INSTITUTE FOR FUSION STUDIES

DOE/ET/53088-438
DOE/ER/53267-77
IFSR #438
FRCR #373

Nonlinear Tearing Mode; Rutherford Regime and Global Characteristics

Y. Z. Zhang, R. Denton, S. M. Mahajan
Institute for Fusion Studies
The University of Texas at Austin, Austin, Texas 78712

Chen Jiayu
Institute of Plasma Physics
Academia Sinica, Hefei, People's Republic of China

A. Wootton
Fusion Research Center
The University of Texas at Austin, Austin, Texas 78712

June 1990

THE UNIVERSITY OF TEXAS

AUSTIN

The nonlinear tearing mode: Rutherford regime and global characteristics

Y. Z. ZHANG, R. DENTON, S. M. MAHAJAN
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712
CHEN JIAYU
Institute of Plasma Physics
Academia Sinica
Hefei, People's Republic of China
and
A. WOOTTON
Fusion Research Center
The University of Texas at Austin
Austin, Texas 78712

Abstract

Experiments on the TEXT tokamak show that the temporal evolution of a single helicity tearing mode is dominated by the Rutherford regime. Further analysis of the Mirnov signal and sawtooth activity combined with numerical simulation emphasizes the global characteristics of the tearing mode.

A normal tokamak discharge is free of the most dangerous magnetohydrodynamic (MHD) instabilities. Even in the best operated tokamaks, however, two MHD instabilities are sometimes observed; the $m = 1$, $n = 1$ (m is the poloidal number and n is the toroidal number) internal mode responsible for sawtooth oscillations (minor disruption), and the $m = 2$, $n = 1$ tearing mode, that always emerges prior to a major disruption. This letter deals with some important aspects of the tearing instability.

Resistive MHD predicts that the temporal evolution of a single helicity tearing mode is characterized by three distinct stages: (1) the linear stage in which the magnetic perturbation ($\mathbf{b}$) grows exponentially with time,[1] (2) the Rutherford regime[2] in which the magnetic island width (W) grows linearly with time, and (3) the regime of nonlinear saturation. When the magnetic island width exceeds the linear tearing layer width $\delta_t \sim (\gamma\tau_A^2/\tau_R)^{1/4}$ (where γ is the linear growth rate, τ_A the Alfvén time, and τ_R is the resistive time) the vorticity equation decouples from the Ohm's law terminating the linear stage (1). Later temporal evolution for a single helicity tearing mode can be described by[3]

$$\frac{d\tilde{W}}{dt} = \frac{\Delta'(W)r_s}{\tau_R} - \alpha W, \tag{1}$$

where $\tilde{W} \equiv W/r_s$ is the normalized island width, r_s is the radial position of the rational surface, $\Delta'(W) \equiv \left[(d\psi/dr)_{r_s+W/2} - (d\psi/dr)_{r_s-W/2}\right]/\psi(r_s)$ is the well-known discontinuity parameter of the kink-tearing mode theory, and ψ is the perturbed flux. If α is sufficiently small, and $\Delta'(W)$ is independent of $W\,[\sim \Delta'(0)]$, then Eq. (1) tells us that $\tilde{W}$ increases linearly with time. The range of time (if any), in which the above-mentioned condition holds, is called the Rutherford regime.

Notice that of the two quantities ($\Delta' r_s$ and τ_R) comprising the Rutherford growth rate, τ_R is local, determined by the local resistivity at the rational surface, whereas $\Delta' r_s$ is controlled by the global current profile, in particular, by the value of the central q, $[q(0)]$. It has been shown that for reasonable current profiles, $\Delta'(0)r_s$ is a sharply increasing function of $q(0)$ beyond marginal stability.[4] As a result, the saturation level of the nonlinear tearing mode (if dominated by the Rutherford regime) could be more sensitive to the variation of $q(0)$ than to the variations in the quantities localized at the magnetic island. However, if α is not small, or $\Delta'(W)$ is not a slowly varying function of W, it is hard to conceive of an extended Rutherford regime with its concomitant characteristics controlling the mode dynamics.

In this letter we report the essential aspects of an experimental and theoretical investigation concerning the evolution of the $m/n = 2/1$ tearing mode in the TEXT tokamak plasma. It is observed that the Rutherford regime, does indeed, dominate the nonlinear evolution of the tearing mode. The experimental results are found to be in agreement with numerical simulations. The existence of an extended Rutherford regime implies that the island saturation width should depend sensitively on $q(0)$. Based on the understanding of the global behavior of the tearing mode, a method for measuring $q(0)$ and $Z_{\text{eff}}(0)$ (central effective charge number) is proposed. To study the behavior of the nonlinear tearing mode, plasma parameters are so adjusted that the discharges can sustain high levels of magnetic perturbations b_θ without disrupting. The amplitude $b_\theta(t)$ is obtained by a time integration of the Mirnov coil signal $\left[\dot{b}_\theta(t)\right]$, i.e.,

$$b_\theta(t) = \int_0^t \dot{b}_\theta(t)dt. \tag{2}$$

A proper integration is necessitated by the fact that the oscillation frequency of $b_\theta(t)$ changes dramatically during the evolution of the mode.

The Mirnov-coil array consists of 24 poloidal and 4 toroidal coils. A spatial Fourier analysis code is used to identify the mode numbers associated with the signal. It is found that, in these discharges, the MHD activity is dominated by the $m/n = 2/1$ mode; this is independently confirmed by measurements of soft x-rays, density and temperature perturbations.

The first step in the unfolding of the experimental data (to compare it with theory) is to calculate [using Eq. (2)] the poloidal magnetic field perturbation b_θ. In vacuum, b_θ equals b_r, the radial component of the perturbation [cylindrical geometry is used throughout this paper]. This sets the stage for calculating the magnetic island width, $W =$

$4\,(qr_sRb_r/m\hat{s}B)^{1/2}_{r=r_s}$ [5] in terms of experimentally determined quantities, i.e., the major radius R, the total field B, $q\,(r_s) = 2 = m$ for the present case, and the radial perturbation b_r at $r = r_s$. The location of the rational surface r_s and the magnetic shear parameter $\hat{s}\,(r_s)\,[= (r/q)_{r_s}(dq/dr)_{r_s}]$ are determined from a q-profile fitted to the measured electron temperature profile [Thomson scattering], using Spitzer resistivity and a constant Z_{eff}-profile. The perturbation $b_r\,(r = r_s)$ is obtained from the extrapolation of the vacuum behavior $b_r \sim r^{-3}$ (for $m = 2$), which has been found to be a good approximation for the actual solution of the global ψ for $r \gtrsim r_s$.[6] The position of the mode rational surface (typically $0.7a < r_s < 0.8a$, a is the minor radius of plasma) is comparatively insensitive to details of the temperature profile near the plasma edge and is consistent with the inversion point of the densities, temperature and soft x-ray perturbation locations that are respectively determined by far infrared interferometry, electron cyclotron emission, and a surface barrier diode array. For discharges with saturated Mirnov signals, the experimental data reveals two different types of temporal behavior. In the first case, W increases linearly with time, and then saturates. In the second case there are two distinct stages of linear growths before the mode saturates. In this letter we simply concentrate on the analysis of the first type, i.e., the modes with a single Rutherford growth rate [Fig. 1]. It is evident from Fig. 1, where examples of the island width time history are shown, that the Rutherford growth rate is strongly correlated with the saturation island width. In general, larger growth rates lead to larger saturated island sizes. Notice that the smallness of the amplitude in the pre-Rutherford stage of this region rules out a quantitative characterization.

In order to further understand these results, we performed numerical simulations for the growth and saturation of the $m/n = 2/1$ mode using a fully nonlinear 2-D reduced magnetohydrodynamic code. The initial q-profile is taken to be $q(r) = q(0)\Big\{1 + (r/a)^{2\lambda}\left[(q(a)/q(0))^{\lambda} - 1\right]\Big\}^{1/\lambda}$ with $q(a) = 3.3$ and with λ so chosen that $q\,(r_s = 0.75a) = 2.0$. With these restrictions, the q-profile is determined by the choice of $q(0)$. The radial dependence of the resistivity is chosen so that $\eta J(r, t = 0)$ is a constant, and the Reynold number at the $q = 2.0$ surface is taken to be 4.44×10^5 (equivalent to the choice of $Z_{\text{eff}}(r_s) \sim 3.5$ for TEXT discharges).

In Fig. 2 we show the evolution of the calculated island width W (the maximum distance between the island separatrices) as a function of time for three different values of initial $q(0)$. As $q(0)$ decreases towards the marginal stability point $\left(q(0) \gtrsim 0.93\right)$, the growth of the island width becomes slower, and the final width is also smaller. The remarkable similarity between the experimental (Fig. 1) and numerical simulation (Fig. 2) results emphasizes the importance of $q(0)$ to the overall dynamics of the tearing mode. It is equally noteworthy that the experimentally observed, and the numerically obtained Rutherford growth rate are of the same order. For example, the typical experimental Rutherford growth rate is about 1 cm/ms for the final island width $\sim a/3$ [Shot No. 129822 in Fig. 1]. This is approximately equal to the Rutherford growth rate curve $a\,(q(0) = 1.2)$ of Fig. 2 calculated for the TEXT parameters: $B \sim 20\,\text{KG}$, $n \sim 2 \times 10^{13}/\text{cm}^3$, $R = 100\,\text{cm}$, and $a = 25\,\text{cm}$.

Since the MHD activity (tearing mode) is favored by $q(0) > 1.0$, it is natural to expect that high Mirnov signals should imply low sawtooth activity favored by $q(0) < 1$. This expectation is indeed consistent with observations in most TEXT discharges. In Fig. 3 we display a typical time history of a high MHD, TEXT shot. The integrated Mirnov coil signal (interpreted as a tearing mode), and the soft x-ray signal (used to monitor sawteeth) are plotted as functions of time. Notice that when the Mirnov coil signal is high, there are no sawteeth, and as soon as the Mirnov signal decreases, the sawtooth activity appears. At no time do we see their simultaneous presence.

It is worth noting that this strong anti-correlation between Mirnov oscillations and sawtooth activity persists in the auxiliary heating experiments. Electron cyclotron resonance heating (ECRH) near the plasma center can be used to trigger sawtooth activity in TEXT. When ECRH is applied to the discharges with large Mirnov signal, the sawteeth appear only after the Mirnov signals are greatly suppressed. In Fig. 4 ECR power is initiated at about 300 ms. At this time, the Mirnov signal is large, indicating a high level of the tearing mode (large island). As the temperature rises indicated by electron cyclotron emission at center, the Mirnov signal falls, disappearing at about 320 ms with the concurrent appearance of strong sawteeth indicated by soft X-ray fluctuations at the central chord.

The sensitivity of the Rutherford growth rate to $q(0)$ can be used either to determine $q(0)$, or as a consistency check on other methods for measuring $q(0)$. For a single helicity tearing mode dominated by the Rutherford regime, Eq. (1) can be approximated by taking $\Delta'(W) \sim \Delta'(0)$ and neglecting the αW term for small island width. Assuming a parametric q-profile, for example, with four independent parameters representing $q(0)$, $q(a)$, r_s and $\hat{s}(a)$ (in limiter discharges with circular cross section $\hat{s}(a)$ can be set equal to 2.0), one can determine $q(a)$, r_s, and $\hat{s}(a)$ by the traditional diagnostics. The right-hand side of Eq. (1) $[\Delta'(0)r_s/\tau_R]$ then becomes a function of only two unknown quantities, viz., $q(0)$ and $Z_{\text{eff}}(r_s)$. The function $\Delta'(0)$ can be expressed in terms of $q(0)$ using the parametric q-profile.[4] It follows then that $q(0)$ could be determined if we could experimentally measure $Z_{\text{eff}}(r_s)$ for example, from visible Bremsstrahlung. However, by definition, $q(0) = 0.327 B_{[T]} Z_{\text{eff}}(0) \ln \Lambda_{[20]} / V_{[\text{Volt}]} T_e^{3/2}(0)_{[\text{kev}]}$, where $B_{[T]}$ the total magnetic field in Tesla, $\ln \Lambda_{[20]}$ is the Coulomb logarithmic divided by 20, $V_{[\text{Volt}]}$ is the loop voltage in volts, $T_{e_{[\text{kev}]}}$ is the electron temperature in keV, with all of these quantities evaluated at the plasma center. Between this expression, and the formula for $q(0)$ supplied by Eq. (1), it becomes possible to calculate $q(0)$ (and then $Z_{\text{eff}}(r_s)$ and $Z_{\text{eff}}(0)$) if just the ratio $Z_{\text{eff}}(r_s)/Z_{\text{eff}}(0)$ is experimentally known. For the high MHD discharges on TEXT, $q(0)$ is a reasonably sensitive function of $Z_{\text{eff}}(r_s)/Z_{\text{eff}}(0)$, implying that the inferred value of $q(0)$ is trustworthy. The method gives $q(0)$ in the correct range; the details will be presented elsewhere.

In summary, experimental measurements of the perturbed magnetic field produced by an $m/n = 2/1$ assumed tearing mode can be interpreted in terms of a magnetic island width.

When this is done, it is found that there is a *significant* period in which the island grows linearly in time, as predicted by Rutherford.[2] Numerical simulation for the growth rate, and the final saturated width are in reasonable agreement with experimental results, and show a strong dependence on the central safety factor $q(0)$. This global nature (i.e., a dependence on $q(0)$, and not just on local parameters at the mode rational surface) is supported by the experimental observations that *strong* tearing mode and sawtooth activity do not usually occur together.

Acknowledgments

We gratefully acknowledge the contributions of the TEXT staff, in particular, to Drs. W. L. Rowan, P. E. Phillips, and S. C. McCool. This work was supported by U. S. Dept. of Energy Contract No. DE-FG05-80ET-53088.

References

1. H. P. Furth, J. Killen, and M. N. Rosenbluth, Phys. Fluids **6**, 459 (1963).

2. P. H. Rutherford, Phys. Fluids **16**, 1903 (1973).

3. R. B. White, D. A. Monticello, M. N. Rosenbluth, and B. V. Waddell, Phys. Fluids **20**, 800 (1977).

4. H. P. Furth, P. H. Rutherford, and H. Selberg, Phys. Fluids **16**, 1054 (1971).

5. J. Wesson, *Tokamaks*, §7-2, Clarendon Press Oxford, 1987.

6. D. C. Robinson and K. McGuire, Nucl. Fusion **19**, 115 (1979).

Figure Captions

1. Temporal evolutions of $(m/n = 2/1)$ magnetic island width in TEXT. Shot numbers are shown in the figure.

2. Temporal evolutions of $(m/n = 2/1)$ magnetic island width from numerical simulation for Reynold's number $= 4.44 \times 10^5$ at $q = 2$ surface. Curve a, b, c refers to the initial $q(0) = 1.2, 1.0, 0.95$, respectively with initial $q(a) = 3.3$, $r_s = 0.75a$. The island width and time is normalized to minor radius a, and Alfvén time τ_A, respectively.

3. A typical discharge in TEXT, showing exclusiveness of Mirnov oscillations to sawtooth activities.

4. A direct experiment for global characteristics of tearing mode. When ECRH at plasma center is turned on, the Mirnov oscillations are suppressed and then disappear with concurrent appearance of strong sawteeth.

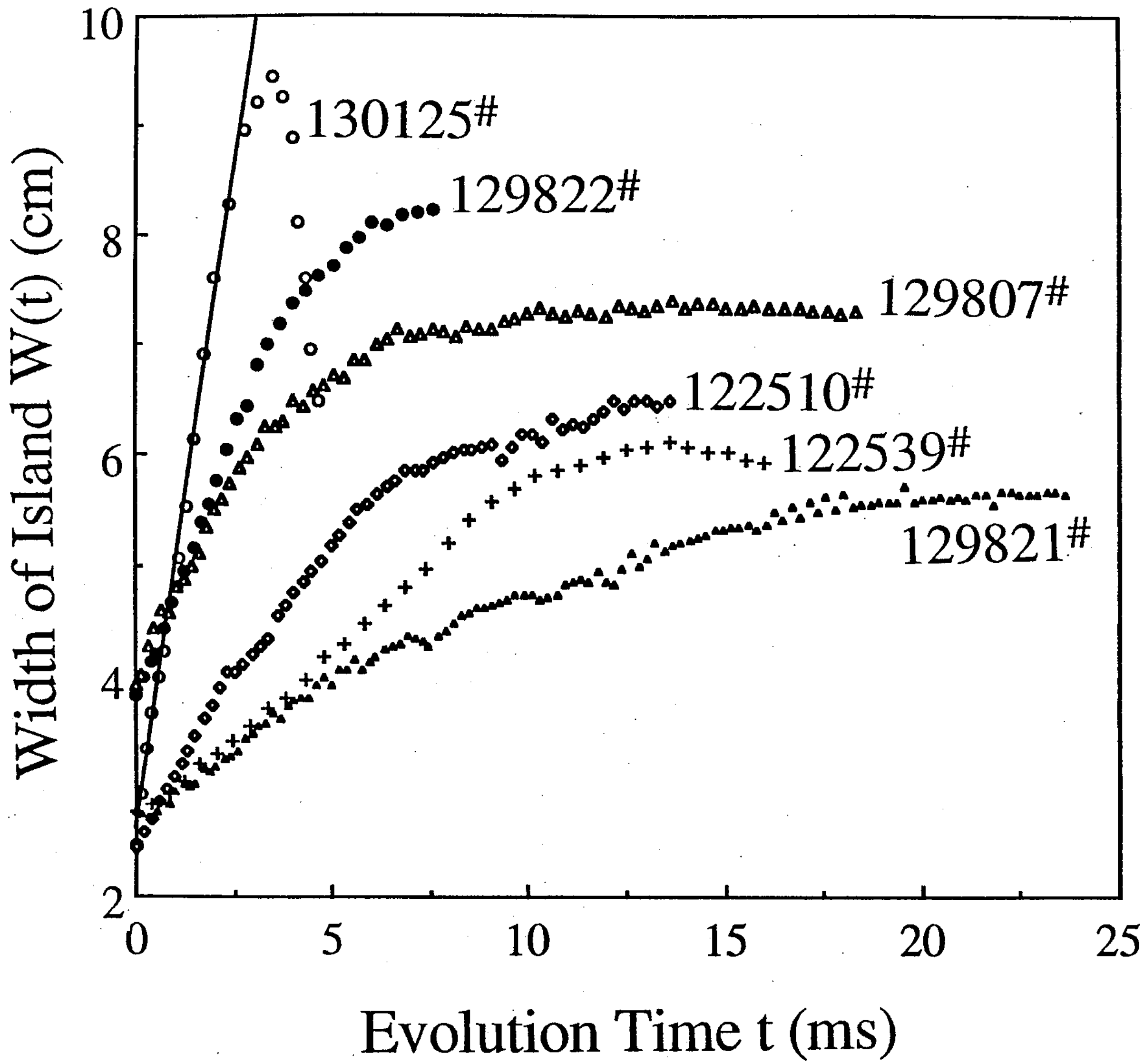

130125#
129822#
129807#
122510#
122539#
129821#
Width of Island W(t) (cm)
Evolution Time t (ms)
10
8
6
4
2
0
5
10
15
20
25

Fig. 1

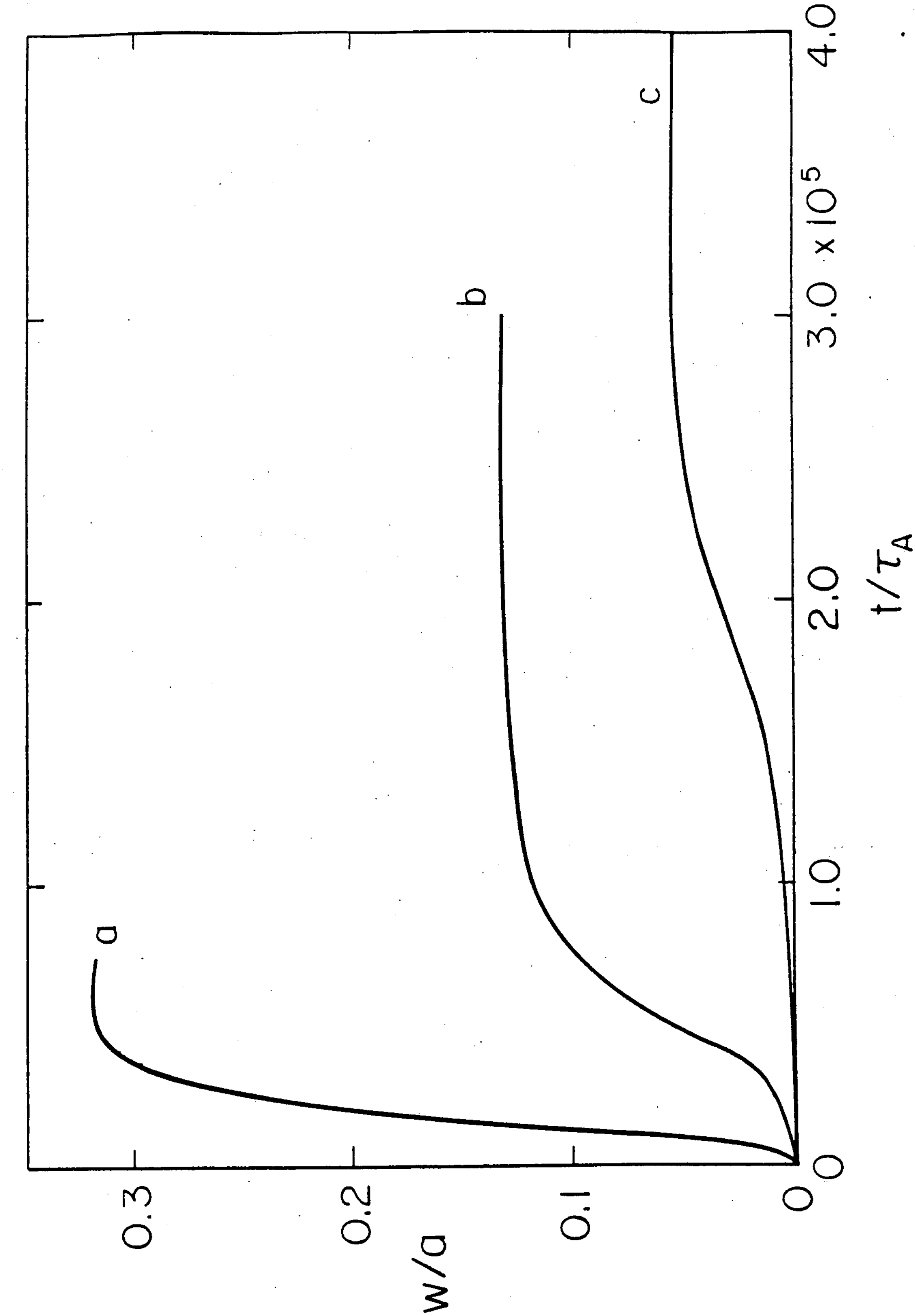
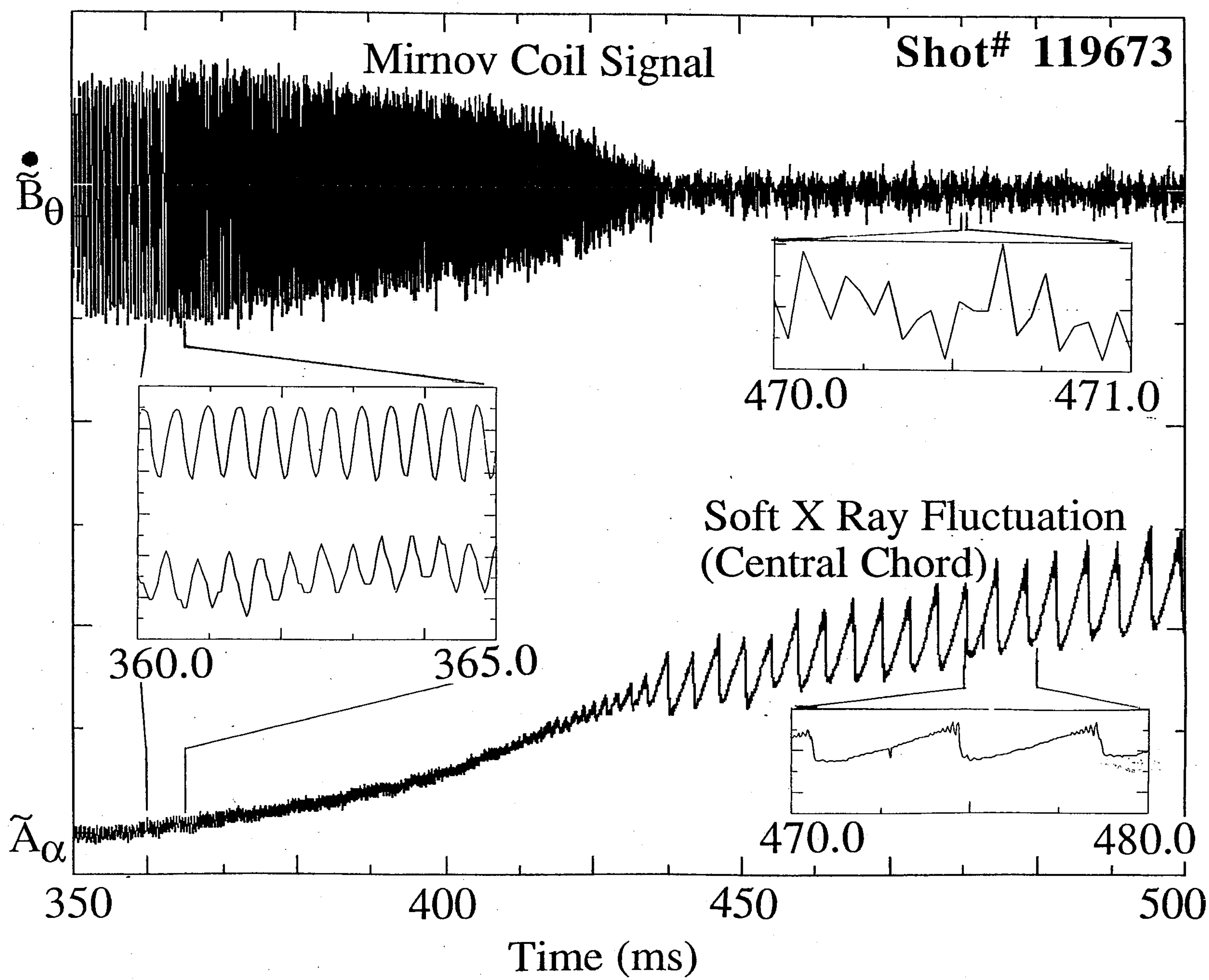


Figure 2



Mirnov Coil Signal
Shot# 119673
$\dot{\tilde{B}}_\theta$
$\tilde{A}_\alpha$
360.0
365.0
470.0
471.0
Soft X Ray Fluctuation
(Central Chord)
470.0
480.0
350
400
450
500
Time (ms)

Fig-3