INSTITUTE FOR FUSION STUDIES

DOE/ET-53088-429 IFSR #429

Bent Crystal Septa For Beam Extraction From Accelerators

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March 1990

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Abstract

In this brief report, we summarize our discussion of the application of the phenomenon of the channeling of charged particles in curved single crystals which has been subject of several recent talks.[1–3]

I. Introduction

In this report, we will briefly discuss the principle issues of physics in the channeling of charged particles in curved crystals. The application of this aspect of crystal channeling to beam optics in high-energy accelerators in general and slow extraction in particular motivates the research. The objective of this report is to put into a more ponderable form the subject matter of several recent lectures.[1,2,3] It is by no means a comprehensive review of crystal channeling in bent crystals or otherwise. Common terminology will be defined, the general state of the art indicated and some recent new results from our own research mentioned. Details of the last item will be provided in a full length paper to be submitted for publication in the archival literature. We will conclude with some discussion of the open issues as they pertain to the slow extraction problem.

The phenomenon of particle channeling in crystals is the tendency of a highly collimated beam of charged particles directed along the axes or planes of high symmetry in crystalline solids to be steered by the atomic potentials of the atoms in the lattice. First suggested by Stark in 1912, this has been known experimentally for about twenty-five years and has become an important diagnostic tool in solid state materials science.[4] That this steering might also occur in crystals which are curved was first suggested by Tsyganov in 1976.[5] He also estimated the limits of curvature beyond which channeling would fail. It is the planar channeling that is interesting. While axial channeling has been observed in bent crystals, it has some features which make it less suitable in our applications. Also of interest here is the reduction, for positively charged channeled particles, of multiple scattering and nuclear background below that in an amorphous solid because of reduction of single particle interactions with the nuclei of atoms of the solid. Collisions with the valence electrons still occur. Although these are weak for heavy projectiles, the amount of transverse energy a particle can acquire in passing through the crystal is restricted by the requirement that it be confined to the potential well formed by atomic axes or planes. Too much transverse energy gain and the particle passes over the top of the well. This is called, in our context, normal dechanneling and will be discussed in Sec. II. In a curved crystal, the centrifugal force skews the potential well. A particle whose transverse energy exceeds the energy of the lower edge of the well will also dechannel. This is called bending dechanneling. It is discussed in Sec. III. Here too is some discussion of initial conditions. A unified treatment of both of these by a Fokker-Planck transport model is described in Sec. IV. Conclusions, some discussion of areas of concern and other open issues, and suggestions for further research comprise the final section.

II. Normal Dechanneling

The charged particles of interest are relativistic and their axial velocity is essentially unchanged in passing through the crystal. Their transverse momenta are small relative to their axial momenta and it suffices to treat their motion in the paraxial approximation. This ratio defines an angle in momentum space and the size of this angle relative to the critical angle for channeling, ψ_c, determines whether a given particle will be channeled. The critical angle is a function of the channel potential and $\psi_c = K/\sqrt{p}$. The constant K depends on the material and includes numerical factors of order unity which reflect the specific model used for the channel potential.[4,7] More important is the decrease in the critical angle with particle momentum, p (in rest momentum units). At SSC energy, this angle for channeling in the (110) planes of Si, is slightly more than a microradian. It increases as $\sqrt{Z}$ of the material.

As the particle proceeds down the channel, it scatters off the valence electrons in the channel and its transverse momentum increases. When the transverse momentum reaches $p\psi_c$, the particle dechannels. This is normal dechanneling. A typical measure of this process is the dechanneling half-length; the length of crystal at which the flux of a beam of channeled particles is reduced by one-half. It has become customary to estimate this using an exponential fit and a phenomenological dechanneling length at a determined momentum.

The dechanneling length at other energies is set by an inverse momentum scaling. We have used results from a Fokker-Planck calculation made in another context to obtain an analytic estimate of the dechanneling half-length. In a Gaussian approximation, the flux in the channel at a position, z is given by:

$$F = \operatorname{erf} \frac{\psi_c}{\theta_{\mathrm{RMS}}}, \tag{1}$$

where erf is the error function and θ_{RMS} is a function of distance into the crystal. The ratio, $(\psi_c/\theta_{\mathrm{RMS}})^2$, has been found[6] to be given by:

$$\frac{\psi_c}{\theta_{\mathrm{RMS}}} = \left(\frac{4a_c^2 kp}{Dz}\right)^{1/2}, \tag{2}$$

where p is the particle momentum in rest momentum units, a_c is the channel half-width, z is the distance into the crystal, D is the multiple scattering coefficient for scattering from valence electrons[6] based on a model in Ref. 7, and $a_c^2 k$ is the depth of the channel potential in rest energy units. It is equal to $4pC\frac{m_e}{m_I}Zr_eNa_c^2$, where r_e is the classical electron radius, Z the atomic number of the channeling material, N the number density of the material, and m_I is the mass of the projectile. Its atomic number has been taken to be one. The constant, C is a numerical factor of order unity which reflects the statistical model of the atom used to construct the channeling potential. For the (110) planes in Si, it is about 1/8 and the numerical value of $a_c^2 k \times m_I c^2$ is about 20 eV.

Inverting Eq. (1) for $F = 1/2$, gives an expression for the dechanneling half-length:

$$z_{1/2} = 16\frac{a_c^2 kp}{D}. \tag{3}$$

This gives, at SSC energy, a dechanneling half-length of about 110 m. This is consistent with the extrapolations from the phenomenological models although perhaps a factor of three larger. In any case, for the lengths of crystal of interest, the normal dechanneling is completely negligible at this energy. However, the interpretation of bent crystal channeling at energies near 100 GeV must account for normal dechanneling.

III. Bending Dechanneling and Channel Acceptance

Because of the skewing of the channel potential by the centrifugal well, some particles will be dechanneled by the bend. This is, perhaps more accurately, an internal acceptance effect. The skewing of the potential reduces the phase space volume of the channel so particles which are channeled in a straight portion of the crystal suddenly find a phase space mismatch when they arrive at the bend. Those particles which occupy a phase volume which matches the channel acceptance continue to be channeled in the bend subject only to normal dechanneling. The phase space reduction under statistical equilibrium has been calculated by Ellison.[8] The concept of statistical equilibrium is ubiquitous in channeling theory but seems to be loosely defined with the definition seemingly application dependent or perhaps better described as operational. In Ref. 8, the operational definition is the average of the distribution function over a period of the particle oscillation in the channel potential. The relation of the statistical equilibrium concept to transport theories with closer ties to first principles has, to my knowledge, not yet been elucidated; a point Ellison[9] himself has made. The results of this calculation are presented graphically. The dechanneled fraction increases with the particle momentum and with the curvature of the channel. It decreases with increasing Z of the material. For 20 TeV protons bent through $100m_{\rm rad}$ by a crystal of Si of length 10 cm corresponding to a radius of curvature of 10^5cm, the bending dechanneling in Ellison's theory is less than ten percent. By using Ge, either the crystal length could be reduced by a factor of about two or the radius of curvature doubled for the same bending.

The same question of phase space acceptance that occurs at the entrance to the crystal bend also obtains for the entrance into the channel at the face of the crystal. Some estimates[10] have appeared in the literature. These seem to apply to beams which are uniformly distributed in divergence angle and aligned with the channel planes. We have made our own estimate for Gaussian beams which may make a mean angle with the channel. This can be expressed in terms of a surface acceptance, A, given by:

$$A = \frac{1}{2}\left(\operatorname{erfc}\frac{\psi_c + \theta}{\theta_{\mathrm{RMS}}} + \operatorname{erfc}\frac{\psi_c - \theta}{\theta_{\mathrm{RMS}}}\right), \tag{4}$$

where erfc is the complimentary error function and θ is the mean angle of the beam relative to the channel. If the rms divergence of the beam is approximately equal to the critical angle with the beam perfectly aligned, about 85% of the beam is accepted into the channel. If an alignment error of the same size as the critical angle is made at the same divergence, the acceptance decreases to about 50%. The alignment of the crystal is important. In addition to the angular acceptance, there is a geometric factor to account for beam particles which impact on an atomic plane within about a Thomas-Fermi distance. This would give an additional multiplicative factor which is about 80% for the (110) planes in Si and about 85% for the same planes in Ge.

IV. Fokker-Planck Transport

We have recently considered,[6] in another context, the charged particle transport in crystal channels without statistical equilibrium assumptions using a Fokker-Planck model. By assuming the channel potential is harmonic, an analytic solution for the Green's function can be found by well-known methods.[11] Details can be found in Ref. 6. This treatment can be extended to the case of a bent crystal in a straightforward way. We have done this and extended the work further by integrating the Green's function over an initial distribution. Two cases have been considered, a uniform beam and a Gaussian beam. These are divergence angle distributions. Over the spatial dimensions of a channel, the beams are uniform. The distribution function so obtained is integrated over the channel and integrated in angle to the critical angle for channeling, ψ_c. That part of the distribution which lies outside of this angle is taken to be unchanneled. The result gives us the relative intensity of the channeled beam as a function of the beam and channel parameters and the distance into the crystal.

It is interesting and satisfying to find that the limit of this result as the distance into the crystal approaches zero goes over to the expression for the channel acceptance given earlier, Eq. (4).

Very good agreement has been found with the channeled particle fluxes found in the bent crystal channeling experiments done at Dubna in 1978, Ref. 12. We find a relative intensity of approximately 4% which agrees very well with the data presented in Ref. 12. Given the uncertainties in entities like the multiple scattering coefficient the agreement is remarkable and to some small extent disconcerting. In this experiment, at 8.4 GeV, the small flux is due to surface acceptance loss and multiple scattering. There is essentially no bending dechanneling even at the largest deflection angles of about 25 mrad in agreement with both the reported results of the experiment and Ellison's theory. The dechanneling half-length is about a factor of two longer than that reported by Sun in his analysis of the Dubna data. The reason is not clear.

We have begun to calculate the channeling fluxes for the experiment[13] of Forster and co-workers using the MB beamline at Fermilab. In this beautiful experiment, they bend the beam through about 32 mrad in a continuously bent Si crystal. Continuous bending is achieved by sputtering a layer of ZnO onto the surface of the crystal. The mismatch in the lattice parameters of the ZnO and Si causes the crystal to bend. This eliminates the confusion in the comparison of theory and experiment due to nonuniform bending and local strain fields which occurs with pin benders. We do find bending loss of flux although it seems to be smaller than observed in the experiment. In this experiment, the surface acceptance, normal dechanneling, and bending dechanneling are important. Data analysis is required to separate the bending dechanneling loss from the others. The detailed comparison of our theory and the experiment is now being done. This and a full description of the model will be prepared for publication in the archival literature.

V. Concluding Remarks

The deflection of beams of charged particles by the channeling effect in bent single crystals of Si has been demonstrated experimentally and the essential features of the process is understood theoretically. There is nothing to suggest that, insofar as the channeling physics is concerned, major refinements of our models will be needed to be able to interpret the outcome of future experiments. In the Fokker-Planck transport theory, the most severe assumption probably is the use of a harmonic well. Nevertheless, the spectrum of channeling radiation produced by positrons in the planar channels of Si provides some justification. This shows that the well is essentially harmonic over much of the channel width.[15] As one would expect, significant deviation occurs near the atomic planes. But, because this represents only a small fraction of the channel width, it would not be expected to significantly modify the transport. Conversely, it might be argued that even a small amount of nonlinearity may, by phase mixing, increase the rate of approach to statistical equilibrium. This could be incorporated, perturbatively[14] or otherwise, into the Fokker-Planck transport model. However, as discussed earlier, our preliminary results indicate the statistical equilibrium bending dechanneling theory is conservative and it suggests that a $100m_{\rm rad}$ bend at SSC parameters is not stressing. By using Ge rather than Si, an additional margin of "comfort" of more than a factor of two can be had.

Most of the loss at SSC parameters occurs from the surface acceptance. As seen earlier, good alignment of the channel planes with the beam direction is needed to minimize the loss. While an absolute calibration is not necessary, a goniometer step size comparable to the critical angle, ψ_c, is. The crystal planes are not necessarily parallel to the crystal face. The planes and beam are brought into alignment by adjusting for maximum transmission. The angle between the face and planes leads to an effective septum width. Crystals can be fabricated with planes and face parallel to better than 1.7 mrad, Ref. 13. This corresponds to an effective septum width of about 85 microns for a 5 cm crystal. This is comparable to the effective width of 3 mils for a conventional electrostatic septum. Furthermore, it is not clear whether the 1.7 mrad represents limits on the fabrication technology, limits on the measurement of the error or both. It seems, in principle, the effective septum width could be reduced substantially. Additional work in this area would be useful.

The surface acceptance is also dependent on the phase space of the large amplitude particles. This is just now being understood on the SSC as the machine design becomes finalized. It also depends on the processes which populate the large amplitude orbits and how these particles are brought onto the crystal face. One scenario might be to incorporate a slow spill onto the crystal septum into a beam cleanup protocol. Defining and executing a set of experiments on existing accelerators will be essential in addressing this issue and may also provide information about beam scraping in general.

Acknowledgments

We wish to thank H. Edwards, R. Carrigan, T. Toohig, T. Murphy, D. Finley, S. Peggs, S. Baker, and Brad Cox for many helpful discussions.

This has been supported by the SSC Laboratory.

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