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FLUCTUATION SPECTRA OF A DRIFT WAVE SOLITON GAS

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Abstract

A theory of drift wave turbulence is presented based on a low density gas of drift wave solitons. The Gibbs's ensemble for the ideal gas is used to calculate the dynamical scattering factor form $S(\underset{\sim}{k},\omega)$. In contrast to renormalized turbulence theory, the spectrum has a broad frequency component with $\Delta\omega$ proportional to the fluctuation level $\delta n_e/n_0$ at fixed k and peaks at a frequency $\omega > k_y v_{de}$.

I. Introduction

The electromagnetic scattering experiments of Mazzucato[1], Surko and Slusher[2], and Koechlin[3] have led to the identification of the micro-turbulence in tokamaks with drift wave turbulence. The general features of the dynamical form factor $S(\underset{\sim}{k},\omega)$ for the electron density fluctuations $\langle|\delta n_e(\underset{\sim}{k},\omega)|^2\rangle$ are interpreted in terms of the frequency $k_y v_{de}$ where v_{de} is the electron diamagnetic drift velocity, and the most unstable wavenumbers of drift wave theory $k_\perp \rho_s \lesssim 1$ where ρ_s is the ion inertial scale length, $\rho_s = c(m_i T_e)^{1/2}/eB$. Efforts to make a detailed comparison of the scattering data with theory, however, have been frustrated by the fact that for a well-defined $\underset{\sim}{k}$ and scattering volume, the distribution of the scattered power has a peak at a frequency which is two to ten times larger than the linear drift frequency, $\omega^\ell_{\underset{\sim}{k}}$, and a width, $\Delta\omega$, larger than the peak frequency. An explanation of the peak has been given in terms of a doppler shift due to a radial electric field by Koch and Tang[4]. This explanation does not account for the width of the spectrum as reported, for example, in Figure 6 of Mazzucato[5]. The broad frequency spectrum lies outside the scope of renormalized weak turbulence theories which typically give $\Delta\omega \lesssim \omega^\ell$ [6]. These theories use an assumption of weak correlations (e.g., "maximal randomness" of the DIA[7]) which predicts, for moderate levels, $\Delta\omega$ proportional to integrals over $I_{\underset{\sim}{k}} \propto \langle|\delta n_e(\underset{\sim}{k})|^2\rangle$.

In this paper we propose, as an alternative theoretical framework for interpretation of the scattering experiments, the concept of an (nearly) ideal gas of drift wave solitons. Similar ideas have been extensively studied in the context of condensed matter physics[8]. These studies show that solitons contribute to the free energy of nonlinear lattices in equilibrium, and furthermore, that their effects are experimentally detectable[9].

While the configuration space phenomenology of Currie et al.[8] has been applied to nonlinear Klein-Gordon equations, very few results have been obtained for the field theories of plasma physics. Kingsep, Rudakov and Sudan[10] have studied the wavenumber spectrum generated by a gas of Langmuir solitons. Zakharov[11] has obtained a kinetic equation for a given set of KdV solitons. He shows that the pairwise interaction of solitons (which causes a phase shift) leads to an effective renormalization of the soliton velocity.

We follow the approach of Matsuno[12], who derived the wavenumber spectrum due to KdV solitons from a particular initial configuration, assuming that soliton overlap can be neglected. We obtain the form factor, $S(\underset{\sim}{k},\omega)$, due to solitons which arise from an ensemble of initial conditions with a given mean square fluctuation level, $\langle(\delta n)^2\rangle$.

For simplicity, we adopt a one-dimensional drift wave theory given by Petviashvili[13] which is obtained from pressureless ion fluid equations with adiabatic electrons. However, the concept of a drift wave soliton gas also applies to many other systems with stable, localized, coherent structures in any number of dimensions.

The principal result of this paper is a formula for the spectrum, $S(\underset{\sim}{k},\omega)$, which is qualitatively different from previous formulas (see Ref. 6) based on weakly correlated linear normal modes. Basically, this result follows from the soliton "dispersion" relation, $\omega = ku$, where soliton velocity, u, depends upon its amplitude. In a system where a large number of solitons are excited with varying amplitudes, the frequency spread for a given k is $\Delta\omega \sim k\langle(\Delta u)^2\rangle^{1/2}$ where Δu is the width of the soliton velocity distribution. Furthermore, for drift waves (as well as many other cases), the allowed soliton velocities fall in a range complementary to the phase velocity of the linear modes. In particular, drift wave solitons have $u > v_d$ or $u < 0$. We will show that for moderate but low fluctuation levels, the $u > v_d$ solitons are preferentially excited, leading to a spectra peaked at $\omega > kv_d$. For larger levels the $u < 0$ solitons become more important, giving spectra peaked at $\omega \lesssim 0$.

II. Drift Wave Solitons

We adopt a simplified two-fluid description of the plasma, where the electrons are adiabatic

$$n_e(\underset{\sim}{r},t) = n(x) \exp\left[\frac{e\Phi(\underset{\sim}{r},t)}{T_e(x)}\right]$$

and the ions are pressureless. The magnetic field is constant, $B = B_o\hat{z}$, while the mean density, n , and electron temperature, T_e , depend only upon the radial coordinate, x. Beginning with quasineutrality and the hydrodynamic equations for the ion momentum and density, one derives a single equation for the electrostatic potential, Φ, which is valid for $\omega/\Omega_i \sim \rho_s/r_n \ll 1$. Here, $\Omega_i = eB/m_i c$, $r_n^{-1} = -\partial_x n/n$, and $\rho_s = c_s/\Omega_i$ with $c_s = (T_e/m_i)^{1/2}$. This equation contains both the $\underset{\sim}{E}\times\underset{\sim}{B}$ nonlinearity[14] and temperature gradient nonlinearity[15]. In this paper we consider the limit $(k_x\rho_s)(k\rho_s)^2 \ll \eta_e(\rho_s/r_n)$ where $\eta_e = -r_n\,(\partial_x T_e/T_e)$ so that the temperature gradient nonlinearity dominates. The opposite limit (which is essentially two-dimensional) will be treated in later work.

The temperature gradient drift wave is governed by the equation

$$\left(1 - \rho_s^2\nabla_\perp^2\right)\partial_t\varphi + v_d\partial_y\varphi - v_d\varphi\partial_y\varphi = 0 \qquad (1)$$

where $\varphi = \eta_e(e\Phi/T_e)$ and $v_d = \rho_s c_s/r_n$ [13].

The Petviashvili equation (1) possesses two-dimensional solitary wave solutions, but for analytical tractability we consider only quasi-one-dimensional solutions. The radial dimension of such solutions is limited by the scale of variation of $v_d(x)$ in Eq. (1). Balancing $v_d \rho_s^2 \partial_x^2$ with the variation of $v_d(x)$ about its maximum, $\Delta x^2 \partial_x^2 v_d$, gives

$$(\Delta x)^2 \sim \rho_s r_n \quad . \tag{2}$$

Thus, the one-dimensional drift wave solitons are taken to extend over the radial region Δx, centered at the maximum of v_d, with an axial length $\Delta z = L_c$.

The one-dimensional version of Eq. (1) has been studied extensively as a "regularized" version of the KdV equation[16], called the regularized long-wave (RLW) equation and seems to have been first derived by Peregrine[17] in the context of tidal waves. Like the KdV equation, Eq. (1) has solitary wave solutions

$$\varphi_s(y,t;y_0,u) = -3\left(\frac{u}{v_d} - 1\right) \operatorname{sech}^2\left[\frac{1}{2\rho_s}\left(1 - \frac{v_d}{u}\right)^{\frac{1}{2}}\left(y - y_0 - ut\right)\right], \tag{3}$$

where the velocity is restricted to the ranges

$$u > v_d \quad \text{or} \quad u < 0 \quad . \tag{4}$$

Unlike the KdV equation, these solutions are not "solitons" in the pure sense, since they are not preserved upon collision[18]. However for collisions of moderate amplitude solitary waves travelling in the same direction, the inelasticity of collisions is extremely difficult to detect. In head-on collisions the inelasticity is more pronounced[18].

Therefore, solitary waves of Eq. (3) will persist for long times and through many collisions. As has been emphasized previously[8], solitary waves which are not strictly solitons still can have an important contribution to the statistical properties of the turbulent fields.

To determine statistical properties we will need the drift wave energy

$$E = \frac{1}{2}\int\left[\varphi^2 + (\rho_s \partial_y \varphi)^2\right] \frac{dy}{\rho_s} \tag{5}$$

which is the physical energy in units of $n_e T_e \rho_s \Delta x L_c/\eta_e^2$. The energy of the solitary wave, Eq. (3), is

$$E_s = \frac{12}{5}\left(\frac{u}{v_d}\right)^2\left(1 - \frac{v_d}{u}\right)^{3/2}\left(6 - \frac{v_d}{u}\right) \tag{6}$$

and is displayed in Figure 1 as a function of u/v_d. For $u/v_d \gg 1$, E_s increases quadratically, while as $u \to 0^-$ Eq. (6) reduces to $E_s \simeq 12/5(v_d/|u|)^{\frac{1}{2}}$. The minimum E_s for $u < 0$ occurs at $u = v_d(2 - \sqrt{10})/12 = -0.096\, v_d$ where $E_s = 14.01$.

III. Solitary Wave Spectrum

A turbulent state described by Eq. (1) will consist of a broad wavenumber spectrum of small amplitude modes together with an ensemble of solitary waves. For each linear mode the frequency spectrum will be peaked about $\omega_k^\ell = kv_d/[1 + (k\rho_s)^2]$ with some width determined, for example, by resonance broadening theory. Each solitary wave, however, contributes frequencies which depend upon its velocity (and hence its amplitude) through $\omega = ku$. By virtue of Eq. (4) these frequencies will range over $\omega > kv_d$ and $\omega < 0$ which is complementary to the range of the linear dispersion relation.

As a first approximation we ignore the small amplitude component supposing that its spectrum can be merely added to that for the solitary waves. Actually we expect[8] that the interaction between solitary waves and linear modes will act to renormalize the solitary wave parameters giving "dressed solitons."

Furthermore, we assume the potential can be written as a superposition of solitary waves

$$\varphi(y,t) = \sum_{n=1}^{N_s} \varphi_s(y,t;y_n,u_n) \ . \tag{7}$$

This ignores the strong interactions between these essentially nonlinear objects which occur whenever they overlap. To the extent that the solitary waves act as KdV solitons (e.g., $(k\rho_s)^2 \ll 1$) the only effect of this interaction is a phase shift of the soliton positions. Zakharov[11] has shown that this phase shift acts to renormalize the soliton velocities. This effect is proportional to the soliton number density which we assume small, and verify *a posteriori*.

The spectral density is the Fourier transform of the two-point correlation function:

$$\begin{aligned} S(\xi,\tau) &= \langle \varphi(x+\xi,\ t+\tau)\varphi(x,t)\rangle \\ &= \frac{1}{(2\pi)^2}\int dk \int d\omega\ S(k,\omega)\, e^{ik\xi - i\omega\tau} \end{aligned}$$

where the average is over the ensemble specified below. Utilizing the complete field from Eq. (7) with the solution Eq. (3), gives

$$S(k,\omega) = \begin{cases} \dfrac{1}{L} \displaystyle\sum_{n=1}^{N_s} \left\langle \left[12\pi k\rho_s \left(\dfrac{u_n}{v_d}\right) \operatorname{csch}\left(\dfrac{\pi k \rho_s}{\sqrt{1 - v_d/u_n}}\right)\right]^2 \delta(\omega - k u_n) \right\rangle \\ 0 \qquad\qquad 0 < \omega < k v_d \end{cases} \tag{8}$$

where we have assumed that the solitary wave positions, y_n, are randomly distributed along the length L (generally, $L = 2\pi r$ the circumference of the confinement device at the radius r which gives the maximum v_d). For a large number of solitary waves, $N_s \gg 1$, the sum in Eq. (8) may be converted to an integral over the distribution function, $f_s(u)$, of solitary waves in u-space:

$$S(k,\omega) = \frac{1}{L} f_s\left(\frac{\omega}{k}\right)\left[12\pi\rho_s \frac{\omega}{v_d} \operatorname{csch}\left(\pi k \rho_s \sqrt{\frac{\omega}{\omega - kv_d}}\right)\right]^2 \tag{9}$$

where $\int_{-\infty}^{\infty} f_s(u)\,du = N_s$. Before determining $f_s(u)$, we can deduce the qualitative shape of $S(k,\omega)$ directly from Eq. (9)

$$S(k,\omega) \sim \begin{cases} \omega^2 f_s\left(\frac{\omega}{k}\right) & \omega < 0 \\ 0 & 0 < \omega < kv_d \\ \exp\left[-2\pi k\rho_s\left(\frac{kv_d}{\omega - kv_d}\right)^{\frac{1}{2}}\right] & kv_d < \omega \ll kv_d\left[1 + (\pi k\rho_s)^2\right] \\ \omega^2 f_s\left(\frac{\omega}{k}\right) & kv_d\left[1 + (\pi k\rho_s)^2\right] \ll \omega . \end{cases} \tag{10}$$

Note that $S = 0$ just in the range where small amplitude excitations contribute. If there is some maximum amplitude solitary wave, $-|\varphi_{max}|$, then Eqs. (9) and (3) imply $S(k,\omega) = 0$ for $\omega > (1 + 1/3|\varphi_{max}|)kv_d$.

IV. Canonical Distribution Function

To obtain the function $f_s(u)$, we suppose that the drift wave disturbances can be characterized by a Gibbs ensemble. The solitary waves, to the extent they resemble solitons, can be thought of as nonlinear normal mode solutions. It is well known that, for a soliton-bearing equation such as KdV, the inverse spectral transform acts as a canonical transform to action-angle coordinates in which each soliton is represented by one degree of freedom (J,θ)[19]. In these coordinates the Gibbs ensemble for a single soliton is

$$P(J,\theta) = \frac{1}{Z} e^{-\beta_s E_s(J)} \qquad (11)$$

where the energy is only a function of the soliton action J, and Z is the partition function (normalization constant). The effective inverse temperature, β_s, fixes the mean soliton energy.

Even though such a transformation may be difficult (or, in the case of RLW, perhaps impossible) we can calculate P by following an argument given by Bolterauer and Opper[20]. Integration of Eq. (1) over θ and a transformation of coordinates from J to E_s gives

$$P(E_s) = \frac{2\pi}{Z} \left(\frac{\partial E_s}{\partial J}\right)^{-1} e^{-\beta_s E_s} .$$

Since J is a canonical variable, the derivative $\partial E_s/\partial J$ is the frequency $\dot{\theta}$. For a soliton, which acts as a free particle

$$\dot{\theta} = 2\pi \frac{u}{L}$$

and therefore

$$P(u) = \frac{L}{Z} \frac{\partial E_s}{\partial u} \frac{1}{u} e^{-\beta_s E_s(u)} . \qquad (12)$$

Note that the soliton energy is now expressed as a function of its velocity, which is an easily calculable function. Since true solitons act as independent degrees of freedom, the N_s soliton probability distribution is just the product of N_s one-soliton probabilities, Eq. (12); therefore the one-soliton distribution function is

$$f_s(u) = N_s P(u) . \qquad (13)$$

The argument used to derive Eq. (13) is not strictly valid for RLW since this equation does not have true soliton solutions. Nevertheless, the great stability of the RLW solitary waves observed in simulations indicates that Eq. (13) is a reasonable approximation.

In Figure 2 we present a plot of $f_s(u)$, Eq. (13), with the energy function of Eq. (6) and $\beta_s = 1/2$. For low temperatures, $\beta_s \gg 1$, the distribution function simplifies to the KdV form since the negative velocities have exponentially small weight

$$f_s(u) \propto \begin{cases} N_s\left(\frac{u}{v_d} - 1\right)^{5/2} \exp\left[-12\beta_s\left(\frac{u}{v_d} - 1\right)^{3/2}\right] & u > v_d \\ 0 & u < v_d \,. \end{cases} \qquad (\beta_s \gg 1)$$

(14a)

Negative velocity solitary waves become significantly excited for $\beta_s \lesssim 1/2$ with the distribution peaked in the region $-0.096\ v_d < u < 0$ and

$$f_s(u) \propto \begin{cases} N_s\left(\frac{v_d}{-u}\right)^{5/2} \exp\left[\frac{12}{5}\beta_s\left(\frac{v_d}{-u}\right)^{1/2}\right] & u < 0 \\ 0 & u > 0 \,. \end{cases} \qquad (\beta_s \ll 1)$$

(14b)

V. Soliton Number Density and Temperature

To utilize the distribution function, Eq. (13), it is necessary to know the effective temperature, $T_s = \beta_s^{-1}$, which fixes the mean energy, $<E_s>$. The relationship between these quantities is obtained through

$$<E_s> = \int_{-\infty}^{\infty} du\, E_s(u) P(u)$$

where $P(u)$ is given by Eq. (12). This integral can be done approximately utilizing Eq. (14), yielding

$$<E_s> \sim \begin{cases} T_s & T_s \ll 1 \\ 3T_s & T_s \gg 1 \end{cases} \qquad (15)$$

where in the upper (lower) relation only the $u > v_d$ $(u < 0)$ solitary waves contribute.

Since the solitary waves represent independent degrees of freedom, energy is equipartitioned and

$$N_s <E_s> \simeq \left(\frac{L}{\rho_s}\right) <\varphi^2> = \left(\frac{L}{\rho_s}\right) \varphi_0^2 \qquad (16)$$

where we assume that the fluctuation energy represented by φ_0^2 is entirely due to the solitary waves which have a number density $n_s = N_s/L$.

The total available thermal energy for the drift wave field may be estimated using thermodynamic arguments. There are two energy sources arising from the background temperature and density gradients: The diamagnetic kinetic energy and the free energy of expansion from relaxation of the gradients[21]. When the radial scale of the drift waves is large, $\Delta x \gg \rho_s$, expansion energy dominates and a thermodynamic bound is[21]

$$\varphi_0^2 \simeq \eta_e^2 \left(\frac{\Delta x}{r_n}\right)^2 \simeq \eta_e^2 \left(\frac{\rho_s}{r_n}\right) . \qquad (17)$$

Equation (2) has been used for Δx, the radial extent of the drift waves.

Once the energy available to the field is known, we only need to calculate N_s to obtain, through Eqs. (16) and (15), the temperature. This requires knowledge of the number of solitons emerging from a particular initial state, $\varphi(x)$. Since the initial value problem for the RLW equation remains unsolved, we turn again to KdV--recalling that the results will be correct for small T_s.

The inverse spectral transform allows the determination of the number of solitons emitted by any particular initial state. For moderate amplitude initial states,

$$\varphi_{max} \gg \left(\frac{\rho_s}{r_n}\right)^2 ,$$

the number of solitons produced is large and a WKB approximation of the inverse problem can be used to obtain[22]

$$N_s[\varphi] = \frac{1}{\sqrt{6}\pi\rho_s} \int_{\varphi<0} dx \sqrt{-\varphi(x)} . \qquad (18)$$

This result is only valid when $\varphi < 0$ for all y: When $\varphi > 0$, non-soliton excitations significantly affect Eq. (18). As a first approximation, we will use Eq. (18) for the general case.

To compute the mean number of solitons, we average Eq. (18) using a Gibbs ensemble with the KdV energy

$$E_{KdV} = \frac{1}{2}\int \varphi^2 \frac{dy}{\rho_s} \tag{19}$$

which is obtained from Eq. (5) when $k\rho_s \ll 1$. The mean square potential is fixed to agree with the availble energy of Eq. (17), $\langle\varphi^2\rangle = \varphi_0^2$. The mean number of solitons is determined by a functional integral which upon discretization becomes

$$\langle N_s\rangle = \frac{1}{Z}\prod_{i=1}^{n}\int d\varphi_i \; N[\varphi] \; \exp\left[-\frac{1}{2}\left(\frac{\varphi_i}{\varphi_0}\right)^2\right] \tag{20}$$

where $Z = (2\pi\varphi_0^2)^{n/2}$ is the normalization and $\varphi_i = \varphi(x_i)$. We then obtain

$$n_s = \frac{\langle N_s\rangle}{L} = \alpha \frac{\varphi_0^{\frac{1}{2}}}{\rho_s} \quad ; \tag{21}$$

$$\alpha = \frac{\Gamma\left(\frac{3}{4}\right)}{\left(12\sqrt{2}\pi^3\right)^{\frac{1}{2}}} = 0.053 \quad .$$

Combining Eqs. (21) and (16) gives the mean energy

$$\langle E_s\rangle = \frac{1}{\alpha}\varphi_0^{3/2} \tag{22}$$

which, in conjunction with Eq. (15), gives T_s.

Using the estimate of ψ_0 in Eq. (17) gives

$$n_s = \frac{\alpha\eta_e^{1/2}}{\rho_s}\left(\frac{\rho_s}{r_n}\right)^{1/4}$$

$$\langle E_s\rangle = \frac{\eta_e^{3/2}}{\alpha}\left(\frac{\rho_s}{r_n}\right)^{3/4} \quad . \qquad (23)$$

For $\rho_s/r_n \simeq 0.01$ and $\eta_e \simeq 1$, which is appropriate for most present day tokamak experiments, $\langle E_s\rangle = 0.5$. For TMX, however, $\rho_s/r_n \simeq 0.1$ and $\langle E_s\rangle = 3.3$. Equation (15) then implies $T_s = 1/2$ and $T_s \sim 2\text{-}3$, respectively. Spectral densities for these temperatures are shown in Figure 3. When $T_s = 1/2$ virtually all the energy is contained in $u > v_d$ modes, while at the higher temperature, a substantial fraction of the energy is in the negative frequency modes. Significant excitation of negative frequency modes occurs when there is sufficient thermal energy to overcome the required creation energy, $E_{min} = 14.01$ (see Figure 1).

The frequency shift of the positive spectral peak, ω_p , is given by $\omega_p/kv_d \simeq 1.25$ for $T_s = 0.5$ and 1.5 for $T_s = 3.0$. The frequency shift relative to the linear mode frequency depends upon k, and increases rapidly as $k\rho_s \to 1$. If we use the finite Larmor radius formula for ω_k^ℓ (see e.g., Ref. 4), a frequency shift of $\omega_p/\omega_k^\ell \simeq 5$ is obtained when $k\rho_s \simeq 1$ and $T_s = 0.5$, which is in quantitative agreement with the experiments[1,2,3,5].

Of course, to apply these results to experiments, the small amplitude continuum should also be included. Furthermore, we expect that solitary wave collisions will contribute to the spectrum by forcing excitations at frequencies $\omega \sim (u_1 - u_2)k$ which do not obey the linear dispersion relation. Inclusion of these non-soliton effects could provide a spectral width comparable to the experimental width.

VI. Conclusion

Recognizing that the nonlinear drift wave equations generally possess finite amplitude coherent solutions, we propose that drift wave turbulence may, in general, contain a coherent component. In this work, we develop the theory for the soliton component of the turbulence based on a one-dimensional drift wave equation given by Petviashvili. At low energies E_s, the drift wave soliton, reduces to a KdV soliton propagating with a velocity u just greater than the electron diamagnetic drift velocity v_d. At higher energies, the equation gives localized solitary waves propagating either faster than v_d or in the direction opposite to electron drift velocity, $u < 0$.

For the root-mean-square fluctuation levels typical of the saturated state of drift wave turbulence, inverse scattering theory for the soliton (KdV) equation is used to show that a large number, $N_s \gg 1$, of solitons can evolve from the drift wave fields. Each drift wave soliton introduces a spatially localized infinite order set of correlations, due to its intrinsic coherence. These correlations are lost in the truncations of renormalized turbulence theory, and give rise to new features in the fluctuation spectrum even in the limit of an ideal gas approximation to the many-soliton system.

To investigate the features in the fluctuation spectrum of the soliton gas, we introduce the Gibbs' ensemble for the ideal gas limit of the N_s soliton system, Eq. (14), and calculate the dynamical form factor $S(k,\omega)$ from the two-point correlation function of the drift wave fields, Eq. (9).

The fluctuation spectrum $S(k,\omega)$ from the drift wave soliton gas has a broad frequency spectrum for fixed k. The width of the frequency spectrum $\Delta\omega$ is directly proportional to the root-mean-square fluctuation level φ_0, in contrast to resonance broadening theory where $\Delta\omega$ is proportional to the intensity φ_0^2, of the fluctuating fields. For a given k component the peak frequency ω_p of the spectrum is greater than kv_d as given by Eqs. (10) and (14), and show in Figure 3. In contrast, the renormalized turbulence theory spectrum peaks near the linear frequency ω_k^ℓ less than kv_d, a condition which has prevented an understanding of the $S(k,\omega)$ observed by electromagnetic scattering experiments.

The observed fluctuation spectra[1-3] are broad and peaked at a frequency ω_p up to five times the ω_k^ℓ as investigated in terms of Doppler shifts due to the ambipolar radial field in Ref. 4. The drift wave soliton gas can easily give rise to $\omega_p/\omega_k^\ell \sim 5$ as well as negative frequency components to the spectrum at fixed k as shown, for example, by Figs. 3a and 3b.

We do not regard the limitation of the results of this paper to the one-dimensional case as fundamental. The Petviashvili equation possesses two-dimensional solitary waves[13] qualitatively similar to Eq. (3), and therefore the frequency spectrum will also resemble ours qualitatively.

In conclusion, we suggest that from both a theoretical and an experimental point of view, a full understanding of drift wave turbulence may require theory for both the continuum component ω_k^ℓ, $I(\underset{\sim}{k},t)$ of conventional turbulence theory and the soliton component $\underset{\sim}{k}\cdot\underset{\sim}{u}$, $f_s(\underset{\sim}{u},t)$ of the turbulent plasma.

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Figure Captions

Fig. 1 Single soliton energy for the RLW equation, from Eq. (6).

Fig. 2 Gibbs ensemble probability density, Eq. (12), as a function of soliton velocity.

Fig. 3 Spectral density of Gibbs ensemble of RLW solitons at fixed wavenumber. Corresponding linear mode frequency is indicated by the arrow. For smaller (larger) fluctuation levels, Fig. 3a (3b), the $u > v_d$ ($u < 0$) solitons are preferentially excited.

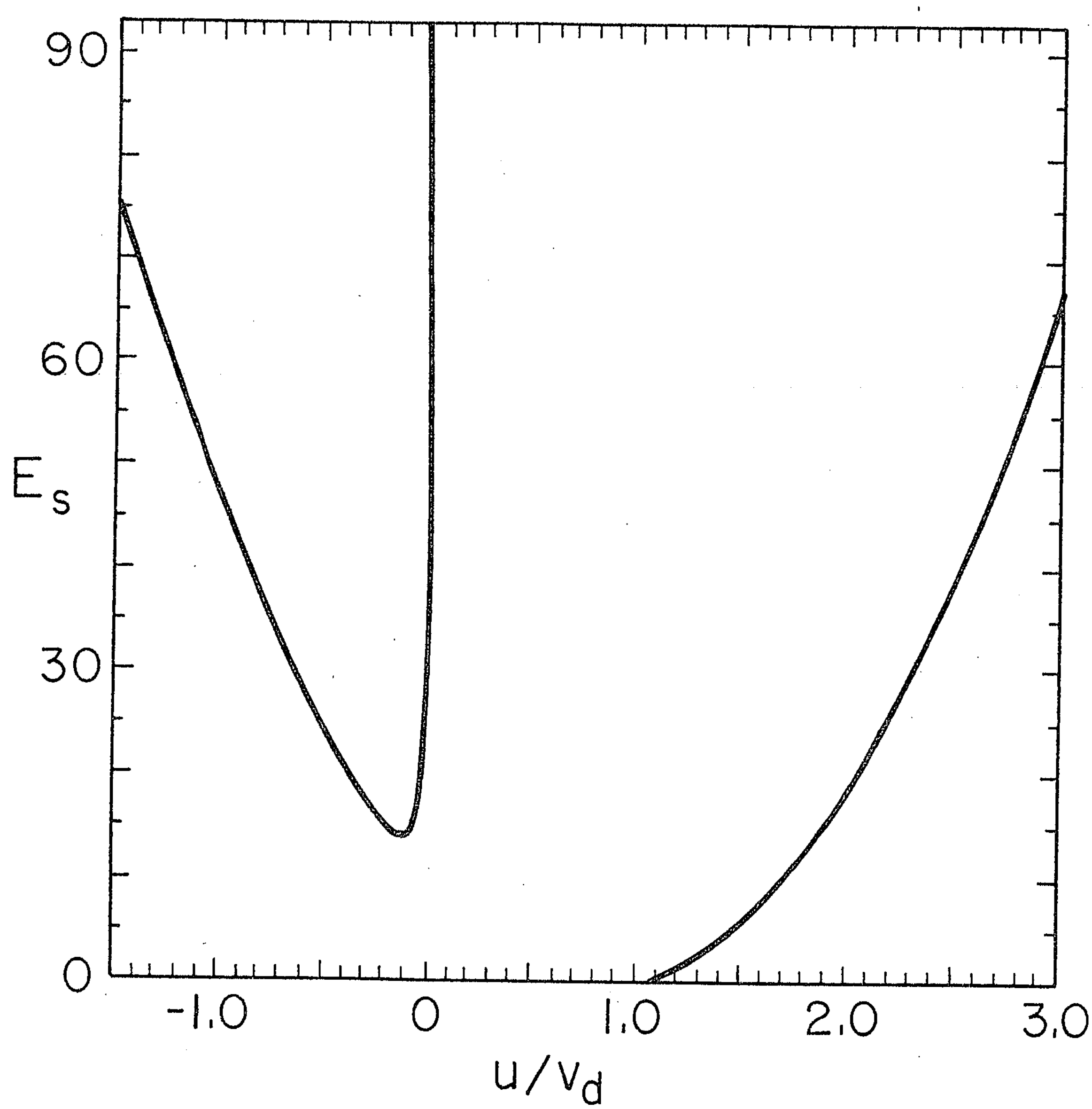

Figure 1

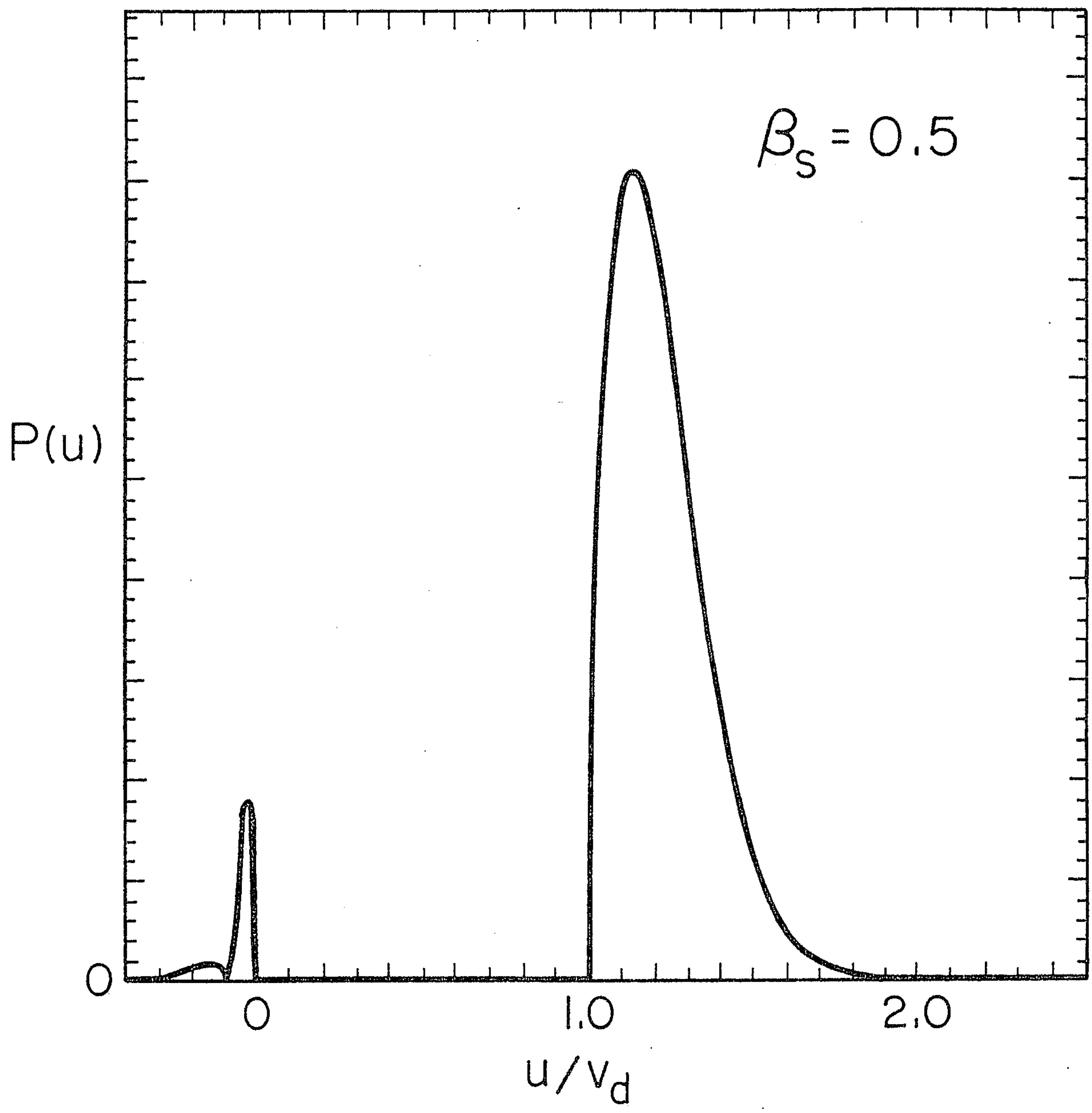

Figure 2

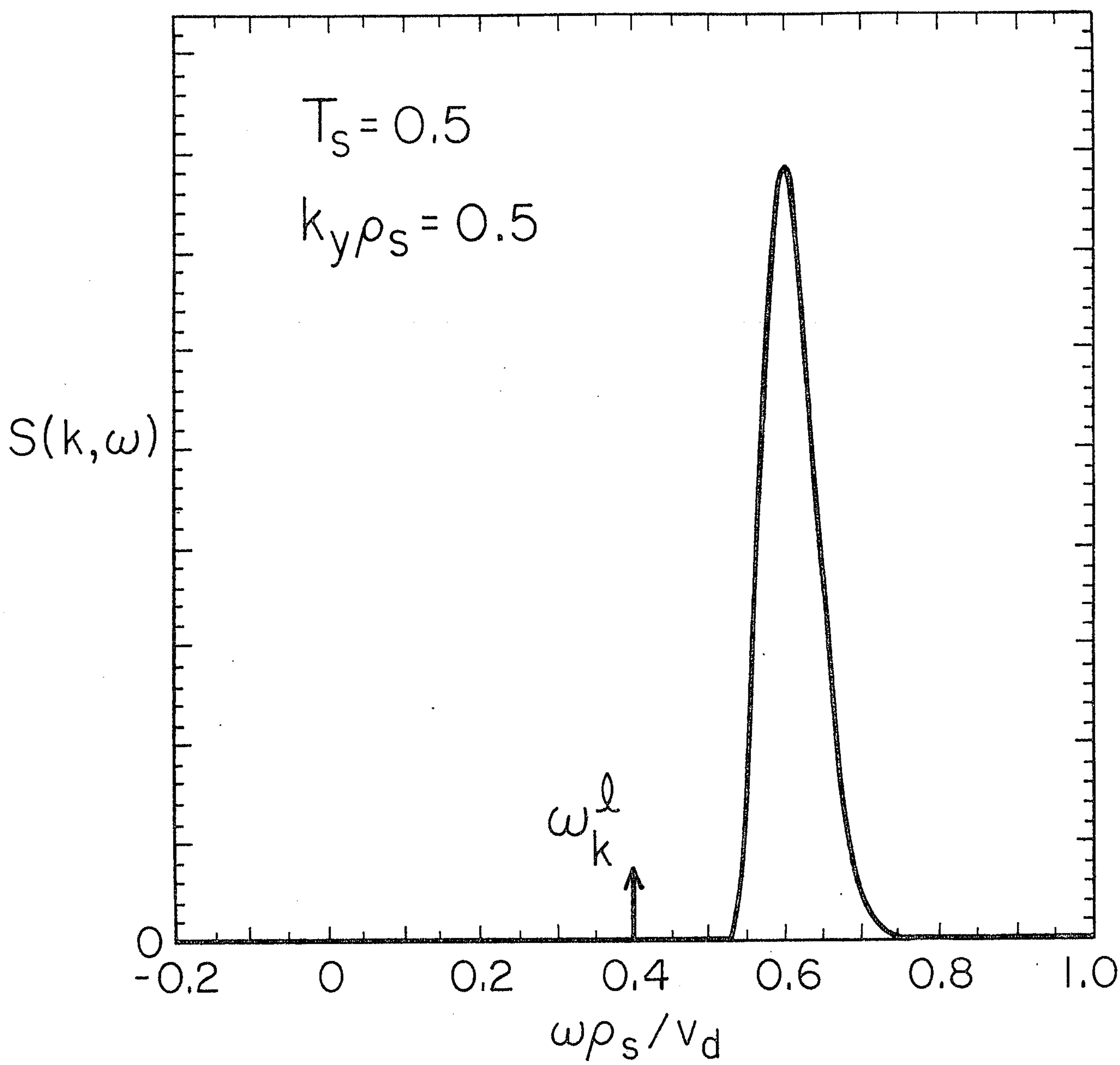
$T_s = 0.5$
$k_y\rho_s = 0.5$
$S(k,\omega)$
ω_k^ℓ
0
-0.2
0
0.2
0.4
0.6
0.8
1.0
$\omega\rho_s/v_d$

Figure 3a

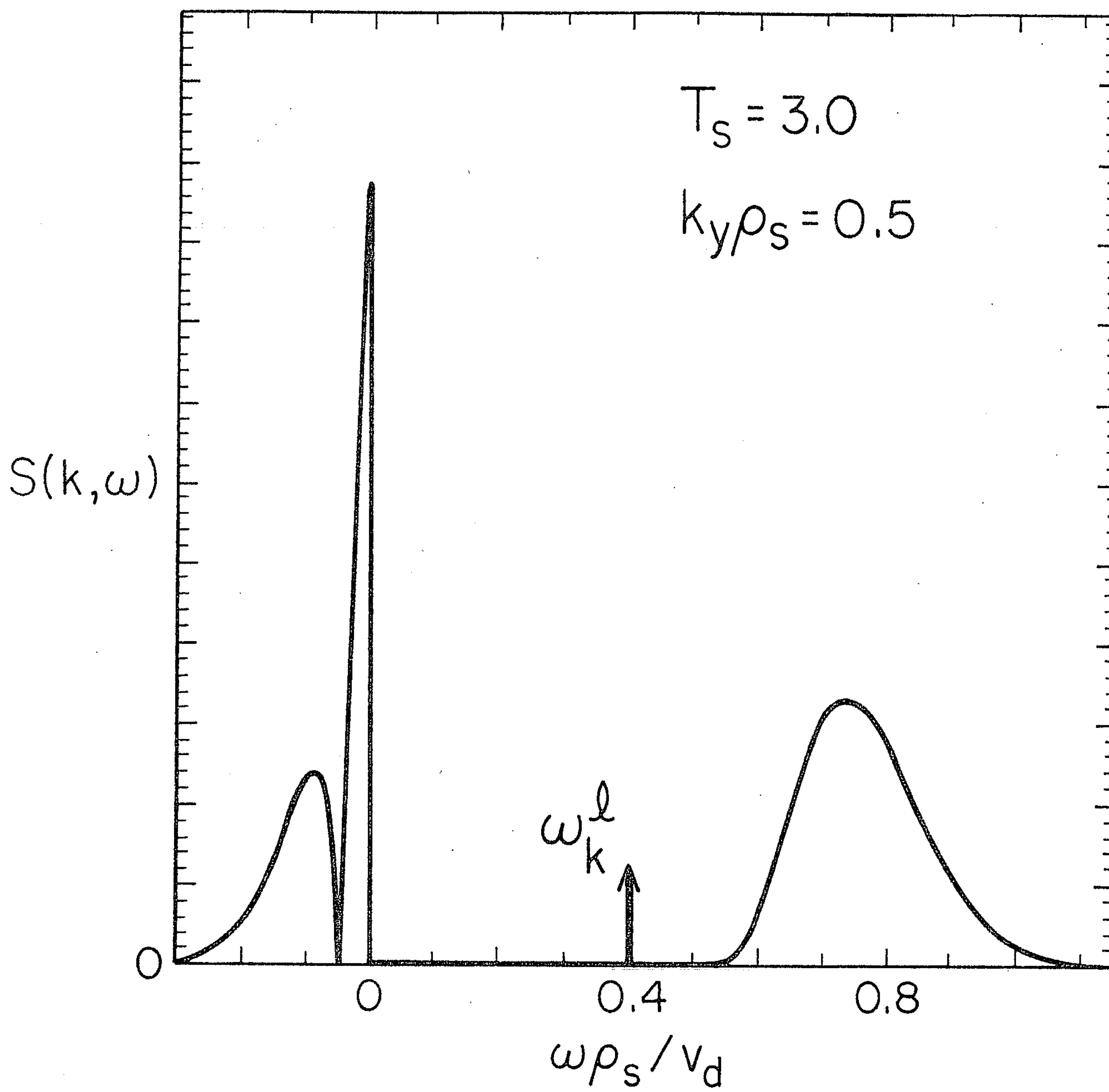

Figure 3b