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IMPURITY FLOW REVERSAL IN TOKAMAKS WITHOUT MOMENTUM INPUT

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ABSTRACT

A new method for impurity flow reversal is suggested, which does not depend upon momentum input. The method uses neutral-beam injection or radio-frequency fields to drive the Pfirsch-Schluter ion current required for toroidal equilibrium. The necessary alteration of the momentum transfer between the hydrogen ions and impurity ions can cause the former to diffuse into the plasma and the impurities to diffuse out.

I. INTRODUCTION

Impurity control is a problem of major importance for tokamak fusion reactors[1]. To date, several active impurity control methods have been proposed, including momentum input from neutral beams or radio-frequency waves[2-8], particle and heat sources[8-12], and plasma rotation[13].

A new method for impurity flow reversal is suggested here, which does not rely specifically on the deposition of particles, momentum or energy. It utilizes neutral beams or radio-frequency fields to drive the "Pfirsh-Schluter current", i.e. the flows of particles and heat, parallel to the magnetic field, which are required in a toroidal equilibrium state. The method is very similar to current-drive methods using neutral beams[14] or radio-frequency fields[15], which do not impart momentum to the plasma. It is shown here that driving a current carried by the ions (rather the electrons) can be used to reverse the flux of impurities into the center of the plasma.

Momentum input can be avoided with neutral beams by using both co-injected and counter-injected beams with different energies and currents[14]. If radio-frequency fields are used, ion cyclotron resonance absorption of long wavelength waves occurs with negligible momentum input[15]. The effect of the beams or fields is to alter the collisional momentum transfer between plasma ions and impurity ions, without changing the total momentum. This collisional momentum transfer between ion species is responsible, in the neoclassical theory[16,17], for the diffusion of impurities into a plasma.

The alteration of the momentum transfer, by the method proposed here, occurs because the plasma ion velocity distribution is skewed, as a result of the neutral beams or radio-frequency fields. In the case of neutral beams, the more energetic beam of ions, produced from the neutral beam injected in one direction, thermalizes more slowly than the oppositely directed ion beam. In the case of radio-frequency fields, ion cyclotron resonance increases the perpendicular energies of particles moving in only one direction. In both cases, the ion velocity distribution becomes skewed.

The skewed nature of the distribution function is similar to that which occurs in the presence of a temperature gradient parallel to the magnetic field, and affects the momentum transfer in a way which is easily understood. Ions with parallel velocities having one sign tend to be more energetic, and hence have lower rates of collisional scattering from impurity ions, than those going in the opposite direction. A resulting net momentum transfer between the ion species occurs in the absence of a relative flow between them. Such a particle flow is thereby established to maintain a steady state with no net momentum transfer, and this is the driven flow; it can be the flow required for toroidal equilibrium. The elimination of the momentum transfer implies that the inward neoclassical diffusion of impurities is also eliminated. Driving more current than is required for a steady state reverses the sign of the momentum transfer and reverses the neoclassical impurity influx.

In Section II, the transport problem is formulated in terms of the drift kinetic equation with source terms corresponding to neutral-beam injection or radio-frequency fields. In the high collisionality limit, the modification of the distribution function due to these sources comes in through the steady-state particle and energy conservation equations. It is shown that the sum of the gradient-driven and source-driven flows must satisfy the toroidal equilibrium condition. The condition for impurity flow reversal is derived in Section III, in terms of the parallel particle flow and heat flow driven by the sources. These flows are assumed to be carried by the suprathermal ions, and are calculated in Section IV. The use of neutral beams and radio-frequency fields is treated separately in Sections V and VI, respectively.

II. NEOCLASSICAL DISTRIBUTION FUNCTIONS

The drift kinetic equation for the plasma ions, to first order in the usual poloidal gyroradius expansion parameter[17], is

$$\frac{\partial f_{io}}{\partial t} + v_{\parallel}\hat{b} \cdot \nabla f_{i1} + \bar{v}_{Di} \cdot \nabla f_{i0} = \sum_k C_{ik}^{(\ell)} f_{k1} + S \quad , \qquad (1)$$

where the independent velocity variables are μ(magnetic moment) and ε(particle kinetic energy). The parallel velocity is $v_{\parallel} = [2(\varepsilon-\mu B)]^{1/2}$, and the unit vector tangent to the field line direction is $\hat{b} = \bar{B}/B$. The axisymmetric magnetic field is $\bar{B} = I(\psi)\nabla\phi + \nabla\phi \times \nabla\psi$, where ϕ is the toroidal angle, ψ is the poloidal flux lable for a magnetic surface, and $I = RB_T$ where R is the major radius and B_T is the toroidal field. The guiding center drift velocity component perpendicular to a magnetic surface can be written as

$$\bar{v}_{Di} \cdot \nabla\psi = \frac{m_i c}{2e} I\bar{B} \cdot \nabla B^{-2}(\mu B + v_{\parallel}^2).$$

The linearized collision operators, containing the effects of like-species as well as unlike-species ion collisional scattering, are denoted by $C_{ik}^{(e)}$. The effects of collisions with electrons, as well as the induced toroidal electric field, are neglected. The electrostatic field is omitted for simplicity, although it is easily included and does not modify the results.

The source term S on the right-hand side of Eq. (1) represents either the creation of energetic ions by neutral-beam injection, or the quasilinear velocity diffusion caused by radio-frequency fields:

$$S = S_1 \delta(\xi-1)\delta(u-u_1) + S_2 \delta(\xi+1)\delta(u-u_2)$$
$$+ \frac{\nu_{RF}}{u_\perp} \frac{\partial}{\partial u_\perp} [u_\perp^{2\ell-1} \delta(u_\parallel - u_0) \frac{\partial f_{i0}}{\partial u_\perp}] \quad . \tag{2}$$

Here all velocities have been normalized to the ion thermal velocity $v_i (= 2T/mi)$. The particle source rates for the two neutral beams are proportional to S_1 and S_2; u_1 and u_2 are their velocities. The first beam has been assumed to be moving parallel to the magnetic field, ($\xi = 1$, where ξ is the cosine of the pitch angle) with a positive component in the direction of the plasma current (co-injected), while the second beam has been assumed to be counter-injected parallel to the field lines ($\xi = -1$). The perpendicular velocity scattering rate ν_{RF} is a measure of the strength of the radio-frequency fields, which have a frequency of ℓ times the ion cyclotron frequency, and are resonantly absorbed by ions with parallel velocity u_0. The strength of the radio-frequency fields has been assumed to be sufficiently small that ν_{RF} is much smaller than the collisional 90^o-scattering rate. An expression for ν_{RF}, in terms of the electric field strength of the waves, as well as other parameters, can be obtained by comparison with the general quasilinear diffusion equation[18]; it is not needed here, however.

The drift kinetic equations for the various impurity ion species are similar to Eq. (1), except that the source term S is absent:

$$\frac{\partial f_{j0}}{\partial t} + v_{\parallel}\hat{b} \cdot \nabla f_{j1} + \bar{v}_{Dj} \cdot \nabla f_{j0} = \sum_k C_{jk}^{(\ell)} f_{k1} \ .$$

The zeroth-order distribution functions f_{j0} for all of the ion species are Maxwellians with the same temperature, with density and temperature constant on a magnetic surface:

$$f_{j0} = n_j(\psi)\pi^{-3/2} \ v_j^{-3} \ \exp(-v^2/v_j^2) \ ,$$

where

$$v_j^2 \equiv 2T(\psi)/m_j \ .$$

In the cool edge region of the plasma, where the mean free paths are short, all ion species may be assumed to be in the collision-dominated regime, $(\nu_j qR/v_j) >> 1$, where ν_j is the collision frequency and v_j the thermal velocity for species j, and qR is the connection length. The standard expansion procedure[17] for the collision-dominated regime leads to ion distribution functions given by

$$f_{j1} = f_j^{(-1)} + f_j^{(0)} + f_j^{(1)} + \ldots$$

where the superscript indicates the order in the small parameter

$(v_j/\nu_j qR)$. One finds that $f_j^{(-1)}$ has the form of the perturbation of a Maxwellian due to pressure and temperature perturbations,

$$f_j^{(-1)} = [\frac{\delta p_j}{p_j} + (\frac{v^2}{v_j^2} - \frac{5}{2}) \frac{\delta T}{T}] f_{jo} \quad . \tag{3}$$

The temperature perturbations of all ion species are equal, in the strong temperature equilibration limit[19]. To next order in the high collisionality expansion, one finds that $f_j^{(0)}$ is the sum of a gradient-driven term and a source-driven term:

$$f_j^{(0)} = f_j^{(G)} + f_j^{(S)}, \tag{4}$$

where

$$\sum_k C_{jk}^{(\ell)} f_k^{(G)} = v_n[\hat{b} \cdot \nabla\delta p_j/p_j + (\frac{v^2}{v_j^2} - \frac{5}{2}) \hat{b} \cdot \nabla\delta T/T] f_{j0} \quad , \tag{5}$$

and

$$\sum_k C_{ik}^{(\ell)} f_k^{(S)} = -S + \frac{\partial f_{i0}}{\partial t} \quad . \tag{6}$$

The time derivative term is included to allow for the heating due to the beam injection or radio-frequency fields.

The equations for the $f_j^{(1)}$'s have as solubility conditions the steady-state particle and energy conservation equations

$$\int d^3v(v_\parallel \hat{b} \cdot \nabla f_j^{(0)} + \bar{v}_{Dj} \cdot \nabla f_{j0}) = \begin{cases} 2\pi v_i^3(u_1^2 S_1 + u^2{}_2 S_2), & j=i, \\ 0, & j \neq i \end{cases} \tag{7}$$

$$\sum_j \int d^3v(\frac{m_j v^2}{2} - \frac{5}{2} T)(v_\parallel \hat{b} \cdot \nabla f_j^{(0)} + \bar{v}_{Dj} \cdot \nabla f_{j0}) = Q \quad , \tag{8}$$

where

$$Q = 2\pi v_i^3 \, T(u_1^4 S_1 + u_2^4 S_2) + \frac{4 n_i T}{\pi^{1/2}} \ell ! \nu_{RF} \exp(- u_0^2) \quad . \tag{9}$$

The right-hand sides of Eqs. (7) and (8) are the rates of particle and energy deposition from the neutral beams or radio-frequency fields.

The particle flux and heat flux moments obtained from the solution of Eqs. (7) and (8) are

$$n_j u_{j\parallel} \equiv \int d^3v \; v_\parallel \; f_j^{(0)} = -(cI/ez_j B) \frac{\partial p_j}{\partial \psi} + K_j B \quad , \tag{10}$$

$$\sum_j q_{j\parallel} \equiv \sum_j \int d^3v \; v_\parallel (\frac{m_j v^2}{2} - \frac{5}{2} T) f_j^{(0)}$$

$$= - \frac{5}{2} (cI/eB)(\sum_j p_j/z_j) \frac{\partial T}{\partial \psi} + LB \quad , \tag{11}$$

where

$$\bar{B} \cdot \nabla K_j = \begin{cases} 2\pi v_i^3(u_1^2 S_1 + u_2^2 S_2), & j = i \quad , \\ 0, & j \neq i \end{cases} \tag{12}$$

and

$$\bar{B} \cdot \nabla L = Q \quad . \tag{13}$$

These are the parallel particle and heat flows required for a toroidal equilibrium, i.e. for a quasi-equilibrium state which changes only on the transport time scale.

The solution of Eq. (5) for the $f_j^{(G)}$'s has been obtained using a moment method[19-21], which is outlined in the appendix. For simplicity, only one impurity mass is included here, although all charge states of the given impurity element are included. Considerable simplification is achieved also by assuming that the impurity mass is much larger than the hydrogen ion mass: $m_I/m_i >> 1$.

By taking moments of Eq. (5), we find the momentum transfer rate, or friction force $F_j^{(G)}$, and the "heat friction" $G_j^{(G)}$, due to the gradients:

$$F_j^{(G)} \equiv \int d^3v \; m_j v_\parallel \sum_k C_{jk}^{(\ell)} \; f_k^{(G)} = \hat{b} \cdot \nabla \delta p_j \quad , \tag{14}$$

$$G_j^{(G)} \equiv \int d^3v \; m_j v_\parallel \left(\frac{v^2}{v_j^2} - \frac{5}{2}\right) \sum_k C_{jk}^{(\ell)} f_k^{(G)} = \frac{5}{2} n_j \hat{b} \cdot \nabla \delta T \quad . \tag{15}$$

By expressing the $f_k^{(G)}$'s in terms of moments, relations between $F_j^{(G)}$,

$G_j^{(G)}$ and these moments may be obtained. The results, for the hydrogen ions, are

$$F_i^{(G)} = -2^{1/2} \frac{m_i n_i \alpha}{\tau_{ii}} [\mu_{00}^i (u_{i\parallel}^{(G)} - \bar{u}_{I\parallel}) - \frac{2}{5} \mu_{01}^i \, q_{i\parallel}^{(G)}/p_i] \quad , \tag{16}$$

$$G_i^{(G)} = 2^{1/2} \frac{m_i n_i}{\tau_{ii}} [\alpha \mu_{01}^i (u_{i\parallel}^{(G)} - \bar{u}_{I\parallel}) - \frac{2}{5} \mu_{11}^i q_{i\parallel}^{(G)}/p_i] \quad , \tag{17}$$

where the μ_{jk}^i's are functions of α which are given in the appendix, with

$$\alpha \equiv \sum_j n_j \, z_j^2/n_i \quad . \tag{18}$$

(The sum running over all charge states of the impurity species) and

$${}_{ii}^{-1} = \frac{16}{3} \left(\frac{\pi}{2}\right)^{1/2} \frac{n_i e^4 \ell n\Lambda}{m_i^2 \; v_i^3} \tag{19}$$

with $v_i \equiv (2T/m_i)^{1/2}$, and $\ell n\Lambda$ the coulomb logarithm. Here $u_{i\parallel}^{(G)}$ and $q_{i\parallel}^{(G)}$ are the parallel particle flow velocity and heat flow moments of the gradient-driven part of the ion distribution function $f_i^{(G)}$. The total moments, given by Eqs. (10) and (11) are related to these moments by

$$u_{i\parallel} = u_{i\parallel}^{(G)} + u_{i\parallel}^{(S)}, \; q_{i\parallel} = q_{i\parallel}^{(G)} + q_{i\parallel}^{(S)} \quad , \tag{20}$$

where $u_{i\parallel}^{(S)}$ and $q_{i\parallel}^{(S)}$ are the moments of the source-driven part of the ion distribution function, $f_i^{(S)}$. The source-driven parts of the impurity flows $u_{j\parallel}$, $q_{j\parallel}$ are neglected; the average appearing in Eqs. (16) and (17) is

$$\bar{u}_{I\parallel} \equiv \sum_j n_j z_j^2 u_{j\parallel}/(n_i \alpha) \quad . \tag{21}$$

Equation (17) may be solved to give $q_{i\parallel}^{(G)}$ in terms of $G_i^{(G)}$ [$= \frac{5}{2} n_i \hat{b} \cdot \nabla\delta T$, from Eq. (15))]. The analogous equation for the impurities, Eq. (A7) in the appendix, may be solved to give $q_{j\parallel}^{(G)}$, also in terms of $\hat{b} \cdot \nabla\delta T$. Then, after summing over all ion species, $\hat{b} \cdot \nabla\delta T$ can be solved for, in terms of $(\sum_j q_{j\parallel}^{(G)} + q_{i\parallel}^{(G)})$, and substituted into the expression for $q_{i\parallel}^{(G)}$ with the result

$$q_{i\parallel}^{(G)} = (\sum_j q_{j\parallel} + q_{i\parallel}^{(G)}) - \frac{5}{2} T \frac{\tilde{\mu}_{01}^{I}}{\tilde{\mu}_{11}^{I}} \alpha(\sum_j n_j u_{j\parallel} - N_I \bar{u}_{I\parallel}), \tag{22}$$

where the sums are over the impurity charge states, the total impurity density is $N_I \equiv \sum_j n_j$, and the $\tilde{\mu}_{jk}^{I}$'s are functions of α which are given in the appendix. Terms which are small in $(m_i/m_I)^{1/2}(N_I/n_i)$ have been neglected. The first terms in parenthesis in the right-hand side of Eq. (22) may be related to the expressions given in Eq. (11).

III. CONDITION FOR IMPURITY FLOW REVERSAL

The ion flux across a magnetic surface is related to the friction force by[17]

$$\Gamma_i^\psi \equiv \langle n_i \, \bar{u}_i \cdot \nabla\psi\rangle = -(cI/e)\langle F_i(\frac{1}{B} - \frac{B}{\langle B^2\rangle})\rangle \quad , \tag{23}$$

where the angular bracket denotes a magnetic-surface average. By momentum conservation,

$$\sum_j F_j = -F_i \quad , \tag{24}$$

the charge-weighted impurity flux is

$$\sum_j z_j \, \Gamma_j^\psi = -\Gamma_i^\psi \quad . \tag{25}$$

The term "impurity flow reversal" is to be interpreted here as $\sum_j z_j \, \Gamma_j^\psi > 0$. The condition for this to occur is therefore the same as for inward ion diffusion: $\Gamma_i^\psi < 0$. Since the source (neutral-beam injected ions or radio-frequency fields) imparts no net momentum to the plasma, the momentum moment of Eq. (6) is

$$F_i^{(S)} \simeq \int d^3v m_i v_\parallel \sum_k C_{ik}^{(\ell)} \, f_k^{(S)} = 0 \quad . \tag{26}$$

The total friction force is therefore $F_i = F_i^{(G)}$, given by Eq. (16).

By combining Eqs. (23) and (16) the ion flux can be written as

$$\Gamma_i^{\psi} = +(cI/e)\left(\frac{2^{1/2} m_i n_i \alpha}{\tau_{ii}}\right)\Big[\mu_{00}^i \left\langle (u_{i\parallel}^{(G)} - \bar{u}_{I\parallel})\left(\frac{1}{B} - \frac{B}{\langle B^2\rangle}\right)\right\rangle$$

$$- \frac{2}{5}\frac{\mu_{01}^i}{p_i}\left\langle q_{i\parallel}^{(G)}\left(\frac{1}{B} - \frac{B}{\langle B^2\rangle}\right)\right\rangle\Big] \quad . \tag{27}$$

Using Eqs. (20) and (22), the condition for impurity flow reversal becomes

$$\left\langle \left(u_{i\parallel}^{(S)} - \frac{2}{5}\frac{\mu_{01}^i}{\mu_{00}^i}\frac{q_{i\parallel}^{(S)}}{p_i}\right)\left(\frac{1}{B} - \frac{B}{\langle B^2\rangle}\right)\right\rangle > R \quad , \tag{28}$$

where

$$R = \left\langle (u_{i\parallel} - \bar{u}_{I\parallel})\left(\frac{1}{B} - \frac{B}{\langle B^2\rangle}\right)\right\rangle$$

$$+ \frac{\mu_{01}^i}{\mu_{00}^i}\frac{\tilde{\mu}_{01}^I}{\tilde{\mu}_{11}^I}\frac{\alpha}{n_i}\left\langle \left(\sum_j n_j u_{j\parallel} - N_I \bar{u}_{I\parallel}\right)\left(\frac{1}{B} - \frac{B}{\langle B^2\rangle}\right)\right\rangle \tag{29}$$

$$- \frac{2}{5}\frac{\mu_{01}^i}{\mu_{00}^i p_i}\left\langle \left(\sum_j q_{j\parallel} + q_{i\parallel}\right)\left(\frac{1}{B} - \frac{B}{\langle B^2\rangle}\right)\right\rangle$$

and $u_{i\parallel}^{(S)}$, $q_{i\parallel}^{(S)}$ are the ion flows driven by neutral-beam injection or radio-frequency fields.

The expressions for $u_{i\parallel}$, $u_{j\parallel}$, $\bar{u}_{I\parallel}$ [defined by Eq. (21) in terms of $u_{j\parallel}$] and $(\sum_j q_{j\parallel} + q_{i\parallel})$ are given by Eqs. (10) and (11). [Note that the sum on j in Eq. (11) includes both hydrogen and impurities.] These expressions contain the terms K_i and L which are related to the particle and energy deposition rates, by Eqs. (12) and (13). We assume here that these deposition rates are up-down symmetric; in terms of the usual poloidal angle ϕ, $\phi = 0$ is the midplane, we assume that S_1, S_2 and ν_{RF} are <u>even</u> functions of ϕ. This is in contrast with the particle or heat source methods[8-12] for impurity flow reversal, in which the sources are assumed to have odd parts; for maximum effectiveness, they would need to have particle or heat sinks as well as sources. These contributions to R, [given by Eq. (29)] from the terms K_i and L in Eqs. (10) and (11), are thus assumed here to be absent. Then Eq. (29) becomes

$$R = -(cI/e)\left(\left\langle\frac{1}{B^2}\right\rangle - \frac{1}{\langle B^2\rangle}\right)\left\{\frac{1}{n_i}\frac{\partial p_i}{\partial\psi} - \frac{1}{n_i\alpha}\frac{\partial}{\partial\psi}\sum_j z_j p_j\right.$$

$$+ \frac{\mu^i_{01}}{\mu^i_{00}}\frac{\tilde{\mu}^I_{01}}{\tilde{\mu}^I_{11}}\frac{\alpha}{n_i}\left[\frac{\partial}{\partial\psi}\sum_j p_j/z_j - \frac{N_I}{n_i\alpha}\frac{\partial}{\partial\psi}\sum_j z_j p_j\right] \qquad (30)$$

$$\left. - \frac{\mu^i_{01}}{\mu^i_{00}p_i}\left(\sum_j p_j/z_j\right)\frac{\partial T}{\partial\psi}\right\} .$$

In the usual large aspect ratio concentric circular magnetic surface model, the factor containing averages in Eq. (30) is

$$\langle\frac{1}{B^2}\rangle - \frac{1}{\langle B^2\rangle} \simeq 2(r/R_0)^2/B_0^2$$

where r is the minor radius. The factor which appears in Eq. (28) is

$$\frac{1}{B} - \frac{B}{\langle B^2\rangle} \simeq [2(r/R_0)\cos + (r/R_0)^2(\frac{1}{2} - \cos^2 \phi)]/B_0$$

where ϕ is the poloidal angle.

In the absence of source-driven flows, i.e. when $u_{i\parallel}^{(S)} = 0$, $q_{i\parallel}^{(S)} = 0$, the ion flux, Eq. (27), can be written as

$$\Gamma_i^{\psi} = (cI/e)(\frac{2^{1/2}m_i n_i \alpha}{\tau_{ii}})\mu_{00}^{i} R \quad , \tag{31}$$

where R is given by Eq. (30). The condition for impurity flow reversal due to source-driven flows is Eq. (28).

IV. SUPRATHERMAL ION DISTRIBUTION

The effect of neutral-beam injection or radio-frequency fields on the hydrogen ion distribution function is described by Eq. (6), where S is given by Eq. (2). We assume that only suprathermal hydrogen ions are strongly affected, so that $u >> 1$ (and we neglect any direct effect on the impurities). We therefore include pitch-angle scattering on the thermal ions and impurities, as well as drag on the thermal ions, but neglect energy diffusion. We assume that the ion energies of interest are less than the "critical energy", at which electron drag and ion drag are equal, so that electron drag can also be neglected. This latter assumption is not critical, but it simplifies the analysis somewhat. We also assume that since the energy delivered to the fast ions is transferred to the thermal ions, a steady state may be assumed for the fast ion distribution function, and the time derivative in Eq. (6) can be dropped. Then Eq. (6) becomes

$$\nu_i\left[\frac{(1+\alpha)}{2u^3} \frac{\partial}{\partial \xi}(1-\xi^2) \frac{\partial f_i^{(S)}}{\partial \xi} + \frac{1}{u^2} \frac{\partial f_i^{(S)}}{\partial u}\right] = -S(u,\xi) \quad , \qquad (32)$$

where

$$\nu_i = \frac{4\pi n_i e^4 \ln\Lambda}{m_i^2 v_i^3} \quad , \qquad (33)$$

and $u = v/v_i$ is the velocity normalized to the thermal value,

$$v_i \equiv (2T/m_i)^{1/2} \quad . \tag{34}$$

We define the function $a_1(u)$ by

$$a_1(u) = \frac{3}{2}\int_{-1}^{1} d\xi\xi\, f_i^{(S)}(u,\xi) \quad , \tag{35}$$

in terms of which the ion flow and heat flow moments of $f_i^{(S)}$ are

$$n_i\, u_{i\parallel}^{(S)} = \frac{4\pi}{3} v_i^4 \int_0^\infty u^3 du\; a_1(u), \tag{36}$$

$$q_{i\parallel}^{(S)} = \frac{4\pi}{3} v_i^4 \;\; T \int_0^\infty u^3 du (u^2 - \frac{5}{2})\; a_1(u) \quad . \tag{37}$$

By multiplying Eq. (32) by ξ and integrating over ξ, an equation for $a_1(u)$ is obtained:

$$-\frac{(1+\alpha)}{u^3} a_1 + \frac{1}{u^2}\frac{da_1}{du} = -b_1/\nu_i \quad , \tag{38}$$

where

$$b_1(u) = \frac{3}{2}\int_{-1}^{1} d\xi\xi\; S(u,\xi) \quad . \tag{39}$$

Using the boundary condition $a_1 \to 0$ for $u \to \infty$, one finds the solution to be

$$a_1(u) = u^{1+\alpha} \int_u^\infty du\; u^{1-\alpha}\; b_1(u)/\nu_i \quad . \tag{40}$$

By substituting this expression into Eqs. (36) and (37), we find

$$n_i u_{i\parallel}^{(S)} = \frac{4\pi}{3} \frac{v_i^4}{\nu_i} \int_0^\infty du \frac{u^6}{(5+\alpha)} b_1(u) \quad , \tag{41}$$

and

$$q_{i\parallel}^{(S)} = \frac{4\pi}{3} \frac{v_i^4 T}{\nu_i} \int_0^\infty du\; b_1(u) [\frac{u^8}{(7+\alpha)} - \frac{5}{2} \frac{u^6}{(5+\alpha)}] \quad . \tag{42}$$

The large powers of u, which appear in the integrands in these equations, justify extending the lower limit of integration below the range $u \gg 1$ needed to justify the form of Eq. (32).

By substituting the definition of S, Eq. (2), into Eq. (39), we find

$$b_1(u) = \frac{3}{2} [S_1 \delta(u-u_1) - S_2 \delta(u-u_2) + \frac{4\nu_{RF} n_i}{\pi^{3/2} v_i^3} \frac{u_0}{u^2} (u^2 - u_0^2)^{\ell-1} (u^2 - u_0^2 - \ell) e^{-u^2} \Theta(u-u_0)] \quad , \tag{43}$$

where

$$\Theta(u-u_0) = \begin{cases} 0, & u < u_0 \\ 1, & u > u_0 \end{cases} \quad . \tag{44}$$

The moments, Eqs. (41) and (42) may now be evaluated; for the terms proportional to ν_{RF}, we evaluate the integrals asymptotically for $u_0 >> 1$. The results are

$$n_i\, u_{i\parallel}^{(S)} = \frac{2\pi v_i^4}{\nu_i(5+\alpha)} [S_1\, u_1^6 - S_2\, u_2^6 + \frac{3\ell!\nu_{RF} n_i}{\pi^{3/2} v_i^3} u_0^2\, e^{-u_0^2}] \quad , \qquad (45)$$

$$q_{i\parallel}^{(S)} = \frac{2\pi v_i^4\; T}{\nu_i}\{S_1[\frac{u_1^8}{(7+\alpha)} - \frac{5}{2}\,\frac{u_1^6}{(5+\alpha)}] - S_2[\frac{u_2^8}{(7+\alpha)} - \frac{5\; u_2^6}{2(5+\alpha)}]$$

$$+ \frac{5\ell!\nu_{RF} n_i}{\pi^{3/2} v_i^3} u_0^2\, e^{-u_0^2}\, [\frac{u_0^2}{(7+\alpha)} - \frac{(3/2)}{(5+\alpha)}]\} \quad . \qquad (46)$$

The source strengths of the two beams, S_1 and S_2, are related through the condition of no momentum input:

$$\int d^3v m_i v_\parallel S = 0 \quad , \qquad (47)$$

which gives

$$S_2 = S_1(u_1/u_2)^3 \quad . \qquad (48)$$

V. IMPURITY FLOW REVERSAL USING NEUTRAL BEAM INJECTION

Since neutral beams have been developed for heating in present tokamaks, their use for impurity control might prove to be convenient. It is of interest to estimate the neutral-beam power required for impurity flow reversal in a large tokamak.

The dimensionless power input, P_d, can be defined in terms of Q, given by Eq. (9):

$$P_d \equiv Q/(2p_i \nu_i) = (\pi v_i^3/n_i \nu_i)\; S_1 u_1^3 (u_1 + u_2) \quad , \tag{49}$$

where Eq. (48) has been used to eliminate S_2, and we have set $\nu_{RF} = 0$. Similarly, the dimensionless ion flow and heat flow are

$$u_{i\parallel}^{(S)}/v_i = (2\pi v_i^3/n_i \nu_i)\; \frac{S_1 u_1^3}{(5+\alpha)} (u_1^3 - u_2^3) \quad , \tag{50}$$

$$q_{i\parallel}^{(S)}/(v_i p_i) = (2\pi v_i^3/n_i \nu_i)\; S_1 u_1^3 \left[\frac{(u_1^5 - u_2^5)}{(7+\alpha)} - \frac{5}{2}\frac{(u_1^3 - u_2^3)}{(5+\alpha)}\right] \quad . \tag{51}$$

The combination of these flows which appears in the impurity flow reversal condition, Eq. (28), is

$$J \equiv u_{i\parallel}^{(S)}/v_i - \gamma_i q_{i\parallel}^{(S)}/(v_i p_i) \quad , \tag{52}$$

where

$$\gamma_i \equiv \frac{2}{5} \frac{\mu_{01}^i}{\mu_{00}^i} \quad , \tag{53}$$

and the $\mu_{j\parallel}^i$'s are defined in the appendix. For greatest efficiency, we need to maximize the ratio J/P_d, which leads to the results

$$u_1 = [\frac{(1 + \frac{5}{2}\gamma_i)}{2\gamma_i} \frac{(7+\alpha)}{(5+\alpha)}]^{1/2}, \; u_2 << u_1 \quad , \tag{54}$$

and

$$(J/P_d)_{max} = \frac{2\gamma_i u_1^4}{(7+\alpha)} \quad . \tag{55}$$

Using, as an example, $\alpha = 1$, we find $u_1 = 1.91$, $J/P_d = 1.12$. For $\alpha = 5$, on the other hand, $u_1 = 1.90$, $J/P_d = 0.62$. Hence, for larger impurity concentrations, corresponding to larger values of α, the efficiency decreases.

In order to estimate the left-hand side of Eq. (28), we use the usual large aspect ratio concentric circular magnetic surface model, and take the ϕ-dependence of J [defined by Eq. (52)] to be proportional to $(1 + \cos\phi)$. Then the average in Eq. (28) is proportional to $r/(R_0B_0)$, where r is the minor radius. The condition for impurity flow reversal, Eq. (28) may then be written as

$$[(\frac{r}{R_0})/B_0]\ Jv_i > R \quad , \tag{56}$$

and the energy deposition requirement becomes

$$Q > 2(R_0/r)(p_i \nu_i/v_i)\ B_0 R/(J/P_d) \quad . \tag{57}$$

The minimum required energy occurs for beam velocities given by Eq. (54), when $J/P_d = (J/P_d)_{max}$, given by Eq. (55).

The right-hand side of Eq. (56), given by Eq. (30), can be estimated as

$$R \simeq (\frac{c}{eB_0^2})\ \frac{r}{R_0}\ qT/\Delta r \quad , \tag{58}$$

where $q \equiv (rB_T)/(RB_\phi)$ is the tokamak safety factor, and Δr is the scale length for the ion density and temperature gradients. Since the current J must flow within a layer of thickness Δr, the total power required is

$$P = Q\Delta V \quad , \tag{59}$$

where $\Delta V = 4\pi^2 R_0 r \Delta r$ is the volume within that layer. Combining Eqs. (57)-(59), the power required for impurity flow reversal is found to be independent of the thickness Δr, and given by

$$P = 8\pi^2 q R_0 r(cT/eB_0)(p_i \nu_i/v_i)/(J/P_d) \quad , \tag{60}$$

or, using Eq. (33) for ν_i,

$$P = 8\pi^3 q R_0 r(c/B_0)\, n_i^2\, e^3 \ell n\Lambda/(J/P_d) \quad . \tag{61}$$

The required power is given in watts by

$$P = 1.24 \times 10^7 (\frac{\ell n\Lambda}{15}) \frac{n_{14}^2 q \gamma_M R_M}{B_T(J/P_d)} \quad , \tag{62}$$

where n_{14} is the ion density in units of 10^{14}cm^{-3}, γ_M and R_M are the minor and major radii in meters, and B_T is the toroidal magnetic field in Teslas.

As an example, with $n_{14} = 0.5$, $\ell n\Lambda = 15$, $q = 4$, $\gamma_M = 1.2$, $R_M = 5.0$, $B_T = 5.0$, and u_1, u_2 given by Eq. (54), with $\alpha = 1$, the required power is

$$P = 13.3 \times 10^6 \text{ watts} .$$

VI. IMPURITY FLOW REVERSAL USING RADIO FREQUENCY HEATING

Since the use of radio-frequency fields for heating is being planned for large tokamaks, the use of such fields for impurity control would seem to be an attractive alternative to neutral-beam methods. An estimate of the required power will now be given.

The dimensionless power input is [using Eq. (9)]

$$P_d \equiv Q/(2p_i \nu_i) = \frac{2}{\pi^{1/2}} \ell!(\frac{\nu_{RF}}{\nu_i}) e^{-u_0^2} \quad . \tag{63}$$

The ion flow and heat flow are given by Eqs. (45) and (46), with $S_1 = S_2 = 0$. The combination of these flows which appears in the impurity flow reversal condition, Eq. (52), is

$$J = \frac{6}{\pi^{1/2}} \frac{u_0^2 e^{-u_0^2}}{(5+\alpha)} [1 + \frac{5}{2} \gamma_i - \frac{5}{3} \gamma_i \frac{(5+\alpha)}{(7+\alpha)} u_0^2] \quad , \tag{64}$$

where γ_i is given by Eq. (53). By maximizing J/P_d, we find

$$u_0 = [\frac{3(1 + \frac{5}{2} \gamma_i)(7+\alpha)}{10\gamma_i(5+\alpha)}]^{1/2}, \; (J/P_d)_{max} = \frac{3(1+ \frac{5}{2} \gamma_i)}{2(5+\alpha)} u_0^2 \quad . \tag{65}$$

As an example, for $\alpha = 1$, this gives $u_0 = 1.48$, $(J/P_d)_{max} = 1.01$, while for $\alpha = 5$ we find $u_0 = 1.47$, $(J/P_d)_{max} = 0.56$.

The same estimates for the terms in the impurity flow reversal condition, Eq. (28), as were made in the last section, can be made here also. Equation (62) applies in this case also. As an example, with $n_{14} = 0.5$, $\ell n\Lambda = 15$, $q = 4$, $\gamma_M = 1.2$, $R_M = 5.0$, $B_T = 5.0$, and u_0 given by Eq. (65), with $\alpha = 1$, the required power is

$$P = 14.7 \times 10^6 \text{ watts} .$$

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APPENDIX

The method of solution of Eq. (5) and the results, for $m_i/m_I \ll 1$, are given here. The distribution functions are expressed in terms of Sonine polynomials[19]

$$f_j^{(G)} \simeq \frac{2v_\parallel}{v_j} \sum_{\ell=0}^{2} u_{j\ell}\, L_\ell^{(3/2)}(x) \quad , \qquad (A1)$$

where the orthogonal polynomials are

$$L_0^{(3/2)}(x) = 1,\ L_1^{(3/2)}(x) = 5/2 - x,\ L_2^{(3/2)}(x) = 35/8 - 7x/2 + x^2/2 \quad ,$$

with $x \equiv v^2/v_j^2$. The $\ell = 0$ and $\ell = 1$ coefficients are proportional to the mean velocity and heat flux:

$$u_{j0} = u_{j\parallel}/v_j \ ,\ u_{j1} = -\frac{2}{5}\frac{q_{j\parallel}}{v_j p_j} \quad . \qquad (A2)$$

By multiplying Eq. (5) by $(v_\parallel/v_j)\, L_m^{(3/2)}(x)$ and integrating over all velocity, one obtains three equations for each ion species. Using Eqs. (14) and (15), these equations are found to have the form

$$\sum_k \sum_{\ell=0}^{2} (M_{jk}^{m\ell}\, u_{j\ell} + N_{jk}^{m\ell}\, u_{k\ell}) = F_j\, \delta_{m0} - G_j\, \delta_{m1} \quad . \qquad (A3)$$

In general, the matrix elements in these equations are complicated functions of the ion masses.

By assuming that there is only one impurity mass, which is much larger than the hydrogen ion mass, these matrix elements simplify considerably. All charge states of the given impurity ion type can still be taken into account easily.

The third equation for each ion species [m = 2 in Eq. (A3)] is solved for u_{j2} in terms of u_{j0} and u_{j1}, and the results are substituted into the other equations. The hydrogen ion equations are then given by Eqs. (16) and (17), where α, τ_{ii} and $\bar{u}_{I\parallel}$ are defined by Eqs. (18), (19) and (21), and

$$\mu^i_{00} = \frac{(2^{1/2} + \frac{52}{45}\alpha)}{(2^{1/2} + \frac{433}{180}\alpha)} , \quad \mu^i_{01} = \frac{(2^{1/2} + \frac{11}{15}\alpha)}{(2^{1/2} + \frac{433}{180}\alpha)} , \tag{A4}$$

and

$$\mu^i_{11} = 2^{1/2} + \frac{13}{4}\alpha - \frac{1}{5}\frac{(2^{1/2} + \frac{23}{4}\alpha)^2}{(2^{1/2} + \frac{433}{180}\alpha)} . \tag{A5}$$

The equations for the j^{th} charge state of the impurity are

$$F_j/(n_j z_j^2) = -F_i/(n_i\alpha) - \frac{2^{1/2} m_i v_i \alpha}{\tau_{ii}} [\tilde{\mu}^I_{00}(u_{j0} - \bar{u}_{I0}) + \tilde{\mu}^I_{01}(u_{j1} - \bar{u}_{I1})] , \tag{A6}$$

$$G_j/(n_j z_j^2) = \frac{2^{1/2} m_i v_i}{\tau_{ii}} [\bar{\mu}^I_{11} \, \bar{u}_{I1} + \alpha \tilde{\mu}^I_{01} (u_{j0} - \bar{u}_{I0})$$

$$+ \tilde{\mu}^I_{11} (u_{j1} - \bar{u}_{I1})] \quad , \tag{A7}$$

where

$$\bar{u}_{I0} = \bar{u}_{I\parallel}/v_I \, , \; \bar{u}_{I1} = \sum_j n_j z_j^2 \, u_{j1} / (\sum_j n_j z_j^2) \quad , \tag{A8}$$

and

$$\tilde{\mu}^I_{00} = 2^{-1/2} - \frac{225}{2048} \alpha/d \quad , \tag{A9}$$

$$\tilde{\mu}^I_{01} = \frac{3}{4} 2^{-1/2} - \frac{6255}{8192} \alpha/d \quad , \tag{A10}$$

$$\tilde{\mu}^I_{11} = \frac{15}{2} (\frac{m_i}{m_I})^{1/2} + \frac{59}{32} 2^{1/2} \alpha - \frac{(417\alpha)^2}{2^{15} d} \quad , \tag{A11}$$

with

$$d = \frac{175}{8} (\frac{m_i}{m_I})^{1/2} + \frac{8385}{2048} 2^{1/2} \alpha \quad , \tag{A12}$$

and

$$\bar{\mu}^I_{11} = \frac{15}{2} (\frac{m_i}{m_I})^{1/2} + 2^{1/2} \alpha - \frac{9}{8} \alpha^2/d_1 \quad , \tag{A13}$$

with

$$d_1 = \frac{175}{8} \left(\frac{m_i}{m_I}\right)^{1/2} + \frac{45}{16} 2^{1/2} \alpha \quad . \tag{A14}$$

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