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M. RYUTOVA,* M. KAISIG, and T. TAJIMA
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

**Institute of Nuclear Physics, Novosibirsk, U.S.S.R.*

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M. Ryutova, M. Kaisig, and T. Tajima
Institute for Fusion Studies
The University of Texas at Austin
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Abstract

Effects of strong and random inhomogeneities of the magnetic fields, plasma density, and temperature in the solar atmosphere on the properties of magnetoacoustic waves of arbitrary amplitudes are studied. The procedure which allows one to obtain the averaged equation containing the nonlinearity of a wave, dispersion properties of a system, and dissipative effects is described. It is shown that depending on the statistical properties of the medium, different scenarios of wave propagation arise: in the predominance of dissipative effects the primary wave is damped away in the linear stage and the efficiency of heating due to inhomogeneities is much greater than that in homogeneous medium. Depending on the interplay of nonlinear and dispersion effects, the process of heating can be afforded through the formation of shocks or through the storing of energy in a system of solitons which are later damped away. Our computer simulation supports and extends the above theoretical investigations. In particular the enhanced dissipation of waves due to the strong and random inhomogeneities is observed and this is more pronounced for shorter waves.

1. Introduction

The studies of large-scale motions of strongly inhomogeneous or multiphase media is the general problem very important for various physical objects, where the parameters of medium are random functions of coordinates. This problem is of particular importance for the physics of the solar atmosphere, which can be simply described as strongly inhomogeneous plasma with random magnetic fields (see, for example, Stenflo, 1989, 1990). A great variety of magnetic structures of the solar atmosphere can be roughly divided into two classes. One class represents magnetic elements which are bundles of field lines penetrating a nonmagnetized plasma and far removed from each other. The transverse dimensions of these elements are, as a rule, much less than their length. These are, for example, pores, or photospheric flux tubes, knots, filigree, spicules and others. A sketch of the cross-section of a region containing such a kind of inhomogeneities is shown in Fig. 1(a). In Fig. 1(b) is a sketch of the other class of magnetic structures in the solar atmosphere, where magnetic elements are tightly settled. In this case all the parameters of medium magnetic field, plasma density, temperature, the size (across the magnetic field) of these elements change from one domain to another by an order of magnitude. From a theoretical point of view we now deal with a plasma containing strong and random inhomogeneities of magnetic field, density, temperature, etc. As observational examples these can be faculae, plages, spots (umbrae and penumbrae), prominences, etc. The interaction of all these magnetic structures with large-scale acoustic and MHD-waves as well as with plasma flows are the most important agent contributing to the energy balance and dynamics of the solar atmosphere. This is the reason why we stress here the importance of the study of strongly inhomogeneous media as a necessary step toward the understanding of real processes in the solar atmosphere. However, the extracted physics is also of interest from the point of view of general physics and it can include different multi-phase media such as liquid crystals, fluids with vortices, accretion discs, molecular clouds, polycrystals, etc.

The propagation of magnetoacoustic waves in a plasma with small-scale inhomogeneities of density, temperature and magnetic field can be described in principle by the averaged equations under the assumption that the characteristic wavelength of magnetosonic wave λ is much larger than the scale length of inhomogeneities R

$$\lambda \gg R \, . \tag{1}$$

Even in this case the explicit procedure of derivation of averaged equations is not trivial.

Some aspects of this problem were considered in previous papers (Ryutov and Ryutova, 1976; Ryutova and Persson, 1984). Ryutov and Ryutova investigated the plasma containing the random ensemble of magnetic flux tubes far removed from each other (the distance between flux tubes assumed to be much larger than their radius (see Fig. 1(a)). It was shown that such systems reveal some very important properties in the processes of transfer of the energy from the convective zone to upper layers of the atmosphere. First of all, sound oscillations excited in the convective zone can be absorbed by magnetic flux tubes due to the effect similar to the Landau damping. This effect consists of resonance excitation of oscillations propagating along the flux tubes. After that, over a considerably longer time than the damping time of the resonance absorption of sound waves, magnetic flux tubes give off their energy in a form of secondary acoustic waves in higher layers of the atmosphere. Beside resonance absorption, the resonant scattering of sound waves by magnetic flux tubes can take place depending on the magnetic filling factor. In this case the energy of primary sound waves is transferred directly to the energy of secondary acoustic waves without the preliminary accumulation of energy in natural oscillations of flux tubes.

Ryutova and Persson (1984) studied the propagation of long-wave magnetohydrodynamic (MHD) oscillations in a plasma containing the ensemble of tightly settled magnetic flux tubes. This collection of dense magnetic tubes may be modelled as a random strongly inhomogeneous plasma (Fig. 1(b)). The averaged linear equations describing large-scale motions of plasma with random inhomogeneities are obtained in two dimensions. It was shown that unlike the propagation of linear magnetoacoustic waves in a homogeneous plasma, in the present problem large local gradients of velocity, temperature, pressure, etc., associated with the presence of small-scale inhomogeneities (flux tubes) appear, which leads to the enhanced dissipation of the energy of magnetoacoustic waves. The enhancement factor of dissipation is of the order of $(\lambda/R)^2$, which is much greater than unity. The physical reason for this effect can be easily understood. Since all the plasma parameters change from tube to tube, the velocity amplitude of perturbations as well as all other perturbed quantities are different from the neighboring tubes. This can be shown to lead to the appearance of vortex part in averaged equations. The characteristic scale of vortex part is of the order of the tube radius R (the scale of inhomogeneities), at which dissipative effects are enhanced.

The general form of the linear dispersion relation is as follows:

$$\omega^2 = 2\left\langle \frac{1}{\left(\frac{\gamma}{2}-1\right)p_0 + P_0} \right\rangle^{-1} \left\{ \left\langle \frac{1}{\rho_0} \right\rangle k^2 + Q_{\alpha\beta} k_\alpha k_\beta \right\} , \tag{2}$$

where $Q_{\alpha\beta}$ is a tensor whose symmetry is determined by the field of density fluctuations, γ is the adiabatic gas constant, $\rho_0 = \rho_0(x,y)$ is the unperturbed background plasma density, $p_0 = p_0(x,y)$ is the gas-kinetic pressure and

$$P_0 = p_0(x,y) + B_0^2(x,y)/8\pi$$

is the total pressure. For some special cases $Q_{\alpha\beta}$ can be found analytically. If, for example, there is no statistically preferred direction in xy-plane (in the plane of wave propagation) then $Q_{\alpha\beta} = Q\delta_{\alpha\beta}$, where Q is constant and $\delta_{\alpha\beta}$ the Kronecker delta. For isotropic fluctuations $Q_{\alpha\beta} = 0$ and the dispersion relation can be written in a usual form

$$\omega = k \cdot v_{ph} \ , \tag{3}$$

with the renormalized phase velocity

$$v_{ph} = \left\langle \frac{1}{\left(\frac{\gamma}{2}-1\right)p_0 + P_0} \right\rangle^{-1/2} \left\langle \frac{2}{\rho_0} \right\rangle^{-1/2} . \tag{4}$$

The present paper is devoted to the further development of the theory which includes the influence of plasma inhomogeneities on the propagation of large-scale magnetoacoustic waves of an arbitrary amplitude and on the dispersive properties of the system. We then further study by removing the restriction (1), allowing both $\lambda \gg R$ and $\lambda \sim R$ by the method of numerical simulation.

We investigate here two problems. First, we obtain the nonlinear hydrodynamic equations describing the evolution of the averaged characteristics of the medium and show that these equations are similar to those of a homogeneous medium presented in Sec. 2. In particular, it remains valid that the finite amplitude perturbation splits into two simple waves propagating in the opposite directions. Each of the simple wave has a tendency of steepening and overturning with the subsequent formation of shocks. This is discussed in Sec. 3. The information on random inhomogeneities is carried in the expression of the averaged (Eq. (4)) pressure and density. We describe the procedure that allows to obtain this dependence in terms of statistic properties of inhomogeneities.

Secondly, we consider the influence of inhomogeneities on the propagation of a small amplitude magnetoacoustic wave and find the higher order effects in the powers of wavevector k. It turns out that the presence of inhomogeneities gives rise to a cubic (in k) correction to the frequency of magnetoacoustic waves, so that the phase velocity of waves possess a finite dispersion. For a small but finite amplitude one should take into account both (a small) nonlinearity and (a small) dispersion. We obtain then a Korteweg-deVries (KdV) type equation with the coefficients determined by the statistical properties of inhomogeneities. The addition of dissipative effects greatly enhance the presence of inhomogeneities which leads naturally to the KdV-Burgers' equation. This is discussed in Sec. 4.

In Sec. 5 we present results of computer simulation which support our theoretical investigations. We then generalize our results by exploring the regime where the wavelength can be as small as the scale length of inhomogeneity; i.e. $\lambda \sim R$, which is beyond the theoretical analysis.

2. Nonlinear Equations

We restrict ourselves to a one-dimensional problem and assume that all plasma parameters are random functions of coordinate x only: $\rho_0(x)$, $p_0(x)$, $B_0(x)$, $T_0(x)$. The lifetime of such inhomogeneities is determined by the thermal conductivity and diffusion which should be small in a strongly magnetized plasma in the direction (x) perpendicular to the flux tube direction. This means that the inhomogeneities belong to the "entropy" class (cf. Ryutova and Persson, 1984) and can be considered as stationary, providing at the same time the constancy of the total pressure

$$P_0 = p_0(x) + \frac{B_0^2(x)}{8\pi} = \text{const.} \tag{5}$$

At the same time we make no assumption that inhomogeneities are small: all parameters can change from one tube to another by an order of magnitude.

To describe the magnetosonic waves, we use the ideal MHD-equations

$$\begin{aligned} &\rho\frac{d\mathrm{v}}{dt} = -\frac{\partial P}{\partial x} , \\ &\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\rho v = 0 , \\ &\frac{\partial B}{\partial t} + \frac{\partial}{\partial x}Bv = 0 , \\ &\frac{d}{dt}\left(\rho^{-\gamma}p\right) = 0 , \end{aligned} \tag{6}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}$. From the second and the third equations of the system (5) there follows the frozen flux condition

$$\frac{B}{\rho} = \frac{B_0}{\rho_0} , \tag{7}$$

where $B_0(x)$ and $\rho_0(x)$ are the values of $B(x)$ and $\rho(x)$ at the point where a given element of the medium was located at the initial instant of time ($t = 0$). Similarly, from the equation of the entropy conservation we have

$$p\rho^{-\gamma} = p_0\rho_0^{-\gamma} \ . \tag{8}$$

We average the first and the second equations of the system (6) over the scale L that is much larger than the size of inhomogeneities R, but much smaller than the length of magnetosonic wave λ: $R \ll L \ll \lambda$. Denoting this averaging by angular brackets, we find

$$\left\langle \rho \frac{dv}{dt} \right\rangle = -\frac{\partial}{\partial x} \langle P \rangle \tag{9}$$

$$\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial}{\partial x} \langle \rho v \rangle = 0 \ . \tag{10}$$

Now, because of the averages $\left\langle \rho \frac{dv}{dt} \right\rangle$ and $\langle \rho v \rangle$ the system (9)–(10) is not a closed one any more. To solve it, we have to find a way to "split" these averages. For this we use the following considerations. Let us return to the exact energy equation presented in a form

$$\frac{\partial v}{\partial x} = -\frac{1}{\gamma + 1} \frac{1}{p} \frac{dp}{dt} \ , \tag{11}$$

which follows from the equations of continuity and the conservation of entropy. Since we are considering the motions of the scale $\lambda \gg R$, the logarithmic derivative $d \ln p / dt$, which can be estimated as v/λ, is small with respect to v/R. So that we have the following estimation from (11):

$$\frac{\partial v}{\partial x} \sim \frac{v}{\lambda} \ll \frac{v}{R} \ . \tag{12}$$

This means that despite the presence of inhomogeneities of density, pressure, and magnetic fields which have the scale R, the velocity v is a "smooth" function, changing only over the scale $\lambda \gg R$. This allows us to write the following relations:

$$\left\langle \rho \frac{dv}{dt} \right\rangle \simeq \langle \rho \rangle \frac{d \langle v \rangle}{dt} \ ,$$

$$\langle \rho v \rangle \simeq \langle \rho \rangle \langle v \rangle \ .$$

These relations are valid with the accuracy of the order of $(R/\lambda) \ll 1$. Recall that the scale L over which the averaging is made is small compared to λ and large compared to R. As a result we obtain instead of equations (9) and (10) the following equations:

$$\langle\rho\rangle \frac{d\langle v\rangle}{dt} = -\frac{\partial}{\partial x}\langle P\rangle \ , \tag{13}$$

$$\frac{\partial\langle\rho\rangle}{\partial t} + \frac{\partial}{\partial x}\langle\rho\rangle \cdot \langle v\rangle = 0 \ . \tag{14}$$

The form of equations (13) and (14) is similar to that of the equations for 1-D gas dynamics. The analogy would become complete if we could find the "closing" relationship between the averaged quantities $\langle\rho\rangle$ and $\langle P\rangle$.

Now we proceed to this part of the problem. First of all, we note that the density ρ of each plasma element can be expressed in terms of its initial density $\rho_0(x)$, pressure $p_0(x)$, and full pressure P at a given spatial point. By using the definition of P and the relationships (5), (7), and (8), we obtain

$$P = p_0\left(\frac{\rho}{\rho_0}\right)^{\gamma} + \frac{P_0 - p_0}{\rho_0^2}\rho^2 \ . \tag{15}$$

This relationship determines implicitly the dependence of ρ on P, ρ_0, and p_0 (P_0 is assumed to be known)

$$\rho = \rho\left(P, \rho_0, p_0\right) \ . \tag{16}$$

In what follows, it is convenient to introduce the distribution function $f\left(p_0, \rho_0\right)$ of the random quantities $p_0(x)$ and $\rho_0(x)$, which is defined as follows: the fraction of those segments of the axis x, where p_0 and ρ_0 take the values in the intervals $(p_0, p_0 + dp_0)$, $(\rho_0, \rho_0 + d\rho_0)$, is proportional to $f\left(p_0, p_0\right) dp_0\, d\rho_0$

$$dx_0 \sim f\left(p_0, \rho_0\right) dp_0\, d\rho_0 \ .$$

We now choose two Lagrangian planes with the distance L

$$R \ll L \ll \lambda \ .$$

The amount of the matter between the Lagrangian planes (which by definition are "sticked" to plasma particles) is constant

$$M = \int_L \rho\, dx = \int_{L_0} \rho_0\, dx_0 = \text{const.}$$

This condition leads to the obvious relation determining the connection of the segment dx in MHD wave with its initial value dx_0

$$dx = dx_0 \frac{\rho_0(x)}{\rho\left(P, p_0, \rho_0\right)} \ . \tag{17}$$

Note now that the total pressure P is a slow varying function with the characteristic scale of the order of λ [see Eq. (13)]. Since the distance L is much less than λ, the change of P between Lagrangian planes is small and we can substitute P by its average value $\langle P \rangle$.

Thus we can find the density of each element of plasma with the help of Eq. (16), which can be written as

$$\rho = \rho\left(\langle P \rangle, p_0, \rho_0\right) \ . \tag{18}$$

In place of (17) we now have

$$dx = dx_0 \frac{\rho_0(x)}{\rho\left(\langle P \rangle, p_0, \rho_0\right)} \ . \tag{19}$$

Taking into account that

$$dx_0 \sim L_0 f\left(p_0, \rho_0\right) dp_0\, d\rho_0 \ ,$$

from (19) we have

$$L = L_0 \frac{\displaystyle\int \frac{\rho_0}{\rho\left(\langle P \rangle, p_0, \rho_0\right)} f\left(p_0, \rho_0\right) dp_0\, d\rho_0}{\displaystyle\int f\left(p_0, \rho_0\right) dp_0\, d\rho_0} \ . \tag{20}$$

The whole mass of the substance between the Lagrangian planes is obviously following

$$L_0 \frac{\displaystyle\int \rho_0 f\left(p_0, \rho_0\right) dp_0\, d\rho_0}{\displaystyle\int f\left(p_0, \rho_0\right) dp_0\, d\rho_0} \ .$$

Dividing this mass by the distance defined by (20), we obtain the expression for the average density

$$\langle \rho \rangle = \frac{\int \rho_0 f(p_0, \rho_0)\, dp_0\, d\rho_0}{\int \frac{\rho_0}{\rho(\langle P \rangle, p_0, \rho_0)} f(p_0, \rho_0)\, dp_0\, d\rho_0} \equiv F(\langle P \rangle) \ . \tag{21}$$

So, in principle, one can find the relationship between $\langle \rho \rangle$ and $\langle P \rangle$ for any distribution function $f(p_0, \rho_0)$. Thus, (21) together with Eqs. (13) and (14) forms a closed system of dynamical equations, describing self-consistently the propagation of long-wave magnetosonic oscillations of a finite amplitude in an inhomogeneous plasma. The specific features of the system are determined by its statistic properties represented by the function f.

3. Formation of Shocks

There is a clear analogy of the problem under consideration to the problem of a one dimensional compressible gas, as it was already mentioned above. But, in general, the conclusion about the steepening of a wavefront and its overturning is connected with the dependence of ρ on P, that has a specific form of a function $\rho = F(P)$. For an ordinary gas this dependence, which is $\rho \sim p^{1/\gamma}$ and $\gamma > 1$, automatically satisfies the condition of overturning (see, for example, Landau and Lifshitz).

$$\frac{du}{d\rho} > 0 \ , \tag{22}$$

where $u = v + c$ and

$$v = \int \frac{c}{\rho} d\rho, \qquad c^2 = \frac{dp}{d\rho} \ .$$

To analyze the shock formation in our problem, first of all we have to represent the condition (22) in a more general form through the relationship between the density and pressure, and then check if the relationship (21) satisfies this condition.

We rewrite the Eqs. (13) and (14) in a form (we omit herewith the brackets and bear in mind that in our problem we deal with the averaged quantities)

$$\rho \frac{dv}{dt} = -\frac{dP}{d\rho} \frac{d\rho}{dv} \frac{dv}{dx} , \tag{23}$$

$$\frac{d\rho}{dv} \frac{dv}{dt} = -\rho \frac{\partial v}{\partial x} . \tag{24}$$

Eliminating dv/dt in these equations, we find that

$$\frac{dP}{d\rho} \frac{d\rho}{dv} \frac{dv}{dx} = \rho^2 \frac{\partial v/\partial x}{d\rho/dv} . \tag{25}$$

Substituting this expression in the right-hand side of Eq. (23), we obtain the general form of nonlinear equation

$$\frac{\partial v}{\partial t} + \left(v + \rho \frac{dv}{d\rho} \right) \frac{\partial v}{\partial x} = 0 . \tag{26}$$

As we introduce now the following notation

$$u = v + c ,$$

the analogy with an ideal compressible gas becomes complete and the condition for overturning (22) of the wavefront remains the same

$$\frac{dv}{d\rho} + \frac{d}{d\rho} \left(\rho \frac{dv}{d\rho} \right) > 0 . \tag{27}$$

We now have to express all terms in this condition through the function $\rho = \rho(P)$. It is easy to find from (25) that

$$\rho \frac{dv}{d\rho} = \sqrt{\frac{dP}{d\rho}} ,$$

or

$$\rho \frac{dv}{d\rho} = \frac{1}{\sqrt{\rho'(P)}} , \tag{28}$$

where the prime means the derivative with respect to P. For the first term in condition (27) this expression yields simply

$$\frac{dv}{d\rho} = \frac{1}{\rho(P)\sqrt{\rho'(P)}} \,. \tag{29}$$

The second term in (27) can be written as follows:

$$\frac{d}{d\rho}\left(\rho\frac{dv}{d\rho}\right) = \frac{dP}{d\rho}\frac{\partial}{\partial P}\frac{1}{\sqrt{\rho'(P)}} \,,$$

or, performing the differentiation

$$\frac{d}{d\rho}\left(\rho\frac{dv}{d\rho}\right) = -\frac{1}{2}\frac{\rho''(P)}{[\rho'(P)]^{5/2}} \,. \tag{30}$$

With the help of (29) and (30) from (27) we obtain the most general form of the condition for steepening of a wavefront in a medium with an arbitrary relationship between the density and pressure (of course, under the condition that nonlinear equation of motion has a similar form as (27))

$$2\left(\frac{d\rho}{dP}\right)^2 > \rho(P)\frac{d^2\rho}{dP^2} \,. \tag{31}$$

We elucidate whether the condition (31) is satisfied in our case when $\rho = F(\langle P\rangle)$ is defined through the expression (21). The first and second derivatives of the function form

$$\frac{dF}{d\langle P\rangle} = \frac{\int \frac{\rho_0}{\rho^2}\frac{d\rho}{d\langle P\rangle}d\Gamma}{\left[\int \frac{\rho_0}{\rho(\langle P\rangle)}d\Gamma\right]^2}\cdot\int \rho_0\, d\Gamma \tag{32}$$

$$\frac{d^2F}{d\langle P\rangle^2} = \left\{\frac{2\left[\int \frac{\rho_0}{\rho^2}\frac{d\rho}{d\langle P\rangle}d\Gamma\right]^2}{\left[\int \frac{\rho_0}{\rho}d\Gamma\right]^3} + \left[\int \frac{\rho_0}{\rho}d\Gamma\right]^{-2}\cdot\int\left[\frac{\rho_0}{\rho^2}\frac{d^2\rho}{d\langle P\rangle^2} - 2\frac{\rho_0}{\rho^3}\left(\frac{d\rho}{d\langle P\rangle}\right)^2\right]d\Gamma\right\}$$

$$\cdot\int \rho_0\, d\Gamma \,, \tag{33}$$

where $d\Gamma = f(p_0, \rho_0)\, dp_0\, d\rho_0$.

Combining the condition (31) with (21), (32), and (33) after some algebra, we have

$$\frac{\left[\int \rho_0\, d\Gamma\right]^2}{\left[\int \frac{\rho_0}{\rho} d\Gamma\right]^3} \cdot \int \left[-2\frac{\rho_0}{\rho^3}\left(\frac{d\rho}{d\langle P\rangle}\right)^2 + \frac{\rho_0}{\rho^2}\frac{d^2\rho}{d\langle P\rangle^2}\right] d\Gamma < 0\ . \tag{34}$$

It is obvious that $\int \rho_0\, d\Gamma > 0$ and $\int \frac{\rho_0}{\rho} d\Gamma > 0$.

The first and second derivatives in (34) can be easily found from the equation (15) where P is substituted by its average value $\langle P\rangle$ (see the explanations before Eq. (18))

$$\frac{d\rho}{d\langle P\rangle} = \frac{\rho}{\gamma\langle P\rangle + (2-\gamma)(P_0 - p_0)\rho^2/\rho_0^2}$$

and

$$\frac{d^2\rho}{d\langle P\rangle^2} = \frac{\frac{d\rho}{d\langle P\rangle}\left[\gamma\langle P\rangle - (2-\gamma)(P_0 - p_0)\rho^2/\rho_0^2\right] - \gamma\rho}{\left[\gamma\langle P\rangle + (2-\gamma)(P_0 - p_0)\rho^2/\rho_0^2\right]^2}\ .$$

These derivatives taken in the point $\langle P\rangle = P_0$ become

$$\left.\frac{d\rho}{d\langle P\rangle}\right|_{P_0,\rho_0} = \frac{\rho_0}{\gamma\rho_0 + 2(P_0 - p_0)}\ ,$$

$$\left.\frac{d^2\rho}{d\langle P\rangle^2}\right|_{P_0,\rho_0} = \frac{\rho_0\left[\gamma\rho_0 - \gamma^2\rho_0 - 2(P_0 - p_0)\right]}{\left[\gamma\rho_0 + 2(P_0 - p_0)\right]^3}\ .$$

Substituting these expressions into the condition (34), we obtain

$$-\int f(p_0, \rho_0)\frac{\gamma(\gamma+1)\rho_0 + 6(P_0 - p_0)}{\left[\gamma\rho_0 + 2(P_0 - p_0)\right]^3} d\rho_0\, d\rho < 0\ . \tag{35}$$

In accordance with the equilibrium condition (5) the magnitude $(P_0 - p_0)$ is always positive

$$P_0 - p_0 = \frac{B_0^2}{8\pi} > 0\ .$$

This means that the integrand in the expression (35) is positive definite. Thus the condition (35) as well as its general form (31) is satisfied for any distribution function $f(p_0, \rho_0)$ and it is valid that the magnetosonic wave of finite amplitude propagating in a plasma with random inhomogeneities can split into two simple waves with subsequent steepening and overturning. Note that when the width of the wavefront becomes comparable with the characteristic scale of inhomogeneities, our assumption is not valid. In this case the dispersion effects play an essential role, since at $\lambda \sim R$ the dispersion of magnetosonic waves becomes nonlinear and steepening of wavefront ceases.

4. The Influence of Inhomogeneities on the Dispersion Properties of the System

As it was mentioned above, the dispersion relation (3) may be considered as a first term in the expansion of ω in series of (kR). Now our goal is to find the next term in this expansion, that is to find nonlinear dependence of the frequency on the wavenumber, which determines the dispersion properties of medium. We consider again 1D-problem and start with the linearization of the system (6) introducing small perturbations (unperturbed quantities are random functions of coordinate x)

$$p = p_0(x) + \delta p(x,t) \ ,$$

$$\rho = \rho_0(x) + \delta \rho(x,t) \ ,$$

$$B = B_0(x) + \delta B(x,t) \ ,$$

$$v = \delta v(x,t) \ .$$

Linearized equations are

$$\rho_0 \frac{\partial \delta v}{\partial t} = -\frac{\partial \delta P}{\partial x} \ , \tag{36}$$

$$\frac{\partial \delta \rho}{\partial t} + \frac{\partial}{\partial x} \rho_0(x) \delta v = 0 \ , \tag{37}$$

$$\frac{\partial \delta B}{\partial t} + \frac{\partial}{\partial x} B_0(x)\delta v = 0 \ , \tag{38}$$

$$\frac{1}{p_0}\frac{\partial \delta p}{\partial t} - \frac{\gamma}{\rho_0}\frac{\partial \delta \rho}{\partial t} + \delta v \left(\frac{1}{p_0}\frac{dp_0}{dx} - \gamma \frac{1}{\rho_0}\frac{d\rho_0}{dx}\right) = 0 \ . \tag{39}$$

Using the equilibrium condition $\delta P = \delta p + \frac{B_0 \delta B}{4\pi}$, from the system (36)–(39) we can get the equation for δP

$$\frac{\partial \delta P}{\partial t} = -2\left[\left(\frac{\gamma}{2} - 1\right) p_0 + P_0\right] \frac{\partial \delta v}{\partial x} \ . \tag{40}$$

Equations (36) and (40) form a closed set of equations describing linear perturbations in an inhomogeneous medium.

All perturbed quantities can be represented in the following way

$$\delta P = \left\langle \delta P^{(0)} \right\rangle + \delta P^{(1)} \ ,$$

$$\delta v = \left\langle \delta v^{(0)} \right\rangle + \delta v^{(1)} \ , \tag{41}$$

where $\left\langle \delta P^{(0)} \right\rangle$, $\left\langle \delta v^{(0)} \right\rangle$, etc., are linear perturbations averaged over the distance L: $R \ll L \ll \lambda$ and $\delta P^{(1)}$, $\delta v^{(1)}$, etc., are fluctuating parts of perturbations caused by the presence of inhomogeneities. For slow motions $\partial/\partial t \sim \varepsilon \ll 1$ we have the following estimations from Eqs. (36) and (40)

$$\frac{\partial \delta v^{(0)}}{\partial x} \sim \varepsilon \ll 1; \qquad \frac{\partial \delta P^{(0)}}{\partial x} \sim \varepsilon \ll 1 \ .$$

Therefore, $\delta P^{(0)} \simeq \left\langle \delta P^{(0)} \right\rangle$ and $\delta v^{(0)} \simeq \left\langle \delta v^{(0)} \right\rangle$. Note that by definition

$$\left\langle \delta P^{(1)} \right\rangle = 0 \quad , \quad \left\langle \delta v^{(1)} \right\rangle = 0 \ .$$

Since we deal with stationary inhomogeneities, we can consider a harmonic wave in time and replace $\partial/\partial t$ by $-i\omega$. It is convenient to introduce the notation

$$\sigma_0(x) = \frac{1}{2}\left[\left(\frac{\gamma}{2} - 1\right) p_0(x) + P_0\right]^{-1} \ .$$

Now Eqs. (36) and (40) have a form

$$i\omega\rho_0(x)\delta v = \frac{\partial \delta P}{\partial x} ,$$

$$i\omega\sigma_0(x)\delta P = \frac{\partial \delta v}{\partial x} . \qquad (42)$$

We can represent $\rho_0(x)$ and $\sigma_0(x)$ as follows:

$$\rho_0(x) = \langle\rho_0(x)\rangle + \rho_1(x) ,$$

$$\sigma_0(x) = \langle\sigma_0(x)\rangle + \sigma_1(x) . \qquad (43)$$

Note that the fluctuations of plasma parameters of background the medium and the average values of these parameters are of the same order: $\rho_1(x) \sim \langle\rho_0(x)\rangle$ and $\sigma_1(x) \sim \langle\sigma_0(x)\rangle$ which reflects strong inhomogeneities we are considering, e.g., that all unperturbed quantities change from one tube to another by an order of magnitude. But, at the same $\langle\rho_1(x)\rangle = 0$, $\langle\sigma_1(x)\rangle = 0$. Introducing into the system (42) a fluctuating part of perturbations and of background inhomogeneities, we have

$$i\omega\left(\langle\rho_0\rangle + \rho_1\right)\left(\delta v^{(0)} + \delta v^{(1)}\right) = \frac{\partial}{\partial x}\left[\delta P^{(0)} + \delta P^{(1)}\right] ,$$

$$i\omega\left(\langle\sigma_0\rangle + \sigma_1\right)\left(\delta P^{(0)} + \delta P^{(1)}\right) = \frac{\partial}{\partial x}\left[\delta v^{(0)} + \delta v^{(1)}\right] . \qquad (44)$$

After averaging, Eqs. (44) become

$$i\omega\left[\langle\rho_0\rangle\,\delta v^{(0)} + \left\langle\rho_1\delta v^{(1)}\right\rangle\right] = \frac{\partial \delta P^{(0)}}{\partial x},$$

$$i\omega\left[\langle\sigma_0\rangle\,\delta P^{(0)} + \left\langle\sigma_1\delta P^{(1)}\right\rangle\right] = \frac{\partial \delta v^{(0)}}{\partial x} . \qquad (45)$$

The second terms on the left-hand side of Eqs. (45) are just those which determine the dispersion of the wave due to the presence of inhomogeneities.

To solve the system (45), we have to express $\delta v^{(1)}$ and $\delta P^{(1)}$ through unperturbed quantities. The equations describing $\delta v^{(1)}$ and $\delta P^{(1)}$ can be obtained by subtraction from the system (44) of the corresponding equations of system (45)

$$i\omega\left[\rho_1\delta v^{(0)}+\langle\rho_0\rangle\,\delta v^{(1)}+\left\langle\rho_1\delta v^{(1)}\right\rangle\right]=\frac{\partial\delta P^{(1)}}{\partial x}\ ,$$

$$i\omega\left[\sigma_1\delta P^{(0)}+\langle\sigma_0\rangle\,\delta P^{(1)}+\left\langle\sigma_1\delta P^{(1)}\right\rangle\right]=\frac{\partial\delta v^{(1)}}{\partial x}\ . \tag{46}$$

Since we are looking for first order corrections, we can omit in Eqs. (46) second and third terms in the left-hand side (these terms give the next order corrections in $\delta P^{(1)}$ and $\delta v^{(1)}$). Then the system (46) becomes

$$i\omega\rho_1\delta v^{(0)}=\frac{\partial\delta P^{(1)}}{\partial x}\ ,$$

$$i\omega\sigma_1\delta P^{(0)}=\frac{\partial\delta v^{(1)}}{\partial x}\ . \tag{47}$$

We introduce the following definitions:

$$\rho_1(x)=\frac{d\eta(x)}{dx}\ ,$$

$$\sigma_1(x)=\frac{d\zeta(x)}{dx}\ . \tag{48}$$

Naturally $\langle\eta\rangle=0$ and $\langle\zeta\rangle=0$. With these definitions the equations (47) are as follows:

$$\delta P^{(1)}\equiv i\omega\eta(x)\delta v^{(0)}\ ,$$

$$\delta v^{(1)}\equiv i\omega\zeta(x)\delta P^{(0)}\ . \tag{49}$$

Substituting (49) into (45), we get the final set of equations containing the corrections which determine the dispersion of the wave due to the presence of inhomogeneities

$$i\omega\left[\langle\rho_0\rangle\,\delta v^{(0)}+i\omega\,\langle\rho_1\zeta\rangle\,\delta P^{(0)}\right]=\frac{\partial\delta P^{(0)}}{\partial x}\ ,$$

$$i\omega\left[\langle\sigma_0\rangle\,\delta P^{(0)}+i\omega\,\langle\sigma_1\eta\rangle\,\delta v^{(0)}\right]=\frac{\partial\delta v^{(0)}}{\partial x}\ . \tag{50}$$

Since the coefficients in the system (50) do not depend on coordinates, we can put $\partial/\partial x = ik$ and obtain the dispersion relation

$$\langle\sigma_0\rangle\langle\rho_0\rangle\omega^2 + \langle\rho_1\zeta\rangle\langle\sigma_1\eta\rangle\omega^4 + ik\omega^2\left(\langle\rho_1\zeta\rangle + \langle\sigma_1\eta\rangle\right) = k^2 \ . \tag{51}$$

Using the definitions (48), we see that the third term in the left-hand side of Eq. (51) is a full derivative whose average is evidently zero $\langle\rho_1\zeta\rangle + \langle\sigma_1\eta\rangle = \left\langle \frac{d\eta}{dx}\zeta + \frac{d\zeta}{dx}\eta \right\rangle = \left\langle \frac{d(\zeta\eta)}{dx} \right\rangle \equiv 0$. The square of this expression determines the coefficient of ω^4 in the dispersion relation, which has a negative definite quadratic form

$$\langle\rho_1\zeta\rangle\cdot\langle\sigma_1\eta\rangle = -\frac{1}{2}\left\{\left\langle\frac{d\eta}{dx}\zeta\right\rangle^2 + \left\langle\eta\frac{d\zeta}{dx}\right\rangle^2\right\} \equiv -a^2 \ . \tag{52}$$

After transformations of (51), the dispersion relation becomes

$$\langle\sigma_0\rangle\langle\rho_0\rangle\omega^2 - a^2\omega^4 = k^2 \ . \tag{53}$$

In the zeroth order approximation (neglecting the dispersion of the wave), ω scales linearly with k

$$\omega = s\left(\rho_0, p_0, B_0\right)\cdot k \ , \tag{54}$$

where "sound" speed has a form (cf. Eq. (4))

$$s\left(\rho_0, p_0, B_0\right) = \frac{1}{\sqrt{\langle\sigma_0\rangle\langle\rho_0\rangle}} = \frac{1}{\sqrt{\langle\rho_0\rangle}}\left\langle\frac{1}{\gamma p_0 + B_0^2/4\pi}\right\rangle^{-1/2} \ . \tag{55}$$

Using linear approximation (54) from (53), we obtain the next approximation in wavevector

$$\omega^2 = k^2 s^2 + a^2 k^4 s^6 \ ,$$

or, finally,

$$\omega = ks\left(1 + \delta^2 k^2\right) \ , \tag{56}$$

where δ^2 is a coefficient which determines the dispersion of the wave due to the presence of inhomogeneities

$$\delta^2 = \frac{1}{2}a^2 s^4 \ . \tag{57}$$

For a wave with a small but finite amplitude one should take into account simultaneously the effects of a weak nonlinearity (described by the Eq. (26)) and that of a finite dispersion. To do so, note that the dispersion relation (56), if written in the velocity frame moving with the "sound speed" s, corresponds to the dynamic equation of the form

$$\frac{\partial v}{\partial t} = s\delta^2 \frac{\partial^3 v}{\partial x^3}. \tag{58}$$

Respectively, the desired nonlinear equation can be written as

$$\frac{\partial v}{\partial t} + \alpha v \frac{\partial v}{\partial x} = s\delta^2 \frac{\partial^3 v}{\partial x^3} \tag{59}$$

(in the reference frame moving with s).

Therefore, we conclude that the evolution of the initial magnetosonic perturbation in a plasma with random density inhomogeneities can be described by a KdV equation whose coefficients are uniquely determined by the statistic properties of random plasma inhomogeneities [see Eqs. (21) and (52)]. Note that the coefficient δ which determines the dispersion, differs from zero only in an inhomogeneous plasma; in a homogeneous case we have $\delta = 0$, and even a small initial perturbation steepens till the formation of weak shocks.

In an inhomogeneous plasma the initial perturbation (if it is not too small) can give rise to a formation of solitons. In this case the width ΔX of leading (the largest) solution may be estimated as

$$\Delta X \sim \sqrt{\frac{s\delta^2}{\alpha V_0}} ,$$

where V_0 is the amplitude of the initial perturbation. In order for our approach to be valid, this width should be much greater than the characteristic scale of inhomogeneities R. Since, according to (52), at large enough inhomogeneities ($\rho_1 \sim \rho_0, p_1 \sim p_0$), δ is of the order of R, and α is of the order of unity, we conclude that our description is adequate for the condition

$$V_0 \ll s ,$$

which is a relatively weak constraint.

As it was mentioned above, a plasma with strong random inhomogeneities of density exhibits an important feature: dissipative effects connected with thermal conductivity and viscosity are enhanced. For the complete description of wave propagation in such a media, Eq. (59) has to contain also the dissipation terms. This leads to KdV-Burgers' equation

$$\frac{\partial v}{\partial t} + \alpha v \frac{\partial v}{\partial x} = s\delta^2 \frac{\partial^3 v}{\partial x^3} + \mu \frac{\partial^2 v}{\partial x^2} \, . \tag{60}$$

Where coefficient $\mu\,(\rho_0, p_0, B_0) = \mu_{\text{visc}} + \mu_{\text{therm}}$ is determined by statistic properties of inhomogeneities and contains parts connected with viscous and thermal losses (Ohmic losses remain the same as in homogeneous medium and are much less than those two). The procedure that allows to find μ is described in the paper by Ryutova and Persson (1984) (as to final expression for damping rate see their Eq. (51)).

The equation (60) allows to make general comments. The equation describes the evolution of the arbitrary initial perturbation (cf. Karpman, 1973): depending on the interplay of the nonlinear, dissipative and dispersive effects, it can evolve either to weak shocks, or be split into a train of some number of independent solitons which then will be damped away, or, in a case of the predominance of dissipative effects, the primary perturbation can be damped away in a linear stage.

5. Numerical Simulations

We study numerically the influence of small scale background fluctuations on the propagation and evolution of long wavelength perturbations by using a one-dimensional code of ideal magnetohydrodynamics. The set of the ideal MHD equations [Eqs. (6)] was numerically solved by computer simulations.

5.1 Initial conditions

We consider an isothermal plasma with temperature T and assume the gas to be a polytrope of index $\gamma = 1.5$. The distribution of magnetic field strength $B(x)$ is given by

$$B(x) = [8\pi p(x)/\beta(x)]^{1/2} \ , \qquad (61)$$

where

$$\beta(x) = \beta_0 \cdot rand(x) \qquad (62)$$

and where β_0 is the maximum of the ratio of the gas pressure to magnetic pressure. We used $\beta_0 = 1.0$ in all our simulations. In Eq. (62) $rand(x)$ describes the randomly distributed small scale inhomogeneities of the background.

The initial density and pressure distributions are calculated by using the equation of state

$$p(x) = \rho(x) \cdot T \qquad (63)$$

and the equation of magnetostatic equilibrium [Eq. (5)].

5.2 Perturbations

Long wavelength acoustic perturbations of the form

$$\begin{aligned} \delta v_x(x) &= A \cdot \sin\left(2\pi \frac{x}{\lambda}\right) \ , \\ \delta \rho(x) &= \left(\frac{\rho_0}{C_s}\right) \delta v_x \ , \qquad (64) \\ \delta p(x) &= \rho_0 \cdot C_s \delta v_x \end{aligned}$$

are initially imposed, where A is the amplitude of the initial perturbation, λ its wavelength, and C_s the speed of sound.

5.3 Boundary conditions and numerical method

We assume periodic boundaries for $x = 0$ and $x = X_{\max}$, where $X_{\max}$ is the size of the computational domain. The set of equations (6) is nondimensionalized by using the following normalizing constants: H the scale height, C_s the sound velocity, and ρ_0 the density. Equations (6) are solved numerically by using a modified Lax-Wendroff scheme (Rubin and Burstein 1967) with an artificial viscosity according to Richtmyer and Morton (1967). The tests and accuracy of such a MHD code have been described by Shibata (1983), Matsumoto *et al.* (1988), and Tajima (1989). The mesh size is $\Delta x = X_{\max}/(N_x - 1)$, where N_x is the number of mesh points in the x-direction.

5.4 Numerical results

The aim of our numerical simulations is to support and extend the theoretical results derived above from an analysis of the averaged equations. In particular, we want to show that long wavelength perturbations steepen and form shock waves even in the presence of small scale background fluctuations and that the energy dissipation due to the small scale inhomogeneities is enhanced with respect to the case of an homogeneous background. Although theory assumes the characteristic wavelength is much greater than the inhomogeneity length scale, Eq. (1), our computation can remove such a restriction and generalize the dissipation effect regardless of the characteristic wavelength.

5.4.1 Formation of shock waves

We assume $A = 0.5$, $N_x = 1001$, $X_{\max} = 100$, and $\lambda = X_{\max}$. Since the length scale of the background fluctuations is Δx, our theoretical assumption $\lambda \gg R$ is amply fulfilled. The magnitude A of the initial pressure perturbation is too large to be realistic. However, our simulations with different values of A show that the amplitude of the perturbation within the regime of $0.1 < A < 0.5$ does not qualitatively affect the overall evolution. Since the time scale for the steepening of the initial wave with wavelength λ is of the order of λ/v, its quantitative effect is that the smaller A is, the larger is the computational time. Thus, this high value of the initial perturbation is chosen simply for computational convenience.

Figure 2 shows the time variation of the velocity field V_x in the strongly nonuniform background medium. The initially sinusoidal perturbation becomes more and more asymmetric ($t \sim 22$), steepens and forms strong shock waves at $t \sim 42$. In Fig. 3 we display the velocity V_x, the total pressure P_{tot}, the density ρ, and the vertical magnetic field strength B_z in the final state at $t = 76$. The shocks are best resolved in V_x, ρ and P_{tot}. The profiles of the density and magnetic field strength are strongly modulated by the small scale background fluctuations.

In comparison to these results we show in Fig. 4 the case of a homogeneous background ($\beta(x) = \beta_0$ in Eq. (62)). The overall evolution is similar to the previous case except for the fact that no small scale variations occur. In both cases the characteristic Burgers' sawtooth shock formation is apparent. Note that the "discontinuities" of shocks in both cases are more than several grid spacings so that they are not beyond the numerical resolution.

At this point, we summarize that even in the presence of strong, small scale background inhomogeneities the propagation of sufficiently strong acoustic waves with wavelengths $\lambda \gg R$ is characterized by steepening and the formation of shocks.

5.4.2 Energy dissipation

We now study the energetics of the system and compare the case of inhomogeneous background fluctuations with that of a homogeneous background. We assume the same amplitude of the initial perturbation $A = 0.5$ as in our previous calculations, but use $X_{\max} = 20$. The smaller wavelength of the initial perturbation reduces the time scale for the steepening by a factor of 5 compared to the simulations discussed in the previous chapter.

We first consider the case of a homogeneous background. Figure 5 shows the time variation of magnetic, thermal, and kinetic energies in the system. It should be noted that $\Delta E = E(t) - E(0)$. The kinetic energy ΔE_k strongly decreases within the first 10 timescales. In this period the waves steepen and form shocks. Within the next 30 timescales 10% of the kinetic energy dissipates and is converted mostly into thermal energy through compressional and shock heating, and partly into magnetic energy. The total energy in this simulation is conserved within less than 1% of its initial value.

In Fig. 6 we show the evolution of the energetics with time for the case of an inhomogeneous background. The kinetic energy decreases even more drastically than in the homogeneous cases. Within the first period ($t \sim 30$) the thermal energy strongly increases partly due to the damping of the kinetic energy and partly due to the dissipation of the magnetic energy. The heating effect is gauged by the increase of the thermal energy $\Delta E_{\rm th}$, or lack of it. For example, $\Delta E_{\rm th}$ in Fig. 6 in the inhomogeneous case is ~ 0.18, while $\Delta E_{\rm th}$ in Fig. 5 for the homogeneous case is ~ 0.09. The main difference between the homogeneous and the inhomogeneous case is that in the latter one the magnetic field releases also its energy into thermal energy through compressional and shock heating.

Finally we study the limiting case $\lambda \to R$ in which the wavelength of the initial perturbation is of the same order as the length scale of the background fluctuations. For this situation our theoretical investigation is unable to make any predictions.

We assume in our numerical simulations: $A = 0.5$, $X_{\rm max} = 20$, and $\lambda = X_{\rm max}/16$. We again witness the strong dissipation of kinetic and magnetic energy in the early stage of the evolution. In Fig. 7 we show the time evolution of the thermal energy $\Delta E_{\rm th}$. Compared to the case $\lambda = X_{\rm max}$, which is also shown in Fig. 7, the heating of the plasma is even larger. Due to the strong dissipation from the very beginning, the initial perturbation is rapidly damped and cannot steepen into shock waves.

In summary, we have shown that the presence of small scale background fluctuations results in a much stronger dissipation of long wavelength perturbations and a larger heating of the plasma compared to the case of an homogeneous medium. Qualitatively, this result is independent of the amplitude A of the initial perturbation, but the higher the amplitude the larger the amount of heating. In the limiting case of $\lambda \to R$ in which the wavelength of the perturbation is of the same order as the length scale of the inhomogeneities, our numerical simulations suggest that the waves do not steepen into shocks but are rapidly damped out. Compared to the case of long wavelength perturbations the heating of the plasma is even larger. Computational investigations of dispersive properties will be reported in future works.

We have numerically studied the influence of small scale background fluctuations on the propagation and evolution of long and short wavelength perturbations. We have shown that in the presence of strong, small scale background inhomogeneities (i.e., in spite of their presence) the propagation of acoustic waves with long wavelengths $\lambda \gg R$ is characterized by a steepening and finally by a formation of shocks. Furthermore, the presence of small scale background fluctuations results in a much stronger dissipation of long wavelength perturbations and a larger heating of the plasma compared to the case of an homogeneous medium. Qualitatively, this result is independent of the amplitude A of the initial perturbation, but the higher the amplitude the larger the amount of heating. In the limiting case of $\lambda \to R$ in which the wavelength of the perturbation is of the same order as the length scale of the inhomogeneities, the case that is beyond the realm of theoretical analysis, dissipates their energy even faster and in fact so fast that they do not steepen into shocks. Compared to the case of long wavelength perturbations, the heating of the plasma is even larger.

6. Summary

In the present paper we studied the problem of hydromagnetic wave propagation in strongly inhomogeneous gas with the focus on the following two problems:

1. the propagation of nonlinear magnetosonic waves in plasma with random and strong inhomogeneities of density, magnetic field, temperature, etc., and

2. the influence of inhomogeneities on the dispersive properties of propagating waves.

It was shown that the magnetoacoustic-waves of arbitrary amplitude can split into two simple (Riemann) waves travelling in the opposite direction. Each wave has a tendency of steepening and overturning with the subsequent formation of shocks.

The presence of small-scale inhomogeneities may lead to the finite dispersion of the wave, and in the case with weak but finite dispersion effects, this gives rise to the cubic dependence on wavenumber of the frequency of shift of acoustic wave.

For the wave of small (but finite) amplitude the procedure which allows to get the averaged equation containing the nonlinearity of a wave, dispersion properties of a system and dissipative effects is described. The coefficients in this equation, which appeared to be a KdV-Burgers' type equation, are determined by the spectral density of fluctuations of plasma density, magnetic field, temperature, etc. That means, that depending on the statistic properties of a certain region of the solar atmosphere, the different scenarios of the energy transfer of primary magnetosonic waves to this region can develop.

1. In predominance of the dissipative effects, the primary wave is damped away in the linear stage and the efficiency of heating due to inhomogeneities is much greater than that in homogeneous medium. Note that strong enhancement of the damping of the wave is mostly provided by the viscosity and thermal losses and is connected with the appearance of strong local gradients of velocity and temperature in neighboring magnetic elements. Ohmic losses remain almost the same as in homogeneous medium however.

2. The wave of an arbitrary amplitude with a sufficiently long wavelength has a tendency of steepening the wavefront and overturning the subsequent formation of shocks. The

shock formation can have a number of peculiarities in such systems, which is determined by the interplay of thermal and viscous losses. For example, in predominance of thermal losses the isothermal jump can occur. The specifics of shock formation in strongly inhomogeneous medium and accompanying phenomena will be presented elsewhere.

3. The wave of a small but finite amplitude can be dispersive due to the presence of inhomogeneities under the conditions which are determined by the spectral density of fluctuations. The sign of dispersion determines the sequence of propagation of perturbations with different wavelengths, which, in principle, can be observed.

Depending on the interplay of nonlinearity and dispersion, solitary waves can appear and the energy of the primary magnetoacoustic waves is stored in the system of solutions which are later damped away.

Our computer simulation corroborates the above theoretical picture. It shows in particular the enhanced energy dissipation of the magnetoacoustic waves due to the strong inhomogeneity of the medium. The shock formation is observed in this case as in a homogeneous medium case, but the energy dissipation rate is much higher. Furthermore, when the wavelength of the waves is of the same order of the inhomogeneity, the wave energy is so quickly dissipated that they cannot form a shock.

Each scenario has a direct relevance to magnetized regions of the solar atmosphere. The results obtained are important in studies of propagation of p-modes, of oscillations in sunspots, of wave phenomena in plages including microflares, etc. To make a reasonable application of the present results obtained, one needs a detailed analysis of observational data of parameters of the region under investigation, such as cross-section of magnetic flux tubes or magnetic "islands," magnetic field strengths inside these structures, densities of plasma and their fluctuations, etc. The appropriate analysis can show which of the processes is responsible for heating of a chosen area and what is the amount of the energy of primary acoustic waves transmitted to this area.

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Figure Captions

1. Classes of ensembles of magnetic structures. (a) far removed elements, (b) tightly packed elements.

2. The distribution in x of the velocity V_x at $t = 0, 22, 42$ and 76 for the case of small scale background fluctuations.

3. The distribution in x of (a) the velocity V_x, the total pressure $P_{\rm tot}$, the density ρ, and the magnetic field strength B_z at $t = 76$ for the case shown in Fig. 2.

4. The distribution in x of the velocity V_x at $t = 0, 20, 38$ and 68 for the case of a homogeneous background.

5. Time variations of magnetic (ΔE_m), thermal ($\Delta E_{\rm th}$), kinetic (ΔE_k) and total ($\Delta E_{\rm Tot}$) energies, where $\Delta E = E(t) - E(0)$, for the case of a homogeneous background.

6. Time variations of magnetic (ΔE_m), thermal ($\Delta E_{\rm th}$), kinetic (ΔE_k), and total ($\Delta E_{\rm Tot}$) energies, for the case of an inhomogeneous background.

7. Time variations of thermal energy $\Delta E_{\rm th}$ for different wavelengths of the initial perturbation in the case of an inhomogeneous background.

Author Addresses

M. Kaisig
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

M. Ryutova
Institute of Nuclear Physics
630090 Novosibirsk, 90
U.S.S.R.

T. Tajima
Institute for Fusion Studies and Department of Physics
The University of Texas at Austin
Austin, Texas 78712

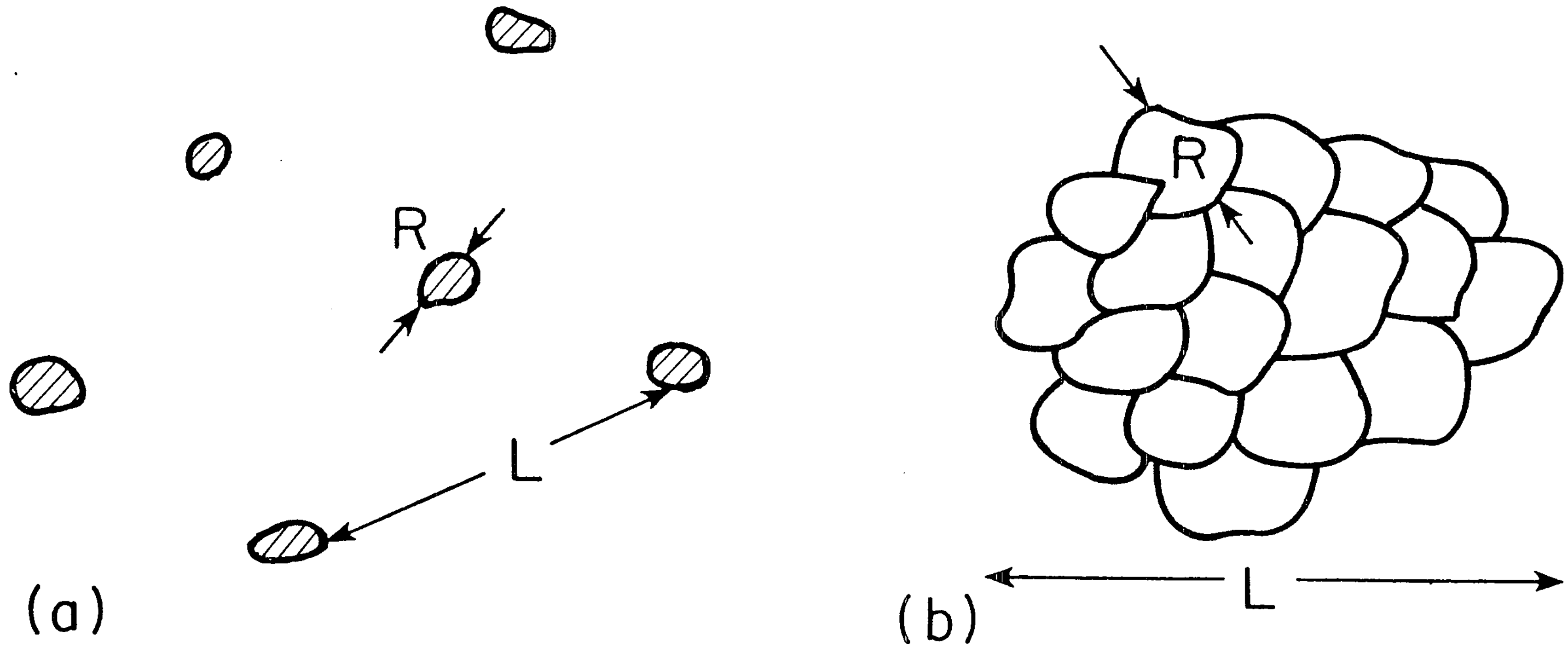


Fig. 1

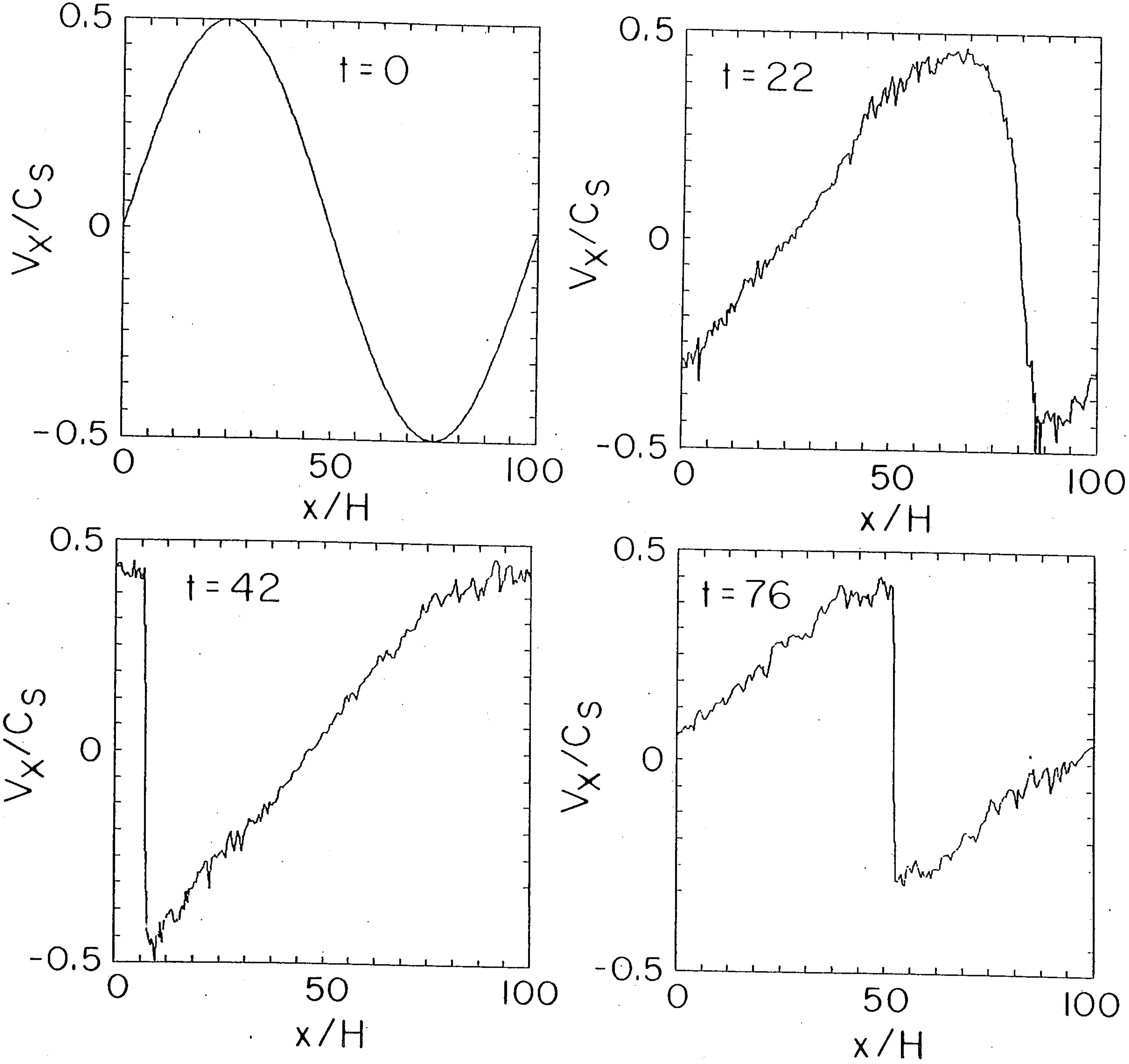
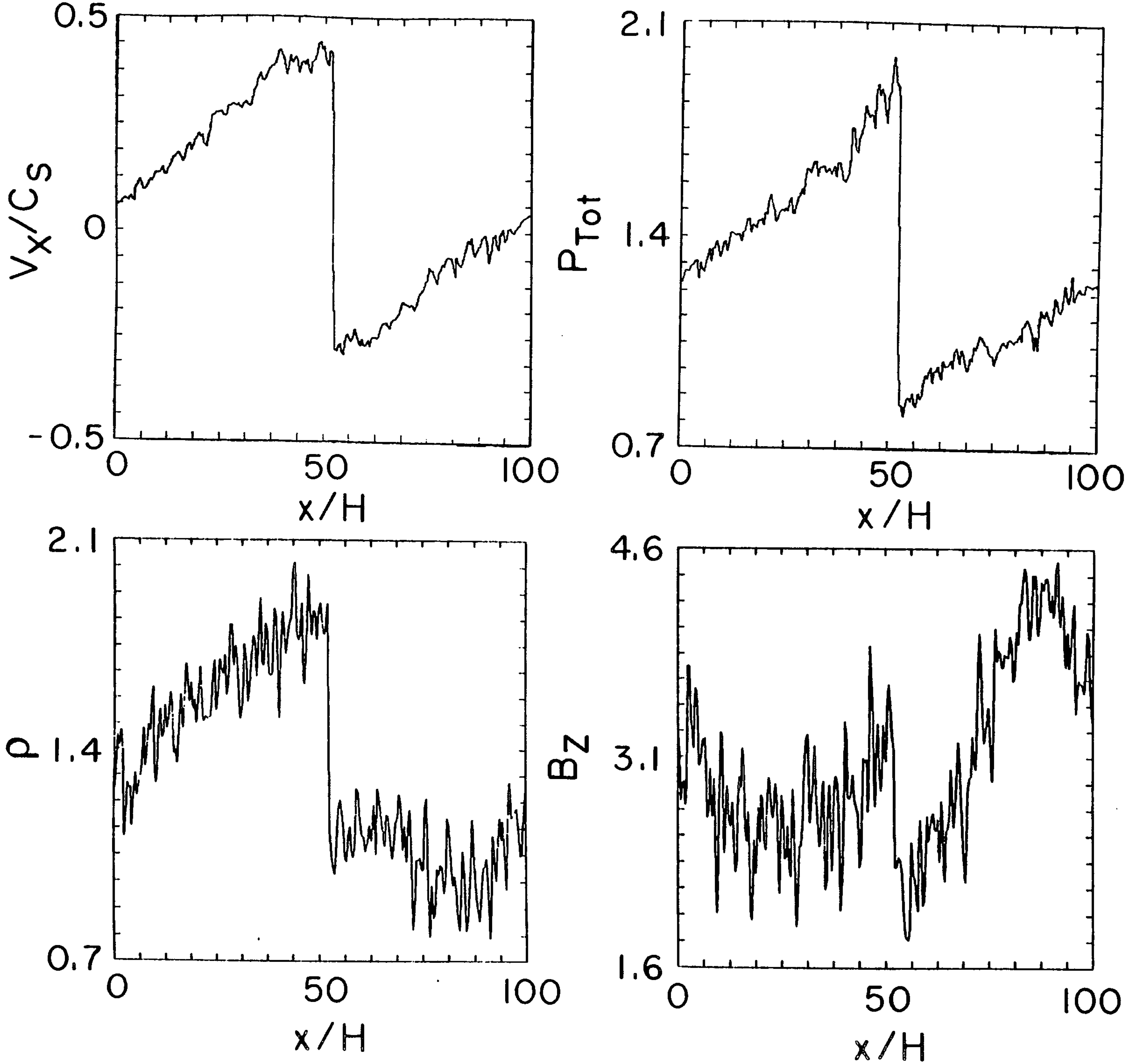


FIG. 2



0.5
0
-0.5
V_x/C_s
0
50
100
x/H
2.1
1.4
0.7
P_{Tot}
0
50
100
x/H
2.1
1.4
0.7
ρ
0
50
100
x/H
4.6
3.1
1.6
B_z
0
50
100
x/H

FIG. 3

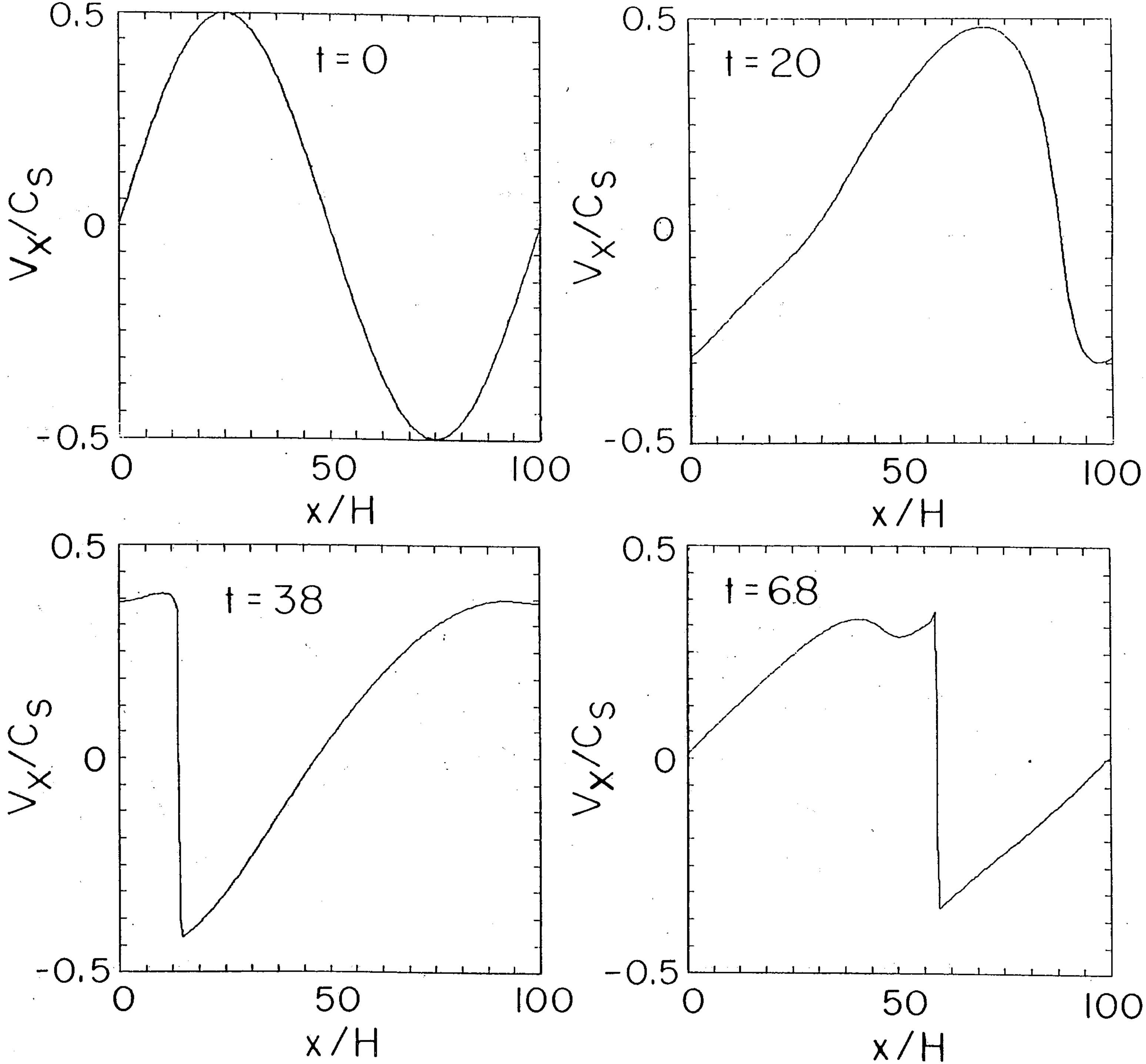
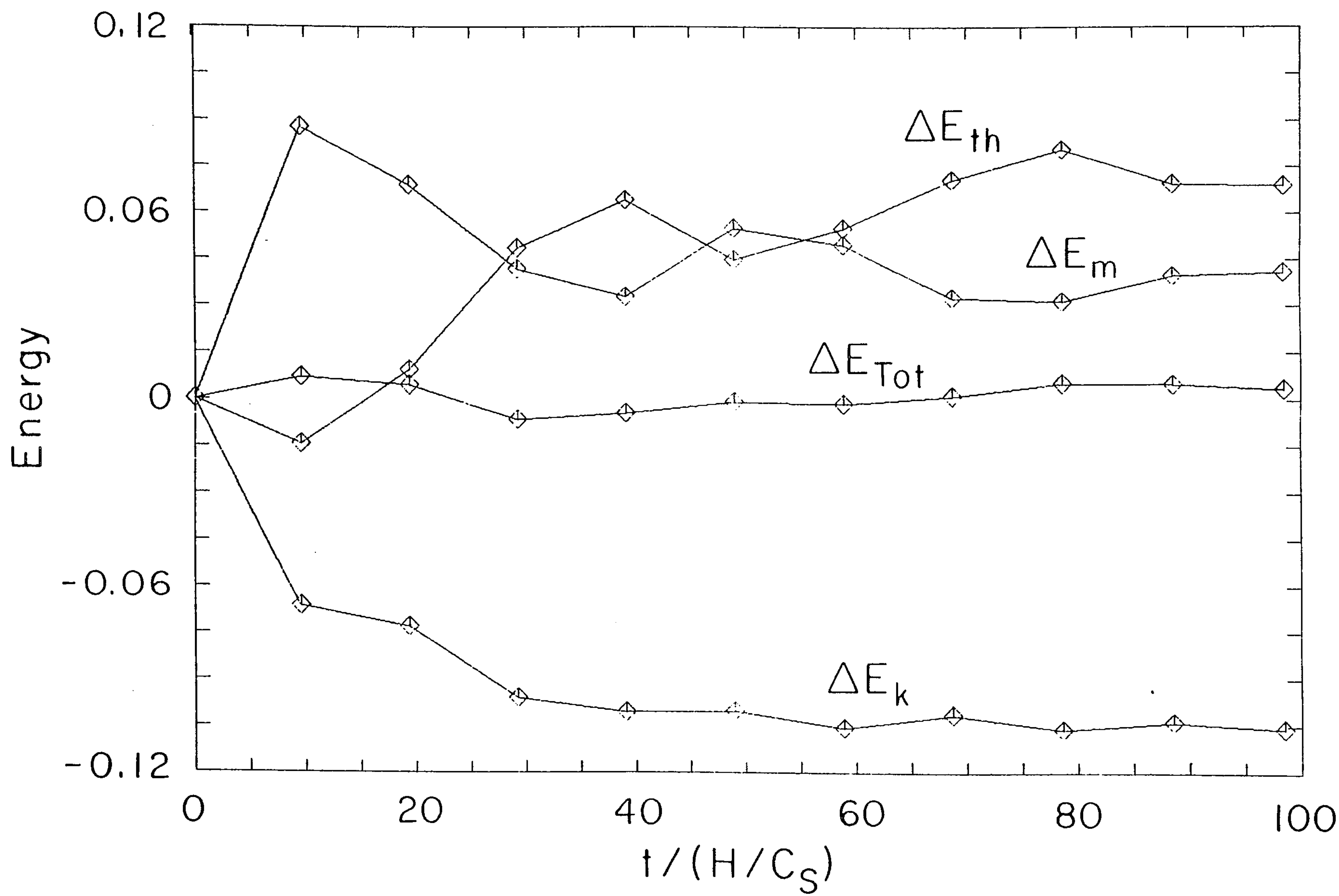


FIG. 4



0.12
0.06
0
-0.06
-0.12
Energy
0
20
40
60
80
100
t/(H/C_S)
ΔE_th
ΔE_m
ΔE_Tot
ΔE_k

FIG. 5

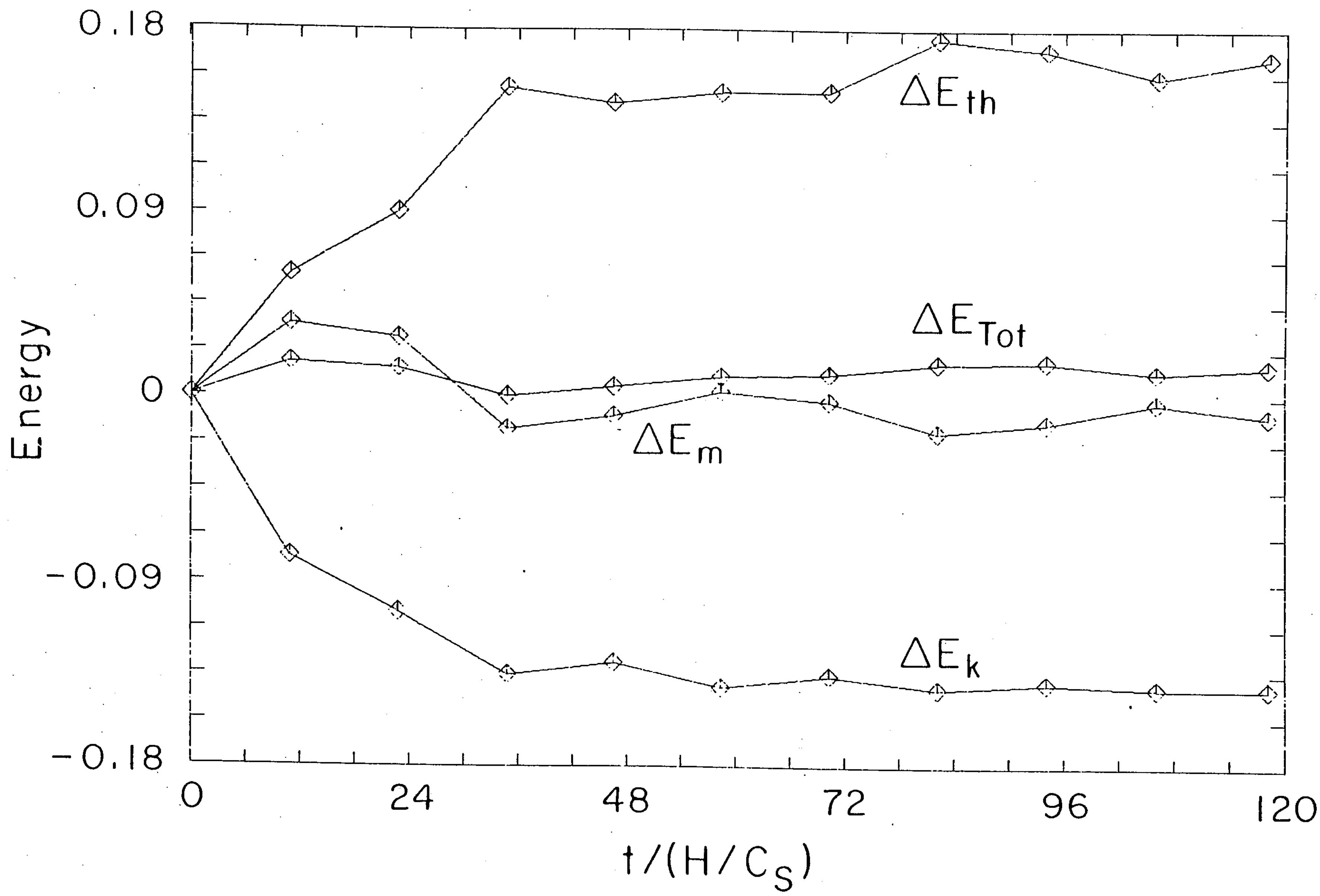

0.18
0.09
0
-0.09
-0.18
Energy
0
24
48
72
96
120
t/(H/C_S)
ΔE_th
ΔE_Tot
ΔE_m
ΔE_k

FIG. 6

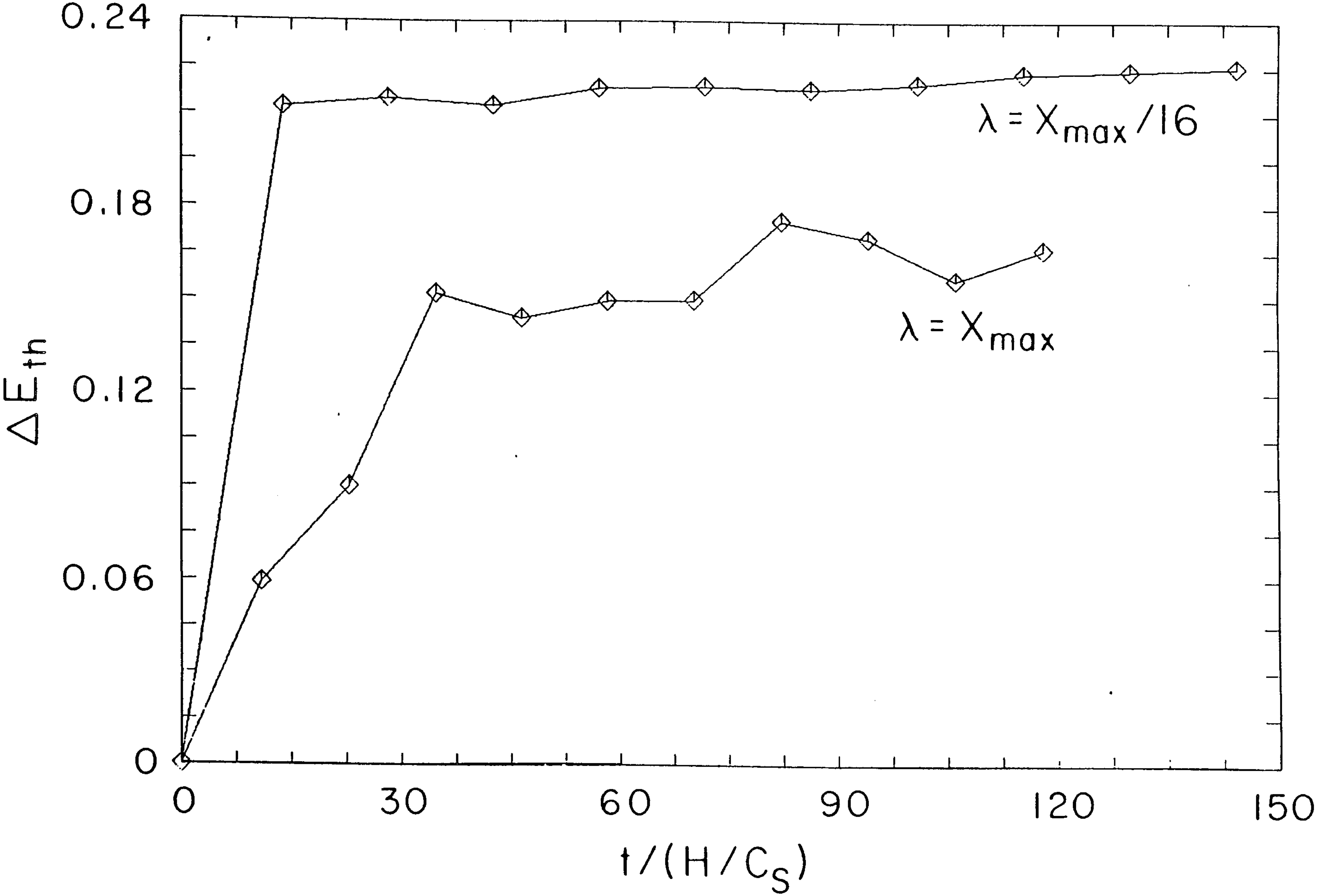


FIG. 7