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Abstract

In this paper, the effect of current drive on the tearing modes in the semi-collisional regime is analyzed using the drift-kinetic equation. A collisional operator is developed to model electron parallel conductivity. For the pure tearing modes the linear and quasilinear growth rates in the Rutherford regimes have been found to have roughly the same forms with a modified resistivity as without current drive. One interesting result is the prediction of a new instability. This instability, driven by the current gradient inside the tearing mode layer, is possibly related to MHD behavior observed in these experiments.

I. Introduction

Low m tearing modes constitute the dominant instability problem in present-day tokamaks. Feedback stabilization of these modes by lower hybrid (LH) current drive has been proposed by several authors.[1,2] However, even with LH current drive, tearing modes are still experimentally observed which saturate at a low fluctuation level, generally well below the onset of a disruption in tokamak discharges.[3]

Tearing mode instabilities in tokamaks with lower hybrid waves are examined in this paper. When the lower hybrid wave is included in the tearing mode theory it is found that the drift tearing mode is linearly destabilized while pure tearing modes have roughly the same forms as without lower hybrid current drive in both linear and quasilinear regimes. In References 1 and 2, the driven current is treated as an additional perturbed current in a resistive MHD analysis, and it is found that the pure tearing modes are possibly suppressed by lower hybrid current drive in the Rutherford regime. In this paper, starting from the drift kinetic equation with collisions and Landau damping from quasilinear LH waves, keeping LH waves both in equilibrium and perturbed state and following the Chapman-Enskog expansion to find the perturbed current, we can obtain explicit dispersion relations for semicollisional tearing modes in the slab model.

The linear destabilizing term arises from the equilibrium current gradient inside the electron current layer. A simple physical estimate of this term gives a lower bound on the order of magnitude of the growth rate of the normal drift tearing mode. A similar destabilizing term has also been found in the standard drift tearing modes when the equilibrium parallel electric field $E_{\parallel}^{0}$ is included inside the electron current layer. Even so, our results suggest that feedback stabilization of tearing modes is possible by a proper combination of direction and magnitude of these two currents.

Recent research on the linear theory of semicollisonal tearing modes in a sheared slab has shown that the width of the layer in which non-ideal effects are important is much smaller than the ion Larmor radius.[4] When the full nonlocal ion response is included, the finite Larmor radius effect (FLR) provides a stablizing effect. Reference 4 introduces an expansion procedure for solving the quasineutrality equation in powers of the ratio of the width of the tearing layer to the ion Larmor radius, and it is found that the modes with $\Delta' a \leq 100$ are stable, where $\Delta' = (\frac{\partial \widetilde{A}_z}{\partial x})^{\Delta}_{-\Delta} / \widetilde{A}_z(0)$ measures the discontinuity of the perturbed parallel vector potential $\widetilde{A_z}$ across the current layer, and a is the minor radius of a tokamak. The calculation done here uses the quasineutrality integral equation which is similar to the early treatment of Antonsen and Coppi[6] in the collisionless case to obtain the same result in the standard drift tearing modes. However, we would like to point out that in the flat density limiting case η_i and $\eta_e \rightarrow \infty$, ion orbit effects also become destablizing if $(dT_i/dr) > (dT_e/dr)$ (see Eq. (41)) in a sheared slab geometry.

The remainder of this paper is organized as follows. Section II. illustrates the basic physical mechanisms which govern the dynamics of the drift tearing modes by presenting a heuristic derivation of the new instability. Section III. gives the equilibrium conditions and basic assumptions. Section IV. discusses the linear tearing mode instability with the full nonlocal ion response. Section V. treats the pure quasilinear tearing modes and Section VI. is devoted to a discussion of the results of this calculation.

II. Heuristic Derivation of the New Tearing Instability

We examine here the tearing mode instability with LH current drive. From the kinetic point of view, LH current drive flattens the Maxwellian distribution function of electrons within a certain range of velocity (u_1, u_2) along the direction of the magnetic field,[5] thereby producing a current. The linearized Ohm's law is written as

$$n_0 e \eta_* (\widetilde{j}_\parallel - \widetilde{n}_e e \overline{v}_D) = n_0 e \widetilde{E}_\parallel - \widetilde{\Delta_\parallel P} - n_0 \alpha \widetilde{\Delta_\parallel T} + \frac{3\widetilde{T}_e}{2T} n_0 e E^0_\parallel, \tag{1}$$

where η_* is the Spitzer-Harm resistivity with the correction from LH current drive (which is approximately 2-3 times smaller than without current drive for LH waves and is the result of pushing more electrons to the high velocity limit by LH waves), and $n_0 e \overline{v}_D$ is the drive current. The derivation of Eq. (1) is given in detail in Eq. $(A.1)$ in the Appendix.

The standard linear tearing mode theory shows that there is a central current layer around the rational surface where nonideal MHD effects such as inertia or resistivity are important. Analysis carried out in a slab model shows that the perturbed current $\widetilde{j}_\parallel$ within this narrow current layer, which results from the field line slipping with respect to the plasma, produces a discontinuity in $\widetilde{B}_y$. The tearing mode equation can be expressed approximately as

$$\Delta' \widetilde{A}_{\|} = -\frac{4\pi}{c}\Delta \widetilde{J}_{\|}(0). \tag{2}$$

If the drive current is assumed to exist in the current layer, the new instability follows immediately once $\widetilde{J}_{\|}(0)$ is evaluated.

For the sake of simplicity, the perturbed temperature $\widetilde{T}_e(0)$ is ignored in this section. The linearized electron continuity equation is

$$-i\omega\widetilde{n}_e + ik_{\|}\frac{\widetilde{j}_{\|}}{e} - i\omega_{*e}^{n}\frac{n_0 e\widetilde{\phi}}{T_e} + \frac{e\widetilde{B}_x}{B}\frac{\partial}{\partial x}J_{\|}^{0} = 0, \tag{3}$$

where

$$\omega_{*e}^{n} = \frac{cT_e k}{eBr_n}, \qquad \frac{1}{r_n} = \frac{1}{n_0}\frac{dn_0}{dx}. \tag{3a}$$

An adiabatic ion response and quasineutrality condition imply that

$$\widetilde{n}_e = \frac{n_0 e\widetilde{\phi}}{T_e}, \tag{4}$$

and

$$\widetilde{n}_e(0) = \frac{\omega_{*e}^{n}}{\omega + \omega_{*e}^{n}}\frac{n_0}{T}\frac{\widetilde{A}_{\|}}{c}\frac{\partial}{\partial n}J_{\|}^{0}. \tag{5}$$

Substituting Eq. (5) into Eq. (1) gives

$$\widetilde{j}(0) = \frac{\omega_{*e}^{n}}{\omega + \omega_{*e}^{n}}\frac{n_0 e\overline{v}_D}{T}\frac{\widetilde{A}_{\|}}{c}\frac{\partial}{\partial n}J_{\|}^{0} + i\sigma_*\{\omega - \omega_{*e}^{n}[1 + \eta_e(1+\alpha)]\}\frac{\widetilde{A}_{\|}}{c}. \tag{6}$$

Combining Eq. (6) with Eq. (2) gives the dispersion relation as

$$\frac{c^2}{4\pi}\frac{\Delta'}{\Delta} + \frac{\omega_{*e}^{n}}{\omega + \omega_{*e}^{n}}\frac{n_0 e\overline{v}_D}{T}\frac{\partial}{\partial n}J_{\|}^{0} = i\sigma_*\{\omega - \omega_{*e}^{n}[1 + \eta_e(1+\alpha)]\}. \tag{7}$$

The real frequency and growth rate of the drift tearing modes follow from Eq. (7) and are

$$\omega_0 = \omega_{*e}^{n}[1 + \alpha(\eta_e)], \tag{8}$$

and

$$\frac{\gamma}{\omega_{*e}^{n}} = i^{3/2}\frac{2}{[1+\alpha(\eta_e)]\beta_p}|\Delta|\Delta' + i\frac{2}{1+\alpha(\eta_e)}\frac{\nu_e}{\omega_{*e}^{n}}\frac{\overline{v}_D}{v_T^2}\frac{d}{dn}(n\overline{v}_D), \tag{9}$$

where $\alpha(\eta_e) = (1+\alpha)\eta_e$. The width of the semicollisional current layer is defined to be

$$\Delta^2 = \frac{\omega\nu_*}{i(k_\parallel' v_T)^2}, \tag{10}$$

and

$$\beta_p = \frac{4\pi n_0 T_e}{B^2}\frac{L_s^2}{r_n^2}. \tag{11}$$

The first term in Eq. (9) is the growth rate of the standard drift tearing mode (with a slight numerical difference). The second term is the new destabilizing term which comes from the current drive. As the local driven current gradient becomes large, the growth rate of this new instability may be comparable to or exceed the previous growth rate.

It should be emphasized that the equilibrium parallel electric field $E_\parallel^0$ essentially plays no role in this new instability because the perturbed temperature $\widetilde{T}_e(0)$ has been neglected. The quantities $\widetilde{n}_e/n$ and $\widetilde{T}_e/T$, however, are the same order in the tearing mode activities. Thus, in a more careful calculation where $\widetilde{T}_e$ is included, the equilibrium electric field $E_\parallel^0$ will drive a similar instability. This calculation is done in section IV.

III. Basic Theory

The electron distribution function f evolves according to the drift guiding-center kinetic equation

$$\frac{\partial f}{\partial t} + \vec{v}_\parallel \cdot \nabla f + \frac{c\vec{E}\times\vec{B}}{B^2}\cdot\nabla f + \frac{eE_\parallel}{m}\frac{\partial f}{\partial v_\parallel} = C(f,f) + \frac{\partial}{\partial v_\parallel} D \frac{\partial f}{\partial v_\parallel}, \tag{12}$$

where $\vec{v}_E = (c\vec{E}\times\vec{B})/B^2$ is assumed to dominate the particle guiding-center drifts. The quantity $C(f,f)$ represents the self-collisions of electrons and the scattering of electrons off the ion distribution f_i. The last term on the right-hand side of Eq. (12) is the lower hybrid current drive term, and $v_\parallel$ is the electron velocity parallel to magnetic field $\vec{B}$.

It is pointed out here that an analytic solution for the tearing modes with current drive is made possible by approximating the collision operator as

$$C(f,f) \approx \frac{\partial}{\partial v_\parallel}\nu' F_0 \frac{\partial}{\partial v_\parallel}\left(\frac{f}{F_0}\right), \qquad \nu' = \nu(v)\frac{v_T^2}{2}, \tag{12a}$$

where it appears in the equilibrium and Ohm's law equations. The full collision operator is used in all other cases. This collision operator is obtained when the full Fokker-Planck collision operator is integrated over the perpendicular velocity direction, where f is assumed to be Maxwellian, and for high parallel electron velocity.[9]

As usual, we consider a one-dimensional sheared slab model with density and temperature gradients in the x direction and external magnetic field $\vec{B} = B_0[\hat{z} + (x/L_s)\hat{y}]$ where L_s is the shear length of the external field. We look for a mode with a rational surface at $x = 0$ and therefore take the mode to have no variation in the z-direction giving $k_z = 0$. It is further assumed that the plasma flow is incompressible so that

$$\nabla \cdot \vec{v} \approx \nabla \cdot \vec{v}_E = 0. \tag{13}$$

The equilibrium situation is a steady state with an electric field $\vec{E}^0 = \hat{z}E^0_\parallel$ and current drive. Then, the equation for the unperturbed distribution function F is obtained from Eq. (12) with collision operator Eq. (12a) as

$$\vec{v}_\parallel \cdot \nabla F + \frac{eE^0_\parallel}{m}\frac{\partial F}{\partial v_\parallel} = \frac{\partial}{\partial v_\parallel}\nu\left[\frac{v_T^2}{2}\frac{\partial F}{\partial v_\parallel} + v_\parallel F\right] + \frac{\partial}{\partial v_\parallel}D\frac{\partial F}{\partial v_\parallel}. \tag{14}$$

Since $\vec{v}_\parallel \cdot \nabla F = 0$, the solution for F is

$$F = \frac{n_0}{(\sqrt{\pi}v_T)^3}\exp\left(-\int dv_\parallel \frac{\nu(v)v_\parallel - \frac{e}{m}E^0_\parallel}{D + \nu'}\right).$$

As a special case, the collision and current drive terms are considered to be dominant so that

$$F = F_0 + F_1, \tag{15}$$

where

$$F_0 = \frac{n_0}{(\sqrt{\pi}v_T)^3}\exp\left(-\int dv_\parallel \frac{\nu(v)v_\parallel}{D + \nu'}\right). \tag{15a}$$

and

$$F_1 = F_0\left(\frac{eE^0_\parallel}{m}\int dv_\parallel \frac{1}{D + \nu'}\right). \tag{15b}$$

Note that where $D(v_\parallel)$ vanishes, F is locally Maxwellian, and where $D(v_\parallel) \gg \nu'(v)$, F is locally flat. A useful model for lower hybrid waves is to take

$$D(v_\parallel) = \begin{cases} D, & u_1 \le v_\parallel \le u_2; \\ 0, & \text{elsewhere.} \end{cases} \tag{16}$$

where $D \to \infty$. The salient features of distribution F_0 are pictorially shown in Fig. 1. The equilibrium current is given as

$$J_{\parallel}^{0} = n_0 e\overline{v} = n_0 e\overline{v}_E + n_0 e\overline{v}_D, \tag{17}$$

The first term in Eq. (17) is the inductive current and $n_0 e\overline{v}_D$ is the drive current. Here,

$$\overline{v}_E = \frac{eE_{\parallel}^{0}}{m\nu_*}, \qquad \overline{v}_D = \frac{1}{\sqrt{\pi}v_T}\frac{u_2^2 - u_1^2}{2}\exp\left(-\frac{u_1^2}{v_T^2}\right), \tag{17a}$$

and

$$\nu_*^{-1} = \frac{v_T^2}{2}\frac{1}{n_0}\int dv_{\parallel}\frac{F_0}{D+\nu'} \tag{17b}$$

is the model collision frequency. Further, we make two more assumptions regarding the electron flow

$$(1) \qquad \left(\frac{\overline{v}}{v_T}\right)^2 \ll \frac{\omega}{\nu_*}, \tag{18a}$$

$$(2) \quad \frac{1}{\overline{v}}\frac{d}{dn}(n\overline{v}) \gg 1, \quad or \quad \frac{n}{Q_{\parallel}^{0}}\frac{d}{dn}Q_{\parallel}^{0} \gg 1, \tag{18b}$$

where $Q_{\parallel}^{0}$ is the equilibrium energy flow defined in Eq. $(33a)$.

IV. Linear Tearing Instability

The linearized version of Eq. (12) for the perturbed electron distribution function is given by

$$-i(\omega - k_{\|}v_{\|})\tilde{f} + (\tilde{\vec{v}}_E + v_{\|}\frac{\tilde{\vec{B}}}{B})\cdot\nabla F + \frac{eE_{\|}^0}{m}\frac{\partial \tilde{f}}{\partial v_{\|}} + \frac{e\tilde{E}_{\|}}{m}\frac{\partial F}{\partial v_{\|}} = C(\tilde{f}) + \frac{\partial}{\partial v_{\|}}D\frac{\partial \tilde{f}}{\partial v_{\|}}. \quad (19)$$

Neglecting the perturbed compressional magnetic field for a low β plasma,

$$\tilde{\vec{A}} = \tilde{A}_{\|}\hat{b}_0, \quad \hat{b}_0 = \vec{B}_0/B \approx \hat{z},$$

the electromagnetic perturbation is given by

$$\begin{aligned}\tilde{\vec{E}} &= -\nabla\tilde{\phi} + \frac{i\omega}{c}\tilde{\vec{A}},\\ \tilde{\vec{B}} &= \nabla\times\tilde{\vec{A}}.\end{aligned} \quad (20)$$

$\tilde{\vec{A}}$ is driven by the perturbed plasma current

$$\nabla^2\tilde{\vec{A}}_{\|} = -\frac{4\pi}{c}\tilde{\vec{J}}_{\|} \quad (21)$$

and $\tilde{\phi}$ is produced by the associate quasineutral condition

$$\tilde{n}_e = \tilde{n}_i \quad (22)$$

since typically $c^2/c_A^2 \gg 1$. The perturbed quantities are assumed to vary as $\exp(-i\omega t + iky)$.

Integrating Eq. (19) once over velocity space gives the electron continuity equation (3).

In order to solve Eq. (19) for $\tilde{f}$, it is useful to note that from Eq. (14)

$$(D+\nu')\frac{\partial F}{\partial v_{\|}} = -\nu v_{\|}F + \frac{e}{m}E_{\|}^0 F.$$

For the special cases given in Eqs. (15) to (16), we obtain

$$v_{\|}F = -\frac{v_T^2}{2}\frac{\partial F_0}{\partial v_{\|}} + v_{\|}\overline{F}$$

and

$$v_\parallel \widetilde{f} = v_\parallel F_0 \widetilde{g} = -\frac{v_T^2}{2}\frac{\partial}{\partial v_\parallel}(F_0\widetilde{g}) + \frac{v_T^2}{2} F_0 \frac{\partial \widetilde{g}}{\partial v_\parallel} + v_\parallel F_D \widetilde{g} \tag{23}$$

where

$$\overline{F} = F_1 + F_D.$$

F_1 is defined by Eq. (15b), and F_D is the distribution function in the plateau region. Outside of the plateau region F_D is zero. F_0 is Maxwellian distribution function with the plateau given by LH current drive.

Substituting Eq. (23) into (19) and rearranging gives

$$-i(\omega F_0 - k_\parallel v_\parallel F_D)\widetilde{g} + \frac{1}{2} i k_\parallel v_T^2 F_0 \frac{\partial \widetilde{g}}{\partial v_\parallel} + \widetilde{v}_{Ex}\frac{\partial F}{\partial x} + \frac{\widetilde{B}_x}{B}\frac{d}{dx}(v_\parallel \overline{F}) + \frac{e\widetilde{E}_\parallel}{m}\frac{\partial F_1}{\partial v_\parallel}$$

$$= C(\widetilde{f}) + \frac{\partial}{\partial v_\parallel} D \frac{\partial \widetilde{f}}{\partial v_\parallel} + \frac{\partial}{\partial v_\parallel} F_0 [(-\frac{eE_\parallel^0}{m} + \frac{1}{2} i k_\parallel v_T^2)\widetilde{g}$$
$$- \frac{e\widetilde{E}_\parallel}{m} + \frac{v_T^2}{2}\frac{\widetilde{B}_x}{B}\frac{1}{F_0}\frac{dF_0}{dx}]. \tag{24}$$

For the semicollisional modes, it is expected that

$$k_\parallel v_T \ll \nu_e, \qquad \omega \approx \frac{k_\parallel^2 v_T^2}{\nu_e} \tag{25}$$

and the ordering becomes

$$\omega \ll k_\parallel v_T \ll \nu_e \ll \omega_{ce}. \tag{26}$$

The Chapman-Enskog expansion technique is now used to solve Eq. (24). Keeping only the leading order terms in Eq. (24) gives

$$C(\widetilde{f}) + \frac{\partial}{\partial v_\parallel} D \frac{\partial \widetilde{f}}{\partial v_\parallel} = 0, \tag{27}$$

which has the solution

$$\widetilde{f} = F_0 \left\{ \frac{\widetilde{n}_e}{n_0} + \frac{\widetilde{T}_e}{T^2}\left[m\left(\int dv_\parallel \frac{v_\parallel \nu'}{D+\nu'} + \frac{v_\perp^2}{2}\right) - \frac{3}{2}T\right]\right\}. \tag{28}$$

The symbol T is defined as

$$T = \frac{2}{3}\left[m\left(\int d^3 v F_0 \int dv_\| \frac{v_\| \nu'}{D+\nu'} + \frac{v_\perp^2}{2}\right)\right],$$

$\widetilde{n}_e$ and $\widetilde{T}_e$ are the electron perturbed density and temperature. These quantities represent particle number and energy conservation in collisions and current drive, where the energy-like quantity is given by

$$m\left(\int dv_\| \frac{v_\| \nu'}{D+\nu'} + \frac{v_\perp^2}{2}\right).$$

Because of

$$m\left(\int dv_\| \frac{v_\| \nu'}{D+\nu'} + \frac{v_\perp^2}{2}\right) = \epsilon - m\int dv_\| \frac{v_\| D}{D+\nu'} \approx \epsilon - \frac{1}{2} m v_\|^2 \frac{D}{D+\nu'}$$
$$= \begin{cases} \frac{1}{2} m v_\perp^2, & u_1 \le v_\| \le u_2; \\ \epsilon, & \text{elsewhere}, \end{cases} \tag{29}$$

outside of the current drive region, ϵ is a solution of Eq. (12a). Inside the current drive region, $mv_\perp^2/2$ is a solution of Eq. (27) with collision operator Eq. (12a).

The right hand side of Eq. (24) is large so that using $\widetilde{g}_0$ to first order and including the model collision operator Eq. (12a), we rewrite it as

$$C(\widetilde{f}_1) = \frac{\partial}{\partial v_\|} F_0 \nu' \frac{\partial \widetilde{g}_1}{\partial v_\|} + \frac{\partial}{\partial v_\|} \nu' \frac{\partial F_0}{\partial v_\|}\left[\frac{\widetilde{n}_e}{n_0} + \left(\frac{\nu_*}{\nu_*^T} - \frac{3}{2}\right)\frac{\widetilde{T}_e}{T^2}\right] \tag{30}$$

and

$$\frac{d\widetilde{g}_1}{dv_\|} = \left(-\frac{eE_\|^0}{m} + \frac{1}{2} i k_\| v_T^2\right)\frac{\widetilde{g}_0}{D+\nu'} +$$
$$+ \left\{\frac{e\widetilde{E}_\|}{m} - \frac{v_T^2}{2}\frac{\widetilde{B}_x}{B}\frac{1}{F_0}\frac{dF_0}{dx} - \frac{eE_\|^0}{m}\left[\frac{\widetilde{n}_e}{n_0} + \left(\frac{\nu_*}{\nu_*^T} - \frac{3}{2}\right)\frac{\widetilde{T}_e}{T^2}\right]\right\}\frac{1}{D+\nu'}. \tag{31}$$

where the last terms in Eqs. (30) and (31) are due to the perturbed collision frequency, and the collision frequency-like quantities ν_* and ν_*^T will be given in Eq. (33a). The quantities $\widetilde{n}_e$ and $\widetilde{T}_e$ are determined by requiring that both the density and energy-like moments of the right hand side of Eq. (24) vanish.

Thus, we obtain the following set of equations to describe the linearized response of the electrons:

$$-i\left(\omega-k_{\|}\overline{v}_D+\frac{ik_{\|}^2v_T^2}{2\nu_*}\right)\widetilde{n}_e+\left(ik_{\|}\overline{v}_1+\frac{k_{\|}^2v_T^2}{2\nu_*^T}\right)\frac{n_0\widetilde{T}_e}{T}+$$
$$+ik_{\|}\left(\frac{n_0e\widetilde{E}_{\|}}{m\nu_*}-\frac{v_T^2}{2}\frac{\widetilde{B}_x}{B}\frac{dn_0}{dx}\frac{1}{\nu'_*}\right)+\widetilde{v}_{Ex}\frac{dn_0}{dx}+\frac{\widetilde{B}_x}{B}\frac{d(n_0\overline{v})}{dx}=0 \tag{32}$$

and

$$-in_0\widetilde{T}_e\left(\frac{3}{2}\omega-k_{\|}\overline{v}_2+\frac{ik_{\|}^2v_T^2}{2\nu_*^{TT}}\right)+\left(-\frac{1}{3}ik_{\|}\overline{v}_D+\frac{k_{\|}^2v_T^2}{2\nu_*^T}\right)\widetilde{n}_eT+$$
$$+ik_{\|}\left(\frac{n_0e\widetilde{E}_{\|}}{m\nu_*^T}-\frac{v_T^2}{2}\frac{\widetilde{B}_x}{B}\frac{dn_0}{dx}\frac{1}{\nu'^T_*}\right)T+$$
$$+\frac{3}{2}n_0\widetilde{v}_{Ex}\frac{dT}{dx}+\frac{\widetilde{B}_x}{B}\left[\frac{dQ_{\|}^0}{dx}-\frac{3}{2}T\frac{d(n_0\overline{v})}{dx}\right]-ne\overline{v}_E\widetilde{E}_{\|}=0, \tag{33}$$

where

$$\frac{1}{\nu_*}=\frac{v_T^2}{2}\frac{1}{n_0}\int d^3v\frac{F_0}{D+\nu'_*},$$
$$\frac{1}{\nu'_*}=\frac{v_T^2}{2}\int d^3v\frac{1}{D+\nu'_*}\frac{dF_0}{dn},$$
$$\frac{1}{\nu_*^T}=\frac{1}{n_0m}\int d^3v\frac{F_0}{D+\nu'_*}\left[m\left(\int dv_{\|}\frac{v_{\|}\nu'_*}{D+\nu'*}+\frac{v_{\perp}^2}{2}\right)-\frac{3}{2}T\right],$$
$$\frac{1}{\nu_*^{TT}}=\frac{1}{n_0Tm}\int d^3v\frac{F_0}{D+\nu'_*}\left[m\left(\int dv_{\|}\frac{v_{\|}\nu'_*}{D+\nu'*}+\frac{v_{\perp}^2}{2}\right)-\frac{3}{2}T\right]^2,$$
$$\frac{1}{\nu'^T_*}=\int d^3v\frac{1}{D+\nu'_*}\frac{dF_0}{dn}[m(\int dv_{\|}\frac{v_{\|}\nu'_*}{D+\nu'*}+\frac{v_{\perp}^2}{2})-\frac{3}{2}T],$$
$$Q_{\|}^0=m\int d^3v v_{\|}\overline{F}\left(\int dv_{\|}\frac{v_{\|}\nu'_*}{D+\nu'*}+\frac{v_{\perp}^2}{2}\right),$$
$$\frac{\chi_{\|}}{n}=\frac{v_T^2}{2}\left[\frac{1}{\nu_*^{TT}}-\frac{\nu_*}{(\nu_*^T)^2}\right],$$
$$\overline{v}_1=-\frac{1}{3}\overline{v}_D+\frac{3}{2}\overline{v}_E,$$
$$\overline{v}_2=\frac{5}{4}\overline{v}_D+\left(\frac{2\nu_*}{v_T^2}\frac{\chi_{\|}}{n}+\frac{3}{2}\frac{\nu_*}{\nu_*^T}\right)\overline{v}_E. \tag{33a}$$

Cowley, Kulsrud, and Hahm point out that in the analysis of the tearing mode stability, the use of the "unmagnetized" ion response is incorrect.[4] When the full nonlocal ion response is included, the ion density becomes

$$\frac{\widetilde{n}_i}{n_i} = -\frac{e}{T_i}[-\widetilde{\phi}(x) + G \cdot \widetilde{\phi}(x)]. \tag{34}$$

The first term is the adiabatic response of the ions. The second term is a convolution (integral) operator which gives the non-adiabatic density response of the ions to all orders in the gyroradius:

$$G \cdot \widetilde{\phi}(x) = \int_{-\infty}^{\infty} dx' \widetilde{\phi}(x') G(|x - x'|), \tag{34a}$$

where

$$G(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \exp(ikx) \left[(\frac{T_i}{T_e} \frac{\omega_{*e}^n}{\omega} + 1) S_0 - \frac{T_i}{T_e} \frac{\omega_{*e}^n}{\omega} \eta_i b (S_0 - S_1) \right], \tag{34b}$$

$S_n = I_n(b) \exp(-b)$, I_n is the modified Bessel function of order n, and $b = \frac{1}{2} k^2 \rho_i^2 = k^2 T_i / (m_i \Omega_i^2)$. In writing Eq. (34) we have assumed that $x \leq \rho_i$ and consequently $k_{\|} v_i / \omega \ll 1$. Thus, only the motion of the ions transverse to the magnetic field is important.

Solving for $\widetilde{n}_e$ using Eq. (32) and (33), and considering the quasineutrality condition as well as Eq. (3), for $T_i = T_e$, we obtain the peturbed electrostatic potential

$$\begin{aligned}
\widetilde{\phi} =& [\frac{3}{2}\omega(\omega + \omega_{*e}^n) - \beta_1 k_{\|} + \beta_2 k_{\|}^2 - i\beta_3 k_{\|}^3 - \beta_4 k_{\|}^4]^{-1} \cdot \{\{\{[\frac{3}{2}\omega^2 - \\
& - \alpha_1 k_{\|} + \alpha_2 k_{\|}^2 - i\alpha_3 k_{\|}^3 - \alpha_4 k_{\|}^4] G \cdot \widetilde{\phi} + \{\{\frac{3}{2}\omega\omega_{*e}^n \frac{d}{dn}(n\overline{v}) + \\
& + \{i\frac{3}{2}\omega(\omega - \omega_{*e}^n \frac{\nu_*}{\nu_*'}) \frac{v_T^2}{2\nu_*} + \omega_{*e}^n [\overline{v}_1 (\frac{1}{T}\frac{dQ_{\|}^0}{dn} - \frac{3}{2}\frac{d}{dn}(n\overline{v})) - \\
& - \frac{d}{dn}(n\overline{v}) \cdot \overline{v}_2]\} k_{\|} - i\frac{v_T^2}{2}\omega_{*e}^n \cdot [\frac{1}{\nu_*^T}(\frac{1}{T}\frac{dQ_{\|}^0}{dn} - \frac{3}{2}\frac{d}{dn}(n\overline{v})) - \\
& - \frac{1}{\nu_*^{TT}}\frac{d}{dn}(n\overline{v})] k_{\|}^2 - \frac{v_T^2}{2}\frac{\chi_{\|}}{n}\frac{\omega}{\nu_*}(1 - \frac{\omega_{*e}^n}{\omega}) k_{\|}^3\}\} \frac{\widetilde{A}_{\|}}{c}\}\}\},
\end{aligned} \tag{35}$$

where

$$\begin{aligned}
\beta_1 =& (\omega + \omega_{*e}^n)\overline{v}_2 + \frac{3}{2}\omega\overline{v}_D - \frac{3}{2}\eta_e\omega_{*e}^n\overline{v}_1, \\
\beta_2 =& i\left[\frac{(\omega + \omega_{*e}^n)}{\nu_*^{TT}} + \frac{3\omega}{\nu_*} - \frac{3}{2}\eta_e\frac{\omega_{*e}^n}{\nu_*^T}\right]\frac{v_T^2}{2}, \\
\beta_3 =& \left(\frac{\overline{v}_D}{\nu_*^{TT}} - \frac{-\overline{v}_D + 4\overline{v}_E}{\nu_*^T} + \frac{2\overline{v}_2}{\nu_*}\right)\frac{v_T^2}{2}, \\
\beta_4 =& \frac{v_T^2}{\nu_*}\frac{\chi_{\|}}{n},
\end{aligned}$$

and

$$
\begin{aligned}
\alpha_1 &= \left(\frac{3}{2}\overline{v}_D + \overline{v}_2\right)\omega, \\
\alpha_2 &= i\omega\left[\frac{1}{\nu_*^{TT}} + \frac{3}{2\nu_*}\right]\frac{v_T^2}{2}, \\
\alpha_3 &= \left(\frac{\overline{v}_D}{\nu_*^{TT}} - \frac{-\frac{1}{3}\overline{v}_D + \overline{v}_1}{\nu_*^T} + \frac{\overline{v}_2}{\nu_*}\right)\frac{v_T^2}{2}, \\
\alpha_4 &= \frac{v_T^2}{2\nu_*}\frac{\chi_\parallel}{n}.
\end{aligned}
\tag{35b}
$$

In Eq. (35), we have used the assumptions given by Eqs. (18 *a*-*b*) to neglect certain small terms.

Substituting Eq. (34) and (35) into the electron continuity equation (Eq. (3)), rearranging the result and then using Eq. (21), we obtain the Ampere's law

$$
\begin{aligned}
\frac{cT_e}{4\pi n_0 e^2}\nabla^2\widetilde{A}_\parallel = &-[\frac{3}{2}\omega(\omega+\omega_{*e}^n) - \beta_1 k_\parallel + \beta_2 k_\parallel^2 - i\beta_3 k_\parallel^3 - \beta_4 k_\parallel^4]^{-1}\cdot \\
&\cdot\{\{\{\{[-\alpha_1(\omega+\omega_{*e}^n)+\omega\beta_1] + [\alpha_2(\omega+\omega_{*e}^n) - \omega\beta_2]k_\parallel + \\
&+[-i\alpha_3(\omega+\omega_{*e}^n) + i\omega\beta_3]k_\parallel^2 + [-\alpha_4(\omega+\omega_{*e}^n)+\omega\beta_4]k_\parallel^3\}G\cdot\widetilde{\phi} - \\
&-\{\{i(\omega+\omega_{*e}^n)\frac{3}{2}\omega(\omega-\omega_{*e}^n\frac{\nu_*}{\nu_*'})\frac{v_T^2}{2\nu_*} + \overline{v}_1(\omega+\omega_{*e}^n)\omega_{*e}^n\cdot \\
&\cdot[\frac{1}{T}\frac{dQ_\parallel^0}{dn} - \frac{3}{2}\frac{d}{dn}(n\overline{v})] + \frac{3}{2}\omega_{*e}^n(\eta_e\omega_{*e}^n\overline{v}_1 + \omega\overline{v}_D)\frac{d}{dn}(n\overline{v})] + \\
&+i(\omega+\omega_{*e}^n)\frac{v_T^2}{2}\{-\frac{\omega_{*e}^n}{\nu_*^T}[\frac{1}{T}\frac{dQ_\parallel^0}{dn} - \frac{3}{2}\frac{d}{dn}(n\overline{v}) - \frac{\nu_*^T}{\nu_*^{TT}}\frac{d}{dn}(n\overline{v})] - \\
&-\beta_2\omega_{*e}^n\frac{d}{dn}(n\overline{v})\}k_\parallel + \{-(\omega+\omega_{*e}^n)\frac{v_T^2}{2}\frac{\chi_\parallel}{n}\frac{\omega}{\nu_*}(1-\frac{\omega_{*e}^n}{\omega}) + \\
&+i\beta_3\omega_{*e}^n\frac{d}{dn}(n\overline{v})\}k_\parallel^2 + \beta_4\omega_{*e}^n\frac{d}{dn}(n\overline{v})k_\parallel^3\}\}\frac{\widetilde{A}_\parallel}{c}\}\}\} = 0.
\end{aligned}
\tag{36}
$$

Eq. (36) may be solved by the consideration of three regions in the variable x, which measures the distance from the singular surface where $\vec{k}\cdot\vec{B}(x) = 0$.[6] The three regions are the fluid region $x > \rho_i > \Delta_e$, the intermediate region $x \approx \rho_i > \Delta_e$ and the inner region $x \approx \Delta_e > \rho_e$ (where $\Delta_e = (L_s\omega)/(k_y v_e)$ and ρ_e is the electron gyroradius). The quantity Δ_e represents the width of the region within which the effects of electron inertia are important. For $(4\pi n T_e/B^2) < (r_n/L_s)^2$, $\widetilde{A}_\parallel$ may be approximated as a constant in the inner and intermediate regimes. The matching condition between the three regions is obtained from the integral of the Ampere's law across the reconnection region. The result is given as

$$\frac{cT_e}{4\pi n_0 e^2}\Delta'\widetilde{A}_\parallel = -\Delta_1' + \Delta_0', \tag{37}$$

where

$$\Delta_0' = \int_{-\infty}^{+\infty} dx[\quad\cdots\quad]\frac{\widetilde{A}_\parallel}{c}, \tag{37a}$$

and

$$\Delta_1' = \int_{-\infty}^{+\infty} dx[\quad\cdots\quad]G\cdot\widetilde{\phi}. \tag{37b}$$

The brackets in Eqs. (37a) and (37b) correspond to the terms which appear as the coefficients of $\widetilde{A}_\parallel/c$ and $G\cdot\widetilde{\phi}$ in Eq. (36).

The calculation of the stabilization term Δ_1' from finite ion Larmor radius response is quite similar to Antonsen's calculation.[6] The result is given here as

$$\Delta_1' = -\frac{\sqrt{\pi}}{4}\left(\frac{\omega}{k_\parallel' v_T^2}\right)^2 \frac{1}{\rho_i}\ln\left(\frac{\rho_i}{\Delta_e}\right)\left(1-\frac{\omega_{*e}^n}{\omega}\right)^2\frac{\omega_{*e}^n}{\omega}\left(1+\frac{\omega}{\omega_{*e}^n}-\frac{\eta_i}{2}\right)\frac{\widetilde{A}_\parallel}{c}v_T^2. \tag{38}$$

In order to calculate the quantity Δ_0', the quantity $-\beta_1 k_\parallel - i\beta_3 k_\parallel^3$ is assumed to be small compared to $(3/2)\omega(\omega+\omega_{*e}^n)+\beta_2 k_\parallel^2-\beta_4 k_\parallel^4$ in the integrand of Δ_0'. This is equivalent to the assumption given by Eq. (18a) . Contour integration of the integral yields

$$\Delta_0' = \frac{\sqrt{i^3}\pi}{(\sqrt{H_1}+\sqrt{H_2})}\frac{1}{\sqrt{H_1 H_2}}\frac{3\omega(\omega+\omega_{*e}^n)v_T^2}{4\nu_*\beta_4 k_\parallel'^4}[G(\omega)-\gamma_N], \tag{39}$$

where

$$G(\omega) = \left(\omega-\omega_{*e}^n\frac{\nu_*}{\nu_*'}\right)+\left[\frac{2}{3}(1+\frac{\omega_{*e}^n}{\omega})\frac{\chi_\parallel}{n}\frac{\nu_*}{v_T^2}\right]^{\frac{1}{2}}(\omega-\omega_{*e}^n),$$

and

$$\begin{aligned}\gamma_N =& i\frac{4\nu_*}{3\omega(\omega+\omega_{*e}^n)v_T^2}\{\{\{\omega(\omega+\omega_{*e}^n)\{\{\overline{v}_1+\frac{\sqrt{H_1H_2}}{2(\sqrt{H_1}+\sqrt{H_2})^2}\cdot\\ &\cdot\{\frac{\beta_1}{2}[\frac{3}{2}\omega(\omega+\omega_{*e}^n)\frac{\chi_\parallel}{n}\frac{\nu_*}{v_T^2}]^{-\frac{1}{2}}+\frac{\beta_3}{2\beta_4}\frac{v_T^2}{\nu_*}\}\frac{\nu_*}{\nu_*^T}\}\}[\frac{1}{T}\frac{dQ_\parallel^0}{dn}-\frac{3}{2}\frac{d}{dn}(n\overline{v})]+\\ &+\omega_{*e}^n\{\{\frac{3}{2}(\eta_e\omega_{*e}^n\overline{v}_1+\omega\overline{v}_D)+\frac{3\beta_3}{2}[\frac{3}{2}\omega(\omega+\omega_{*e}^n)\frac{\chi_\parallel}{n}\frac{\nu_*}{v_T^2}]^{\frac{1}{2}}\frac{n}{\chi_\parallel}+\\ &+\frac{\sqrt{H_1H_2}}{2(\sqrt{H_1}+\sqrt{H_2})^2}\{\frac{3\beta_3}{2}[\frac{3}{2}\omega(\omega+\omega_{*e}^n)\frac{\chi_\parallel}{n}\frac{\nu_*}{v_T^2}]^{-\frac{1}{2}}(\omega-\frac{\eta_e}{2}\omega_{*e}^n\frac{\nu_*}{\nu_*^T})+\beta_1+\\ &+\frac{3\beta_3}{2\beta_4}\frac{v_T^2}{\nu_*}(\omega-\frac{\eta_e}{2}\omega_{*e}^n\frac{\nu_*}{\nu_*^T})+\beta_3 k_\parallel'^2\sqrt{H_1H_2}\}\}\frac{d}{dn}(n\overline{v})\}\}\}.\end{aligned} \tag{39b}$$

iH_1 and iH_2 are the two roots of the equation

$$(3/2)\omega(\omega+\omega_{*e}^n)+\beta_2 k_{\|}^{\prime 2}x-\beta_4 k_{\|}^{\prime 4}x^2=0.$$

For the case in which $\Delta_e \ll \rho_i$ and $(\overline{v}/v_T)^2 \ll \omega/\nu_*$, the integral of Δ_0' and therefore $G(\omega)$ in Δ_0' dominates in Eq. (37), so that the mode frequencies ω_0 are found by requiring $G(\omega_0)=0$, which gives a real root

$$\omega_0=\omega_{*e}^n[1+\alpha(\eta_e)] \tag{40}$$

corresponding to the drift tearing mode frequency.

Substituting Eq. (38) and (39) back into Eq. (37), the Δ' integral, the Δ' contribution from the outer region, and the equilibrium gradient term in Δ_0' then determine the instability:

$$\gamma = i^{\frac{3}{2}}\frac{d^2}{2a}\left\{\Delta' a-\frac{\sqrt{\pi}}{4}\beta_p\frac{a}{\rho_i}\ln(\frac{\rho_i}{\Delta_e})\frac{\alpha^2}{[1+\alpha(\eta_e)]^3}[2+\alpha(\eta_e)-\frac{\eta_i}{2}]\right\} \times\frac{4\nu_*\beta_4 k_{\|}^{\prime 4}(\sqrt{H_1}+\sqrt{H_1})\sqrt{H_1H_2}}{3\pi\omega(\omega+\omega_{*e}^n)}+\gamma_N. \tag{41}$$

where $d=c/\omega_p$ is the collisionless skin depth.

Current drive tends to push electrons to higher velocities which has the effect of decreasing the collision frequency. However, we may assume that the ratios ν_*/ν_*^T and ν_*/ν_*^{TT} are not changed by the current drive, and use the numbers given by Eq. (A.4). In this case, Eqs. (40) and (41) can be greatly reduced. The frequency and growth rate of the modes are given by

$$\omega=\omega_{*e}^n(1+0.4\eta_e), \tag{42}$$
$$\frac{\gamma}{\omega_{*e}^n}=i^{\frac{3}{2}}0.48\frac{|\Delta|}{\beta_p a}\left\{\Delta' a-\frac{\sqrt{\pi}}{4}\beta_p\frac{a}{\rho_i}\ln\left(\frac{\rho_i}{\Delta_e}\right)\frac{\alpha^2}{[1+\alpha(\eta_e)]^3}\left[2+\alpha(\eta_e)-\frac{\eta_i}{2}\right]\right\}+\frac{\gamma_N}{\omega_{*e}^n}.$$

For simplicity, we take η_e=2, γ_N for which has the form as follows:

$$\frac{\gamma_N}{\omega_{*e}^n}=i\frac{\nu_*}{\omega_{*e}^n}\frac{1}{v_T^2}\left[(0.12\overline{v}_D+1.36\overline{v}_E)\frac{1}{T}\frac{dQ_{\|}^0}{dn}+(5.67\overline{v}_D-0.74\overline{v}_E)\frac{d(n\overline{v})}{dn}\right].$$

Comparing this result with the heuristically derived growth rate of Eq. (9), we see that the standard drift tearing modes look quite similar. The characteristics of the destabilizing terms are the same but the equilibrium electric field now also contributes to the present instability. The stabilizing effect from FLR is the same as Cowley's result.[4] We consider the new destabilizing terms in four cases:

Case I. There is no equilibrium electric field: $E_{\parallel}^{0} = 0$. In this case, $Q_{\parallel}^{0} = n\overline{v}_D T$

$$\frac{\gamma_N}{\omega_{*e}^{n}} = i5.80\frac{\nu_*}{\omega_{*e}^{n}}\frac{\overline{v}_D}{v_T^2}\frac{d(n\overline{v}_D)}{dn}. \tag{43}$$

If D is assumed to be constant inside the tearing mode layer, and $u_1/v_T = 3$, using Eq. (17a) gives

$$\frac{\gamma_N}{\omega_{*e}^{n}} = i104.54\frac{\nu_*}{\omega_{*e}^{n}}(\frac{\overline{v}_D}{v_T^2})^2. \tag{43a}$$

Case II. There is no current drive: $\overline{v}_D = 0$. For this model, $Q_{\parallel}^{0} = 4.2n\overline{v}_E T$

$$\frac{\gamma_N}{\omega_{*e}^{n}} = i26.41\frac{\nu_*}{\omega_{*e}^{n}}(\frac{\overline{v}_E}{v_T^2})^2. \tag{44}$$

Case III. Both electric field and current drive coexist. Taking the same assumptions as in case I for the drive current we obtain

$$\frac{\gamma_N}{\omega_{*e}^{n}} = i(104.54\overline{v}_D^2 + 33.52\overline{v}_E\overline{v}_D + 26.41\overline{v}_E^2)\frac{\nu_*}{\omega_{*e}^{n}}\frac{1}{v_T^2}. \tag{45}$$

In each of above three cases, the growth rate of the new instability is much larger than the standard drift tearing mode growth rate and comparable to the damping rate given by the FLR effect from the ions. In a discharge with $1keV < T_e < 8keV$, $\beta_p \approx 1/4$ and $n_e \approx 2 \times 10^{13} cm^{-3}$, a typical experimental regime for present-day large tokamaks, the growth time of the new instability is on the order of several hundred milliseconds.

Case IV. The current profile is flattened at the $m = 2$ rational surface. By making the same assumptions as in case I for the drive current we get

$$\frac{d(n\overline{v})}{dn} = 0,$$
$$\frac{\gamma_N}{\omega_{*e}^{n}} = i(0.244\overline{v}_D^2 + 4.911\overline{v}_E\overline{v}_D + 24.544\overline{v}_E^2)\frac{\nu_*}{\omega_{*e}^{n}}\frac{1}{v_T^2}. \tag{46}$$

If we take the ratio of the drive current $\overline{v}_D$ to the inductive current $\overline{v}_E$ to be

$$\overline{v}_E = (1-\Sigma)\overline{v}, \qquad \overline{v}_D = \Sigma\overline{v},$$

and keep the total current $\overline{v}$ constant, the stabilizing effects of current drive are maximized at

$$\overline{v}_E = -0.11\overline{v}, \qquad \overline{v}_D = 1.11\overline{v},$$

The damping rate of Eq. (46) thus becomes

$$\frac{\gamma_N}{\omega_{*e}^n} = i0.002\frac{\nu_*}{\omega_{*e}^n}\left(\frac{\overline{v}}{v_T}\right)^2. \tag{46a}$$

Noticing that $G \cdot \widetilde{\phi}(x) \to (\omega_{*e}^n/\omega + 1)\widetilde{\phi}(x)$ as $x \to \infty$, it is clear that the inner and outer regions match asymptotically. Eqs. (34), (35), and (36) approach the ideal MHD approximation for the perturbed density $\widetilde{n}$, electric field $\widetilde{E}_\parallel = 0$, and the equation of motion for displacement $\xi = \widetilde{A}_\parallel/B_y$. Thus , it is not surprising that the growth rate of the new instability is on the order of or larger than one of the standard drift tearing modes. While both instabilities have the same structure, the present case shows that without current drive, the growth rate is dominated by the gradient of the energy flow rather than by the current gradient.

V. Nonlinear Tearing Mode Instability

The quasilinear stabilization of a single linearly growing drift-tearing mode has been calculated analytically in the semicollisional limit using kinetic theory.[7] The basic argument is that the electron orbits are strongly altered by the total magnetic field configuration. In the same way, we can treat the nonlinear tearing modes instability with current drive. Neglecting the equilibrium gradients, and equilibrium electric field, and using Eq. (12) with the model collisional operator (Eq. (12a)), we have

$$\frac{\partial \widetilde{f}}{\partial t} + v_\parallel \frac{\partial \widetilde{f}}{\partial s} - \frac{\partial}{\partial v_\parallel}(D + \nu')F_0 \frac{\partial \widetilde{g}}{\partial v_\parallel} = -\frac{e\widetilde{E}_\parallel}{m}\frac{\partial F_0}{\partial v_\parallel}. \tag{47}$$

When $\gamma < \nu_*$, $F = F_0 + \widetilde{f}$ is still constant along a given field line as long as the electron completes many orbits around the magnetic island during a time γ^{-1}. Averaging $\widetilde{f}$ over a field line and eliminating the operator $v_\parallel(\partial/\partial s)$ gives

$$-\frac{\partial}{\partial v_\parallel}(D + \nu')F_0 \frac{\partial \widetilde{g}}{\partial v_\parallel} = -\frac{\partial F_0}{\partial v_\parallel}\frac{e}{m}\frac{1}{s}\int_0^s ds \widetilde{E}_\parallel,$$

where s is the length of one period of the field line. Integrating twice, we have

$$\widetilde{g} = \left(\frac{e}{m}\frac{1}{s}\int_0^s ds\widetilde{E}_{\|}\right)\int dv_{\|}\frac{1}{(D+\nu')} + c_1.$$

Particle conservation implies that

$$\int d^3v\widetilde{f} = 0$$

The expression for c_1 is now

$$c_1 = -\frac{1}{n_0}\left(\frac{e}{m}\frac{1}{s}\int_0^s ds\widetilde{E}_{\|}\right)\int d^3vF_0\int dv_{\|}\frac{1}{(D+\nu')},$$

and the perturbed current is

$$\frac{\widetilde{j}_{\|}}{e} = \int d^3vv_{\|}\widetilde{f} = \left(\frac{e}{m}\frac{1}{s}\int_0^s ds\widetilde{E}_{\|}\right)\int d^3vv_{\|}F_0\int dv_{\|}\frac{1}{(D+\nu')} + c_1n\overline{v}_D.$$

As $D \to \infty$, and for $u_1 \leq v_{\|} \leq u_2$, the second term can be neglected since it is smaller than the first term by $\overline{v}_D/u_1$, this leads to

$$\widetilde{j}_{\|} = \left(\frac{e}{m\nu_*}\frac{1}{s}\int_0^s ds\widetilde{E}_{\|}\right). \tag{48}$$

where ν_* is defined by Eq. ($33a$).

The usual nonlinear tearing mode assumption gives

$$\widetilde{A}_{\|}(x,y,t) = -\frac{B_0x^2}{2L_s} + \widetilde{A}_{\|}\cos\theta, \tag{49}$$

where $\widetilde{A}_{\|}$ is essentially constant across the layer, and $\theta = k_y(s)$. For this model,

$$W = 2\sqrt{\left(\frac{\widetilde{A}_{\|}L_s}{B_0}\right)}, \tag{50}$$

where W is the half-width of the magnetic island. Substituting Eq. (48)-(50) into Eq. (21) (Ampere's law) and integrating over θ gives

$$\frac{dW}{dt} = \frac{1}{2}W\frac{d\ln\widetilde{A}_{\|}}{dt} = \frac{\Delta' c^2}{16\pi G}\eta_*, \tag{51}$$

where

$$G = \frac{1}{2\pi SW}\int dx\int_0^{2\pi} d\theta\cos\theta\int_0^s ds\cos[\theta(s)], \qquad \eta_* = \frac{m\nu_*}{n_0e^2}.$$

With the replacement of $\eta_{sp} \to \eta_*$, Eq. (51) agrees with Drake and Lee's result.[7]

VI. Summary and Conclusions

The physics of the tearing instability in the presence of lower hybrid current drive has been examined. We have shown that for pure tearing modes, the linear growth rate and the quasilinear growth rate in the Rutherford collisional regime of the island have roughly the same form as without the lower hybrid RF current drive except with a slightly modified resistivity. For the drift tearing modes the situation is quite different. A new instability is found which is driven by the current and equilibrium energy flow gradient and this is predicted to occur under the experimental conditions prevalent, for example, in the Petula Tokamak in France and in the PLT experiment at Princeton. The growth rate of this instability is comparable to or larger than the standard drift tearing mode growth rate which depends on the gradient of the equilibrium current. We emphasize that by making LH waves drive plasma current and equilibrium electric field drive current in opposite directions with the appropriate ratio, and keeping total current constant and flattened inside the tearing layer, the standard linear tearing modes can be stabilized.

VII. Appendix

In the case without current drive we calculate the perturbed current and energy flow from the perturbed electron distribution function $\tilde{f}_e$ given by Eq. (31). These quantities are

$$n_0 e\eta_*(\tilde{j}_\parallel - \tilde{n}_e e\bar{v}_D) = n_0 e\tilde{E}_\parallel - \widetilde{\nabla_\parallel P} - n_0\alpha\widetilde{\nabla_\parallel T} + \frac{3\tilde{T}_e}{2T} n_0 e\tilde{E}_\parallel, \qquad (A.1)$$

$$\tilde{Q}_\parallel = T(\alpha + \frac{7}{2})\frac{\tilde{j}_\parallel}{e} - \chi_\parallel\widetilde{\nabla_\parallel T} + \chi_\parallel eE_\parallel^0\frac{\tilde{T}_e}{T}, \qquad (A.2)$$

where

$$\alpha = \frac{\nu_*}{\nu_*^T} - 1, \qquad \eta_* = \frac{m\nu_*}{n_0 e^2}, \qquad (A.3)$$
$$\chi_\parallel = \frac{n_0 T}{m}[\frac{1}{\nu_*^T T} - \frac{\nu_*}{(\nu_*^T)^2}].$$

By comparing with the fluid model in Ref. 8 we obtain the values

$$\frac{\nu_*}{\nu_*^T} = 1.71, \qquad \eta_* = \eta_{sp}, \qquad \frac{\chi_\parallel}{n}\frac{\nu_*}{v_T^2} = 1.6. \qquad (A.4)$$

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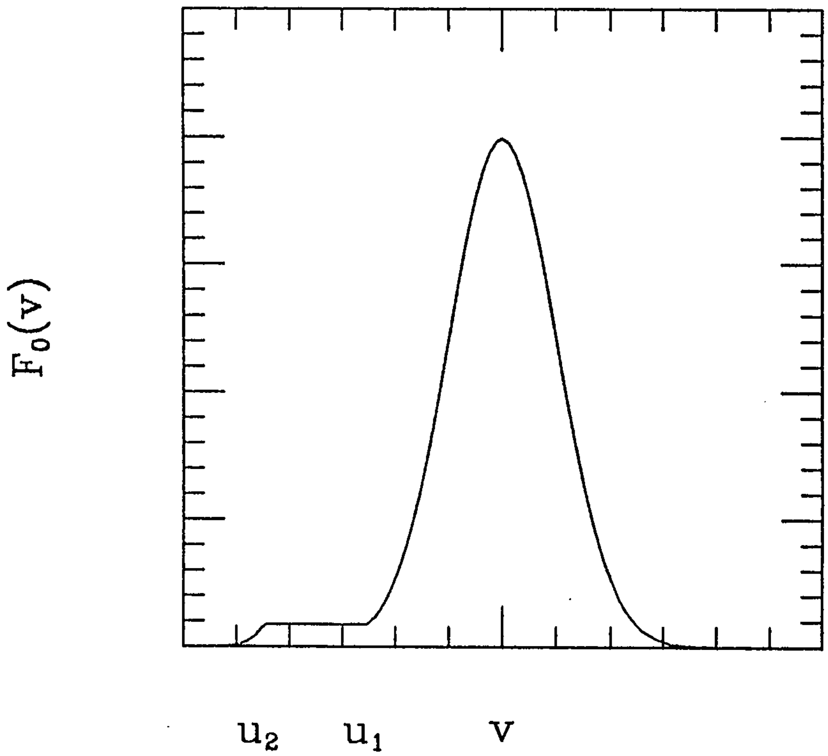

FIG. 1. Flattened Maxwellian Distribution.