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Temperature-Gradient Instability Induced by Conducting End Walls

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Abstract

A new rapidly growing electron temperature gradient instability is found for a plasma in contact with a conducting wall. The linear instability analysis is presented and speculations are given for its nonlinear consequences. This instability illustrates that conducting walls can produce effects that are detrimental to plasma confinement. This mode is of importance in open-ended systems such as mirror machines and relevant to the edge of tokamaks where field lines are open and are connected to limiters or divertors and astrophysical plasmas like the ones of the flux tubes in a solar atmosphere, with the footpoints on the photospheric level.

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I. Introduction

In plasmas on open field lines, e.g., in mirror machines and in the edge regions of toroidal devices, the plasma density along a magnetic field line is usually finite (though possibly small) up to the material walls that terminate the plasma. It is generally believed that in such situations high electrical conductivity of the walls improves plasma stability; e.g., the stability of the flute-interchange mode. However, as has recently been pointed out,[1] there exists a situation where the high conductivity of the end walls has an adverse effect compared to insulating walls: the conducting end plates in the so-called "gas-dynamic trap"[2] (GDT) give rise to a considerable reduction of the stabilizing contribution from the regions of favorable magnetic field curvature adjacent to the wall. The fact that the conducting walls do not completely stabilize the flute interchange mode was known for some time[3,4] but in the models studied previously the high conductivity of the walls did not affect the sign of the overall stabilizing or destabilizing contribution of the field line curvature, only the response rate of the mode was changed. In contrast, in the problem considered in Ref. 1, a switch from insulating to conducting end plates could bring the system from a robustly stable to a strongly unstable state.

In the present work we give another example of the destabilizing effect of conducting end plates. We show that the electron temperature gradient transverse to the magnetic field in a plasma bounded by conducting end plates gives rise to a rapid instability whose growth rate can be considerably larger than that of a flute instability. A remarkable feature of this new instability is that unlike the usual curvature driven flute instability, it can develop even in the systems with straight field lines.

The paper is organized as follows. In Sec. II we describe a model used in the stability analysis. In Sec. III the basic equations for temperature-gradient instability are derived. Section IV contains the analysis of the dispersion equation. Section V discusses the anomalous transport caused by this instability. In Sec. VI finite Larmor radius (FLR) effects are considered. Section VII contains discussion of results. The equations describing the instability for end plates of arbitrary resistivity are presented in Appendix A while Appendix B contains a detailed derivation of the equation describing the FLR effects.

II. The Model

In order to easily visualize the main features of the instability we consider a simplest possible model: the plasma in a homogeneous magnetic field (directed along the z axis) is confined at the ends by two conducting plates (Fig. 1), and the ion temperature T_i and density n are spatially uniform. As we only consider large-scale motion, we assume the quasineutrality of electrons and ions in the bulk of the plasma; hence we do not need to use subscripts "e" and "i" to distinguish between electron and ion density. The electron temperature T_e, which is taken to vary transverse to the magnetic field lines (in x direction), is the only spatially varying equilibrium parameter. We assume the properties of the end plates are such as to reflect most of the ions impinging upon them, and absorb only a small fraction $\varepsilon(\ll 1)$ of the incoming ion flux. The model of reflecting walls simulates to some extent the effect of magnetic mirrors. To maintain the stationary state of the plasma, sources of particles and energy are present in the plasma volume. The condition $\varepsilon \ll 1$ guarantees that the plasma equilibrium parameters are nearly uniform along the field lines and that the sources are small to $\mathcal{O}(\varepsilon)$.

The plasma lifetime τ in each flux tube is given by the ratio of the number of particles to the source strength, and is

$$\tau = \frac{\sqrt{\pi}\, L}{\varepsilon v_{Ti}} \tag{1}$$

where $v_{Ti} \equiv \sqrt{2T_i/m_i}$ is the ion thermal velocity. When $\varepsilon \ll 1$, τ is much greater than the ion transit time L/v_{Ti}.

It will be shown below that the instability growth rate Im ω can be much shorter than the ion bounce frequency v_{Ti}/L; i.e.

$$\frac{|\omega|L}{v_{Ti}} \gg 1 \ . \tag{2}$$

We will consider only that domain of plasma and instability parameters where condition (2) indeed holds. This allows us to neglect the source term in the analysis. We also assume that the equilibrium electron temperature does not vary appreciably along a field line; a condition that will be established if $\lambda_e/L \ll \left[\left(\frac{m_e}{m_i}\right)^{1/2} \frac{e\varphi}{T_e}\right]^{1/2}$ where λ_e is the electron mean free path, φ the plasma potential, and m_e and m_i the electron and ion masses.

Along with condition (2), we assume that the electron transit time L/v_{Te}, is much shorter than $|\omega|^{-1}$,

$$\frac{|\omega|L}{v_{Te}} \ll 1 \ . \tag{3}$$

The condition (3) means that electrons have a Boltzmann distribution along a field line.

As already noted, the plasma in a stationary state is nearly homogeneous along the field lines and therefore over the bulk of the plasma there is negligible equilibrium electric field parallel to the magnetic. Under such conditions, the balance of the electron and ion flow to the wall is provided by the formation of a Debye sheath which reflects the majority of the electrons.

We take the potential of the end plates to be zero; then the plasma potential $\varphi(x)$ is positive. To determine φ, one sets to zero the total current density due to electrons and ions impinging on each end wall. The ion current does not depend on φ and is equal to

$$j_{\|i}\Big|_{z=\frac{L}{2}} = \frac{\varepsilon e n \, v_{Ti}}{2\sqrt{\pi}} \tag{4}$$

where n is the plasma density. The electron current density is

$$j_{\|e}\Big|_{z=\frac{L}{2}} = -\frac{e n \, v_{Te}}{2\sqrt{\pi}} \, G(\varphi) \tag{5}$$

where $G(\varphi)$ is a dimensionless function whose details are determined by the electron distribution function in the energy range close to $e\varphi$. If in this range the electron distribution function only weakly deviates from Maxwellian (as is the case of sufficiently short mean free path λ_e or even long mean free path if $\omega > \nu$, where ν is the electron collision frequency), then

$$G = \exp(-e\varphi/T_e) \ . \tag{6}$$

In what follows we shall consider a situation that is typical for the gas-dynamic trap where λ_e is relatively small, so that G is determined by expression (6). Then the condition $j_{\|e} = j_{\|i}$ gives the following expression for the equilibrium plasma potential:

$$\varphi(x) = \Lambda \frac{T_e(x)}{e} \tag{7}$$

with

$$\Lambda = \ell n \left(\frac{1}{\varepsilon} \sqrt{\frac{T_e m_i}{T_i m_e}} \right) \ . \tag{8}$$

Typically, Λ is $5-7$. A broader discussion of the Debye sheath properties required for the stability analysis can be found in Refs. 5–10.

The presence of the equilibrium electric field $E = -d\varphi/dx$ causes the plasma to drift in y direction with the velocity

$$v_E = \frac{c}{B} \frac{d\varphi}{dx} \ . \tag{9}$$

III. Basic Equations for Linear Instability

We consider a low-β plasma and neglect the perturbations of the magnetic field. The perturbations of the electric field are then curl-free and can be described by an electrostatic potential $\delta\varphi$. The motion of the plasma will be treated as two-dimensional in the x-y plane, with very weak dependence of the perturbations on z (caused by the coupling of the system to the end walls). More generally, this flute-like response can be justified if $L < v_A/\omega$, where

L is the axial length, v_A the Alfvén speed and ω the mode frequency (typically we will treat ω greater than the diamagnetic frequency).

To obtain a self-consistent equation for $\delta\varphi$, we first consider the motion of the bulk of the plasma and express the perturbations of electron and ion current density at the wall in terms of $\delta\varphi$ by means of the continuity equation. Then we match these perturbations with the perturbed boundary conditions given by (4) and (5).

For perturbations of the form $\exp(-i\omega t + iky)$ the transverse ion motion is described by the equation

$$-i\Omega\, m_i\, \delta\mathbf{v}_{\perp i} = \frac{e}{c}\,\delta\mathbf{v}_{\perp i} \times \mathbf{B} - e\boldsymbol{\nabla}_\perp\,\delta\varphi \tag{10}$$

where $\Omega = \omega - kv_E$. For a low frequency instability with $\Omega \ll \omega_{Bi} \equiv eB/m_i c$ we have:

$$\delta\mathbf{v}_{\perp i} \simeq -c\frac{\boldsymbol{\nabla}_\perp\,\delta\varphi \times \mathbf{B}}{B^2} + ic\frac{\Omega}{\omega_{Bi}}\,\frac{\boldsymbol{\nabla}_\perp\,\delta\varphi}{B}\ . \tag{11}$$

The terms of order ω_{Bi}^{-2} and higher are neglected. As the electron inertia is much smaller than the ion, we retain only the leading term in the expression for $\delta\mathbf{v}_{\perp e}$:

$$\delta\mathbf{v}_{\perp e} \simeq -c\frac{\boldsymbol{\nabla}_\perp\,\delta\varphi \times \mathbf{B}}{B^2}\ . \tag{12}$$

For the localized perturbations with characteristic scale much smaller than the scale-length $a \equiv |d\ell n\, T_e/dx|^{-1}$ of the equilibrium state; i.e., with

$$k, \left|\frac{\partial}{\partial x}\right| \gg a^{-1} \tag{13}$$

we obtain from (11) that

$$\boldsymbol{\nabla}\cdot\delta\mathbf{v}_{\perp i} = ic\frac{\Omega}{\omega_{Bi}}\,\frac{\boldsymbol{\nabla}_\perp^2\,\delta\varphi}{B} \tag{14}$$

while from Eq. (12) we find $\boldsymbol{\nabla}\cdot\delta\mathbf{v}_{\perp e} = 0$. The ion and electron continuity equations are of the following form:

$$-i\Omega e\delta n + icen\,\frac{\Omega}{\omega_{Bi}}\,\frac{\boldsymbol{\nabla}_\perp^2\,\delta\varphi}{B} + \frac{\partial\delta j_{\|i}}{\partial z} = 0 \tag{15}$$

$$i\Omega e\delta n + \frac{\partial\delta j_{\|e}}{\partial z} = 0\ . \tag{16}$$

By summing these two equations, we obtain that

$$\frac{\partial}{\partial z}\left(\delta j_{\|e}+\delta j_{\|i}\right)=-icen\,\frac{\Omega}{\omega_{Bi}}\,\frac{\nabla_\perp^2\,\delta\varphi}{B}\,. \tag{17}$$

Note that we have omitted the pressure terms in Eqs. (10) and (12). This is because the essentially 2D motions that we are considering are almost incompressible in a low-β plasma and therefore the density perturbations (in the case of an initially homogeneous plasma) are small. Under such conditions the additional pressure terms in Eqs. (10) and (12) do not contribute to $\nabla\cdot\delta\mathbf{v}_\perp$.

Because of the symmetry of the system with respect to $z=0$ plane, both $\delta j_{\|e}$ and $\delta j_{\|i}$ vanish at $z=0$. On the other hand, as the right-hand side of Eq. (17) is independent of z, the integration of (17) along z gives (accounting for two end walls)

$$\delta j_{\|e}+\delta j_{\|i}\Big|_{z=\frac{L}{2}}=-\frac{icen\,\Omega L}{2\omega_{Bi}\,B}\,\nabla_\perp^2\,\delta\varphi\,. \tag{18}$$

Now we use the boundary conditions (4) and (5), with $G(\varphi)$ defined by Eq. (6). This yields

$$\delta j_{\|e}+\delta j_{\|i}\Big|_{z=\frac{L}{2}}=-\frac{\varepsilon en\,v_{Ti}}{2\sqrt{\pi}}\left[\left(\frac{1}{2}+\Lambda\right)\frac{\delta T_e}{T_e}-\frac{e\delta\varphi}{T_e}\right] \tag{19}$$

with Λ given by Eqs. (7) and (8).

To find δT_e, we use the equation

$$-i\Omega\,\delta T_e+\delta\mathbf{v}_{\perp e}\cdot\nabla T_e=0\,. \tag{20}$$

This equation neglects the direct axial energy loss and assumption that is justified if $1/\Omega\tau\ll 1$. Using Eqs. (12) and (20), we find for δT_e:

$$\delta T_e=-\frac{kc\,\delta\varphi}{\Omega B}\,\frac{dT_e}{dx}\,. \tag{21}$$

We have neglected the adiabatic contribution to δT_e (which is of order of $T_e\,\delta n/n$), as it is much smaller than the perturbation determined from (21). As the equilibrium ion temperature is independent of x, its perturbation is negligible. Of course, we have neglected in (21) the thermal flux to the walls and the volume heat sources which both are of order $\varepsilon v_{Ti}/|\omega|L$ as compared with the retained term.

Finally, combining Eqs. (18), (19), and (21) we obtain the equation describing a flute-like (where $\delta\varphi$ is independent of z in the bulk of the plasma) perturbation:

$$\Omega^2 \left(k^2 - \frac{d^2}{dx^2}\right) \delta\varphi + \frac{i}{\tau}\frac{\omega_{Bi}^2 m_i}{T_e} \Omega\delta\varphi + \frac{i}{\tau}(\Lambda + 1/2)\frac{k\omega_{Bi}}{a}\delta\varphi = 0 \tag{22}$$

with $a \equiv (d\ell n\, T_e/dx)^{-1}$ and τ is given by Eq. (1).

For localized perturbations, one can assume that $\delta\varphi$ is proportional to e^{iqx}, where q is an x component of a wavenumber. The most unstable perturbations correspond to $q \ll k$. Then Eq. (22) reduces to the following dispersion relation:

$$\Omega^2 + \Omega\frac{i\nu}{\mathrm{w}^2} + i\frac{\Gamma^2}{\mathrm{w}} = 0 \tag{23}$$

where

$$\mathrm{w} = ka\ ;\ \nu = \frac{\omega_{Bi}^2 m_i a^2}{\tau T_e}\ ;\ \Gamma^2 = \frac{\omega_{Bi}}{\tau}(\Lambda + 1/2)\ .$$

In dimensionless form Eq. (23) can be written as

$$\tilde{\Omega}^2 + i\frac{\tilde{\Omega}}{\eta^2} + \frac{i}{\eta} = 0 \tag{24}$$

where $\tilde{\Omega} = \Omega/(\Gamma^4/\nu)^{1/3}$ and $\eta = \mathrm{w}(\Gamma/\nu)^{2/3}$.

IV. Analysis of the Dispersion Relation

The first term in the dispersion relation describes the inertia of the flute: it arises from the inertial drift of the ions (the last term in Eq. (11)). The second term, linear in Ω, corresponds to the finite impedance of the Debye sheath; it is similar to the one considered by Kunkel and Guillory.[4] When the electron temperature is homogeneous, and the last term in the dispersion relation vanishes, the balance of the first two terms causes an exponentially decaying mode. The instability described in this paper is caused by the last term in the dispersion relation which is proportional to dT_e/dx.

A somewhat similar instability of the gas-discharge plasma was considered by B.B. Kadomtsev[5] who discussed a model from which our results can in principle be extracted. However, the discussion in Ref. 5 was concentrated on the instabilities caused by a large axial variation of the unperturbed plasma parameters (that is typical for the gas discharges) while in the solenoids of mirror devices with relatively good axial confinement (whose properties are the main object of our paper), the axial density variations are negligible. In contrast to Ref. 5 we stress in this paper the large effect due to the electron temperature gradient.

Note that in the case of an insulating wall our instability disappears. Indeed, in this case the boundary condition for the current (18) flowing out from the plasma volume should be

$$\delta j_{\|e} + \delta j_{\|i}\Big|_{z=\frac{L}{2}} = 0 \ , \tag{25}$$

instead of the boundary condition (19) . By comparing (18) and (25) we immediately find that in the case of insulating wall the solution of the dispersion relation is just $\Omega = 0$. A more general case of a resistive end wall is described in Appendix A.

To take into account the usual curvature driven flute instability one has to add a $-\Gamma_f^2$ term to Eq. (23), where Γ_f is the growth rate of the flute interchange instability; in paraxial systems it is of the order of v_{Ti}/L, i.e., of the order of the inverse ion transit time. It will be shown below that the growth rate of our instability can be much larger than Γ_f so that for such cases the interchange term is small.

Now we proceed to a formal analysis of the dispersion relation (25). Using the property $\mathrm{Re}\,\omega(-k) = -\mathrm{Re}\,\omega(k)$, $\mathrm{Im}\,\omega(-k) = \mathrm{Im}\,\omega(k)$, we shall consider only the positive values of ka. The explicit expression for the unstable root of Eq. (23) at $ka > 0$ can be written in the following form:

$$\mathrm{Re}\ \Omega = -\left(\frac{\Gamma^4}{\nu}\right)^{1/3} \frac{1}{2\sqrt{2}\,\eta^2}\ \left[(1+16\eta^6)^{1/2}-1\right]^{1/2} \tag{26}$$

$$\mathrm{Im}\ \Omega = \frac{\left(\dfrac{\Gamma^4}{\nu}\right)^{1/3}}{2\sqrt{2}\,\eta^2}\quad \left\{\left[(1+16\eta^6)^{1/2}+1\right]^{1/2}-\sqrt{2}\right\} \tag{27}$$

Dependence of Im $\tilde{\Omega}$ and Re $\tilde{\Omega}$ on η is illustrated by the solid curve in Fig. 2. At large k, $k \gtrsim a^{-1}(\nu^2/4\Gamma^2)^{1/3}$,

$$\mathrm{Im}\ \Omega \simeq \frac{\Gamma}{\sqrt{2ka}}\ ; \tag{28}$$

at small k, $k \lesssim a^{-1}(\nu^2/4\Gamma^2)^{1/3}$

$$\mathrm{Im}\ \Omega \simeq \frac{\Gamma^4}{\nu^3}\,(ka)^4\ . \tag{29}$$

The maximum growth rate is attained at

$$k = k_0 \cong 1.8a^{-1}\left(\frac{\nu^2}{\Gamma^2}\right)^{1/3} \tag{30}$$

and is equal to

$$(\mathrm{Im}\ \Omega)_{\mathrm{max}} = 0.38\left(\frac{\Gamma^4}{\nu}\right)^{1/3}\ . \tag{31}$$

At the same k,

$$\mathrm{Re}\ \Omega = 0.52\left(\frac{\Gamma^4}{\nu}\right)^{1/3} \tag{32}$$

and is of the order of $(\mathrm{Im}\ \Omega)_{\mathrm{max}}$.

The condition $(\mathrm{Im}\ \Omega)_{\mathrm{max}} > \Gamma_f \sim v_{Ti}/L$ yields the following constraint on the plasma parameters:

$$\left(\Lambda + \frac{1}{2}\right)^2 \frac{T_e}{T_i}\frac{L^2}{a^2} > \frac{v_{Ti}\tau}{L}\ . \tag{33}$$

If we apply this condition to a gas-dynamic trap plasma[11] ($L/a \sim 50$, $T_e/T_i \sim 1$, $\Lambda \sim 5$, $\tau \sim 30\,L/v_{Ti}$) we see that it is satisfied by a very large margin. The k value corresponding to the maximum growth rate

$$k \sim 2.5\rho_i^{-1}(T_i/T_e)^{2/3}(a/\Lambda v_{Ti}\tau)^{1/3} , \tag{34}$$

for the same numerical example is still much smaller than $\rho_i^{-1} \equiv \omega_{Bi}/v_{Ti}$ (viz. $k\,\rho_i \sim 0.13$), so that the fluid approximation used in our analysis is well satisfied. This k-value also corresponds to only a moderate poloidal number $m \sim ka$ in GDT. Using $T_i = 100\,\mathrm{eV}$, $B = 10^3$ gauss, $a = 50\,\mathrm{cm}$. We find for a hydrogen plasma $m \simeq 2-3$. For more quantitative predictions a non-eikonal mode analysis is needed.

An interesting feature of the instability under consideration is that, like the interchange mode, its maximum growth rate

$$(\mathrm{Im}\ \Omega)_{\max} \simeq 0.30\,\frac{v_{Ti}}{L}\left(\frac{\Lambda L}{a}\right)^{2/3}\left(\frac{L}{v_{Ti}\tau}\right)^{1/3}\left(\frac{T_e}{T_i}\right)^{1/3} \tag{35}$$

does not depend on the magnetic field. If τ is not too large, the growth rate of Eq. (35) considerably exceeds the interchange growth rate, which scales as v_{Ti}/L.

Let us now consider the limitations on the plasma parameters that follow from our assumption that the magnetic field perturbations are negligible. The source that produces these perturbations is just the plasma current $\delta\mathbf{j}$. Maxwell equations show that the perturbations of the magnetic field at $k \gg q$ is directed along x axis and can be estimated by the formula

$$\delta B_x \sim \frac{2\pi}{kc}\delta j_{\|} .$$

Using Eqs. (12) and (17) one can easily show that

$$\delta B_x \sim 2\pi\, Lmn\,\Omega\delta v_{xi}/B .$$

The presence of δB_x causes the deviation of the magnetic field from z direction by the value of $L\delta B_x/B$. This deviation is significant when it is of the order of the displacement of the flute $\delta v_x/\Omega$. From this consideration we obtain the criterion for neglecting the magnetic field perturbations:

$$(\Omega L)^2 \ll B^2/2\pi\, n m_i \, . \tag{36}$$

Choosing the Ω corresponding to the maximum growth (see Eq. (32)) rate gives the following constraint on the $\beta(\equiv 8\pi n\, T_i/B^2)$ value:

$$\beta < 12 \left(\frac{a}{\Lambda L}\right)^{4/3} \left(\frac{v_{Ti}\tau}{L}\right)^{2/3} \left(\frac{T_i}{T_e}\right)^{2/3} \, . \tag{37}$$

For the numerical example considered above ($L/a \sim 50$, $T_e/T_i \sim 1$, $\Lambda \sim 5$, $\tau \sim 30\, L/v_{Ti}$) this inequality gives $\beta < 7 \times 10^{-2}$. In the experiments[11] on GDT this condition was satisfied as $\beta < 2 \times 10^{-2}$.

V. Enhanced Transport Caused by a Temperature Gradient Instability

The development of the instability to a nonlinear stage gives rise to an enhanced transverse thermal transport. As we assume T_i and n to be constant transverse to the magnetic field line, the transport only causes electron heat flux. For a strong instability with Im $\Omega \sim$ Re Ω, the anomalous thermal diffusivity χ_{anom} can be estimated as (cf. Ref. 10)

$$\chi_{\text{anom}} \sim \left(\frac{1}{k^2}\, \mathrm{Im}\, \Omega\right)_{\max} \, . \tag{38}$$

The relationships (30) and (31) show that the maximum of Im Ω/k^2 is attained at $k \sim k_0 \sim 1.8 a^{-1}(\nu^2/\Gamma^2)^{1/3}$ so that for χ_{anom} we have

$$\chi_{\text{anom}} \sim .1\, \frac{a^2}{\tau} \left(\frac{T_e}{T_i}\right)^{5/3} \left(\frac{\Lambda}{\varepsilon}\, \frac{L}{a}\right)^{4/3} \left(\frac{\rho_i}{a}\right)^2 \, . \tag{39}$$

Assuming as previously $L/a \sim 50$, $T_e/T_i \sim 1$, $\Lambda \sim 5$, $\varepsilon \sim 1/30$ and taking $\rho_i/a \sim 1/30$ we obtain

$$\chi_{\text{anom}} \sim 16\, \frac{a^2}{\tau} \, .$$

This means that anomalous heat transport would make significant changes in the T_e profile within $\sim 1/16$ of the axial confinement time τ. This implies that the plasma can not bear large gradients of T_e, and thus experimentally T_e should be constant over the major part of the plasma cross-section. The problem of the narrow boundary layers that are then formed at the radial plasma edge (the edge is usually determined by material limiters) is beyond that scope of this paper. We just note that observations made in the Novosibirsk gas-dynamic trap experiment[13] show that T_e is almost constant transverse to the magnetic field.

VI. Finite Larmor Radius Effects

FLR effects are present only if equilibrium spatial gradients are present in the density and ion temperature. The following notation for the various scale lengths are introduced:

$$a_n^{-1} = \frac{d}{dx}\ell n\, n \;,\; a_{Ti}^{-1} = \frac{d}{dx}\ell n\, T_i \;,\; a_{Pi}^{-1} = \frac{d}{dx}\ell n(n_i T_i) \;. \tag{40}$$

The treatment of finite Larmor radius effects is standard and details are presented in Appendix A. The principal result is an extra characteristic finite Larmor radius term in the dispersion relation, which now takes the form

$$\Omega^2 + \left(\frac{i\nu}{\mathrm{w}^2} - \omega^* \mathrm{w}\right) + i\,\frac{\Gamma^2}{\mathrm{w}} = 0 \tag{41}$$

or in dimensionless variables

$$\tilde{\Omega}^2 + \tilde{\Omega}\left(\frac{i}{\eta^2} - \eta\,\delta\right) + \frac{i}{\eta} = 0 \tag{42}$$

where

$$\Gamma^2 = \frac{\omega_{Bi}}{\tau}\left[\Lambda + \frac{1}{2}\left(1 - \frac{a}{a_{Ti}}\right)\right]$$

$$\delta = \frac{T_i}{T_e}\frac{a}{a_{Pi}}\left[\Lambda + \frac{1}{2}\left(1 - \frac{a}{a_{Ti}}\right)\right]^{-1}$$

$$\omega^* = \frac{cT_i}{eBaa_{pi}}$$

and all other quantities are defined after Eqs. (23) and (24) (note that Γ^2 has been slightly generalized).

This dispersion relation shows that the driving force of the instability under consideration remains the gradient of the electron temperature (combined with the high conductivity of the end walls). The density gradient alone does not contribute to the driving force while the presence of the ion temperature gradient gives rise to the standard FLR contribution plus only a small correction to Γ^2.

To evaluate the importance of FLR we estimate the size of the ω^* term at the maximum growth that was evaluated without this term. Hence with $\mathrm{w} = 1.8(\nu^2/\Gamma^2)^{1/3}$ we find that the FLR term is of importance if

$$\omega^* \gtrsim \frac{\nu}{\mathrm{w}^3} \approx \frac{.17\Gamma^2}{\nu} \,. \tag{43}$$

Assuming that $a/a_{Ti} \approx 1$ and that the scale lengths of T_e, T_i and n are comparable, Eq. (43) is equivalent to the condition

$$1 \gtrsim .17 \, \frac{a_{P_i}}{a} \frac{T_e}{T_i} \Lambda \,. \tag{44}$$

Thus for $a_{Ti} \approx a$, the maximum growth rates may be somewhat affected by FLR effects.

The dashed curve in Fig. 2 gives the real and imaginary frequencies with FLR present (for completeness damped solutions are plotted as well) when $a_{Ti} = a_{P_i} = a$, and $T_i = T_e$ and $\Lambda = 5$ (i.e. $\delta = 0.2$). We see that the maximum growth rate is practically unaffected by FLR effects. For larger k, the maximum growth is considerably reduced, and when $\mathrm{w} \gg (\nu/\omega^*)^{1/3}$ and $\left(\frac{\nu}{\Gamma}\right)^{2/3}$ the growth rate, which is always present if $a_{pi}\, a > 0$, is given by

$$\Omega = \frac{i\Gamma^2}{\mathrm{w}^2 \, \omega^*} \,. \tag{45}$$

In Fig. 3 we plot the real and imaginary frequencies for different values of δ. Note that the system is stable for $\delta < -1$; for $\delta = -1$ the roots of the dispersion relation are $\tilde{\Omega} = -\eta$ (the marginal mode) and $\tilde{\Omega} = -i/\eta^2$ (the damped mode).

VII. Discussion

We have considered a simple model for a plasma in a straight magnetic field where the current flow to the wall is important and determined by Eqs. (4) and (5). We have shown that an electrostatic instability arises from the effect of the cross-field electron temperature gradient on the outflow current. Despite its simplicity, the model and the resulting instability should be applicable to many diverse systems, including mirror machines, the edge of toroidal devices and flux tubes connected to boundaries of the ionosphere in planetary magnetospheres and the photosphere in stars.

We have also considered the effect of ion temperature gradients, density gradients, and FLR on the instability. When there is an electron temperature gradient, the magnetic field curvature is found to often be insignificant in affecting the dispersion relation (this is true if Eq. (34) is satisfied, which is likely unless axial confinement is very good). Further, the presence of the density and ion temperature gradients does not qualitatively change the overall picture of instability. Due to FLR effects the maximum growth rate is somewhat reduced. The growth rate at large k is reduced significantly, but this part of the spectrum is not expected to be crucial to the mode's identification or nonlinear evolution.

In mirror machine fusion devices, like GDT or tandem mirrors, it is the long central solenoid that is simulated by the model of a homogeneous magnetic field. For the application of our model we consider the plugs or plasma expanders as a place where the potential structure is established, but otherwise does not affect the plasma dynamics. This is justified if these structures are sufficiently short, which is the case (for a flute perturbation) if the plasma inertia of the end region is small compared to the central region and ion transit time, τ_s, through this region is sufficiently short, $\omega\tau_s \ll 1$. When these conditions are fulfilled, the role of end structure is the same as the Debye sheath in our model; only the total potential drop between the wall and the plasma is important and it determines the current $\delta j_{\|}$ through the end structure as a function of the plasma potential $\delta\phi$ in the central solenoid. For GDT, for which a strongly collisional regime is typical, we can use our results virtually without change. We need only replace the loss coefficient ε by the inverse mirror ratio $1/R$. We have already noted that significant transverse electron temperature gradients have not been observed in GDT when they were expected. The absence of such temperature gradients may be due to the nonlinear plasma relaxation resulting from the linear instability described here.

In well-confined tandem mirror devices of low collisionality, Eqs. (2) and (5) may need to be modified. This would then lead to a significantly different response for $\delta j_{||}$ than the one given by Eq. (19) (cf. Refs. 5, 14,15) and consequently one will obtain an altered dispersion relation. In addition, due to the smaller plasma losses, one can expect more significant contributions from FLR and curvature effects in the dispersion relation. The study of this problem is beyond the scope of the present paper.

Our results can also be applied to a plasma in a toroidal device on the open field lines that cross the surface of a limiter. As the limiter is probably absorbing a good fraction of the ions hitting it, we should consider ε to be of order of unity, and the ion confinement time of order of the ion transit time. Our instability can still play an important role, as its growth rate can strongly exceed the inverse transit time. The most obvious consequence of the instability should be a considerable broadening of the transition region beyond the limiter's edge.

The response function $\delta j_{||}$ can be changed if there is a strong secondary emission from the limiter. If the secondary emission coefficient η exceeds unity, then the potential drop in the Debye sheath decreases from a few T_e/e to a value that is closer to a fraction of T_e/e. However, as the potential drop is still present (see, e.g., Ref. 10) the response function remains similar to Eq. (19), but with $\Lambda \simeq 1$, rather than $\Lambda \simeq 6$. Instability is still likely but as Λ is no longer large, other competing effects such as ion temperature gradient may be important. The accurate description of this regime needs further investigations.

Acknowledgments

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Appendix A: Finite Resistivity of Walls

When the walls have a finite resistivity, the potential at the conducting plate will vary and the electron end current depends on the difference of the conductor and plasma potential. In terms of our Maxwellian model, the electron end current is

$$j_{\|e} = \frac{-\varepsilon en\, v_{Te}}{2\sqrt{\pi}} \exp\left[\frac{e(\varphi_c - \varphi)}{T_e}\right] \tag{A-1}$$

where φ_c is potential on the conductor and φ the plasma potential. The ion end current is $j_{\|i} = \varepsilon en_0\, v_{Ti}/2\sqrt{\pi}$. In equilibrium we take $\varphi_c = 0$, and φ is the plasma floating potential, and $j_{\|i} + j_{\|e} = 0$ the perturbed electron current is then

$$\delta j_{\|e} = -\frac{\varepsilon en_0\, v_{Ti}}{2\sqrt{\pi}} \left[\frac{\delta n}{n_0} - \frac{e(\delta\varphi - \delta\varphi_c)}{T_e} + (\Lambda + 1/2)\frac{\delta T_e}{T_e}\right] , \tag{A-2}$$

while the perturbed ion current is

$$\delta j_{\|i} = \frac{\varepsilon en\, v_{Ti}}{2\sqrt{\pi}} \frac{\delta n}{n} .$$

Summing the two currents then gives the end current

$$\delta j_{\|} = -\frac{\varepsilon en\, v_{Ti}}{2\sqrt{\pi}} \left[\left(\Lambda + \frac{1}{2}\right)\frac{\delta T_e}{T_e} - \frac{e(\delta\varphi - \delta\varphi_c)}{T_e}\right] . \tag{A-3}$$

We shall assume that the skin effect is negligible so that the current being fed by the plasma penetrates the end plates to give rise to a wall current density that uniformly fills the thickness of the plate whose width is b. The current in the endplate (this current is obtained by integrating the current density in the wall over its thickness) is related to the potential ϕ_c through Ohm's law

$$\mathbf{I} = -\sigma b\, \nabla_\perp \varphi_c . \tag{A-4}$$

The current continuity equation determines $\nabla \cdot \mathbf{I}$ in terms of the plasma current entering the plate

$$\delta j_{\|}\Big|_{z=\frac{L}{2}} = \nabla \cdot \mathbf{I} = -\sigma b\, \nabla_\perp^2 \varphi_c . \tag{A-5}$$

Now substituting (A-5) into Eq. (A-2) yields

$$\delta j_{\|}\Big|_{z=\frac{1}{2}} = -\frac{\dfrac{\varepsilon en\, v_{Ti}}{2\sqrt{\pi}}\left[-\dfrac{e\delta\varphi}{T_e}+\left(\Lambda+\dfrac{1}{2}\right)\dfrac{\delta T_e}{T_e}\right]}{1+\dfrac{\varepsilon ne^2\, v_{Ti}}{2\sqrt{\pi}\,T_e\,\sigma\, b\, k_\perp^2}} \tag{A-6}$$

where $k_\perp^2 = k^2 + q^2$. If we use Eq. (A-6), instead of Eq. (19) for $\delta j_{\|}$, and substitute into Eq. (18) we obtain a generalization to Eq. (22) (we also use Eq. (21) to obtain δT_e in terms of $\delta\varphi$),

$$\Omega^2\left(k_\perp^2+\frac{\varepsilon n\, v_{Ti}\, e^2}{2\sqrt{\pi}\,T_e\,\sigma b}\right)+\frac{i}{\tau}\frac{\omega_{Bi}^2\, m_i}{T_e}\Omega+\frac{i}{\tau}\left(\Lambda+\frac{1}{2}\right)\frac{k\omega_{Bi}}{a}=0\ . \tag{A-7}$$

In the limit $\sigma \to \infty$ the high conductivity result is recovered, while if $\sigma \to 0$ only the solutions where $\Omega \to 0$, expected for the insulation boundary condition case, is present.

The wall resistance affects the conducting boundary conditions when

$$\sigma < \frac{e^2\, n\, L}{2\tau\, T_e\, k_\perp^2\, b}\ . \tag{A-8}$$

To roughly estimate the effect of resistance, we choose $n_0 = 10^{13}\,\mathrm{cm}^{-3}$, $T_e = T_i = 100\,\mathrm{eV}$, $b = 0.1\,\mathrm{cm}$, $L = 10^3\,\mathrm{cm}$, $\tau = 10^{-3}\,\mathrm{sec}$, $k_\perp\,\rho_i = .13$, $B = 10^3\,\mathrm{gauss}$ and we find that the conducting boundary condition is altered only if $\sigma \lesssim 10^{13}\,\mathrm{sec}^{-1}$. The electrical conductivities of metals are typically $\sigma \sim 10^{17}\,\mathrm{sec}^{-1}$. Hence, the resistance of conducting walls is typically too small to change the results given in the text. To prevent the instability described in the text, true insulation boundary conditions must be established, either by segmenting end plates, finding materials that remain insulators under plasma bombardment or by allowing for surface chemistry effects that can cause the surface of a conductor to behave as an insulator.

Appendix B: Effect of Finite Larmor Radius (FLR)

When the ion pressure varies transverse to the magnetic field lines FLR effects may be important. The treatment of the FLR response is well established in the literature.[16,17] Here we include details for the completeness of our discussion.

We now need to account for an equilibrium density gradient dn/dx and an equilibrium ion temperature gradient dT_i/dx. To account for the FLR effect, we add to Eq. (10), the diamagnetic drifts and a vector $\mathbf{f} = -\boldsymbol{\nabla}\cdot\boldsymbol{\Pi}$ with the gyrotropic tensor[18] $\boldsymbol{\Pi}$ defined as

$$\begin{aligned}\boldsymbol{\Pi} &= \frac{nT_i}{2\omega_{ci}}\left\{-\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)(\hat{\mathbf{x}}\hat{\mathbf{x}}-\hat{\mathbf{y}}\hat{\mathbf{y}})+\left(\frac{\partial v_x}{\partial x}-\frac{\partial v_y}{\partial y}\right)(\hat{\mathbf{x}}\hat{\mathbf{y}}+\hat{\mathbf{y}}\hat{\mathbf{x}})\right\}\\ &= \frac{nT_i}{2\omega_{ci}}\left[\mathbf{b}\times\boldsymbol{\nabla}\mathbf{v}+\boldsymbol{\nabla}\mathbf{b}\times\mathbf{v}\right]\ .\end{aligned} \tag{B-1}$$

The unperturbed drift velocity in the y direction is then found to be of the form

$$\begin{aligned} v_i &= v_E + v_i^{(T)} + v_i^{(n)}\\ v_e &= v_E - v_e^{(T)} - v_e^{(n)}\end{aligned} \tag{B-2}$$

where v_E is the electric drift velocity given by Eq. (9)

$$v_{i,e}^{(T)} = \frac{c}{eB}\frac{\partial T_{i,e}}{\partial x} \quad , \quad v_{i,e}^{(n)} = \frac{cT_{i,e}}{eBn}\frac{\partial n}{\partial x}\ . \tag{B-3}$$

For convenience we consider a system without velocity shear so that for the equilibrium $\boldsymbol{\Pi}_0 = 0$ (it turns out that our final results apply to a system with shear, but our assumption avoids some complicated algebraic details).

The perturbed equation of motion, written with the Lorentz force on the left-hand side, acquires the form

$$\begin{aligned} -\frac{e}{c}\,\delta\mathbf{v}_{\perp i}\times\mathbf{B} &= \frac{-\boldsymbol{\nabla}\delta p_i + \dfrac{\delta n}{n}\boldsymbol{\nabla}p_i}{n}\\ &\qquad - e\boldsymbol{\nabla}\delta\varphi + \frac{\delta\mathbf{f}}{n} + i(\omega - kv_i)\delta\mathbf{v}_{\perp i}\,m_i \end{aligned} \tag{B-4}$$

$$\frac{e}{c}\,\delta\mathbf{v}_{\perp e}\times\mathbf{B} = \frac{-\boldsymbol{\nabla}\delta p_e + \dfrac{\delta n}{n}\boldsymbol{\nabla}p_e}{n} + e\boldsymbol{\nabla}\delta\varphi\ . \tag{B-5}$$

Taking the cross product with $\mathbf{B}$ gives for $\delta\mathbf{v}_{\perp e,i}$

$$\delta\mathbf{v}_{\perp i} = \frac{c}{B^2}\left(-\nabla\delta\varphi - \frac{\nabla\delta p_i - \dfrac{\delta n}{n}\nabla p_i}{en}\right)\times\mathbf{B}$$

$$+ i\,\frac{m_i}{e}(\omega - kv_i)c\,\frac{\delta\mathbf{v}_{\perp i}\times\mathbf{B}}{B^2} + \frac{c}{en\,B^2}\,\delta\mathbf{f}\times\mathbf{B} \qquad \text{(B-6)}$$

$$\delta\mathbf{v}_{\perp e} = \frac{c}{B^2}\left(-\nabla\delta\varphi + \frac{\nabla\delta p_e - \dfrac{\delta n\,\nabla p_e}{n}}{en}\right)\times\mathbf{B}\;. \qquad \text{(B-7)}$$

The first term on the right-hand side of Eq. (B.6) is much larger than the second and third terms (the inertial and gyroviscous terms). However, as we shall see, we have to retain the subdominant terms since the dominant terms will automatically cancel in the charge neutrality condition.

The continuity equations for ion and electrons are given by

$$-i\omega\,\delta n + \nabla\cdot(\delta n\,\mathbf{v}_{\perp i} + n\delta\,\mathbf{v}_{\perp i}) + \tfrac{1}{e}\tfrac{\partial\delta j_{\|i}}{\partial z} = 0$$

$$-i\omega\,\delta n + \nabla\cdot(\delta n\,\mathbf{v}_{\perp e} + n\delta\,\mathbf{v}_{\perp e}) - \tfrac{1}{e}\tfrac{\partial\delta j_{\|e}}{\partial z} = 0\;.$$

Now inserting Eqs. (A.6) and (A.7) into these equations exactly yield after some algebra,

$$-i\Omega\,\delta n - \frac{c}{B^2}\nabla\delta\varphi\times\mathbf{B}\cdot\nabla n =$$

$$+\nabla\cdot\left[\frac{cn}{eB^2}\mathbf{B}\times(im_i(\omega - kv_i)\delta\mathbf{v}_i + \delta\mathbf{f}/n)\right] - \frac{1}{e}\frac{\partial\delta j_{\|i}}{\partial z} \qquad \text{(B-8)}$$

$$-i\Omega\delta n - \frac{c}{B^2}\nabla\delta\varphi\times\mathbf{B}\cdot\nabla n = \frac{1}{e}\frac{\partial\delta j_{\|e}}{\partial z} \qquad \text{(B-9)}$$

with $\Omega = \omega - k_y\,v_E$.

The large terms are on the left-hand side of this equation. We note that $\delta j_{\|i,e}$ is a small term, e.g., $\delta j_{\|i} \approx \varepsilon e\delta n\,v_{Ti}$, so that the ion current term is a factor $\varepsilon v_{Ti}/L\Omega$ smaller than the left-hand side. We estimate the large terms by subtracting Eq. (B-8) from Eq. (B-9). If we then integrate over z, assume $\delta j_{\|}(L/2) = -\delta j_{\|}(-L/2)$, we find

$$\delta j_{\|i} + \delta j_{\|e}\Big|_{L/2} = \frac{eL}{2}\nabla\cdot\left[\frac{cn}{eB^2}\mathbf{B}\times\left(im_i(\omega - kv_i)\delta\mathbf{v}_i + \frac{\delta\mathbf{f}}{n}\right)\right]\;. \qquad \text{(B-10)}$$

We have used that the perturbations are independent of z. Further, as Eq. (B-10) is now in terms of only small quantities, one may calculate to lowest order $\delta \mathbf{v}_{\perp i}$, $\delta \mathbf{f}$, as well as δT and δn (upon which $\delta j_{||}$ is dependent on). Thus from Eq. (B.8) we have

$$\delta n = -\frac{ck\delta\varphi \, dn/dx}{\Omega B} \; . \tag{B-11}$$

The temperature perturbation is determined from the linearized adiabatic gas law, which gives

$$-i(\omega - kv_{i,e}) \left(\frac{\delta T_{i,e}}{T_{i,e}} - \frac{2}{3}\frac{\delta n}{n} \right) + \delta \mathbf{v}_{\perp i,e} \cdot \left(\frac{\nabla T_{i,e}}{T_{i,e}} - \frac{2}{3}\frac{\nabla n}{n} \right) = 0 \; . \tag{B-12}$$

If we use only the leading order terms in δn and $\delta \mathbf{v}_{\perp i,e}$, we find after some algebra

$$\delta T_{i,e} = -\frac{c}{\Omega B} \, k\delta\varphi \, \frac{\partial T_{i,e}}{\partial x} \; . \tag{B-13}$$

Combining Eqs. (B-12) and (B-13) then give

$$\delta p_{i,e} = -\frac{c\, k\delta\varphi}{\Omega B} \frac{\partial p_{i,e}}{\partial x} \; . \tag{B-14}$$

The perturbation of the current density at the wall is determined from the equation analogous to Eq. (19)

$$\delta j_{||e} + \delta j_{||i} \Big|_{z=\frac{L}{2}} = \frac{-\varepsilon n\, v_{Ti} e}{2\sqrt{\pi}} \left[\Lambda \frac{\delta T_e}{T_e} + \frac{1}{2} \left(\frac{\delta T_e}{T_e} - \frac{\delta T_i}{T_i} \right) - \frac{e\delta\varphi}{T_e} \right] \; . \tag{B-15}$$

The leading order terms of Eq. (B-6) together with (B-1), (B-10), (B-11), (B-13), and (A-15) constitute a closed set of equations that describes the evolution of the linear system.

To simplify the analysis further, we use the approximation $k \gg q \equiv \frac{\partial}{\partial x} \gg a^{-1}$. Equation (B-10) then reduces to

$$\delta j_{||e} + \delta j_{||i} \Big|_{z=\frac{L}{2}} = \frac{icen\, k^2 L}{2\omega_{Bi} B} (\Omega - \omega^* \mathrm{w}) \delta\varphi \tag{B-16}$$

where $\omega^* = (v_i^{(T)} + v_i^{(n)})/a$, $\mathrm{w} = ka$. The left-hand side of Eq. (B-16) is now expressed in terms of $\delta\varphi$, using Eqs. (B-13) and (B-15). This then gives rise to the following dispersion relation

$$\Omega^2 + \left(\frac{i\nu}{\mathrm{w}^2} - \omega^* \mathrm{w} \right) \Omega + \frac{i\Gamma^2}{\mathrm{w}} = 0 \tag{B-17}$$

where

$$\nu = \frac{\omega_{Bi}^2\, m_i\, a^2}{\tau\, T_e} \ , \ \Gamma^2 = \frac{\omega_{Bi}}{\tau}\left[\Lambda + \frac{1}{2}\,(1 - a/a_{Ti})\right] \ , \ \mathrm{w} = ka \ , \ a_{Ti} = (d\,\ell n\, T_i/dx)^{-1} \ . \tag{B-18}$$

Using the dimensionless variables of Eq. (24), Eq. (B-17) can be rewritten as

$$\widetilde{\Omega}^2 + \widetilde{\Omega}\left(\frac{i}{\eta^2} - \eta\delta\right) + \frac{i}{\eta} = 0 \tag{B-19}$$

with $\delta = \dfrac{T_i}{T_e}\dfrac{a}{a_{Pi}}\left[\Lambda + \dfrac{1}{2}\,(1 - a/a_{Ti})\right]^{-1}$ and $a_{Pi}^{-1} = \dfrac{d}{dx}\,\ell n(n_i\, T_i)$.

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Figure Captions

1. Geometry of plasma. The hatched region represents the end plates.

2. Real and Imaginary Frequencies

   Figure (a) is the normalized real frequency and (b) is the normalized imaginary frequency. The $\delta = 0$ case corresponds to the zero FLR case of Sec. IV while the $\delta = 0.2$ curves are for finite FLR discussed in Section VI.

3. Real and imaginary normalized frequencies for various value of δ. Figure (a) is the real frequency and Fig. (b) the imaginary frequency.

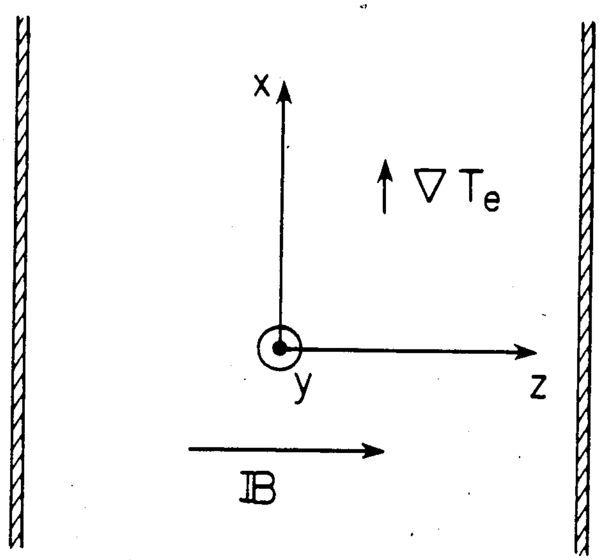


Fig. 1

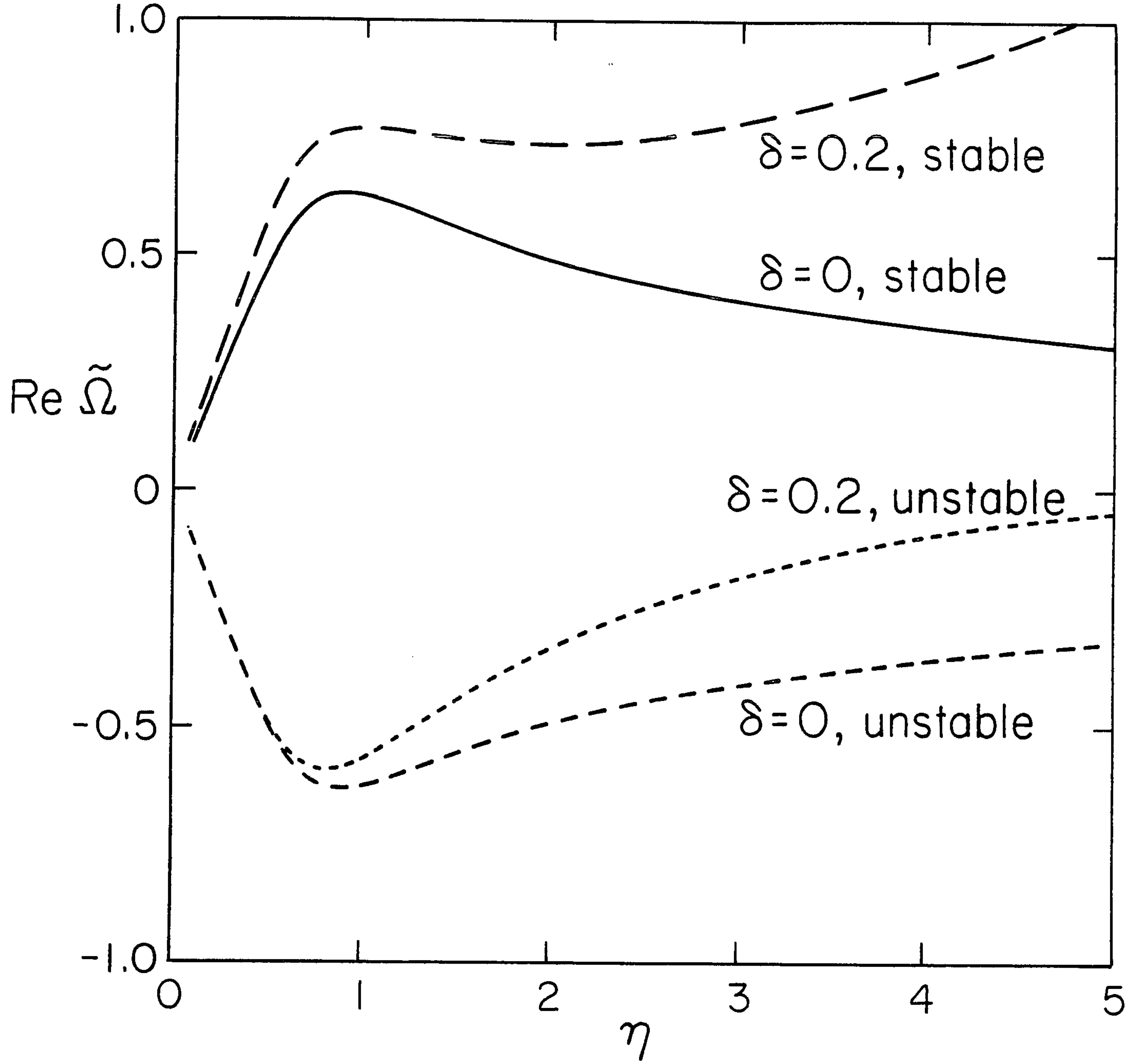

1.0
0.5
0
-0.5
-1.0
Re $\tilde{\Omega}$
0
1
2
3
4
5
η
δ=0.2, stable
δ=0, stable
δ=0.2, unstable
δ=0, unstable

Fig. 2a

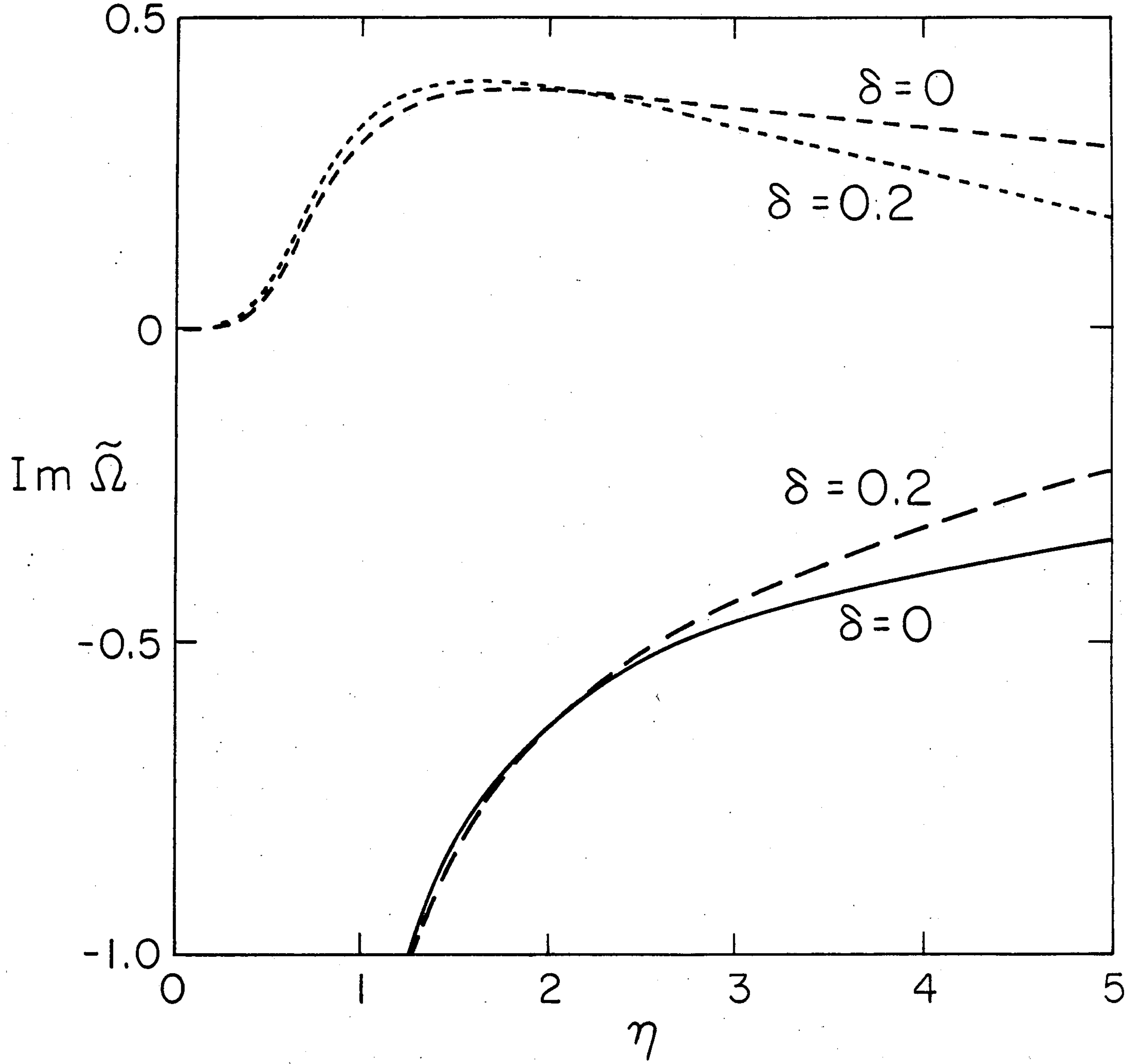
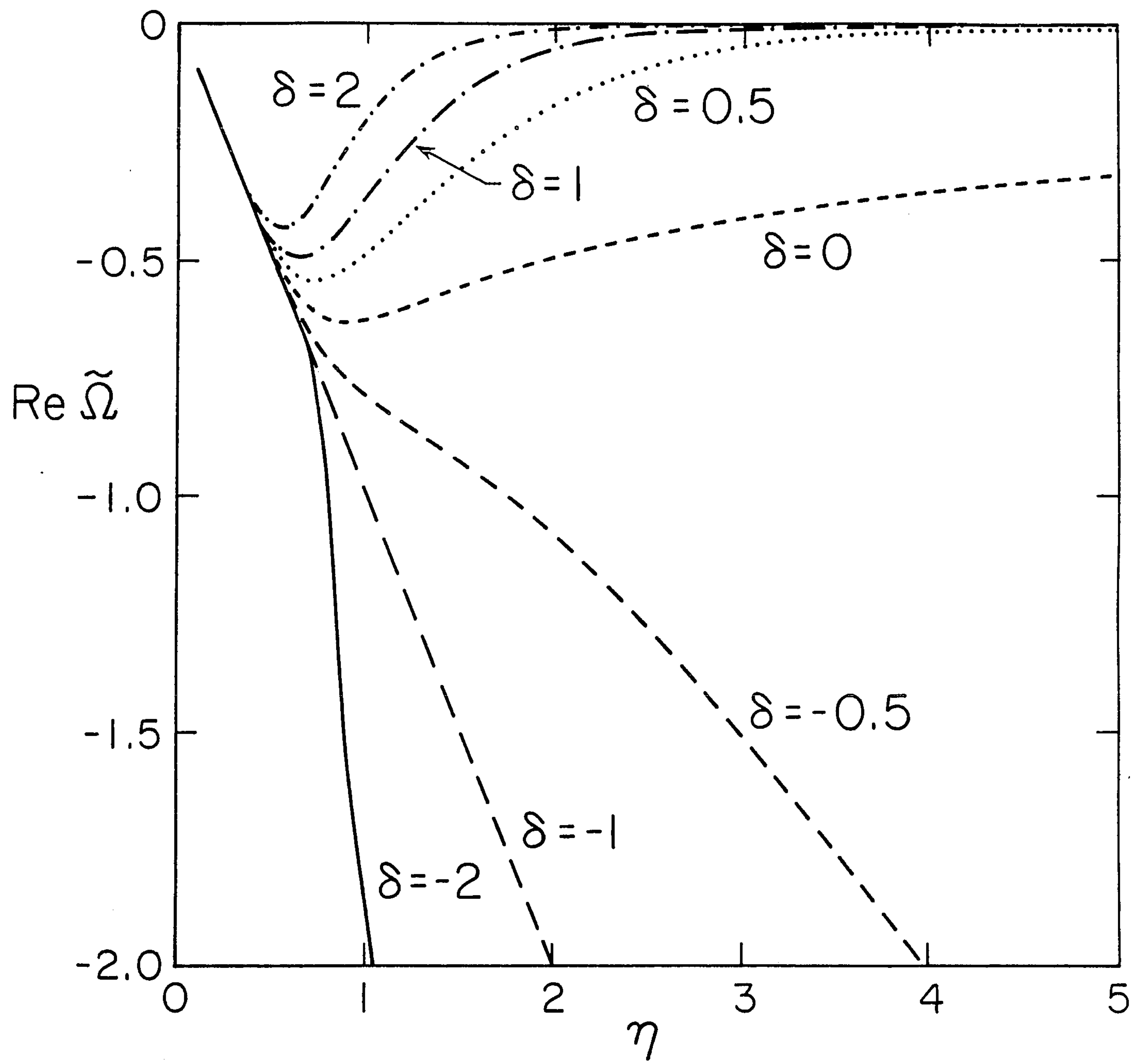

0.5
0
-0.5
-1.0
Im $\tilde{\Omega}$
$\delta = 0$
$\delta = 0.2$
$\delta = 0.2$
$\delta = 0$
0
1
2
3
4
5
η

Fig. 2b



0
-0.5
-1.0
-1.5
-2.0
Re $\tilde{\Omega}$
0
1
2
3
4
5
η
$\delta = 2$
$\delta = 0.5$
$\delta = 1$
$\delta = 0$
$\delta = -0.5$
$\delta = -1$
$\delta = -2$

Fig. 3a

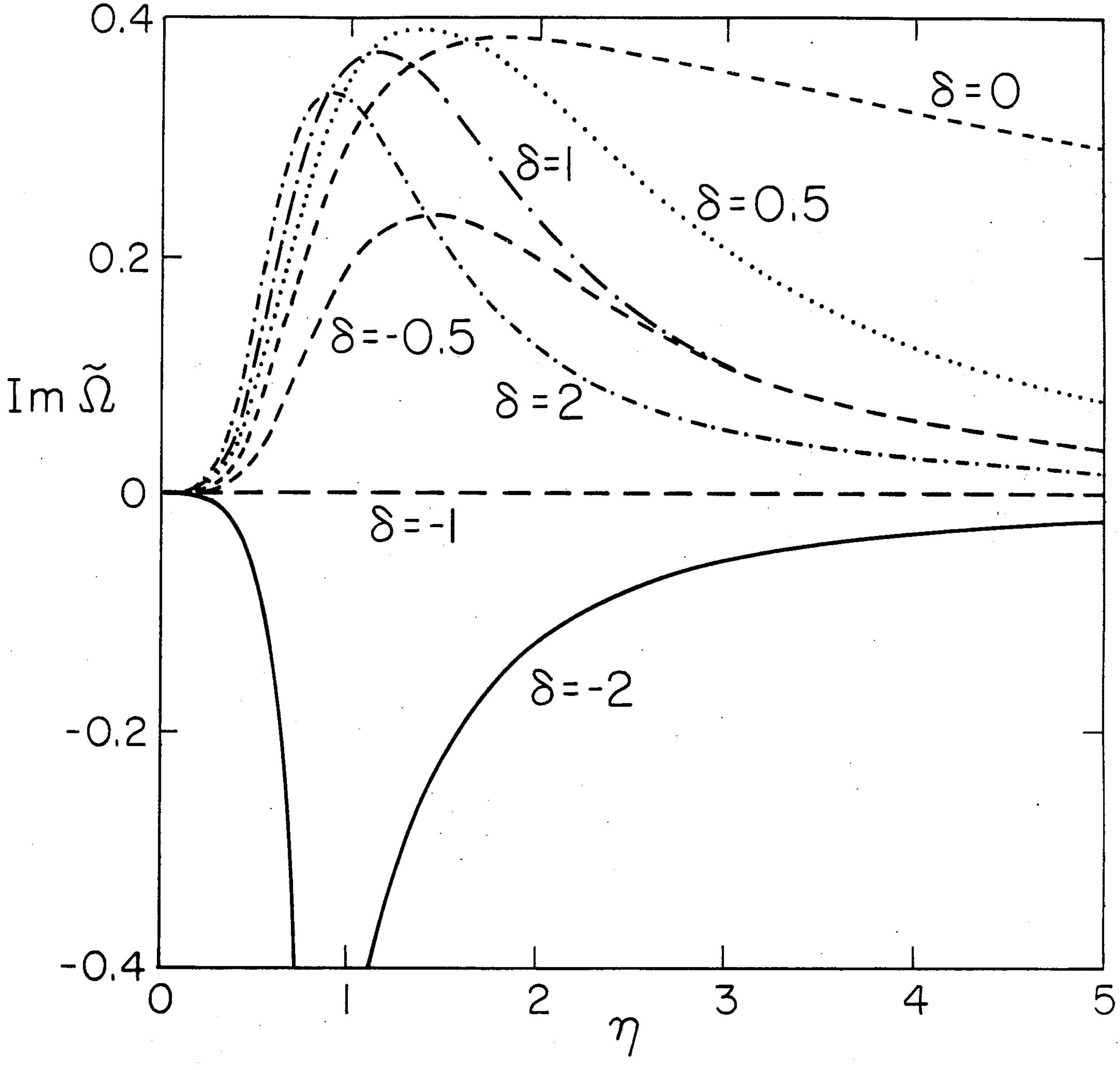

0.4
0.2
0
-0.2
-0.4
Im $\tilde{\Omega}$
0
1
2
3
4
5
η
$\delta = 0$
$\delta = 1$
$\delta = 0.5$
$\delta = -0.5$
$\delta = 2$
$\delta = -1$
$\delta = -2$

Fig. 3b