perp = (x, y) \ ,$$

which describes a well known diffusional walk of magnetic lines in the x-direction (across the non-perturbed flux surfaces). Depending on the longitudinal coordinate z, the average transverse displacement of field lines is governed by

$$\left\langle x^2(z) \right\rangle = \begin{cases} b_0^2 \, z^2 \ , & z < L_0 \ , \\ 2 D_m z \ , & z > L_0 \ , \end{cases} \qquad (4)$$

with D_m denoting the magnetic line diffusivity (Rosenbluth *et al.* 1966, Stix 1973, Kadomtsev & Pogutse 1978).

The calculation of D_m depends significantly on the amplitude of magnetic perturbations, which is convenient to describe in terms of the dimensionless parameter (2). The quantity R can be expressed as the ratio of the original longitudinal correlation length L_0 and the distance $L_\delta = \delta / b_0$. The parameter L_δ is an effective longitudinal correlation length in the frame of a given magnetic line, due to the finite transverse inhomogeneity scale δ, in the formal limit $L_0 \to \infty$. The weak turbulence, or, equivalently, quasi-linear case $R \ll 1$ outlines the predominance of longitudinal magnetic inhomogeneities, thus making the magnetic diffusion insensitive to the transverse correlation scale δ:

$$D_m = (1/2) \int_{-\infty}^{\infty} \langle b_x(\mathbf{r}_\perp, z) b_x(\mathbf{r}_\perp, 0) \rangle \, dz \approx b_0^2 L_0 \ . \tag{5}$$

However, as shown subsequently, δ does enter expressions for the exponentiation length l.

In this section we assume the magnetic field to be stationary, with angular brackets in Eqs. (4),(5) implying the averaging over magnetic lines.

It is emphasized that $\mathbf{r}_\perp$ in the integral expression in (5) depends on z according to Eq. (3). Nevertheless, the estimate of D_m assumes that one may neglect the implicit dependence of b on z through $\mathbf{r}_\perp(z)$, in comparison with the explicit one. In fact, this assumption defines the quasi-linear approximation. (Indeed, if one neglects the dependence $\mathbf{b}$ on $\mathbf{r}_\perp$, the magnetic line equation (3) becomes linear.) In the absence of shear this requirement is equivalent to the inequality (2). However, when shear is taken into consideration, additional restrictions arise. Equation (3) yields $dx/dz \approx b_0$, $dy/dz \approx x/l_s$, and the increment of y, as z passes the longitudinal correlation length L_0, is of the order of $y \approx L_0 x/l_s \approx L_0^2 b_0/l_s$. The quasi-linear approximation holds provided that this increment is much smaller than the transverse correlation length δ, i.e. the shear length l_s should not be too small:

$$l_s \gg L_0 R \ . \tag{6}$$

This inequality, in addition to (2), provides a necessary condition for the quasi-linear geometry of a "braided" magnetic field.

In fact, the problem of a globally stochastic magnetic field with shear is further complicated by that any displacement of a magnetic line in x-direction gives rise to a zeroth order dependence $y(z)$. At the very least, this implies tha connection between correlation lengths L_0, δ and the shear length l_s, as discussed in Krommes, Oberman & Kleva (1983). However, as can be seen from Eq. (3), the shear term, however small, will asymptotically result in an accelerating dependence $y(z)$ and the subsequent accelerating frequency modulation of

$b_x(\mathbf{r}_\perp(z), z)$ through its first argument. These oscillations might result in a non-diffusional (namely, sub-diffusional) dependence of $x(z)$. Yet, this behavior can be unimportant if a test particle decorrelates from the magnetic line before the line manifests this behavior. Furthermore, a diffusional walk of magnetic lines in x-direction could, and probably should, be recovered when formulating the problem in more suitable coordinates, instead of (x, y, z). This issue remains beyond the scope of the present paper.

Along with the tangled walk of a line, the stochastic feature of the magnetic field leads to an exponential scattering of a pair of close lines:

$$\langle \delta r(z) \rangle = \delta r(0) \exp(|z|/l) \ , \tag{7}$$

where $\delta \mathbf{r} = (\delta x, \delta y)$ is the distance between close (i.e. $\delta r \ll \delta$) magnetic lines. This effect is known as stochastic instability (Lichtenberg & Lieberman 1983). It is described by the linearized equation (3):

$$\begin{aligned} d\delta x/dz &= b_{xx}(z)\delta x + b_{xy}(z)\delta y \\ d\delta y/dz &= b_{yx}(z)\delta x + b_{yy}(z)\delta y + \delta x/l_s \ , \end{aligned} \tag{8}$$

where $b_{\alpha\beta}(z) \equiv \partial b_\alpha / \partial r_\beta$, and the shear slab approximation is used: $1/l_s \equiv d\alpha/dx = \text{const.}$.

For the calculation of the Kolmogorov entropy (average growth rate of stochastic instability) let us take advantage of the technique developed by Krommes, Oberman & Kleva (1983). Multiplying the equations (8) by $\delta x, \delta y$, and adding them, one obtains the following system for $\rho^T = \{\langle(\delta x)^2\rangle,\ \langle(\delta y)^2\rangle,\ \langle\delta x \delta y\rangle\}$:

$$d\rho/dz = \mathbf{A}\rho, \ \mathbf{A} = \begin{pmatrix} 4/l_{xx} & 2/l_{xy} & 0 \\ 2/l_{yx} & 4/l_{yy} & 1/l_s \\ 1/l_s & 0 & 1/l_{zz} \end{pmatrix} , \tag{9}$$

Here we have assumed the following correlation properties:

$$\langle b_{\alpha\beta}(z) b_{\gamma\delta}(z') \rangle = \delta_{\alpha\gamma}\delta_{\beta\delta} f_{\alpha\beta}(z - z') \ , \tag{10}$$

where $\alpha, \beta, \gamma, \delta = x, y, z, \delta_{\alpha\beta}$ is the Kronneker symbol, b_z means the longitudinal perturbation $\delta B_z/B_0$. The correlation functions $f_{\alpha\beta}(z)$ decay on the correlation length L_0, and are normalized as

$$(1/2)\int_{-\infty}^{\infty} f_{\alpha\beta}(z)dz = l_{\alpha\beta}^{-1} \approx \begin{cases} b_0^2 L_0/\delta^2 \ , & \alpha, \beta = x, y \ , \\ b_z^2/L_0 \ , & \alpha = \beta = z \ . \end{cases} \quad (11)$$

The average exponentiation rates of magnetic lines $1/l$ (see Eq. (7)) are now defined as the eigenvalues of the matrix $\mathbf{A}$. When the shear is absent $l_s = \infty$) we obtain

$$l_1 = l_{zz}, l_{2,3} = 2 \Big/ \left[l_{xx}^{-1} + l_{yy}^{-1} \pm \sqrt{\left(l_{xx}^{-1} - l_{yy}^{-1}\right)^2 + l_{xy}^{-1} \cdot l_{yx}^{-1}} \right] \ . \quad (12)$$

Usually, the longitudinal component of magnetic perturbation is much smaller than the transverse one: $b_z \ll b_0$. Besides, if $b_z < R = b_0 L_0/\delta$, then the scattering lengths $l_{2,3} \approx l_0$ become much smaller than l_1, thus yielding the fastest mode of the stochastic instability:

$$l \approx l_0 \equiv \delta^2/(L_0 b_0^2) = L_0/R^2 \quad , \quad l_s > l_0 \ . \quad (13)$$

The effect of shear is significant at $l_s < l_0$; in that case we have

$$l \approx (l_0 l_s^2)^{1/3} \quad , \quad l_s < l_0 \ . \quad (14)$$

Formula (13) agrees with results of Kadomtsev & Pogutse (1978), while (14) is exactly the same as the exponentiation length reported in papers Krommes (1978) and Krommes, Oberman & Kleva (1983). The above results (13),(14) show how these limits match each other and when the effect of shear becomes significant.

Let us note that at any l_s the both inequalities (2) and (6), specifying the quasi-linear approximation, can be rewritten in the form $l \gg L_0$, as pointed out in Krommes, Oberman & Kleva (1983).

Let us now imagine what a flux tube of the magnetic field looks like. Owing to the magnetic flux conservation, ($\text{div}\mathbf{B} = 0$), the cross-section area of the tube is nearly constant.

Further, the boundary of this cross-section, being composed of close points, rises in length exponentially, according to (7).[1] This leads to a growing convolution of the side surface of the tube (see Fig. 1), and the exponential decrease of its width:

$$\widetilde{h}(z) \approx h \exp(-|z|/l) , \tag{15}$$

where h is the initial diameter of the magnetic tube. This observation (Rechester & Rosenbluth 1978) is of great importance and we shall return to it later on.

[1]In fact, the elongation of a contour by the map (3) is defined not by the mean but rather by the maximum exponentiation rate. However, in the quasi-linear limit every magnetic line behaves qualitatively in the same manner; hence, the mean and the maximum stochastic growth rates are of the same order of magnitude.

III. Effective Heat Conductivity as Diffusivity of Test Particles

In this section we express test particle transport in terms of the time of decorrelation, and introduce the regime of quick decorrelation and the regime of magnetic line diffusion.

The test particle arguments are commonly used for the evaluation of particle and heat transport. However, one must remember that under plasma quasi-neutrality restraints, electrons cannot diffuse as free test particles. Nevertheless, due to the thermal spread of energies of electrons the energy can be transferred while the center of charge is motionless. That is, heat transport can be treated as the diffusion of test particles that are not actually particles but, rather, quasi-particles that describe the elementary excitations of the thermal state of the plasma. For example, in the hydrodynamic limit one can solve the heat transport equation

$$\partial T/\partial t = \mathrm{div}(\chi_{\parallel}\nabla_{\parallel}T + \chi_{\perp}\nabla_{\perp}T) \tag{16}$$

by the path integration method. The latter is a mathematical technique corresponding to the physical analogy, when (16) is treated as the diffusion equation with T standing for the density of a "substance". So, in this case our test particle is a particle of this "substance".

Thus, the effective heat conductivity can be estimated as the average radial diffusivity of a test particle. To the first approximation, the test particle moves along the magnetic line according to

$$z(t) = \begin{cases} (\chi_{\parallel}t)^{1/2} , & \nu_e t > 1 , \qquad (17a) \\ v_e t , & \nu_e t < 1 , \qquad (17b) \end{cases}$$

which correspond to hydrodynamic (collisional) and kinetic (collisionless) limits, respectively. In Eq. (17) v_e means the average thermal velocity of electrons, $\nu_e = \tau_e^{-1}$ is the collision frequency. In fact, the test particle moves along a specified magnetic line only during a finite time of decorrelation t_d (Krommes, Oberman & Kleva 1983), distinguishing the reversible and irreversible processes.

The cross-field diffusivity of a test particle can now be obtained from (4) as

$$\chi_{\text{eff}} \approx \frac{\langle x^2(z(t_d))\rangle}{t_d} \approx \begin{cases} b_0^2 z_d^2/t_d\ , & z_d > L_0 \qquad \text{(QD)} \qquad (18a) \\ D_m z_d/t_d\ , & z_d > L_0 \qquad \text{(MD)} \qquad (18b) \end{cases}$$

where $z_d \equiv z(t_d)$ is defined from Eq. (17). In the expression (18) we distinguish between two distinct regimes: the regime of quick decorrelation (QD) and the magnetic line diffusion (MD) regime. Hence, in the hydrodynamic limit ($\nu_e t_d > 1$) we have:

$$\chi_{\text{eff}} \approx \begin{cases} \chi_{\|} b_0^2\ , & t_d < L_0^2/\chi_{\|} \qquad \text{(QD)} \qquad (19a) \\ D_m(\chi_{\|}/t_d)^{1/2}\ , & t_d > L_0^2/\chi_{\|}\ , \qquad \text{(MD)} \qquad (19b) \end{cases}$$

and the kinetics ($\nu_e t_d > 1$) leads to

$$\chi_{\text{eff}} \approx \begin{cases} b_0^2 v_e^2 t_d\ , & t_d < L_0/v_e\ , \quad \text{(QD)} \qquad (20a) \\ D_m v_e\ , & t_d > L_0/v_e\ . \quad \text{(MD)} \qquad (20b) \end{cases}$$

Krommes, Oberman & Kleva (1983) describe the hydrodynamic regimes (19a) as "fluid" and (19b) as "double diffusion" (emphasizing that at $t_d \to \infty$ $x \propto t^{1/4}$ and thus $\chi_{\text{eff}} \to 0$). To the kinetic regimes (20a) and (20b) they are referred to as "double streaming" and "collisionless", respectively. Although these terms are widely accepted in the literature, I believe that some of them are physically misleading. For instance, "fluid" (i.e. collisional) description not only applies to to the regime (19a), but to (19b) as well. (However, the "double diffusion" notation for the collisional MD regime still remains quite sensible.) Analogously, the "collisionless" limit is actually the case for both regimes (20a) and (20b), not only for the latter. To avoid this ambiguity, the above classification has been introduced, based on whether a test particle decorrelates before (Quick Decorrelation) or after (Magnetic Diffusion) the magnetic line exgibits the diffusional walk. The advantage of this nomenclature will be further seen in the strong turbulence limit (Part II) where the number of transport regimes is greater.

Now, to obtain the result, we must evaluate the decorrelation time t_d.

IV. Decorrelation in Stationary Stochastic Magnetic Field

In this section we derive expressions for t_d which yield transport different from the results by Rechester & Rosenbluth (1978) only by notations, here being more universal and convenient for further arguments.

One can define t_d as the time it takes the test particle to get to another magnetic line which is separated from the first one by the distance of order of δ. Or, speaking more accurately, t_d is the time in which the test particle leaves the magnetic flux tube whose cross-section $z = 0$ is the circle of radius δ, when the particle starts from its center. Due to the diffusivity and the stochastic instability of magnetic lines, the appearance of this flux tube is rather complicated (see Fig. 1).

First, let us examine the collisional limit $\nu_e t_d > 1$. The transverse decorrelation time is evidently not to exceed the quantity $t_\perp = \delta^2/\chi_\perp$, which, having been substituted into (19a), would yield a result less than $\chi_\perp$, which would be not interesting. Had we put $t_d = t_\perp$ and used double-diffusion expression (19b), then we would obtain the Kadomtsev & Pogutse (1978) result $\chi_{\text{eff}} \approx (\chi_\parallel \chi_\perp)^{1/2} D_m/\delta$. Yet, owing to the special behavior of the magnetic flux tube (Fig. 1) the decorrelation becomes earlier. At very small $\chi_\perp$ the easiest way for the test particle to leave the flux tube is to move along the magnetic line several scattering lengths l and then to diffuse transversely through the short distance $\tilde{h}(z)$. Using the expression (15) we can now solve the equation

$$\delta \exp(-z(t_d)/l) = (\chi_\perp t_d)^{1/2} , \tag{21}$$

for t_d to obtain

$$t_d \approx (l^2/\chi_\parallel) \ln^2[(\chi_\parallel/\chi_\perp)(\delta/l)^2] . \tag{22}$$

Substituting (22) into (19b), one gets the Rechester-Rosenbluth formula

$$\chi_{\text{eff}} \approx \chi_{\|}(D_m/l)\ln^{-1}((\chi_{\|}/\chi_{\perp})(\delta/l)^2] \,. \tag{23}$$

When the characteristic correlation lengths along x and y directions are different, δ should be understood as the shorter of them.

The expression (23) is valid provided that the quantity under the logarithm is large. Otherwise we arrive formally at the Kadomtsev-Pogutse regime (i.e. (19b) at $t_d = t_\perp$). However, any physically interesting regime of anomalous heat conduction should assume $\chi_{\text{eff}} \gg \chi_\perp$. If we demand that in the marginal point between Kadomtsev-Pogutse and Rechester-Rosenbluth regimes $(\chi_\perp/\chi_\| = \delta^2/l^2)$ $\chi_{\text{eff}} > \chi_\perp$, then we come to the inequality $D_m l > \delta^2$ which, as a consequence of Eqs. (5), (13), and (14), holds neither in strong nor in weak shear limits. This point was overlooked by Krommes, Oberman & Kleva (1983), possibly because those authors did not consider the case of very weak shear.

In sum, transport contributions given by both the QD regime (19a) and the "Kadomtsev-Pogutse" regime $\chi_{\text{eff}} \approx (\chi_\|\chi_\perp)^{1/2}D_m/\delta$ are small compared to $\chi_\perp$ and thus uninteresting in the stationary magnetic field. So, the Rechester-Rosenbluth (MD) regime (23) exhausts the hydrodynamic anomalous transport in the quasi-linear approximation.

In the kinetic limit $\nu_e t_d < 1$ the decorrelation occurs faster than a collision, and the background cross-field diffusivity $\chi_\perp$ no longer makes sense. Here, one must replace the test particle transverse "uncertainty" $(\chi_\perp t)^{1/2}$ by some other quantity, for example, the gyroradius r_e. It can be argued that when such scales are taken into account, the essentially three-dimensional particle motion may have its own stochasticity and give a real decorrelation, even in a stationary magnetic field in the absence of collisions. This uncertainty enters the expression for t_d only logarithmically (Eq. (24)) while the expression for χ_{eff} is independent of t_d (Eq. (20b)).

Having introduced this change into the previous arguments and using (17b) instead of

(17a), we can obtain the collisionless decorrelation time

$$t_d = (l/v_e)\ln(\delta/r_e) \ . \tag{24}$$

Under the quasi-linear condition $l_s > L_0 R$ assumed above, the decorrelation time (24) belongs to the case (20b), giving the collisionless MD result $\chi_{\text{eff}} \approx D_m v_e$. So, the "double streaming" regime (20a) is also not available in the stationary quasi-linear limit.

We thus come to the conclusion that there exist no non-trivial quick decorrelation regimes with $\chi_{\text{eff}} \gg \chi_\perp$, given the time-independent magnetic perturbation. As we will see in the next section, this restriction can be abolished in a non-stationary stochastic magnetic field.

V. Non-Stationary Decorrelation

Besides finite $\chi_\perp$ and r_e, the decorrelation may be caused also by non-stationarity of magnetic perturbations, giving rise to new transport regimes.

Let us put $\chi_\perp = 0$, $r_e = 0$ and ω be the characteristic frequency of $\delta\mathbf{B}(x,y,z,t)$. The definition of ω implies that in the time of ω^{-1} the spatial distribution of $\mathbf{b}(\mathbf{r},t)$ changes unrecognizably. It is quite clear that until the stochastic instability has an effect, i.e. under the condition $\chi_\parallel/\omega < l^2$ (in hydrodynamic limit), or $v_e/\omega < l$ (in kinetics), the decorrelation time equals the inverse frequency: $t_d = \omega^{-1}$. At very low frequency, that the test particle has time to pass the distance l in nearly unchanged magnetic field, the effect of magnetic lines exponentiation comes into play, decreasing the decorrelation time with respect to ω^{-1}. For treating this limit we can account for time dependence in Eq. (3) as a small perturbation:

$$d\mathbf{r}_\perp/dz = \mathbf{b}(\mathbf{r}_\perp, z, 0) + t(z)\mathbf{b}_1(\mathbf{r}_\perp, z) \ ,$$
$$\mathbf{b}_1(\mathbf{r}_\perp, z) \equiv \partial\mathbf{b}(\mathbf{r}_\perp, z, t)/\partial t\Big|_{t=0} \approx \omega b_0 \ , \tag{25}$$

$$t(z) = \begin{cases} z^2/\chi_\parallel \ , & \nu_e t_d > 1 \ , \quad (26a) \\ z/v_e \ , & \nu_e t_d > 1 \ . \quad (26b) \end{cases}$$

The first term in the right-hand side of Eq. (25) generates the motion of the particle along a stationary magnetic line, which makes the particle approach exponentially the boundary of the magnetic flux tube (see (15)). The second term represents a non-correlated, with respect to this motion, slow drift with the correlation length $z = L_0$. This implies that the two-point correlation function $\langle \mathbf{b}_1(z')\mathbf{b}_1(z'')\rangle$ decays at $|z' - z''| > L_0$. Then we can write the following estimate for the square-average displacement component $\mathbf{r}_{\perp\omega}$ due to the non-stationarity:

$$\left\langle \mathbf{r}^2_{\perp\omega}(z) \right\rangle = \int_0^z \int_0^z dz' dz'' t(z') t(z'') \langle \mathbf{b}_1(z')\mathbf{b}_1(z'')\rangle \approx \omega^2 b_0^2 L_0 \int_0^z t^2(z') dz' \ . \tag{27}$$

The decorrelation time t_d may now be obtained from the equation similar to (21):

$$\delta \exp[-z(t_d)/l] = \left\langle \mathbf{r}^2_{\perp\omega}(z(t_d)) \right\rangle^{1/2} . \tag{28}$$

Resolving Eq. (28) in every limit (collisional (17a), and collisionless (17b)) we find the expressions for t_d in hydrodynamics ($\nu_e t_d > 1$):

$$t_d \approx \begin{cases} \omega^{-1} , & \omega > \chi_{\|}/l^2 , \quad (29a) \\ l^2/\chi_{\|} \ln^2[(\chi_{\|}/\omega l^2)\delta(lD_m)^{-1/2}] , & \omega < \chi_{\|}/l^2 , \quad (29b) \end{cases}$$

and in kinetics ($\nu_e t_d < 1$):

$$t_d \approx \begin{cases} \omega^{-1} , & \omega > v_e/l , \quad (30a) \\ l/v_e \ln(\delta/r) , & \omega < v_e/l , \quad (30b) \end{cases}$$

Substitution of (29), (30) into (19), (20), respectively, yields the effective thermal conductivity in the non-stationary stochastic magnetic field. In the collisional limit we have

$$\chi_{\text{eff}} \approx \begin{cases} \chi_{\|} b_0^2 , & \omega > \chi/L_0^2 , \quad \text{(QD)} \quad (31a) \\ D_m(\chi_{\|}\omega)^{1/2} , & \chi_{\|}/L_0^2 > \omega > \chi_{\|}/l^2 , \quad \text{(MD)} \quad (31b) \\ \chi_{\|}(D_m/l) \ln^{-1}[(\chi_{\|}/\omega l^2)\delta(lD_m)^{-1/2}] , & \chi_{\|}/l^2 > \omega \quad \text{(MD)} . \quad (31c) \end{cases}$$

Correspondingly, in the collisionless case,

$$\chi_{\text{eff}} \approx \begin{cases} b_0^2 v_e^2/\omega , & \omega > v_e/l , \quad \text{(QD)} \quad (32a) \\ D_m v_e , & \omega < v_e/l , \quad \text{(MD)} \quad (32b) \end{cases}$$

We see that in non-stationary magnetic field, when a free parameter ω appears, there can arise the non-trivial (i.e. $\chi_{\text{eff}} > \chi_{\perp}$) quick decorrelation regimes: (31a) ("fluid"), and (32a) ("double streaming"). The new regime (31b) is also present which is somewhat similar to the Kadomtsev-Pogutse regime. The expression (31c) is the non-stationary analog of the Rechester-Rosenbluth regime (23).

VI. Summary and Discussion

Depending upon the relation between the parameters, several effective transport regimes are possible. Their number increases as new effects are taken into account, e.g. the non-stationarity. When both stationary and non-stationary decorrelating mechanisms are present, one should choose one that produces the shortest time of decorrelation t_d.

Since the multidimensional parameter space is divided into many characteristic domains, it is useful to present interpolation formulas which are valid in each regime. First of all, let us note that in the collisional limit the seed transverse heat conductivity $\chi_\perp$ and the finite frequency ω of magnetic perturbations produce similar decorrelating effects. This justifies the introduction of the notations

$$\begin{aligned} \chi'_\perp &= \chi_\perp + \omega\delta^2 \ , \\ \chi''_\perp &= \chi_\perp + \omega\delta(l\mathrm{D}_m)^{1/2} \ . \end{aligned} \tag{33}$$

Equations (33) allow one to combine Eqs. (22) and (29), yielding a general expression for the collisional decorrelation time:

$$t_d = \min\left\{\delta^2/\chi'_\perp \ , \ (l^2/\chi_\parallel)\ln^2\left[(\chi_\parallel/\chi''_\perp)(\delta/l)^2\right]\right\} \ , \ \nu_e t_d > 1 \ . \tag{34}$$

It is pointed out that in the weak shear limit $(l_s > l_0)$ $\chi'_\perp = \chi''_\perp$.

Similarly, in the collisionless limit we have

$$t_d = \min\left\{\omega^{-1} \ , \ l/v_e \ln\left[v_e\delta/(\omega l(D_m l)^{1/2})\right] \ , \ l/v_e\ln(\delta/r_e)\right\} \ , \ \nu_e t_d < 1 \ . \tag{35}$$

The expressions (19), (34) (in hydrodynamics) and (20), (35) (in kinetics) solve the problem stated.

Despite the fact that in most papers the possible non-stationarity of magnetic perturbations is ignored, the parameter of the characteristic frequency ω appears to be rather important, affecting the number of transport regimes available. Moreover, it is convenient to follow the transition between regimes as ω is changed. A straightforward but cumbersome analysis shows that the collisional (19)–(34) and the collisionless (20)–(35) scaling laws for χ_{eff} are smoothly sewed together on margins between them. This indicates that no substantially new regimes were omitted. Further, this is demonstrated for the case of small shear ($l_s > l_0$, i.e. $l = l_0$) that $\chi'_\perp = \chi''_\perp$. Shown in Fig. 2 are transitions between various transport regimes, as ω varies, for two of several possible cases: $\chi_\parallel/\chi_\perp < R^2 b_0^2$ where (a) $\lambda_e < L_0$ and (b) $L_0 < \lambda_e < l$. Here $\lambda_e = v_e/\nu_e$ is the mean free path of electrons.

The above discussion can be summarized as follows.

(i) The quasi-linear limit of transport in a stochastic magnetic field is outlined by the inequalities $R \ll 1$, $l_s \gg L_0 R$. The magnetic shear affects the quasi-linear transport via the length of exponentiation l under the condition $l_s < L_0/R^2$.

(ii) There are two physically distinct transport regimes: the quick decorrelation (QD), that a test particle decorrelates before the magnetic line exhibits its diffusive behavior, and the magnetic line diffusion (MD), in the opposite case. In a stationary "braided" magnetic field only MD regimes of anomalous transport are present. The non-stationarity of magnetic perturbations can give rise to non-trivial QD regimes.

Acknowledgments

I thank Richard Mett for discussions and suggestions in language corrections. This work was supported in part by U.S. Department of Energy Contract No. DE-FG05-80ET53088.

References

Cook, I., Thyagaraja, A. and Haas, F.A. (1982) J. Phys. D: Applied Physics, **15**, L137.

Galeev, A.A. and Zelenyi, L.M. (1981) Physica, **2D**, 90.

Horton, W. (1983) in *The Principles of Plasma Physics*, Sagdeev, R.Z. and Rosenbluth, M.N., eds., (North-Holland, 1983) Vol. 2.

Haas, F.A. and Thyagaraja, A. (1986) Phys. Rep. **143**, 240.

Isichenko, M.B. (1990) *Effective Plasma Heat Conductivity in "Braided" Magnetic Field. Part II. The Percolation Limit.* IFSR # 464, Univ. of Texas, Austin.

Kadomtsev, B.B. and Pogutse, O.P. (1978) In Plasma Physics and Controlled Nuclear Fusion Research Proc. 7-th Int. Conf. (Innsbruck, 1978), Vol. 1, IAEA, Vienna, 1979, p. 649.

Krommes, J.A., Oberman, C. and Kleva, R.G. (1983) J. Plasma Phys. **30**, 11.

Lichtenberg, A.J. and Lieberman, M.A. (1983) *Regular and Stochastic Motion,* Springer-Verlag, New York.

Liewer, P.C. (1985) Nucl. Fusion **25**, 543.

Rechester, A.B. and Rosenbluth, M.N. (1978) Phys. Rev. Lett. **40**, 38.

Rechester, A.B., Rosenbluth, M.N. and White, R.B. (1979) Phys. Rev. Lett. **42**, 1247.

Rosenbluth, M.N., Sagdeev, R.Z., Taylor, J.B., and Zaslavsky, G.M. (1966) Nucl. Fusion **6**, 297.

Sagdeev, R.Z., Usikov, D.A. and Zaslavsky, G.M. (1988) *Non-Linear Physics from Pendulum to Turbulence and Chaos,* Harwood, London.

Stix, T.H. (1973) Phys. Rev. Lett. **30**, 833.

Stix, T.H. (1978) Nucl. Fusion **18**, 353.

Figure Captions

1. The magnetic flux tube in a "braided" magnetic field.

2. Dependence of $\chi_{\text{eff}}(\omega)$ at $\chi_\perp/\chi_\| < R^2 b_0^2$ for (a) $\lambda_e < L_0$ and (b) $L_0 < \lambda_e < 1$. Near the curves the corresponding formulae numbers are shown. (a): $\omega' = \chi_\|/L_0^2$, $\omega'' = \nu_e$. (b): $\omega' = \nu_e$, $\omega'' = v_e/L_0$. At $\omega' < \omega < \omega''$ χ_{eff} defined by (a) Eq. (31a), (b) Eq. (32b).

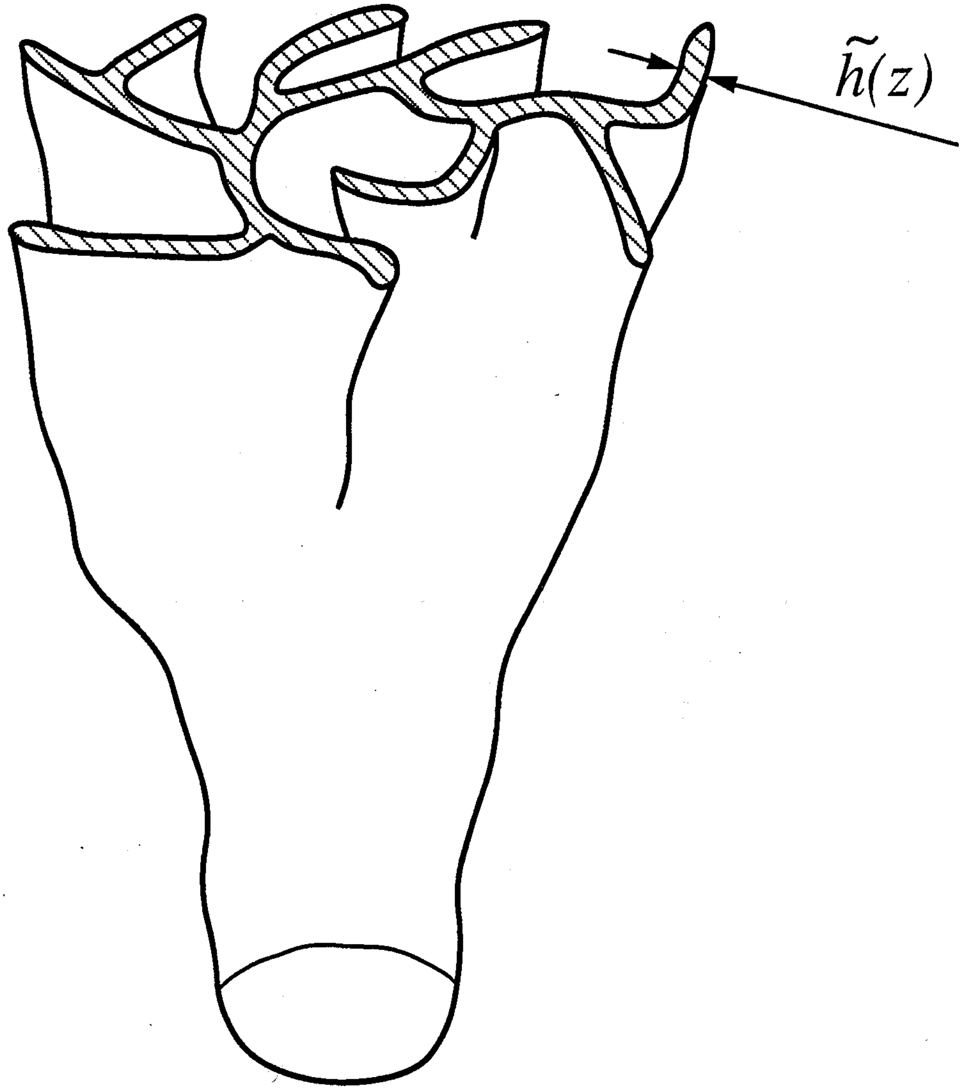

Figure 1.

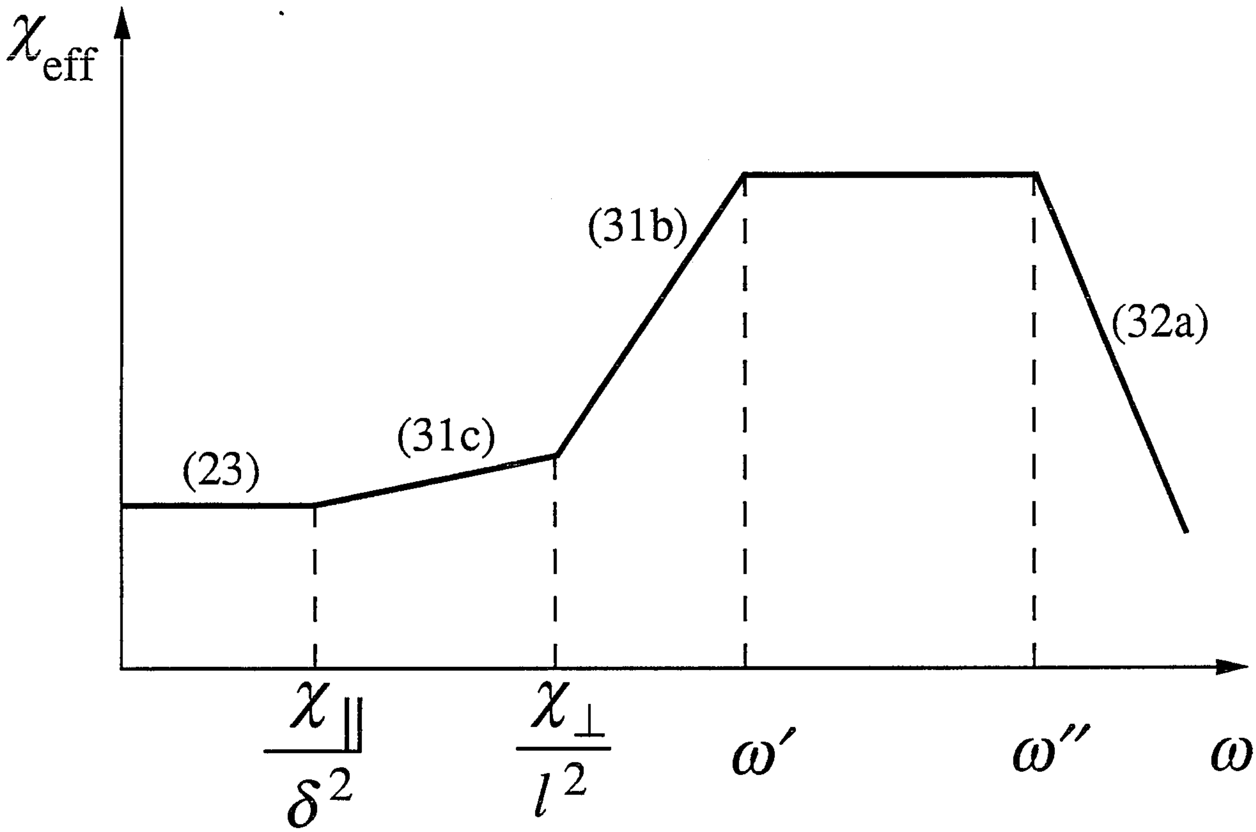
χ_{eff}
(31b)
(32a)
(31c)
(23)
$\frac{\chi_{\parallel}}{\delta^2}$
$\frac{\chi_{\perp}}{l^2}$
ω'
ω''
ω

Figure 2.