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Stability of the Gas Dynamic Trap

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Abstract

The description of stability of the gas dynamic trap is shown to be extremely sensitive to the nature of the boundary conditions. Two model boundary conditions in a moderately long mean free path limit are considered: insulating boundary conditions and conducting boundary conditions. The former boundary condition reproduces the magnetohydrodynamic (MHD) results of previous studies, where the outflow of ions contribute to the MHD stabilizing properties of the system. However, with conducting boundary conditions it is shown that the outflowing ions do not contribute to the system's stability. In this case, which is likely to be physically relevant, the MHD stabilizing term only comes from the electron pressure in the expansion region, and the gas dynamic trap would not be as stable as previously envisioned. The physical difference in these two boundary conditions is attributed to a passive feedback mechanism from the oscillating edge potentials that is only present in the insulating boundary case.

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I. Introduction

Stability of the gas dynamic trap (GDT) has previously been studied based on the magnetohydrodynamic (MHD) model.[1,2] This analysis showed that flute modes in the trap can be stabilized by the expander regions. The curvature of the magnetic field lines in the expanders is favorable and large; large enough to overcome the destabilizing contribution of the axisymmetric mirror cell. One of the results of Refs. 1 and 2 is that stability is proportional to the total pressure gradient in the expanders which includes the density of ion momentum flux arising from ions streaming along field lines as they escape the expander. The increased energy is especially important for GDT because in the expanders the ions are accelerated by the ambipolar potential to an energy several times the central temperature thus increasing their momentum flux.

Kinetic treatments often reproduce similar results as MHD[3,4] predictions for isolated plasmas in mirror geometry or tokamaks. However, kinetic treatments allow for additional modes or effects; e.g. trapped particle curvature driven modes,[5,6] hot particle effects where decoupling can arise if the curvature drift is larger than the mode frequency, and charge uncovering.[7] In the GDT, the geometry also differs from usual considerations in that the plasma is not isolated from the wall; instead it is essential to understand the dynamics of the plasma that flows to the wall. Here we develop a kinetic treatment to describe a model of a mirror trapped plasma satisfying the conditions of the GDT, including the flow to the wall. We find that the description of the stability is highly dependent on boundary conditions. If one takes insulating boundary conditions, where the parallel current density vanishes at the wall, then the MHD result of Refs. 1 and 2 are recovered. However, if one takes an opposite extreme, conducting boundary conditions where parallel current density at the wall is not constrained to vanish, one finds the ion outgoing momentum flux in the expander does not contribute to stabilization, although the electron pressure in the expander does contribute.

This latter result is particularly important for the stabilization principle of the GDT. Our treatment of the conducting boundary condition assumes that line bending effects that would otherwise give rise to line tying at conducting boundaries is not significant for sufficiently low beta. Instead, the mode is assumed flute-like over most of the axial length but varies rapidly near the end walls where in a Debye length the potential can change rapidly and vanish at the wall. However, because of the low density and short length of this region, the effect of this transition from a flute response of the mode is negligible, as can be shown using the analysis given for electrostatic fast growing trapped particle modes described in Ref. 6.

This paper will develop in detail the formalism to describe the theory with conducting boundary conditions for the GDT which has not been previously developed. The result with insulating boundary conditions reproduces the criteria of Refs. 1 and 2, and a short derivation that follows from kinetic equations is given in Appendix A. The physical difference that gives rise to the two different responses is explained in Appendix B.

We find that the two systems are different with regard to the perturbed electron density response, but the ion density response of the two systems is identical. This appears paradoxical since it is the outward ion momentum flux in the expanding region that is responsible for the MHD stabilization. However, for the insulating boundary case we find that the electron density response couples to the ion outward momentum flux through the constraint that axial flow at the ends for these two species must be identical. The perturbed ion current end flow is proportional to the products of the outward ion momentum flux and field line curvature in the expander (i.e. the MHD stabilization term). The electron end flow can match this by having the end-potentials modulate in just the right way. Through the electron continuity equation, the perturbed electron density in the bulk is then proportional to the electron end-flow which in turn is proportional to the MHD stabilization term. If there is a conductor at the ends the end potential can not change and one loses the passive feedback mechanism on the electrons. With the conducting boundary the interchange instability driven from the central region can only be slowed down by dissipative effects as described by Kunkel and Guillory.[8]

II. Equilibrium

The magnetic field configuration of a gas dynamic trap is shown in Fig. 1. The trap consists of the central cell and two expanders terminated by the end walls which absorb plasma flowing out of the mirror cell. The favorable curvature in the expanders is supposed to provide overall stability of the whole machine. Typically the expander is much shorter than the central part $L_A \ll L_c$, and the mirror ratio in the central cell is large, $R \equiv B_{\max}/B_0 \gg 1$, with $B_{\max}$ the peak magnetic field, B_0 the midplane fields and the subscripts "A" and "c" refer to the expander and central cell respectively.

In the central cell both electrons and ions are described by the Maxwell distribution functions

$$F_{jc} = F_{jM} = n_0 \left(\frac{M_j}{2\pi T_j}\right)^{3/2} e^{-E/T_j} \,, \tag{1}$$

where n_0 is the plasma density, $E = \frac{1}{2} M_j v^2 + q_j\varphi$ is the energy, M_j and T_j are the mass and the temperature of species j. We consider the plasma with singly charged ions, $q_i = -q_e$, so that equilibrium density n_0 is the same for both species.

For the sake of simplicity we will assume that $T_e \ll T_i$ in order to neglect the effect of ambipolar potential on ions. Note that in the central cell this potential is constant along field lines almost everywhere except for regions close to the mirror throats where $B \sim B_{\max}$. In this small region ions also depart from its Maxwellian form.

Since the electron collision frequency is assumed to be large enough the electron distribution function in the expander is the same as in the central cell. The ion collision frequency ν_i is assumed to satisfy

$$\frac{v_{Ti}}{R\,L_c} \ll \nu_i \ll \frac{v_{Ti}}{L_c} \tag{2}$$

where L_c is the central cell length and v_{Ti} is about the ion thermal velocity (see Eq. 7 below). In such a regime of collisionality the ions have a long mean free path compared to the machine size, but can still maintain a Maxwellian distribution (with a filled loss region) over most of the central cell. Only near the mirror peak does the ion distribution distort from a Maxwellian and eventually transform to an escaping half Maxwellian at the point where $B = B_{\max}$. In the expanders the ion population consists of particles with $\mu < E/B_{\max}$ which can escape through the mirrors. Their distribution function in the right expander (see Fig. 1) is

$$F_{iA} = F_{iM}\,\theta(E - \mu\, B_{\max})\,\theta(v_{\|})\ , \tag{3}$$

where $\theta(x)$ is the step function. For the left expander one should change $v_{\|} \rightarrow -v_{\|}$ in Eq. (3). Using Eq. (3) to calculate the ion density in the expanders we find

$$n_{iA} = \frac{n_0}{2}\left[1 - \left(1 - \frac{B}{B_{\max}}\right)^{1/2}\right] \longrightarrow \frac{1}{4}\,n_0\,\frac{B}{B_{\max}}\left[1 + \mathcal{O}\left(\frac{B}{B_{\max}}\right)\right]\ . \tag{4}$$

Since electrons here are distributed according to the Boltzmann law, $n_e = n_0 \exp(-q_e\,\Phi/T_e)$, it follows from quasineutrality that when $B \ll B_{\max}$ the ambipolar potential Φ is given by

$$\Phi_A = -\frac{T_e}{q_e}\,\ell n\,\frac{B}{4B_{\max}}\ . \tag{5}$$

This potential extends to the end wall which absorbs plasma flow streaming from the central cell. At the wall there exists a Debye sheath that reflects electrons and thereby adjusts the electron loss flux to the end walls to equal the ion flux.

Using Eq. (3) we can evaluate the particle flux from the trap and find the loss rate ν_p, which is defined as the ratio of number of particles lost per unit time per unit magnetic flux tube to the number of particles in the tube:

$$\nu_p = \frac{2\displaystyle\int d^3v\, v_{\|}\, F_{iA}\Big|_{\text{wall}}}{B_w \displaystyle\int \frac{ds}{B}\int d^3v\, F_i} \tag{6}$$

where B_w is the magnetic field at the end walls. In the integral in the denominator we can neglect the contribution of the expanders because of their small length and low density. Then we find

$$\nu_p = v_{Ti} \left(B_{\max} \int_{L_c} \frac{ds}{B} \right)^{-1} \doteq v_{Ti}\, R\, L_c \;, \tag{7}$$

where $v_{Ti} = \sqrt{2T_i/\pi M_i}$ and the subscript L_c indicates that the integration is taken between the mirror points of the central cell. The inverse quantity, $\nu_p^{-1} \sim R\, L_c / v_{Ti}$, gives the particle lifetime in the gas dynamic trap.

An extensive detailed discussion of the GDT concept is given in Ref. 9.

III. Basic Linear Equations

We consider an electrostatic perturbation which is easiest to describe in the eikonal representation; i.e., the perturbed potential is expressed $\varphi(s)\exp[-i\omega t + iS(\psi, \theta)]$ with $\mathrm{k}_\perp = \nabla\psi(\partial S/\partial\psi) + \nabla\theta(\partial S/\partial\theta) \equiv k_\psi\, \nabla\psi + k_\theta\, \nabla\theta$, where ψ is the magnetic flux, θ is the azimuthal angle and s is the arc length of a field line. The perturbed distribution function f_j satisfies the following small Larmor radius kinetic equation

$$v_\| \frac{\partial f_j}{\partial s} - i(\omega - \omega_{dj})\, f_j - iq_j(1\text{-z})\varphi \left(\omega \frac{\partial F_j}{\partial E} + \omega_j^* \frac{F_j}{T_j} \right) = C_j(f_j) \;, \tag{8}$$

where

$$\omega_j^* = \frac{ck_\theta\, T_j}{q_j} \frac{\partial \ell n\, F}{\partial \psi} \tag{9}$$

is the diamagnetic drift frequency,

$$\omega_{dj} = \frac{ck_\theta\, M_j}{r\, q_j\, B} \left(v_\|^2 + \frac{1}{2} v_\perp^2 \right) \left(\kappa \frac{\nabla\psi}{|\nabla\psi|} \right) \;, \tag{10}$$

κ is the field line curvature and $C_j(f_j)$ is the linearized collision operator. The parameter $z = \frac{1}{2}\,(k_\perp\, v_\perp\, cM_i/q_i B)^2$ is assumed to be small, $z \ll 1$; it describes finite Larmor radius effect (and has to be retained for ions only). In the paraxial approximation z is constant along a field line because $k_\perp \propto r^{-1} \propto (\psi/B)^{1/2}$ and $v_\perp \propto (\mu B)^{1/2}$. In Eq. (10) we neglected plasma rotation and also β effects that alter the drift motion. We shall also neglect temperature gradients so that

$$\omega_j^* = \frac{k_\theta\, cT_j}{q_j\, n_0}\,\frac{dn_0}{d\psi}\,. \tag{11}$$

Assuming that ω is smaller than the bounce frequency, $\omega \ll v_{Ti}/L_c$, it follows that to lowest order $f_e(E,\mu)$ is constant along a field line. We average Eq. (7) along the line and obtain

$$\omega f_e - \overline{\omega}_{de}\, f_e - \frac{q_e\overline{\varphi}}{T_e}\,(\omega - \omega_e^*)F_{eM} = i\overline{C}_e(f_e) \tag{12}$$

with

$$\overline{\varphi} = \frac{1}{\tau}\int \frac{ds}{v_\parallel}\varphi \quad , \qquad \tau = \int \frac{ds}{v_\parallel}\,, \tag{13}$$

where the integrals are taken between the turning points. Note that electrons are trapped by the magnetic field in the central cell and the ambipolar potential in the expanders. Most "passing" electrons are reflected by the potential in the Debye layer at the wall.

When averaging Eq. (8) for ions which are confined in the central cell we will obtain nearly the same equation as (12). However, ions having $\mu < E/B_{\max}$ are not reflected by the magnetic mirrors and produce an important endpoint effect. Hence ions are treated slightly differently than electrons. We divide Eq. (8) by $v_\parallel$ and average the integral between the points of maximum magnetic field keeping the endpoint contributions. For $\mu > E/B_{\max}$ we have that $f^+ = f^-$ at the turning point (the superscripts + and − refer to the parallel velocity direction of the particles), but for $\mu < E/B_{\max}$ $f_i^+ \neq f_i^-$. Then the endpoint contribution on the right end for f_i^+ is finite, while f_i^- vanishes, and the left end gives the same result for particles going in the opposite directions. We shall assume an even mode in ϕ so that by symmetry $f^+(v_\parallel, s) = f^-(|v_\parallel| - s)$. Now subtracting the contributions from the f^+ and f^- averaged equations, keeping in mind that $v_\parallel$ has opposite signs for the two directions, (and introducing the notation $f_i \to \frac{1}{2}(f_i^+ + f_i^-)$ where $\pm$ signs are for positive and negative velocity particles) we obtain

$$\omega f - \overline{\omega}_{di}\, f_i - \frac{q_i\overline{\varphi}}{T_i}\,(1-z)(\omega-\omega_i^*)F_{iM} = i\overline{C}_i(f_i) - i\,\frac{f_i(s=L_c)}{\tau}\,\theta(E-\mu\, B_{\max}) \tag{14}$$

The last term on the right-hand side comes from the end point contributions and describes ion losses from the central cell due to the ion flow through the mirrors.

We will assume that $\omega \gg \nu_i$ so that ions are almost collisionless. Still we should account for collisions for small pitch angles because the last term on the right-hand side of Eq. (14) is a peaked function which angular spread in the midplane is of the order of $R^{-1/2} \ll 1$. The pitch angle scattering dominates collisions in this case and keeping only this term, the collision operator is taken as

$$C_i(f_i) \simeq \nu_i\,\frac{1}{B}\,\frac{\partial}{\partial\lambda}\,\lambda\,\frac{\partial f_i}{\partial\lambda} \tag{15}$$

where $\lambda = \mu/E$,

$$\nu_i = \nu_i(E) \equiv \frac{2^{1/2}\pi\, q_i^4\, n_0}{M_i^{1/2}\, E^{3/2}}\,\Lambda \tag{16}$$

and Λ is the appropriate Coulomb logarithm. (Note that since the equilibrium potential Φ is assumed small compared to the ion energy, E, in Eq. (16) is the kinetic energy, $E = M_i\, v^2/2$). Multiplying Eq. (15) by $\frac{1}{\tau}\, ds/v_{\|}$ and integrating over a field line, and using that λ in this operator corresponds to small pitch angles, we find

$$\overline{C}_i(f_i) = \overline{\nu}\,\frac{\partial}{\partial\lambda}\,\lambda\,\frac{\partial f_i}{\partial\lambda} \tag{17}$$

with

$$\overline{\nu}_i = \frac{\nu_i}{L_c}\int\frac{ds}{B}\,. \tag{18}$$

IV. Solution of the Kinetic Equation

We now proceed to solve the ion kinetic equation assuming the following ordering

$$\omega\ ,\ \omega_j^* \gg \overline{\omega}_{dj}\ . \tag{19}$$

We shall also assume that electrons are highly collisional and ions nearly collisionless

$$\nu_i \ll \omega \ll \nu_e\ . \tag{20}$$

Together with Eq. (2) this means that we are considering an instability which grows faster than the plasma lifetime $L_c\, R/v_{Ti}$. Note also the inequality $\omega \ll \nu_i\, R$, which follows from Eq. (2) and $\omega \ll v_{Ti}/L_c$.

For the central cell we are looking for the perturbed distribution function of ions in the following form

$$f_i = h_i + g_i\ , \tag{21}$$

where g_i is the perturbation due to the loss term in Eq. (14). Since the losses are small we first neglect second term on the right-hand side of Eq. (14). Using Eqs. (17) and (19), we find the following equation for

$$h_i - \frac{i\overline{\nu}}{\omega}\frac{\partial}{\partial\lambda}\lambda\frac{\partial h_i}{\partial\lambda} = \frac{q_i}{T_i}(1-z)\overline{\varphi}\left(1-\frac{\omega_i^*}{\omega}\right)\left(1+\frac{\overline{\omega}_{di}}{\omega}\right)F_{iM}\ . \tag{22}$$

Because the equilibrium potential is constant along field lines in the central cell the averaged potential $\overline{\varphi}$ can be considered as a function of $\lambda \equiv \mu/E$. Estimating the collision operator in Eq. (22) we find that if $\overline{\varphi}(\lambda)$ is a smooth function whose width $\Delta\lambda \gtrsim \sqrt{\lambda\overline{\nu}/|\omega|}$ (or $\Delta\lambda \gtrsim \overline{\nu}/|\omega|$ if $\lambda \lesssim \Delta\lambda$) then collisions can be neglected and

$$h_i = \frac{q_i}{T_i}(1-z)\overline{\varphi}\left(1-\frac{\omega_i^*}{\omega}\right)\left(1+\frac{\overline{\omega}_{di}}{\omega}\right)F_{iM}\ . \tag{23}$$

In the case where $\overline{\varphi}(\lambda)$ has rapid structure, the collisions modify the solution given by Eq. (23). Equation (22) can then be solved using a Green's function and it is found that

$\overline{\varphi}(\lambda)$ in Eq. (23) should be replaced by $\overline{\overline{\varphi}}(\lambda)$ where

$$\overline{\overline{\varphi}}(\lambda) = \int_0^{B_0^{-1}} G(\lambda, \lambda')\overline{\varphi}(\lambda')d\lambda' \ . \tag{24}$$

The complex function $G(\lambda, \lambda')$ is the Green's function

$$G(\lambda, \lambda') = \frac{\pi\omega}{\overline{\nu}} \begin{cases} H_0(\xi)J_0(\xi') \ , & \lambda > \lambda' \\ H_0(\xi')J_0(\xi) \ , & \lambda < \lambda' \end{cases} \tag{25}$$

where J_0 is the Bessel function H_0 is the Hankel function that vanishes for $|\xi| \to \infty$, and $\xi = (4i\omega\lambda/\overline{\nu})^{1/2}$, $\mathrm{Im}\,\xi > 0$. The function G is symmetric, $G(\lambda, \lambda') = G(\lambda', \lambda)$, and has the property

$$\int_0^{B_0^{-1}} G(\lambda, \lambda')\, d\lambda' = 1 \ . \tag{26}$$

This function vanishes for large $|\lambda - \lambda'|/\Delta\lambda$, where $\Delta\lambda \sim \sqrt{\lambda\overline{\nu}/|\omega|}$ when $\lambda \gtrsim \overline{\nu}/\omega$ and $\Delta\lambda \sim \overline{\nu}/|\omega|$ if $\lambda \lesssim \overline{\nu}/|\omega|$ (the width of the Green's function corresponds to a pitch angle spread of the order of $\sqrt{\nu_i/|\omega|}$ which is the angle at which a particle scatters during the wave period $\sim \omega^{-1}$). Recalling the definition of $\overline{\nu}$ in Eq. (18) and using (20) it is easy to show that $B_{\max}^{-1} \ll |\lambda - \lambda'| \ll B_0^{-1}$.

For a smooth function $\overline{\varphi}(\lambda)$ it follows from (24) and (25) that $\overline{\overline{\varphi}}(\lambda) \simeq \overline{\varphi}(\lambda)$ and we obtain the previous result (23).

In what follows we will assume that the width of $\overline{\varphi}(\lambda)$ is larger than the width of the Green's function G and then the use of $\overline{\varphi}$ in Eq. (23) is justified.

In order to evaluate g_i we assume that $g_i \ll h_i$ and substitute h_i for f_i in the last term on the right-hand side of Eq. (14) keeping only the largest terms in h_i. This yields the following equation

$$g_i - \frac{i\overline{\nu}}{\omega}\frac{\partial}{\partial\lambda}\lambda\frac{\partial g_i}{\partial\lambda} = \frac{iq_i\overline{\varphi}}{T_i}\left(\frac{\omega_i^*}{\omega} - 1\right)\frac{F_{iM}}{\omega\tau}\,\theta\,(E - \mu\, B_{\max}) \ . \tag{27}$$

Comparing this with (22) we find

$$g_i = \frac{iq_i(\omega_i^* - \omega)}{\omega^2 T_i} F_{iM} \int_0^{B_0^{-1}} G(\lambda, \lambda')\,\frac{\overline{\varphi}(\lambda')}{\tau(E, \lambda')}\,\theta\,\left(B_{\max}^{-1} - \lambda'\right) d\lambda'$$

$$\simeq i\,\frac{q_i\overline{\varphi}(0)}{T_i}\;\frac{(\omega_i^* - \omega)}{\omega^2\tau_0}\;\frac{G(\lambda,0)}{B_{\max}}\,F_{iM} \tag{28}$$

where $\tau_0 = \tau(E, \lambda = 0) = L_c(M_i/2E)^{1/2}$. In performing the integration in Eq. (28) we have taken into account that the integrand does not change appreciably in the interval $0 < \lambda' < B_{\max}^{-1}$.

For the expanders we make an assumption that due to their small length the ion time of flight through them is much smaller than any inverse frequency entering the kinetic equation. With this ordering, the largest term in Eq. (8) is the first one. In zeroth order we have $\partial f_i/\partial s = 0$ from which it follows that f_i is constant along a field line and hence equals its value at the mirror throat given by the distribution function in the central cell (23). To find this function we note that in the perturbed state only those particles reach the mirror throat whose energy satisfies the inequality $E - q_i\,\varphi_{\rm mir} > \mu\, B_{\max}$ where $\varphi_{\rm mir}$ is the perturbed potential at the mirror peak. Using Eq. (3) for the unperturbed distribution function and neglecting g_i in Eq. (21) we find

$$\begin{aligned} f_{iA} &= F_{iM}\,\theta(E - q_i\,\varphi_{\rm mir} - \mu\, B_{\max})\theta(v_{\|}) - F_{iM}\,\theta(E - \mu\, B_{\max})\theta(v_{\|}) + h_i\,\theta(E - \mu\, B_{\max})\theta(v_{\|}) \\ &= -q_i\,\varphi_{\rm mir}\,F_{iM}\,\delta(E - \mu\, B_{\max})\theta(v_{\|}) + \frac{\omega - \omega_i^*}{\omega}\,\frac{q_i\overline{\varphi}(0)}{T_i}\,F_{iA} \end{aligned} \tag{29}$$

where we have neglected z and the drift term is $\overline{\omega}_{di}$ in Eq. (23).

We see that the ion distribution functions both in the central cell and the expanders, given by the formulas (23), (28), and (29), do not depend on the boundary conditions at the end walls. Electron losses, on the contrary, are very sensitive to these boundary conditions. Below we consider in detail the case of perfectly conducting walls.

In this case, the potential at the wall is fixed and does not change in time. We can then solve this problem in accordance with a method developed in Refs. 10 and 11. The motion of electrons in a given magnetic field and wall potential determines a separatrix in the phase space which bounds the confinement region. We are required to solve Eq. (12)

with f_e vanishing on the separatrix. Since electrons are collisional the bulk of the distribution function $F_{eM} + f_e$ (except near the separatrix) has to be Maxwellian with a density $n_0(1+\alpha)$ and a temperature $T_i(1+\beta)$, where α and β are normalized density and temperature perturbations. For f_e that gives

$$f_e = \left[\alpha + \beta\left(\frac{E}{T_e} - \frac{3}{2}\right)\right] F_{eM} + \delta f \tag{30}$$

with $\delta f \ll \alpha\, F_{eM}$. Substituting this form in Eq. (12), and neglecting δf everywhere except in the collision operator, we find

$$\overline{C}_e(f_e) = -i(\omega - \overline{\omega}_{de})\left[\alpha + \beta\left(\frac{E}{T_e} - \frac{3}{2}\right)\right] F_{eM} + i\,\frac{q_e\overline{\varphi}}{T_e}(\omega - \omega_e^*)F_{eM} \equiv -S_{\text{eff}}\ . \tag{31}$$

The solution to Eq. (31) can be expressed in terms of the Pastukhov problem[12] which is formulated as

$$\overline{C}_e(f) = -S(\mathrm{v})\ . \tag{32}$$

The particle loss rate ν_p and the energy loss rate ν_E, is defined by

$$\nu_p = \frac{\int \frac{ds}{B}\int d^3v\, S}{\int \frac{ds}{B}\int f\, d^3v}\quad , \quad \nu_e = \frac{\int \frac{ds}{B}\int d^3v\, Sv^2}{\int \frac{ds}{B}\int d^3v\, fv^2}\ . \tag{33}$$

Taking the moments of Eq. (30), neglecting δf, we can solve for α and β in terms of ν_p and ν_E. The calculations show that $\beta/\alpha \sim \mathcal{O}(\overline{\omega}_{de}/\omega, \nu_p/\omega)$. Then neglecting β we obtain for the density perturbation

$$\alpha = \frac{q_e\,\langle\varphi\rangle}{T_e}\,\frac{\omega - \omega_e^*}{\omega - \langle\omega_{de}\rangle + i\nu_p} \simeq \frac{q_e\,\langle\varphi\rangle}{T_e}\left(1 - \frac{\omega_e^*}{\omega}\right)\left(1 + \frac{\langle\omega_{de}\rangle}{\omega} - \frac{i\nu_p}{\omega}\right) \tag{34}$$

where the angle brackets denote the averaging

$$\langle\omega_{de}\rangle = \frac{\int \frac{ds}{B}\int d^3v\, F_{eM}\,\omega_{de}}{\int \frac{ds}{B}\,n}\ . \tag{35}$$

It is important to note that when β can be neglected, the problem posed by Eq. (31) is the same as the equilibrium loss problem of electrons and then ν_p in Eq. (33) is equal to the electron equilibrium loss rate, which is the same as the ion equilibrium loss rate given by Eq. (6) (since the net current at the wall in equilibrium vanishes). With this observation, Eq. (34) gives the solution to Eq. (31) in the case of conducting boundaries. We emphasize that the perturbed losses of electrons and ions, in general, are different producing an oscillating current density to the wall, though the total current integrated over the azimuthal angle is zero.

We note that if we had insulating boundary conditions, an additional potential structure at the wall would need to be considered to constrain the parallel current to vanish. One can generalize our method to take this case into account. However, the constraint of zero parallel current density at the boundary allows for a simpler derivation for the response which is given in the Appendix.

The eigenmode equation in our problem follows from the quasineutrality condition

$$\sum_j q_j \widetilde{n}_j = 0 \ , \tag{36}$$

where $\widetilde{n}_j$ is the perturbed density of the species j. With the distribution functions found in the previous section we can now evaluate $\widetilde{n}_j$ using the formula

$$\widetilde{n}_j = q_j \varphi \int d^3v \, \frac{\partial F_j}{\partial E} + \int d^3v \, f_j \ . \tag{37}$$

For ions in the central cell F_j is Maxwellian and the perturbed distribution function is given by Eqs. (23) and (28). To integrate Eq. (37) over velocity space we convert energy and pitch angle ($\lambda \equiv E/\mu$) variables. To account for the E and λ dependence of ϖ_{di} and we define a new variable $\widehat{\omega}_{di}$ which is a function of λ only

$$\widehat{\omega}_{di}(\lambda) = \frac{T_i}{E} \varpi_{di}(E, \mu) \ . \tag{38}$$

Now using the symmetry of the Green's function G and the normalization in Eq. (26) we find

$$\tilde{n}_{ic} = \frac{q_i n_0}{T_i}\left[-\varphi + \frac{1}{2} B\left(1-\frac{\omega_i^*}{\omega}\right)\int_0^{B_0^{-1}} \frac{d\lambda\overline{\varphi}(\lambda)}{\sqrt{1-\lambda B}} \right.$$
$$-\frac{3}{4} b B^2 \left(1-\frac{\omega_i^*}{\omega}\right)\int_0^{B_0^{-1}} \frac{\lambda\overline{\varphi} d\lambda}{\sqrt{1-\lambda B}} + \frac{3B}{4\omega}\left(1-\frac{\omega_i^*}{\omega}\right)\int_0^{B_0^{-1}} \frac{d\lambda\overline{\phi}\,\widehat{\omega}_{di}}{\sqrt{1-\lambda B}} \tag{39}$$
$$\left. + i\overline{\varphi}(0)\left(\frac{\omega_i^*}{\omega}-1\right)\frac{B\, v_{Ti}}{\omega L_c B_{\max}}\right]$$

where $b = (ck_\perp/q_i B)^2 M_i T_i$.

In the expander the distribution function is given by Eq. (29). For $B \ll B_{\max}$ the integration yields

$$\tilde{n}_{iA} = -\frac{n_0 q_i}{2T_i}\left\{ \left[1-\left(1-\frac{B}{B_{\max}}\right)^{1/2}\right]\left[\varphi - \overline{\varphi}(0)\left(1-\frac{\omega^*}{\omega}\right)\right] + \frac{B(\varphi_{\rm mir}-\varphi)}{B_{\max}\left(1-\dfrac{B}{B_{\max}}\right)^{1/2}} \right\}, \tag{40}$$

where n_{iA} is the local density in the expander.

For electrons the perturbed density is

$$\tilde{n}_e = -n\,\frac{q_e\varphi}{T_e} + \alpha n = \frac{q_e n_0}{T_e}\left[-\varphi + \langle\varphi\rangle\left(1-\frac{\omega_e^*}{\omega}\right)\left(1+\frac{\langle\omega_{de}\rangle}{\omega} - \frac{i\nu_p}{\omega}\right)\right]. \tag{41}$$

Proceeding now to the solution of Eq. (36), we first note that since we assume, $b \ll 1$, $\omega_d/\omega \ll 1$ and $\nu_p/\omega \ll 1$, we can to lowest order neglect these terms in Eqs. (39), (40), and (41). Substituting the remaining terms in Eq. (36), allows for a lowest order flute solution of the form

$$\varphi = \langle\varphi\rangle \equiv \varphi_0\,. \tag{42}$$

Thus the potential is constant along a field line corresponding to a flute mode. In the next approximation we put $\varphi = \varphi_0 + \delta\varphi$, where $\delta\varphi \ll \varphi_0$, and use the quasineutrality equation

$$\mathcal{L}\,\phi_0 + \frac{q_e^2 n_0}{T_e}(-\delta\varphi + \langle\delta\varphi\rangle\left(1-\frac{\omega_e^*}{\omega}\right) + \frac{q_i^2 n_0}{T_i}\left[-\delta\phi + \frac{1}{2}B\left(1-\frac{\omega_i^*}{\omega}\right)\int_0^{B_0^{-1}}\frac{d\lambda\,\delta\overline{\varphi}(\lambda)}{(1-\lambda B)^{1/2}}\right]\theta(L_c - s)$$

$$+\frac{q_i^2}{T_i} n_{iA}\left[\delta\phi - 2\delta\phi_{\rm mir} + \overline{\delta\phi(0)}\left(1-\frac{\omega_i^*}{\omega}\right)\right]\theta(s-L_c) = 0\ , \tag{43}$$

where $\mathcal{L}\,\phi_0$ is

$$\mathcal{L}\,\phi_0 = \frac{q_e^2 n_0}{T_e}\left(1-\frac{\omega_e^*}{\omega}\right)\left(\frac{\langle\omega_{de}\rangle}{\omega}-\frac{i\nu_p}{\omega}\right)\phi_0 + \frac{q_i^2\, n_0}{T_i}\left[-\frac{3}{4}\, b\, B^2\left(1-\frac{\omega_i^*}{\omega}\right)\int_0^{B_0^{-1}}\frac{\lambda d\lambda \varphi_0}{(1-\lambda B)^{1/2}}\right.$$

$$\left. + \frac{3B\left(1-\frac{\omega_i^*}{\omega}\right)}{\omega}\int_0^{B_0^{-1}}\frac{d\lambda\widehat{\omega}_{di}\,\phi_0}{(1-\lambda B)^{1/2}} + i\phi_0(0)\left(\frac{\omega_i^*}{\omega}-1\right)\frac{B\,v_{Ti}}{\omega\, L_c\, B_{\max}}\right]. \tag{44}$$

The dispersion relation can be obtained from Eq. (43) by dividing by B and averaging in s which annihilates all but the $\mathcal{L}\phi_0$ term in Eq. (43), yielding $\langle\mathcal{L}\phi_0\rangle = 0$.

For what follows it is convenient to divide the averaging over the full length of the machine into two parts: averaging over the central cell and averaging over the expanders,

$$\langle\omega_{de}\rangle_c = \left(\int_{L_c}\frac{ds}{B}\int d^3v\, F_{EM}\,\omega_{de}\right)\left(n_0\int_{L_c}\frac{ds}{B}\right)^{-1},$$

$$\langle\omega_{de}\rangle_A = \left(\int_{L_A}\frac{ds}{B}\int d^3v\, F_{EM}\,\omega_{de}\right)\left(\int_{L_A}\frac{nds}{B}\right)^{-1}.$$

With this definition

$$\langle\omega_{de}\rangle = \frac{\langle\omega_{de}\rangle_c + \varepsilon\,\langle\omega_{de}\rangle_A}{1+\varepsilon} \simeq \langle\omega_{de}\rangle_c + \varepsilon\,\langle\omega_{de}\rangle_A\ , \tag{45}$$

where, $\varepsilon = \left(\int_{L_A}\frac{nds}{B}\right)\left(n_0\int_{L_c}\frac{ds}{B}\right)^{-1}$, is of the order of magnitude $\varepsilon \sim L_A/R\,L_c \ll 1$. In Eq. (45) we neglect $\varepsilon\,\langle\omega_{de}\rangle_c$ because the curvature in the expanders is much larger than the curvature in the central cell as the gas dynamic trap is designed so that $\langle\omega_{de}\rangle_A \gg \langle\omega_{de}\rangle_c$.

Using $\langle\mathcal{L}\varphi_0\rangle = 0$ and the identity

$$\frac{3}{4}\int_{L_c}dx\int\frac{d\lambda\,\widehat{\omega}_{di}}{\sqrt{1-\lambda B}} = \langle\omega_{di}\rangle_c\left(\int_{L_c}\frac{ds}{B}\right)^{-1}$$

and Eq. (45) with $T_i\,\langle\omega_{de}\rangle_c = -T_e\,\langle\omega_{di}\rangle_c$ one finds the following dispersion relation

$$A\omega^2 + C\omega + D = 0 \tag{46}$$

where

$$
\begin{aligned}
A &= \langle b \rangle_c \\
C &= -\langle b \rangle_c \, \omega_i^* - \varepsilon \frac{T_i}{T_e} \langle \omega_{de} \rangle_A + i\nu_p \frac{T_i}{T_e} \left(1 + \mathcal{O}\left(\frac{T_e}{T_i} \right) \right) \\
D &= \omega_i^* \langle \omega_{di} \rangle_c + \frac{T_i}{T_e} \omega_e^* \langle \omega_{de} \rangle \; .
\end{aligned} \tag{47}
$$

The first term in Eq. (46) is due to plasma inertia. The third one gives the conventional driving force for the flute instability. We emphasize that in this coefficient the ion drift frequency is only averaged over the central cell, which means that ions in the expanders do not contribute to stability. This prediction is in contrast with the results of the MHD theory developed in Ref. 2 for insulating end walls. In Appendix A we present a short derivation of the dispersion relation for the insulating boundary case. In Appendix B we explain the mechanism that allows the two cases to give different results.

As is seen from Eq. (47) the electron drift frequency is averaged over the whole machine including the expanders regions. Hence we conclude that the stability in GDT is provided only by electrons. Note that even in the case $T_e \sim T_i$ the electron pressure in the expanders is several times smaller than the ion one because ions are accelerated by an ambipolar potential.

The linear term in Eq. (46) includes FLR effect to which a charge uncovering term (the second term in the coefficient C) is added. The imaginary part of C is due to the electron losses to the walls. It describes the line-tying effect through the Debye sheath at the walls.[8] Note that in the case $D > 0$ a real C can stabilize the flutes (if $C^2 > 4AD$), but with a complex C instability remains but with a reduced growth rate proportional to ν_p.

V. Conclusion

We have analyzed the stability of the GDT in two extreme limits, that of insulating boundary conditions and conducting boundary conditions. In addition our analysis was limited to low beta, $T_e/T_i \ll 1$ so that ions are unaffected by equilibrium electric fields, long mean free path compared to the system length (though the mean free path was short enough that the ion distribution is still near Maxwellian in the high mirror ratio central cell), and we assumed that electrons were Maxwellian at a single temperature over the entire device including the expander. These two limiting cases give dramatically different stability criteria. In particular, with insulating boundary conditions the outflow of the ions can contribute to the stabilization of the system, as is assumed in the design of GDT, while with conducting boundary conditions the outflow of ions does not contribute to the stabilization. With conducting boundary conditions the electron pressure in the expander does contribute to the stabilization, but in practice, this pressure is several times less than the equivalent pressure due to the ion outflow. Thus if conducting boundary conditions are appropriate, the stability of the GDT may be considerably weaker than originally envisaged.[1,2]

Several unanswered questions still need to be addressed in considering the stability of GDT. The first is to note that metallic end-plates, which would give a response close to conducting boundary conditions, is a fairly natural experimental situation. Using real insulating end-plates as an end condition would likely lead to the problem of maintaining the materials's insulation integrity due to electrical breakdown under the flux of high energy particles. Perhaps effective insulation boundary conditions can be established by radially slotted plates, which can then effectively serve as insulators for low mode numbers, if electrical breakdown does not occur in the slot gaps. Experimental studies may be able to address this problem.

The other important issue is whether the plasma model used with the conducting boundary condition theory gives results that are too pessimistic or difficult to verify under conditions of the present GDT experiment. It should be noted that under present experimental conditions of GDT,[13] the mean free path of electrons is short in the expander, which prevents direct comparison of our theory with existing experiment. However, it is the experimental goal to increase the electron temperature, so that the long mean free path assumption for electrons should be appropriate for a high temperature GDT. In addition, in a high temperature GDT it is predicted[8] that the electron temperature will significantly decrease close to the expander walls due to the plasma outflow in a highly diverging expander. This change has not been taken into account in our theory. This may lead to even less stabilization from electrons.

Probably the most important neglect in our theory is the effect of plasma beta, which can conceivably decouple the dependence of the plasma from the boundary condition, and perhaps yield the more optimistic prediction in accordance to the original MHD analysis.[1,2] This result is predicted in Ref. 2 when the outflow speed exceeds the local Alfvén speed.

We have also neglected the effect of line tying in the expander. This amounts to neglecting the stabilizing line bending energy term in MHD theory. At low beta such neglect can be justified using the theory of trapped particles modes in tandem mirrors.[6] Additional terms due to the non-flute nature of ϕ near the walls are negligible in the dispersion relation (Eq. (46)) if $T_i\, B_c\, \lambda_{dw}\, r_p^2 / B_{\max}\, L_c\, r_L^2\, T_e \ll 1$, where λ_{dw} is the electron Debye length at the wall, B_c the magnetic field in the central cell, and r_L the ion Larmor radius in the central cell. However, the low beta assumption near the walls, where the magnetic field is low, may be violated in experiment. A theory allowing for beta effects may force line bending terms to enter the theory, thereby leading to a synchronization to a stabilizing MHD response insensitive to boundary conditions. Further theory needs to be developed to address these issues and to determine if there are critical beta parameters.

Appendix A: Derivation of Dispersion Relation with Insulating Boundary Conditions

We start from Eq. (8) for f_j for each species, and we assume ϕ is a flute mode so that it is independent of s. We now multiply by q_j/B, integrate over velocity space and a field line and sum over species. Using the relations

$$\int d^3v\, C(f_j) = 0 \qquad \text{(the particle conservation property of the collision operator)}$$

$$\int d^3v\, f_j = \tilde{n}_j + q_j\phi \int d^3v\, \frac{\partial F_j}{\partial E}\ ,$$

$$q_e\,\tilde{n}_e + q_i\,\tilde{n}_i = 0\ ,$$

$$\sum_j q_j \int d^3v\, v_\parallel\, \frac{\partial f_j}{\partial s} = B\,\frac{\partial}{\partial s}\,(j_\parallel/B)\ ; \qquad (j_\parallel \text{ is the parallel current})$$

$$\sum_j q_i \int d^3v\, \frac{\partial F_j}{\partial \psi} = \sum_j \frac{\partial}{\partial \psi}\, q_j\, n_j = 0\ ;$$

we find

$$2i\, j_{\parallel E}/B_E = \sum_j \left[q_j \int \frac{ds}{B}\, f_j\, \omega_{d_j} + q_j^2 \phi \int \frac{ds}{B} \int d^3v\, z \left(\omega\, \frac{\partial F_j}{\partial E} + \frac{ck_\theta}{q_j}\, \frac{\partial F_j}{\partial \psi} \right) \right] \tag{A-1}$$

where $j_{\parallel E}$ and B_E is the current density and magnetic field of the boundary $s = \pm(L_A + L_c)$.

We now need only apply our constraint condition $j_{\parallel E} = 0$ and determine f_j to lowest order (here we assume $\omega_{d_j} \ll 1/\tau_j$, where τ_j is the transit time of species j across the system). In the text it was found that for a flute mode f_j, to lowest order, is given by

$$f_j = \phi \left(q_j\, \frac{F_j}{T_j} - \frac{ck_\theta}{\omega}\, \frac{\partial F_j}{\partial \psi} \right)\ . \tag{A-2}$$

Now substituting Eq. (A-2) into (A-1), and using the quasi-neutrality of the equilibrium, gives the usual paraxial flute dispersion with finite Larmor radius effects,

$$\omega^2 \int \frac{ds}{B^3}\, n_i\, m_i \left(\frac{k_\theta^2}{r^2} + r^2 B^2 k_\psi^2 \right) - \frac{\omega\, ck_\theta}{q_i}\, m_i \int \frac{ds}{B^3}\, \frac{\partial}{\partial \psi}\, P_{\perp i} \left(\frac{k_\theta^2}{r^2} + r^2 B^2 k_\psi^2 \right)$$

$$+k_\theta^2 \int \frac{ds}{B^2 r}\, \kappa \frac{\partial}{\partial \psi} (P_\perp + P_\parallel) = 0 \tag{A-3}$$

where $P_\perp$ and $P_\parallel$ are the perpendicular and parallel "pressures" which includes the directed ion flow velocity.

For the GDT the contribution from the expander is only important in the last term where $P_{\perp i} \approx 0$ and $P_{\parallel i}$ arises from the outflow ions flow out of the system. It is assumed $\frac{\partial}{\partial \psi}(P_\perp + P_\parallel) < 0$, and the positive value of the curvature, κ, in the anchor region is designed to dominate the third integral. We also note that the middle term, linear in ω, is the finite Larmor radius term. In a non-eikonal approximation, where the finite Larmor radius of the system is significant, it is well known that this term will vanish for a displacement mode. The overall stability is then basically determined by the sign of the last term, which for the GDT is designed to be negative due to the ion outflow.

Appendix B: Explanation of Passive Feedback Mechanism for Insulating Boundary Conditions

The dispersion relation for the insulating boundary condition can also be derived from the quasineutrality condition

$$\sum_j \int ds \, \frac{q_j \tilde{n}_j}{B} = 0 \; . \tag{B-1}$$

The perturbed ion density is the same as derived in the text for the conducting boundary condition. The perturbed electron density will be of the form $\tilde{n}_e = \tilde{n}_{ec} + \tilde{n}_{\rm ext}$, where $\tilde{n}_{ec}$ is the perturbed electron density derived in the text using conducting boundary conditions where the end potential vanishes, while $\tilde{n}_{\rm ext}$ is the extra perturbed electron density directly induced from the regulation of the electron outflow by end potentials. This potential variation constrains the total end current, due to the outflow of both electrons and ions, to be zero on each field line. Now $\tilde{n}_{\rm ext}$ satisfies the continuity relation

$$- i\omega q_e \, \tilde{n}_{\rm ext} + B \, \frac{\partial}{\partial s} \left(\frac{j_{\|\rm ext}}{B} \right) = 0 \tag{B-2}$$

where $j_{\|\rm ext}$ is the additional parallel current density carried by electrons which responds to the end potential perturbations such as to ensure that

$$j_{\|{\rm ext},E} + j_{\|ec,E^+} + j_{\|i,E} = 0 \tag{B-3}$$

where $j_{\|ec}$ is the parallel current carried by electrons with zero perturbation of the end potentials and $j_{\|i}$ is the current carried by ions which is quite insensitive to end potentials and the subscript E denotes evaluation at the end wall. Hence if we integrate Eq. (B-2) over the plasma length, use Eq. (B-3) and substitute in Eq. (B-1) we have

$$\int \frac{ds}{B} \, [q_e \, \tilde{n}_{ec} + q_i \, \tilde{n}_i] = -\frac{2i}{\omega} \, \left(j_{\|ec,E} + j_{\|i,E} \right) / B_E \; . \tag{B-4}$$

The electron end current in the absence of end potentials is readily evaluated (see discussion between Eqs. (30)–(34)) as

$$\frac{2j_{\|ec,E}}{B_E} = \nu_p\, q_e^2\, \frac{\varphi}{T_e} \int ds\, \frac{n(s)}{B} \left(1 - \frac{\omega_e^*}{\omega}\right) . \tag{B-5}$$

The ion end current can be found starting from Eq. (8). This equation is divided by $v_\|$, then integrated from just inside the peak of the mirror to the end walls, yielding

$$f_i(s = L_A + L_c) = f_i(s = L_A^-) + i \int_{L_c}^{L_c + L_A} \frac{ds}{v_\|} \left[(\omega - \omega_{di}) f_i + q_i \varphi \left(\omega \frac{\partial F_i}{\partial E} + \omega^* \frac{F_i}{T_i} \right) + C(f_i) \right] \tag{B-6}$$

where $L_A^- = L_A(1 - \epsilon)$ where $\epsilon \ll 1$. For f_{iA} we use Eq. (29). Upon substitution we find for the ion end current (note that the current from the "adiabatic" ion term is zero),

$$\begin{aligned} \frac{2j_{\|iE}}{B_E} &= 2 \int dE\, d\mu\, f_i(s = L_A + L_c) = \frac{q_i^2\, n_0\, v_{Ti}}{T_i\, B_{\max}} \left(1 - \frac{\omega_i^*}{\omega}\right) \varphi \\ &+ 2i\, \frac{k_\theta^2 \varphi}{\omega} \int_{L_c}^{L_c + L_A} \frac{ds}{B^2 r}\, \kappa\, \frac{\partial}{\partial \psi} P_{\|i} \left(1 + \mathcal{O}\left(\frac{P_{\perp i}}{P_{\|i}}\right)\right) \\ &+ 2i q_i^2 \varphi \int_{L_c}^{L_c + L_A} \frac{ds}{B} \int d^3 v\, \omega_{di}\, \frac{\partial F_i}{\partial E} . \end{aligned} \tag{B-7}$$

Now the left-hand side of Eq. (B-4) is the plasma density response without an oscillating end potential and it is equal to the left-hand side of Eq. (46) multiplied by the factor $-q_i^2(\omega^2 T_i)^{-1} \int ds\, n_i/B$. Now we substitute Eqs. (B-5) and (B-7) into the right-hand side of Eq. (B-4), using the relationships

$$\frac{v_{Ti}}{B_{\max}} = \nu_p \int ds\, n/B \ ,$$

$$q_i^2 \int d^3 v\, \omega_{Di}\, \frac{\partial F_i}{\partial E} \doteq -q_i\, c\, k_\theta\, \frac{\kappa n_{iA}}{Br} = \frac{q_e^2}{T_e} \int d^3 v\, \omega_{de}\, F_e$$

$$q_e^2\, \frac{n_e\, \omega_e^*}{T_e} + q_i^2\, \frac{n_i\, \omega_i^*}{T_i} = 0 \ ,$$

we find that Eq. (B-6) reduces to the dispersion relation with insulating boundaries, Eq. (A-3). We interpret the right-hand side of Eq. (B-4) as the induced perturbed electron density produced by the passive feedback arising from the oscillating end potentials that regulates the electron flow. We see from Eq. (B-7) that it contains the MHD stabilization term proportional to the outgoing ion momentum flux $P_{\|i} = nm_i \bar{v}_i^2$ as well as terms that annihilate the charge uncovering and dissipative terms in Eqs. (46) and (47).

A discussion quite similar to the above one is given in Ref. 14.

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Figure Caption

1. Schematic of a gas dynamic trap. The top figure shows magnetic field lines. The bottom figure is the variation of the magnetic field along the axis. The central region has a length L_c, and the expanders length is $L_A, L_A \ll L_c$. The minimum magnetic field in the central cell is B_0, the maximum magnetic field is $B_{\max}$, and the field at the walls is B_w. Plasma flowing from the central cell is absorbed by the end walls.

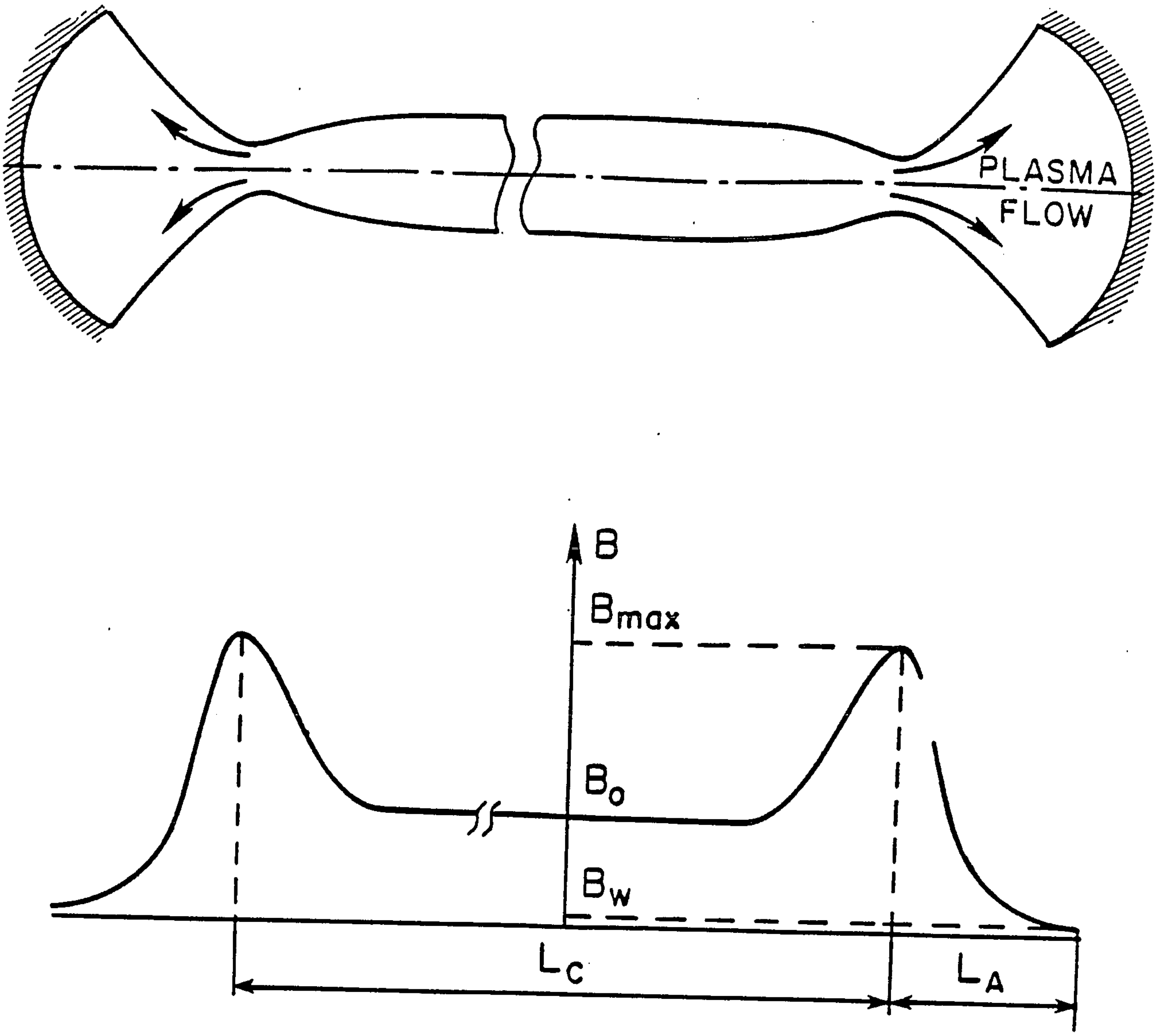

Fig. 1