The Effect of Hot Inhomogeneous Plasma on Geomagnetic Micropulsations

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Abstract:- The hydromagnetic waves generated in the magnetosphere propagating along the magnetic field lines to the ground travels through the medium in which the Alfven velocity increases along the radial direction more rapidly than the variation along the wave vector. We study the effect of the inhomogenity perpendicular to the field line on the polarisation properties of the magnetic variations of various types of waves as the field lines corresponding to lower shells are excited by the impulse transmitted from the higher shell field lines Even though the magnetic variation is predominantlylongitudinal in the generation point, after a short distance in the equatorial plane, a transverse component will develop as a result of the plasma inhomogeneity in the direction normal to the magnetic field. This may be connected to the observed phenomenon that micropulsations detected on the ground at different latitudes have different polarisation characterestics.

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1. Introduction

It is well-known that the low frequency variations in the geomagnetic field are hydromagnetic waves generated in the magnetosphere by various types of plasma instabilities. They are mostly guided along the field lines to reach the ionosphere and then detected on the ground. The energy source of these waves is either near the magnetopause $(L \simeq 6R_E)$ or near the plasma pause $(L \simeq 4R_E)$ where the enhanced ring current plasma is responsible for their excitation. Sometimes the low frequency waves are generated inside the magnetosphere by the resonance interaction of the Alfven waves with the whistlers or by drift mirror instability (Lanzerotti et al, 1974) as suggested by Southwood (1968) arising from the coupling between the drift waves and the mirror instability. All these excitation mechanisms take place in the magnetosphere or at the plasma pause region whose field lines are connected to the high latitude ionosphere. However pulsations are observed at all latitudes as low as the equator. Thus the lines with low shell values ($L = 1.01R_E$) are also excited although the source region is generally at a higher altitude. The dominant period of these low frequency pulsation shows a latitude dependence suggesting the effectiveness of the field-line resonance theory [Dungey (1954); Hasegawa et al, (1983)], where the eigenfrequencies are different for different field lines. But on many occasions similar events with the same periods but different amplitudes are concurrently observed over a wide area extending from high latitude to the equatorial region [Jacobs and Sinnok (1960); Matuura (1961); Kato and Saito (1962)]. According to Kato and Tamao(1962) these hydromagnetic waves have a source at the magnetopause where they are generated by the velocity shear associated with the solar wind and magnetopause discontinuity. The normal component of the wave propagates towards the earth and after some distance these are converted into the Alfven modes and travel along the field lines to have access to the lower part of the atmosphere. However, the proposed theories of Hasegawa (1971) or Lin and Park (1973) indicate that the source of these waves can also be the ring current region. They are basically coupled modes between the Alfven and the compressional waves being triggered by the inhomogeneity of the hot component of the plasma. Even the localized ballooning interchange instability (due to the curvature of the field lines) has been shown [Ohtani, Miura, Tamao (1989)] to excite these waves in this region. The polarization properties are different for different mechanisms. So far as the observation is concerned, these waves have comparable transverse and compressional magnetic perturbations [Barfield, (1971);Barfield et al,1972]. Sometimes the transverse component is seen

to be polarized in the meridian plane. Whatever be the source of these waves, they are generated mostly on the equatorial plane of the magnetosphere with high shell values ($L = 10R_E$ to $4R_E$) for the field lines. They must propagate normal to the field lines in the meridian plane in order to excite the low lying field lines. Since the Alfven velocity changes more rapidly in this direction, it is expected that the polarization properties will be different for different field lines even though the frequency and the excitation mechanism may be the same. In that case the approach of deducing the magnetospheric parameters from the latitudinal structure (i.e. different field lines) of these waves need to be modified. It was already shown by Southwood and Hughes (1983) that in a cold inhomogeneous plasma the Alfven mode couples with the consecutive field lines. In the magnetosphere, however, the inhomogeneity has many forms. First of all, the ambient Alfven velocity increases towards the earth. In the altitude under consideration the cold plasma is not exactly a zero temperature fluid. Most of the time the temperatures at the higher shells are of the order of a few tenths of an electron volt to a few electron volts. Moreover, during some storms when energetic particles are injected in the ring current region, a very hot component of plasma does exist with a temperature of tens of kilovolts. This component may have a very short scalelength, but it can change the character of a wave which passes through this region.

The purpose of this paper is to investigate the effect of the inhomogeneity of the ambient plasma in a direction normal to the field line (but in the meridian plane) on the properties of the low frequency waves, as they propagate from the higher shells to the lower heights in the equatorial plane of the magnetosphere. In the next section we shall derive the general expression for the wave equation in an inhomogeneous medium. Then we shall solve some particular cases under a suitable initial condition and investigate how the polarization characterization changes as the wave propagates perpendicular to the field lines in the equatorial plane towards lower altitudes.

2. Theoretical Formulation

We consider Alfven wave propagation in an inhomogeneous plasma appropriate for the earth's magnetosphere and ionosphere. We assume that a wave with frequency ω and with wave vector k_x and k_z along the x and the z directions, respectively, has been generated by any mechanism and is passing through the plasma in the equatorial plane region of the magnetosphere. The ambient magnetic field $\mathbf{B_0}$ is in the \mathbf{z} -direction in the meridian plane. The x direction is the east-west direction. Both B_0 and the ambient plasma density n_0 are functions of y, the altitude coordinate. The perturbed velocity of a particle (ion or electron) can be written as

$$-i\omega \mathbf{v} = (q/M)[\mathbf{E} + (\mathbf{v} + \mathbf{v_0}) \times (\mathbf{B_0} + \mathbf{B})] - \frac{V^2 \nabla n}{n_0}$$
(1)

Here ∇ is the gradient operator; q and M are respectively the charge and the mass of the particle. The unpertured density $n_0(y)$ has a scale length κ in the y direction. $n, \mathbf{B}, \mathbf{E}$ are the perturbed density, magnetic field and the electric field respectively.

The equilibrium drift velocity v_0 is in the x direction and is given by

$$v_0 = -\frac{(\kappa + 3r^{-1})V^2}{\Omega},\tag{2}$$

where κ is the scale length inverse of the density gradient n_0 . The second term in v_0 comes from the inhomogeneity of the dipole magnetic field at the equator, where r is the geocentric distance. V and Ω are the thermal velocity and the cyclotron frequency respectively. Since there is a current in the x direction because of the density gradient, the unperturbed magnetic field B_0 is of the form (Lin and Park, 1978):

$$B_0(y) = B_d(y)(1 - \frac{\kappa}{2}\beta y)$$

where β is the ratio of the kinetic pressure to the magnetic field pressure. B_d is the dipole field in the equatorial plane.

The continuity equation is

$$-i\omega n = \nabla(n_0 + n)(v_0 + \mathbf{v}),\tag{3}$$

which gives the density fluctuation n/n_0 . Combining equations (1) and (3), we obtain

$$\mathbf{v} = (q/M)\rho^{-1}(\mathbf{E} + v_0 \times \mathbf{B}),\tag{4}$$

where the matrix ρ is given by,

$$\rho_{xx} = -i\left(\omega - \frac{k_x^2 V^2}{\omega - \omega^*}\right),$$

$$\rho_{yy} = -i\left(\omega + \frac{\kappa^2 V^2}{\omega - \omega^*}\right),$$

$$\rho_{zz} = -i\left(\omega - \frac{k_z^2 V^2}{\omega - \omega^*}\right),$$

$$\rho_{xy} = -\Omega + \frac{\kappa k_x V^2}{(\omega - \omega^*)},$$

$$\rho_{yx} = \Omega + \frac{\kappa k_x V^2}{(\omega - \omega^*)},$$

$$\rho_{xz} = \rho_{zx} = \frac{ik_x k_z V^2}{\omega - \omega^*},$$

$$\rho_{yz} = \rho_{zy} = \frac{\kappa k_z V^2}{\omega - \omega^*},$$
(5)

with

$$\omega^* = v_0 k_x$$
.

The Maxwell's equations are:

$$abla imes \mathbf{E} = i(\omega/c)\mathbf{B},$$
 $abla imes \mathbf{B} = -i(\omega/c)\mathbf{E} + 4\pi nq\mathbf{v}.$

Thus the electric field equation is:

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} (\epsilon + 1) \mathbf{E}, \tag{6}$$

where ϵ is the dielectric tensor and for low frequencies is given by

$$\epsilon_{xx} = (c^2/V_A^2)(1 - \frac{\omega^*}{\omega}) + \frac{\omega^*}{\omega} \frac{\kappa V^2 c^2}{V_A^2 \omega^2} \frac{d}{dy},$$

$$\epsilon_{yy} = \epsilon_{xx} + (1 - \frac{\omega^*}{\omega}) \frac{c^2 k_x^2 V^2}{V_A^2 \omega^2} - i \frac{\omega^*}{\omega} \frac{c^2}{V_A^2 k_x} \frac{d}{dy},$$

$$\epsilon_{xy} = \epsilon_{yx}^* = i \frac{\kappa k_x V^2 c^2}{V_A^2 \omega^2},$$
(7)

where V_A^2 is the square of the Alfven velocity $\frac{\Omega^2 M c^2}{4\pi n q^2}$. Thus the dielectric tensor ϵ is a differential operator because of the y dependence of B.The part of ϵ which is

independent of y is similar to that deduced by Lin and Park(1978). Since for low frequency waves E_z can be regarded as zero, substituting these values of ϵ in the Maxwell's equation above one gets the coupled differential equations for E_x and E_y as:

$$-E_x'' + ik_x E_y - \frac{\omega^2}{c^2} \epsilon_{xy} + (k_z^2 - \frac{\omega^2}{c^2} \epsilon_{xx}) E_x = 0,$$
 (8a)

and

$$ik_x E_x' - \frac{\omega^2}{c^2} \epsilon_{yx} E_x + (k^2 - \frac{\omega^2}{c^2} \epsilon_{yy}) E_y = 0, \tag{8b}$$

where $k^2 = k_x^2 + k_z^2$, and the primes denote $\frac{d}{dy}$.

These two equations involving $E_x(y)$ and $E_y(y)$ can be converted into one equation as:

$$-(k_{z}^{2} - e_{yy} - k_{x}d_{2})E_{x}^{"}$$

$$+[(k^{2} - e_{yy})d_{1} + ie_{xy}d_{2} + k_{x}d_{2}^{'} + \frac{k_{x}(k_{x} + d_{2})e_{yy}^{'}}{k^{2} - e_{yy}}]E_{x}^{'}$$

$$+[(k_{z}^{2} - e_{xx})(k^{2} - e_{yy}) - e_{xy}e_{yx} + \frac{ik_{x}k^{2}e_{xy}^{'}}{k^{2} - e_{yy}}]E_{x} = 0$$

$$(9).$$

Here we have defined $e_{ij} = \frac{\omega^2}{c^2} \epsilon_{ij}$. But e_{ij} does not contain the $\frac{d}{dy}$ part of ϵ_{ij} . The effect of this part is represented by the two factors d_1 and d_2 and are respectively given by:

$$d_1=rac{\omega^*}{\omega}rac{\kappa V^2}{V_A^2}, ext{ and } d_2=rac{\omega^*\omega}{V_A^2k_x}.$$

In solving the above equation for E_x numerically one has to sum over the contributions from both the cold and the hot components of the plasma in ϵ_{ij} , d_1 , and d_2 .

Once the field E_x is evaluated by solving the above equation with some suitable initial conditions, the magnetic perturbation fields can be written as:

$$B_x(y) = -(ck_z/\omega)E_y(y),$$
 $B_y(y) = (ck_z/\omega)E_x(y),$ $B_z(y) = (ck_x/\omega)E_y + (ic/\omega)E_x'(y)$

where

$$E_{y}(y) = \frac{\frac{\omega^{2}}{c^{2}} \epsilon_{xy}}{k^{2} - \frac{\omega^{2}}{c^{2}} \epsilon_{yy}} E_{x} - \frac{i(k_{x} + d_{2})}{k^{2} - \frac{\omega^{2}}{c^{2}} \epsilon_{yy}} E'_{x}.$$
 (10)

When there is no y dependence, the above equation reduces to the conventional dispersion relation

$$(k_z^2 - e_{xx})(k^2 - e_{yy}) - e_{xy}e_{yx} = 0$$

For a plasma with density gradient κ^{-1} , e_{xy} is the coupling term between the Alfven mode ($k_z^2 = e_{xx}$) and the compressional mode ($k^2 = e_{yy}$).

For a dipolar magnetic field Ω varies as r^{-3} where r is the geocentric distance. But the plasma density n_0 varies approaximately as r^{-3} to r^{-4} (Hess,1968). Therefore we have assumed V_A^{-2} as $(1-y/y_0)^N$, where y_0 is some initial reference point in the magnetosphere. Here y is positive towards the the center of the earth in the equatorial plane. The value of the exponent N can vary from 1.1 to 1.35. As for the initial condition we have chosen $E_x = 1$; $E_x' = 0$ at y = 0.

3. Results

We solve Eq.(9) in this section. But before doing so, let us first note that if the medium were uniform the Alfven mode and the compressional mode would have been uncoupled. Looking at the equation for E_y one can see that these two modes will be coupled through the E'_x term in the inhomogeneous medium. The field will pass through a resonance (Southwood and Hughes, 1983) near a point where $\omega = k_z V_A$. E_y will change sign on either side of this point. But in a warm plasma the coupling also takes place through nonzero ϵ_{xy} . If the temperature is reasonably high, as is the case in the ring current region, not only ϵ_{xy} contributes to this coupling but also the inhomogeneity of the medium introduces another term d_2 , which also contributes to this coupling.

In the numerical solution we assume that $E_x'=0$ at y=0. This, however, may not always produce a physically acceptable solution when it diverges in y. Generally in numerical solutions this problem is controlled by insisting that the solution vanishes at a suitably located boundary, i.e. treating the problem as a boundary value problem rather than an initial value problem. If the medium were uniform, E_x' would have been zero. Therefore, we want to look at the situation where a $E_x=$ constant solution evolves as the wave passes through the medium.

We have assumed that 10% of the plasma is hot with a scale length of $0.1R_E$. But the temperature also decreases with a scale length of $1R_E$. This type of plasma is known to excite coupled modes with frquency near the local Alfven mode $(k_z V_A)$ (Hasegawa,1971;Lin and Park, 1978) with high azimuthal wavenumber k_x even with isotropic temperature. We, therefore, assume $\omega = k_z V_A(0)$. The value of the exponent N for the y variation of V_A^{-2} is taken to be 1.25.

In the following graphs we have presented the results of the polarizations as a function of y. Here we refer the wave magnetic field B_z , in the direction of geomagnetic field, as longitudinal polarization and the field B_T , which is perpendicular to it as transverse polarization respectively. Therefore, $B_T = \sqrt{B_x^2 + B_y^2}$. The results are plotted in terms of the angles $tan^{-1}\frac{B_x}{B_y}$ and $tan^{-1}\frac{B_{-z}}{B_T}$. The solid line $\frac{(B_z}{B_T}$ and the dashed line $\frac{(B_x}{B_y}$ in these figures are, therefore, the measure of the parallel to transverse polarization and azimuthal to meridional polarization, respectively.

Figure 1 presents the case of moderately hot (T = 100eV.,Fig.1) plasma. The initial polarization is primarily along the field line,showing the mainly compressional nature, and the transverse polarization in the meridian plane. But after some distance $\sim 2 R_E$, longitudinal component is converted into the transverse component.

However, in the transverse direction B_y dominates over B_x , but after some distance relative strength of B_x gradually increases.

For slightly hotter plasma (T = 400eV.,Fig.2) also the same pattern is preserved. Only in this case the relative strength of B_x is stronger than the previous case.

Waves in a very hot plasma (keV, Fig.3), however, decay very fast in the radial direction. But the transverse polarization is mainly in the azimuthal direction. This is because of two reasons. First ϵ_{xy} contributes more to E_y (and hence to B_x) for a hotter plasma. Secondly, E'_x contribution also increases through the d_2 term in Eq.(10). Since ϵ_{xy} is a function of y, $\frac{B_x}{B_y}$ will change wity y even if E_x were constant. Therefore, the effect of E'_x can be seen from the ratio $\frac{B_z}{B_x}$ which would have been just $\frac{k_x}{k_z}$. In Fig.4 we plot the ratios $\frac{B_z}{B_x}$ (the dashed line) and $\frac{E'_x}{E_x}$ as functions of y for one case only(i.e. for T=100ev.). We can see that $\frac{B_z}{B_x}$ is no longer a measure of the ratio of the azimuthal wave vector (k_x) to the longitudinal wave vector (k_z) . Because these very hot plasma waves are localized in the y direction their latitudinal extent on the ground will be very narrow.

To see what happens in a cold plasma, we know that $\omega = k_z V_A(0)$ has a singularity at $y = y_0$ (Southwood and Hughes, 1983). Therefore, we choose a frequency in the range $kV_A(0) > \omega > k_z V_A(0)$ in Fig.5. The amplitude E_x gradually decreases and after some distance becomes negative (phase change). In this neighborhood E_x'/E_x is very large and therefore B_z/B_T also assumes a very large value. Therefore an initial mixed polarization will be totally converted into a longitudinal polarization and again revert back to the mixed polarization state.

Thus we can see that the mode coupling in an inhomogeneous medium is enhanced by the presence of a hot plasma. This is not only because of a very small value of ϵ_{xy} but also because of finite E'_x which is no longer zero. Even if E'_x is zero initially (at y = 0), within a short distance E'_x becomes nonzero because of the variation of $V_A(y)$. This nonzero E'_x builds up a finite E_y . This changes the initial polarisation character of the waves.

4. Discussion

According to Hagege et al (1973) PC4-5 micropulsations are produced by high frequency Whistler noise and the drift waves. These waves, as well as those excited by bounce resonance as proposed by Southwood (1968), have zero radial component at the magnetic equator. The same is true for the drift mirror instability as proposed by Lanzerotti et al (1974). On the other hand, observational data show a finite transverse amplitude. This apparent contradiction can now be understood. Even

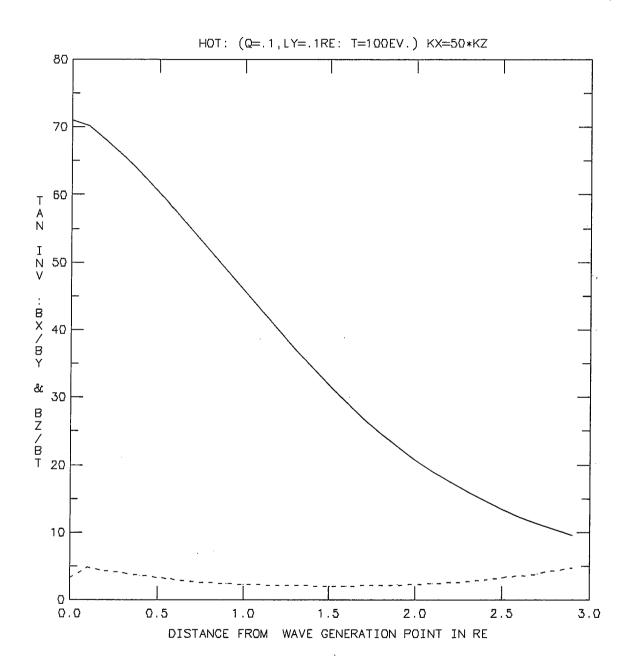
though the magnetic variation is longitudinal in the generation point, after a short distance in the equatorial plane, a transverse component will develop as a result of the medium inhomogeneity in the y direction. Sometimes very low frequency compressional magnetic signals are observed in the ring current region (Saunders etal, 1981; Hedgecock, 1976; Barfield etal, 1972). The difficulty of explaining them in terms of the slow compressional sound wave is the theoretical reality of (Southwood and Hughes, 1983) heavy Landau damping for such waves. On the other hand very long periods suggest them to be unlikely to be fast mode waves. But here we can see that a pure transverse Alfven mode at a higher shell can really be converted into a compressional mode when they are seen at another shell as they travel along the y direction. Therefore these waves need not originate from a fast mode. Observationally also it has been indicated (Stuart etal, 1971; Samson etal, 1971) that when multifrequency signals are seen on a latitudinal chain of stations all the stations agree as to the frequencies at which they spectral peak occurs. However, they vary in polarization character from latitude to latitude. It will be interesting to analyze simultaneous ground micropulsations starting from high latitude to the equatorial region. On many occasions very low frequency fluctuations in the geomagnetic fields on the ground are observed at latitudes as low as the geomagnetic equator (Jain, 1977). From an analysis of the ground magnetic variations from a low latitudinal chain of stations in the Indian suncontinent it was found (Agarwal et al, 1978) that waves in various frequency ranges associated with IMF can find their way to the low latitude ionosphere. Substorm associated hydromagnetic waves also have access to the mid and low latitude ionosphere (Wang et al, 1977). A study of the polarization characteristics of these events, after filtering out the ionospheric inversion effect, can reflect light on the y variation of the waves in the equatorial plane of the magnetosphere.

In order to see this effect of polarization inversion of the same wave at different shells, it is necessary to observe an event at different heights in the equatorial plane which is rather a very difficult proposition. One can however attempt to see them on the ground magnetic pulsation events. Although it is true that at different latitudes on the ground, different-dominant frequencies are observed suggesting that the signals are due to the eigen oscillation of the shells, on many occasions same frequency (although with varying amplitude) is detected over a wide range of latitudes. In such cases the ground observations can be traced back to the equatorial magnetosphere, since the electric fields E_x and E_y can be mapped back along the field line from the equatorial magnetosphere at different heights to the ionosphere at different latitudes.

However the finite and anisotropic conductivity of the ionosphere will rotate (by about 90°) the plane of polarization from the top of the ionosphere to the ground (Ellis and Southwood, 1983). Some early observations on the latitudinal profile of the H and D component of PC5 amplitudes do show (Matuura, 1961; Kato 1962) that the ratio of the north-south to east-west magnetic variations is different at different latitude. When it is translated back to the equatorial plane, it means that the ratio (B_x/B_y) is really different at different heights in the equatorial plane. Hence a coordinated study of the ground magnetic pulsation together with a theoretical calculation to filter out the ionospheric modification should reveal the correct y variation of a pulsation event.

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Figure caption

Fig.1: The angles $tan^{-1}\frac{B_x}{B_y}$ (dashed line) and $tan^{-1}\frac{B_z}{B_T}$ (solid line) in degrees as functions of y. Q is the ratio of hot plasma density to cold plasma density. L_Y is the scale length(κ^{-1}) of the hot plasma. W is the angular frequency ω .

Fig.2: The same as Fig.1 but with temperature T=400 ev.

Fig.3: The same as Fig.1 but with temperature T=1 Kev.

Fig.4: The ratios $\frac{B_z}{B_x}$ and $\frac{E'_x}{E_x}$ as functions of y for the case with T = 100ev.

Fig.4: The same as in Fig.1 for a cold plasma ; i.e. Q=0.

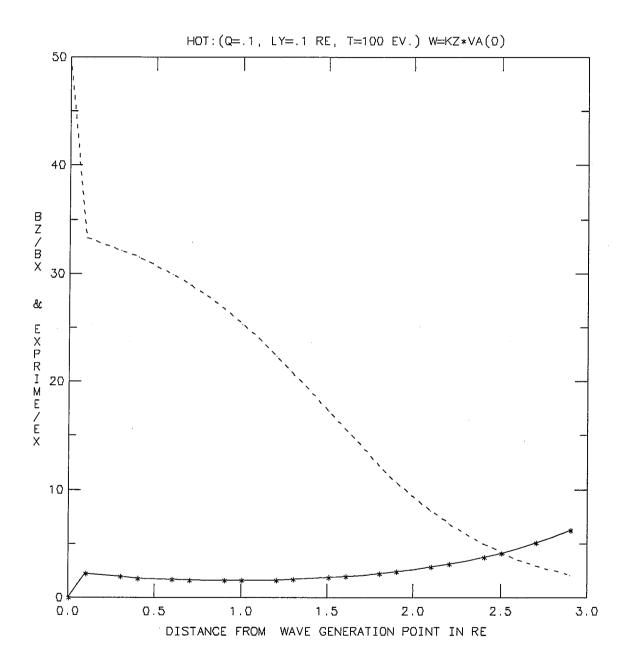


Fig. 1

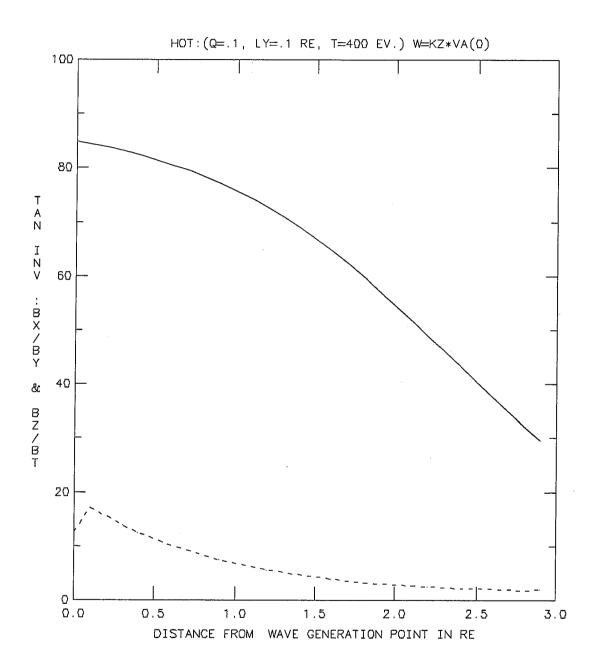


Fig. 2

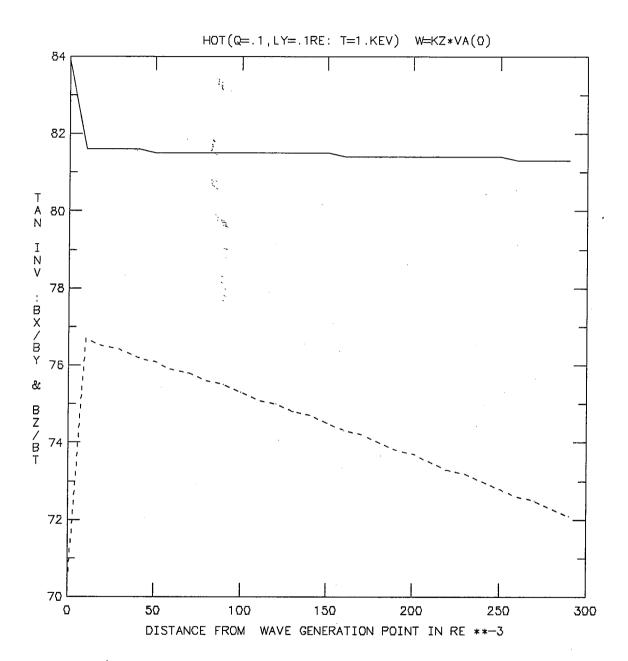
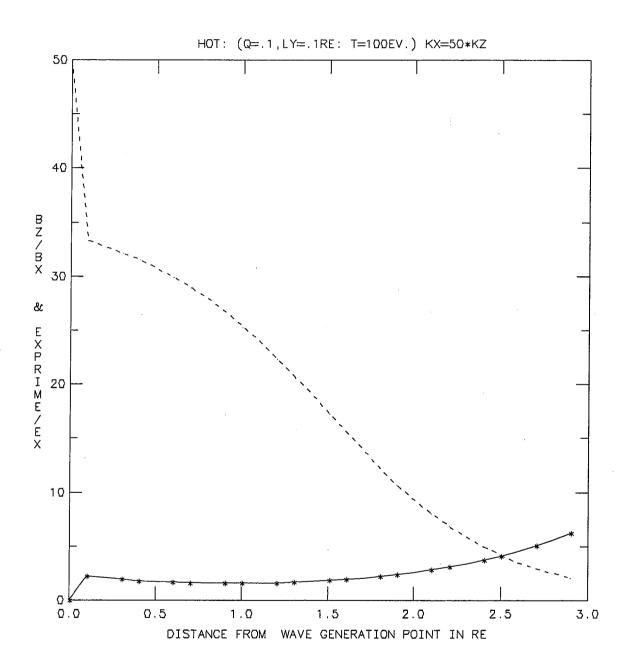


Fig. 3



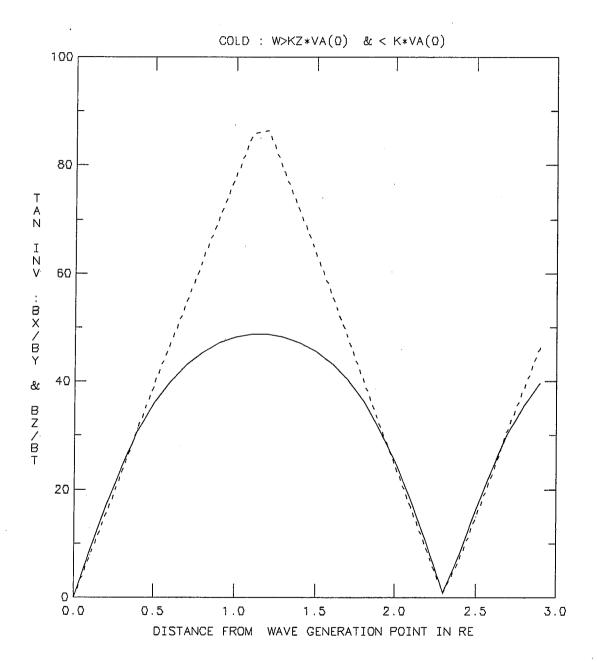


Fig.5