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Direct Conversion of Muon Catalyzed Fusion Energy

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#### Abstract

In this paper a method of direct conversion of muon catalyzed fusion (MCF) energy is proposed in order to reduce the cost of muon production. This MCF concept is based on a pellet composed of many thin solid deuterium-tritium (DT) rods encircled by a metallic circuit immersed in a magnetic field. The direct energy conversion is the result of the heating of the pellet by beam injection and fusion alphas. The expanding DT rods causes the change of magnetic flux linked by the circuit. Our calculation shows that the direct conversion method reduces the cost of one muon by a factor of approximately 2.5 over the previous methods. The present method is compatible with a reactor using the pellet concept, where the muon sticking is reduced by the ion cyclotron resonance heating and the confinement of the exploding pellet is handled by magnetic fields and the coronal plasma.

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#### I. Introduction

Muon absorption in matter and the induced fusion of deuterium and tritium<sup>1-5</sup> is a remarkable phenomenon since the negative muon is capable during its lifetime ( $\tau_{\mu} = 2.2 \times 10^{-6} \, \mathrm{sec}$ ) of inducing about 150 nuclear dt fusion reactions in a liquid density medium. This end result of nuclear fusion occurs after a chain of atomic and molecular processes. The most crucial step in the physics of muon catalysis cycle is the resonant formation of the dt $\mu$  molecule which increases the probability of the end result by at least two orders of magnitude (relative to nonresonant dt $\mu$  formation). However, this high production rate of the dt $\mu$  molecule is disturbed by muons lost in the catalysis cycle. There seems to be a probability of about 0.4% that the helium ion created during the nuclear fusion interaction will capture the negative muon.

The "energy cost" for the creation of a muon is one of the most important practical parameters in analyzing the relevance of muon catalyzed fusion for energy production. The muons are produced during the decay of pions which can be created in nucleon-nucleon collisions. The energy threshold for pion generation is about 500 MeV of projectile kinetic energy and the process of negative muon generation seems to be effective for nucleonic projectiles with a kinetic energy of 1 GeV per nucleon for single pass collisions. An optimistic estimate requires 4 GeV of energy to produce one negative muon. Therefore if one muon catalyzes about two hundred dt fusions, the energy output is  $\approx 3.5$  GeV (one dt fusion gives 17.6 MeV) per muon, so that no energy gain seems to be possible from "pure" fusion nuclear reactions.

For a pure fusion reactor one can define the scientific gain G, as the ratio between the output energy  $X_{\mu}\mathcal{E}_{\text{fus}}$  and the input energy  $E_{\mu}$ 

$$G = \mathcal{E}_{\text{fus}} X_{\mu} / E_{\mu} , \qquad (1)$$

where  $X_{\mu}$  is the number of fusion reactions per muon,  $\mathcal{E}_{\text{fus}} = 17.6 \, \text{MeV}$  (for a dt reaction),

and  $E_{\mu}$  is the energy invested in creating one muon. Taking into account the thermal to electricity efficiency  $\eta_{\rm th}$ , the accelerator efficiency  $\eta_a$  and the fraction  $\eta_r$  of the total electric power required to recirculate (in order to operate the accelerator and the auxiliary facilities) one gets the basic equation for the energy balance that gives the critical  $X_{\mu}^{c}$ 

$$X_{\mu} \ge X_{\mu}^{c} = \left[\frac{E_{\mu}}{17.6 \,\text{MeV}}\right] (\eta_{a} \eta_{r} \eta_{\text{th}})^{-1} .$$
 (2)

For example, taking optimistic efficiencies  $\eta_a=60\%$ ,  $\eta_{\rm th}=35\%$  and assuming  $\eta_r=15\%$  is an economic possibility, one needs  $X_\mu^c=9000$  in a muon catalyzed reactor. This number seems to be unrealistic for the present status of knowledge. However, by improving technologies  $\eta_{\rm th}=80\%$ ,  $\eta_a=70\%$  and  $\eta_r=20\%$  together with  $E_\mu=2\,{\rm GeV}$ , one reduces  $X_\mu^c$  to 1000.

Thus the strategy of the muon catalyzed fusion research may be put in a succinct way: either to increase  $X_{\mu}$  or to reduce  $E_{\mu}$  (or both). In the present paper we explore a method to reduce  $E_{\mu}$ . Before we investigate this, however, let us survey the efforts of  $\mu \, cF$  reactor studies of Petrov,<sup>6</sup> Eliezer, Tajima, and Rosenbluth,<sup>7</sup> and Tajima, Eliezer, and Kulsrud.<sup>8</sup> The last one is based on the idea of Kulsrud,<sup>9</sup> which is the effort to reduce  $X_{\mu}$ .

Since pure fusion devices using muon catalyzed fusion seem to lose rather than to gain energy, Petrov<sup>6</sup> suggested using muon catalysis in a hybrid fusion-fission reactor. This reactor scheme includes an accelerator (of tritium or deuterium), a target (of tritium or beryllium) where the pions are created, a synthesizer with the d-t fuel and a blanket where the fusion and fissile materials are produced. Following Petrov's concept of a muon catalyzed fusion-fission reactor, Eliezer, Tajima and Rosenbluth<sup>7</sup> suggested an improved reactor concept based on two new main ideas.

#### Petrov's Hybrid Reactor<sup>6</sup>

In 1980, Petrov suggested a power reactor based on muon catalyzed fusion combined with nuclear fission processes. This reactor scheme includes an accelerator (of tritium or

deuterium), a target where pions are created, a converter where the pions are confined in vacuum by strong magnetic fields, a synthesizer with the d-t fuel and a a blanket where the fusion and fissile materials are produced. The converter is a cylinder about 40 m long and 20 cm radius (5 m<sup>3</sup> volume) having a longitudinal magnetic field of 11 to 16 Tesla and an applied d.c. electric field of  $7.5 \times 10^5$  volt/m along the converter. Inside this vessel there is a cylinder 2m long with a 2cm radius target where the pions are created. The synthesizer is a second cylinder about 20 m long with an average radius of 20 cm surrounded by a (longitudinal) magnetic field coil of 11 Tesla. The density of the dt fuel is 0.5 liquid hydrogen density with 30% tritium, so that the synthesizer has 80 kg of tritium. The main result of the converter-synthsizer is the conversion of about 75% of the created pions into muons that participate in the catalyzed dt fusion. This means that the energy cost of a stopped negative muon in the dt mixture is about 6 GeV (using 4.5 GeV to create one negative pion<sup>10</sup>) for tritium projectiles and 8 GeV for a deuterium projectile beam. These results seem to be optimistic in this model reactor, mainly because pion and muon drifts to the wall due to collisions were neglected. Also, the influence of the magnetic field on the scattered proton beam were not considered. In fact, the proton Larmor radius is of the same order of magnitude as that of pions, therefore a significant portion of the proton beam is lost in the vessel in this concept reactor. The fissions and fissile material are produced in the blanket not only by the neutrons derived from d-t fusion but also from the fast nucleons of the incident beam which have about 80% of their initial energy after the creation of the pions. Taking into account the losses of the projectiles in the converter-synthesizer vessels due to the magnetic field will reduce the energy and the fissile material obtained from direct beam-blanket collisions. Moreover, by taking into account the diffusion due to collisions, the necessary quantity of tritium might increase significantly.

Petrov's reactor is actually a "neutron factory" which breeds a thermal fissile isotope <sup>233</sup>U or <sup>239</sup>Pu in order to use these materials in satellite fission reactors. In this scheme it

was estimated that a commercial reactor would require about 100 fusion per muon.

#### Eliezer-Tajima-Rosenbluth (ETR) Hybrid Reactor<sup>7</sup>

The ETR reactor concept is based on three ideas: (a) The high energy beam of tritium or deuterium is injected into the deuterium-tritium (d-t) fuel and, after passage through the fuel, part of the beam is collected for reuse while the portion of the beam which suffered a strong interaction is directed into an electronuclear blanket. (b) The pions created in the target are surrounded by the fuel of deuterium and tritium and are magnetically confined until they slow down and decay into muons which catalyze the fusion in situ: (c) The fusion created neutrons are absorbed by blankets to breed fissile matter for energy production. In comparison to Petrov's concept the highlights of this concept are (a) and (b). Instead of a separate target, converter, and synthesizer, these three functions are combined into one. The d-t fuel is the target and the produced pions are trapped in the fuel, which slows them down before they decay into muons. The muons are created in the fuel and trapped there, catalyzing the d-t fusion through the atomic and molecular processes until they decay. In this way, one of the difficult problems of muon catalyzed fusion is solved; i.e., efficient trapping of muons in a relatively small physical volume. The present concept solves this problem by creating mesons in the fuel and by confining them magnetically. The driver is a tritium (or deuterium) beam which is retrieved in part after passing through the target. The significantly scattered portion of the beam feeds into the electronuclear blanket, the fusion created neutrons are captured in the fissile blanket surrounding the fusion fuel. The mirror is filled with a pressurized gas mixture of deuterium and tritium gas. The gas is circulated through the mirror, with a cooling section between traversals. The mirror is enclosed by <sup>238</sup>U or <sup>232</sup>Th blankets with admixed lithium for breeding of Pu, <sup>233</sup>U and tritium. Magnets provide a magnetic field configuration with a mirror ratio  $R_m$ . The field at the mirror throats is typically 10 T.

An accelerated tritium beam of approximately 1 GeV per nucleon and a current of about 0.1 Ampére is injected through a small tube perpendicular to the axial magnetic field into the fuel. When the high energy beam (1 GeV/nucleon) strongly interacts with the target, in effect proton-proton, proton-neutron, neutron-neutron scatterings take place.

The total gain factor is given by

$$K = \frac{E_{\text{blanket,out}}}{E_{\text{target.in}}} = \frac{E_{\text{blanket,out}}(EN) + E_{\text{blanket,out}}(\mu c)}{y_{\mu}^{-1} y_{\pi}^{-1} T_0} . \tag{3}$$

EN denotes the energy gain from "direct" beam-blanket interaction and  $\mu c$  denotes the energy gain from muon catalytic processes. The value of K is given in terms of the physical quantities  $X_{\mu}$ ,  $y_{\mu}$ , and  $y_{\pi}$  by the formula

$$K = \frac{Z_e T_{0f} \varepsilon_{\text{fis}}}{T_0} + \frac{y_\mu y_\pi X_\mu \delta_f \varepsilon_{\text{fis}}}{T_0} = K_{EN} + K_{\mu c}$$
 (4)

where  $y_{\mu}$  is the  $\pi^-$  to  $\mu^-$  conversion efficiency ( $\sim 50\%$ );  $y_{\pi}^{-1}$  is the number of nucleonic projectiles<sup>6</sup> necessary to produce a  $\pi^-$ , the EN contribution is  $K_{EN} = Z_e T_{0f} \, \varepsilon_{\rm fis}/T_0$  and the muon catalyzed gain is given by the second term,  $\varepsilon_{\rm fis} \approx 0.2 \, {\rm GeV}$  is the uranium fission energy (for <sup>238</sup>U blanket),  $Z_e \approx 20 \, {\rm fissions/GeV}$  in the "direct" beam-blanket interaction (i.e., the EN process),  $T_{0f}$  is the beam kinetic energy before colliding with the blanket while  $T_0$  is their initial kinetic energy before hitting the target (for pion  $\rightarrow$  muon creation).  $\delta_f \approx 1$  is the number of fissions in the blanket caused by one 14 MeV neutron from the fusion process.  $X_{\mu}$  was measured experimentally to be about 150 for liquid hydrogen d-t targets and  $y_{\pi}$  and  $y_{\mu}$  were estimated in Ref. 7.

#### Tajima-Eliezer-Kulsrud (TEK) Fusion Reactor<sup>8</sup>

In order to achieve a thermodynamically meaningful energy gain by muon catalyzed DT fusion alone without resorting to breeding fissile matter by fusion neutrons, it is believed that the fusion catalysis cycling rate in a muon lifetime,  $X_{\mu}$ , needs to exceed 1000. Neglecting faster intermediate processes, nowadays we can identify two crucial bottlenecks for muon

catalyzed fusion energy production. The one is the  $dt\mu$  mesomolecular formation process (rate  $\lambda_{dt\mu}$ ) and the other is the muon sticking with fusion  $\alpha$  particles (probability  $\omega_{s0}$ ):

$$X_{\mu}^{-1} \approx \omega_s + (\lambda_{\rm dt\mu} \, \tau_{\mu})^{-1} \,\,\,\,(5)$$

where

$$\omega_s = \omega_{s0}(1 - R) , \qquad (6)$$

 $au_{\mu}$  is the lifetime of a muon, R is the stripping rate of muon from the  $(\alpha\mu)^+$  atom, and  $\omega_{s0} \approx 0.008$ ,  $R \approx 40\%$ ,  $\lambda_{\mathrm{dt}\mu} \approx 10^9 \, \mathrm{sec}^{-1}$ . The sticking rate has now been experimentally and theoretically settled to  $\omega_s \approx 0.0045$ . This means that if  $\lambda_{\mathrm{dt}\mu} \to \infty$ , the catalysis cycle rate  $X_{\mu}$  is merely 220.

The effective sticking probability  $\omega_s$  can be reduced by increasing the stripping coefficient R. The stripping (or reactivation) coefficient R may be written as

$$R = 1 - \exp\left[-n\int_{E_i}^{E_b} \frac{\sigma_{\rm st}(E)dE}{F(E)}\right] , \qquad (7)$$

where  $\sigma_{\rm st}$  is the stripping cross-section, F is the stopping power of  $(\alpha\mu)^+$  in matter, and  $E_b$  and  $E_i$  are the birth energy of  $(\alpha\mu)^+(\approx 3.5\,{\rm MeV})$  and the  $(\alpha\mu)^+$  ionization potential  $(\approx 10\,{\rm keV})$ . The cross-section peaks around  $\approx 10\,{\rm keV}$  and quickly decreases below this energy. In order to make the reactivation coefficient large (i.e., close to unity), Bracci and Fiorentini<sup>11</sup> argued that  $(\alpha\mu)^+$  should be kept as long as possible in the higher velocity range where the stripping probability is higher. They suggested to apply accelerating electric fields in the matter. Unfortunately, the effective slowdown field in the solid matter is of the order of  $45\,{\rm MeV/cm}$ , with which, either by dc or ac fields, breakdown of the target arises.

Kulsrud<sup>9</sup> proposed a clever scheme to reduce the sticking coefficient by reaccelerating the  $(\alpha\mu)^+$  ions by ion cyclotron resonance mechanism. In this scheme the target D-T is in solid (or liquid) form, say at about 25° K, and the target is divided into many rods (or pellets) of the order of 100 microns (each) in radius. By imposing a magnetic field and a rotating

electric field the  $(\alpha\mu)^+$  is kept at a constant (average) velocity until stripping occurs. The drag force in the rods (while the stripping occurs) is balanced by the acceleration force in the vacuum between the rods (or the pellets). The acceleration is achieved by the ion cyclotron resonance heating in vacuo with the frequency  $\omega = \Omega_{\alpha\mu} = eB/4m_pc$ , where  $m_p$  is the proton mass. In his calculation the necessary strength of rf electric field  $E_{\rm st}$  for stripping is

$$eE_{\rm st} \approx 5 \times 10^4 \, f \, \rm kV/cm \; , \tag{8}$$

where f is the filling factor of the DT target. On the other hand, the breakdown field for ac field  $E_{bd}$  (which is much higher than the dc case) is approximately given<sup>12</sup> by

$$eE_{bd} \approx m\omega c$$
 , (9)

where m is the electron mass and  $\omega$  is the rf frequency. At the frequency range of  $\Omega_{\alpha\mu} \approx 10^{10}\,\mathrm{Hz}$  (with 1 M Gauss magnetic field) the field in Eq. (8) is tolerable against breakdown. Unfortunately, Kulsrud's idea does not work as he put it, since the alpha heating of the target cannot be tolerated, for the following reason: Imagine that we want to let fusion happen in DT ice bars (below 25° K). Per a dt fusion reaction, a neutron of 14.1 MeV and an alpha particle of 3.5 MeV are created. The neutron leaves the ice and presumably hits a surrounding blanket (or worse, a metal), while the alpha particle is likely to stay in the DT ice and heats it. If we are to insist on keeping the low temperature of the specimen, every time a fusion alpha particle heats the target, we have to cool (or extract the energy of that amount from) the material. That is, we have to remove 3.5 MeV energy per fusion. With the cooling efficiency  $\eta_e$  (because of the second law of thermodynamics, it is not unity and in fact a miniscule number typically of the order of)  $\approx 0.1$ , the necessary energy to keep the material at a desired cool temperature is 3.5 MeV/ $\eta_e$ ( $\approx 35\,\mathrm{MeV}$ ) per fusion. On the other hand, the total fusion energy production is of course only 17.6 MeV per fusion. No energy gain is possible. We call this problem "too cold fusion — no energy production".

The TEK fusion reactor concept is based on the following configuration (Fig. 1). A simple mirror ratio  $R_m$  with magnetic field **B** generated by superconductors is surrounded by neutron blankets such as Li etc. Although the strength of the magnetic field is not uniquely fixed, we may use a typical number of 1 MG. A pellet (or pellets) of DT ice is repetitively injected either mechanically or gravitationally into the mirror vessel. The pellet consists of a series of ribbons of DT ice. The minimum overall size is  $\approx (2 \text{ cm})^3$ , which is determined by the two conditions, the inertial confinement condition of the overall pellet and the magnetic confinement condition of  $\pi^-$ . These DT ice ribbons fill the pellet with the filling factor f in order to accommodate sufficient acceleration time, while keeping enough matter to strip  $(\alpha\mu)^+$  according to Kulsrud's mechanism of stripping. These DT ice specimens are penetrated by a high energy beam of d or t particles ( $\approx 1 \,\mathrm{GeV/nucleon}$ ) injected perpendicular to the mirror axis, which coincides with the (longitudinal) axis of the specimens. Beam particles collide with the target nuclei, creating  $\pi^-$ . Pions are created in situ and confined by the mirror magnetic field, as in the earlier hybrid reactor concept. The portion of the beam that did not suffer strong interaction will be collected, cooled, accelerated, and reused for the next pass. At this energy of the d or t the beam cooling may be provided by appropriate methods such as Budker's electron cooling method $^{13}$  or Van der Meer's stochastic cooling method.14

The ribbons have the following characteristics. The thickness  $\ell_1$  of a ribbon should be smaller than the range of  $(\alpha\mu)^+$  at the 3.5 MeV energy at birth. Typically we choose  $100\mu m$ . The width  $\ell_2$  is typically several times  $\ell_1$ . The length  $\ell_3$  is determined by the larger of the inertial confinement length and a few  $\pi^-$  Larmor radius:

$$\ell_3 = \max(c_s \tau_\mu, 4\rho_\pi) , \qquad (10)$$

where  $c_s$  is the sound speed of the DT specimen and  $\rho_{\pi} = \beta c/\Omega_{\pi}$ . For 1 MG field and 1 eV DT specimen, these two are in the same range of  $\approx$  1 cm. The distance  $d_2$  between two

ribbons in the width direction is

$$d_2 = \min(2\rho_{\alpha\mu}, \ell_2/\sqrt{f}) , \qquad (11)$$

and the distance  $d_1$  between two ribbons in the thickness direction is

$$d_1 = \max\left[\ell_1/\sqrt{f}\,,\,\,\frac{\ell_1\ell_2}{2f\rho_{\alpha\mu}}\right]\,\,,\tag{12}$$

where  $\rho_{\alpha\mu}$  is the Larmor radius of  $(\alpha\mu)^+$ . Typical numbers for these are 0.4 cm and 0.2 cm, respectively. Radio frequency electromagnetic waves with frequency  $\omega = \Omega_{\alpha\mu}$  are applied with the electric field rotating.

When the beam is injected into these ribbons and fusion reactions begin, fusion alpha particles are created and all their energy is deposited in the heating DT ice. In order to avoid the dilemma of too cold fusion — no energy production mentioned before, the DT ribbons are heated up to temperature  $T_h$  which is much higher than the room temperature  $T_r$ , so that we can now extract energy from the DT ribbon instead of investing energy to cool it. This condition is written as

$$\eta_h \Delta T_h - \Delta T_e / \eta_c > 0 , \qquad (13)$$

where  $\eta_h$  and  $\eta_c$  are the thermodynamical efficiencies of the heat energy extraction from the hot material  $(T_h)$  into electricity and the cooling, and  $\Delta T_h = T_h - T_r$  and  $\Delta T_e = T_r - T_{\rm ice}$ . From this  $T_h$  has to be larger than 1-2eV for  $\eta_c \approx 0.3$  and  $\eta_c \approx 0.1$ . This will resolve the difficulty of Kulsrud's idea. Of course, this brings in a new difficulty of confining hot DT ribbons at nearly the solid density to cope with an enormous pressure.

In order to confine these DT ribbons, we introduce the concept of coronal confinement. We surround each ribbon with a coat of coronal plasma with density  $n_c$  and temperature  $T_c$  such that

$$n_h T_h = n_c T_c (14)$$

where  $n_h$  and  $T_h$  are the heated DT ribbon density  $(\phi \approx 1)$  and temperature  $(< 1 \, \text{eV})$ .

When the DT ribbon expands after alpha heating, the coronal pressure balances it, while the coronal pressure is now borne by the magnetic pressure outside of it. Thus, the corona is magnetically confined:

$$n_c T_c + \frac{B_c^2}{8\pi} = \frac{B^2}{8\pi} \,\,\,\,(15)$$

where  $B_c$  is the magnetic field which penetrates into the corona and B is the external field. Details of the creation of such a corona will not be discussed here. Some possibilities include the alpha heating of the ribbon itself with or without a sponge-like surface, injection of very hot and very tenuous plasma to evaporate the surface, surface current application, laser surface ablation, etc.

The magnetic field tends to penetrate into the corona (and the ribbon) by the resistive process. The velocity of such penetration,  $v_p$ , is given by

$$v_p = \frac{\eta_e \nabla P_c}{B^2} \approx \frac{\eta_0 c(n_h T_h)/\ell_c}{B^2 (T_c/T_0)^{3/2}},$$
 (16)

where  $\eta_e$  is the collisional resistivity of the corona and an explicit dependence of the coronal temperature is written in. This velocity should be sufficiently small such that

$$\ell_c/v_p > \tau_\mu \ . \tag{17}$$

This equation imposes a constraint on  $B, \ell_c, T_h$ , and  $T_c$ . Besides the classical resistive penetration, there may arise anomalous penetration.

The above described TEK fusion reactor concept via muon catalysis tried to reduce the effective sticking probability to be less than  $10^{-3}$ . The TEK concept relies on the hot coronal pressure. It is, however, this lopsided energy content in the corona, some  $10^2$  times that in the pellet that causes difficulty in this concept. Unless one can keep the coronal energy from shot to shot, an unlikely technical feat, the amount of energy needed to heat the corona is too energetically expensive. To date we are unable to devise a method enabling fusion alphas to heat preferentially the corona rather than the pellet.

In the present paper we suggest a new idea of direct conversion of energy produced by fusion alpha particles and by collisions of the high energy (~ 2 GeV) beam with target particles. This technique, we shall show, is capable of reducing the energy cost of muon creation by a considerable amount. In Sec. II we present this idea, followed by the energy cost estimate for muon creation in Sec. III. We briefly comment on implications of the present method on reactor considerations in Sec. IV, and finally Sec. V is given for discussion.

## II. Direct Energy Conversion

The schematic configuration of our direct energy conversion method for muon catalyzed fusion is described in Fig. 2. The pellet is composed of many DT rods.8 The pellet is immersed in a magnetic field, which confines created  $\pi^-$  and  $\mu^-$ . The typical dimensions of a rod are: the thickness  $\ell_1 = 100 \mu m$ , the width  $\ell_2$  is several times  $\ell_1$ , the length  $\ell_3$  is larger of the inertial confinement length  $c_s \tau_\mu$  and a few  $\pi^-$  Larmor radii  $\rho_\pi$ , where  $\tau_\mu$  is the lifetime of  $\mu^- c_s$  is the sound speed, and  $\rho_{\pi} = 100 \, \mathrm{cm}/B(\mathrm{Tesla})$ . The filling factor f of rods in the pellet is related to the requirement that the stuck muon  $(\alpha \mu)^+$  can be appropriately accelerated by the microwave heating, i.e., the ion cyclotron resonance heating (ICRH).9,8 The rods in the pellet can be the target of the beam projectile creating  $\pi^{-0.7}$  A part of the beam which suffers electromagnetic collisions is recirculated. The pellet in Fig. 2(a) is encircled by a circuit that extracts electromotive energy created during the expansion of the rods. The expansion is due to both the heating by projectile particle collisions and by fusion produced alpha particles. The alphas are produced by fusion catalyzed by muons that were created by the projectile within the rods. The pellet is so designed that when rods are heated up to  $1-3\,\mathrm{eV}$  they explode in a matter of a few  $\tau_{\mu}$ . The cross-section of the final stage of the explosion is schematically depicted in Fig. 2(b).

A simple model describing the direct energy conversion is given in Fig. 3. The external magentic field  $B_0$  is assumed not to penetrate the by now hot gas/plasma of the rods.

Whatever penetrates the rods  $B_1$  and plasma  $B_2$  is much smaller than  $B_0$ . The piston in this model is simply the boundary between the vacuum and the DT material (gas/plasma). The initial temperature before explosion but after heating is  $T_1 \sim 1-3 \,\mathrm{eV}$  with volume  $V_1$  and density  $n_1$  ( $\sim$  liquid density) [Fig. 3(a)]. The final temperature, volume, and density of the gas/plasma are denoted by  $T_2$ ,  $V_2$ , and  $n_2$ . The exact optimal temperature  $T_1$  depends on several factors, including the possible mesomolecule formation rate of  $(\mathrm{dt}\mu)$  via the plasma mechanism. The flux  $\psi$  contained in vacuum within the circuit is squeezed by this expansion process. This results in the change of flux  $\partial \psi/\partial t$  and thus the electromotive force  $\mathcal E$  and current I in the circuit  $\mathcal R$ , which is extracted directly. This explosion is most likely adiabatic and thus it is our basic assumption in the model. In the discussion we shall point out possible cases when the assumption breaks down,

The work W done by the DT rods after heating on the external  $B_0$  magnetic field during the expansion is given by

$$W = \int_{V_1}^{V_2} p \, dV = W_B + W_T \,\,, \tag{18}$$

where  $W_B = \int p_B dV$  and  $W_T = \int p_T dV$  and  $p_B$  and  $p_T$  are the magnetic and thermal pressures. An operational assumption, though not necessary, is introduced: the external magnetic field  $B_0$  is kept constant and the flux  $\psi$  inside the DT material is kept constant. The magnetic pressure is

$$p_B = \frac{B^2}{8\pi} = \left(\frac{\psi_1^2}{8\pi L^2}\right) V^{-2} , \qquad (19)$$

where  $\psi_1 = B_1 S_1 = B_1 V_1 / L$ ; the length L of the system is so long that it is assumed to be constant during the expansion.

$$W_B = \frac{B_1^2}{8\pi} V_1 (1 - \varepsilon) , \qquad (20)$$

where

$$\varepsilon = \frac{V_1}{V_2} \,\,, \tag{21}$$

and  $\varepsilon$  is much smaller than unity in our interest. This would correspond to adiabatic expansion with "gas constant" for magnetic field"  $\gamma_B = 2$ . The thermal work  $W_T$  for adiabatic expansion

$$W_T = \frac{p_1 V_1}{(\gamma - 1)} \left( 1 - \varepsilon^{\gamma - 1} \right) , \qquad (22)$$

where  $\gamma$  is the gas constant and is assumed to be  $\gamma = (f+2)/f = 5/3$  with f being the number of degrees of freedom. Thus the total work W is

$$W = \frac{B_1^2}{8\pi} V_1(1 - \varepsilon) + \frac{2n_1 T_1 V_1}{\gamma - 1} \left( 1 - \varepsilon^{\gamma - 1} \right) , \qquad (23)$$

where we assumed electron and ion pressures are  $n_1 T_1$ . The total initial energy Q in the system

$$Q = \frac{2}{\gamma - 1} n_1 T_1 V_1 + \frac{B_1^2}{8\pi} V_1 . {24}$$

The efficiency of the direct conversion of energy Q into electricity in the circuit is

$$\eta_{\rm dc} \equiv \frac{W}{Q} = \frac{\frac{1}{\gamma - 1} \left( 1 - \varepsilon^{\gamma - 1} \right) + \frac{1}{2} (1 - \varepsilon) \beta^{-1} - W_r}{\frac{1}{\gamma - 1} + \frac{1}{2} \beta^{-1} + Q_c} , \tag{25}$$

where  $Q_c$  is the energy needed to cool the DT rods to the solid temperature and  $W_r$  is the work done by the system returning from the final expanded state Fig. 3(b) back to the initial state Fig. 3(a), and  $\beta = n_1 T_1/(B_1^2/8\pi)$ . In most cases of interest  $\beta$  is much larger than unity and, therefore, we neglect the second terms in both the numerator and the denominator in Eq. (25) herewith. The energy necessary for cooling the DT material to the solid temperature is estimated to be

$$Q_c = \frac{MC\Delta T}{\eta_c} \simeq \frac{MCT_2}{\eta_c} \,, \tag{26}$$

where M is the mass of the rods, C the heat capacity  $\sim \frac{fk_B}{2M}$  ( $k_B$  = Boltzmann's constant),  $\Delta T = T_2 - 24$  K, and  $\eta_c$  the cooling efficiency. The work  $W_r$  is calculated along the isotherm  $T = T_2$  yielding

$$W_r = p_2 V_2 \ln(V_2/V_1) = -NT_2 \ln \varepsilon , \qquad (27)$$

where  $N = n_2 V_2$ . Thus we obtain the direct conversion efficiency as

$$\eta_{\rm dc} = \frac{1 - \varepsilon^{\gamma - 1} \left[ 1 + \frac{\gamma - 1}{2} \left| \ell n \varepsilon \right| \right]}{1 + \frac{1}{2\eta_c} \varepsilon^{\gamma - 1}} \ . \tag{28}$$

For example, for  $\gamma = 5/3$ ,  $\varepsilon = 10^{-3}$ , and  $\eta_c = 0.1$ , one obtains  $\eta_{\rm dc} \cong 0.92$ . Here we assumed that when we return from the expanded final state back to the initial state, the temperature of the plasma  $T_2$  does not change. This is a pessimistic assumption. It is possible to cool by (natural) radiation of heat from the temperature  $T_2$  at the final stage of expansion down to the room temperature  $T_r = 0.03 \, \text{eV}$  and then compress isothermally (see Fig. 4). If this process is taken, Eq. (28) replaced by

$$\eta_{\rm dc} = \frac{1 - \varepsilon^{\gamma - 1} \left[ 1 + \frac{\gamma - 1}{2} \frac{T_r}{T_2} |\ell n \varepsilon| \right]}{1 + \frac{1}{2\eta_c} \frac{T_r}{T_2} \varepsilon^{\gamma - 1}} \,. \tag{28'}$$

## III. Energy of Muon Creation

Utilizing the present direct conversion method in Sec. II, we demonstrate that the energy cost of muon production for MCF is significantly reduced. We shall estimate the energy cost for three MCF reactor schemes: (A) the TEK scheme,<sup>8</sup> (pellet with a recirculating beam)
(B) the MIGMA<sup>16</sup> accelerator with the pellet, and (C) the Petrov's target and convertor<sup>6</sup> with the pellet synthesizer.

#### (A) The TEK scheme

In this reactor concept the muon sticking is reduced by the acceleration or heating by ICRH in the pellet with the structure described in Fig. 1. In concepts (A) and (B) the target and the synthesizer (along with the convertor) are one and the same (the synergetic). Furthermore, a portion of the beam energy is recovered for recirculation. With the direct energy convertor

upon this reactor concept, however, the recovered energy  $E_r$  is increased. The net energy cost  $E_{\mu}$  for muon is given by

$$E_{\mu} = E_0 - E_r \tag{29}$$

where  $E_0$  is the energy needed to create one  $\pi^-$  without energy recovery

$$E_0 = y_{\pi}^{-1} T_0 \tag{30}$$

with  $y_{\pi}$  being the yield of  $\pi^-$  (given for example in Ref. 7) and  $T_0$  is the energy of the projectile per nucleon. The recovered energy  $E_r$  by the direct conversion, the recirculation and the conventional thermal conversion can be estimated in the domain  $E_{r\min} \leq E_r \leq E_{r\max}$ , where

$$E_{r\min} = y_{\pi}^{-1} \left[ (T_0 - \mathcal{E}_{tot}) \eta_a + \mathcal{E}_{em} \eta_{dc} + \mathcal{E}_{nuc} \eta_{th} \right] , \qquad (31)$$

$$E_{r\max} = y_{\pi}^{-1} \left\{ (T_0 - \mathcal{E}_{tot}) \eta_a + \mathcal{E}_{em} \eta_{dc} + \mathcal{E}_{nuc} \left[ r \eta_{th} + (1 - r) \eta_{dc} \right] \right\} . \tag{32}$$

 $\mathcal{E}_{\rm tot} = \mathcal{E}_{\rm em} + \mathcal{E}_{\rm nuc}$ ,  $\mathcal{E}_{\rm nuc} = \mathcal{E}_{\rm el} + \mathcal{E}_{\rm in}$ , where  $\mathcal{E}_{\rm em}$ ,  $\mathcal{E}_{\rm el}$ , and  $\mathcal{E}_{\rm in}$  are the energy losses during one mean free path of the projectile in the target due to the electromagnetic, elastic nuclear and inelastic nuclear collisions and estimates of  $\mathcal{E}_{\rm em} \sim 100 \, {\rm MeV}$ ,  $\mathcal{E}_{\rm el} \sim 100 \, {\rm MeV}$ ,  $\mathcal{E}_{\rm in} \sim 300 \, {\rm MeV}$  may be used. The efficiencies  $\eta_{\rm dc}$ ,  $\eta_a$ , and  $\eta_{\rm th}$  are for the direct conversion, for the reacceleration (and cooling) of the recirculated beam, and for the thermal conversion, respectively. Estimates of  $\eta_{\rm dc} \sim 0.9$ ,  $\eta_a \sim 0.6$ , and  $\eta_{\rm th} \sim 0.35$  may be used. We define r as the neutron ratio in the collisions

$$r = \frac{A_1 + A_2 - Z_1 - Z_2}{A_1 + A_2} \,, \tag{33}$$

where A and Z are the atomic and charge numbers for the projectile and target.

For d + d collisions r = 0.5,  $E_0 = 5.9 \,\text{GeV}$  ( $T_0 = 1 \,\text{GeV}$ ), we get

while for d+t collisions  $r=0.6,\,E_0=4.8\,\mathrm{GeV}$   $(T_0=1\,\mathrm{GeV}),$ 

$$1.8 \,\text{GeV} \le E_{\mu} \le 2.3 \,\text{GeV} \ .$$
 (35)

For a d projectile on a 50% D+50% T target this constitutes a reduction of muon energy by a factor in the range of  $2.1 \le \frac{E_0}{E_{\mu}} \le 2.7$ .

#### (B) The MIGMA scheme

This scheme introduces a self-colliding projectile configuration on the pellet. The energy cost for muon production may be estimated in the domain of

$$E_{\mu \min} \le E_{\mu} \le E_{\mu \max} \tag{36}$$

where

$$E_{\mu \min} = \tilde{E}_0 - [r\eta_{\text{th}} + (1-r)\eta_{\text{dc}}] \tilde{E}_0 ,$$
 (37)

and

$$E_{\mu \text{max}} = E_0 - [r\eta_{\text{th}} + (1 - r)\eta_{\text{dc}}] E_0 , \qquad (38)$$

where  $\tilde{E}_0$  is the muon energy cost with multiple collisions allowed, while  $E_0$  is one with single collisions as defined by Eq. (30).

For d+d collisions  $\tilde{E}_0$  is estimated to be 4.5 GeV, while for d+t  $\tilde{E}_0 \simeq 3.7$  GeV.<sup>10</sup> Thus, for d+d collisions

$$1.7 \,\text{GeV} \le E_{\mu} \le 2.2 \,\text{GeV} \,\,, \tag{39}$$

and for d+t collisions

$$1.7 \,\mathrm{eV} \le E_{\mu} \le 2.1 \,\mathrm{GeV} \ . \tag{40}$$

The reduction factor for 50% D + 50% T target with d projectiles is  $E_0/E_\mu \simeq 2.5$ .

### (C) Petrov's production

In this method we combine Petrov's original method<sup>6</sup> of muon production by injection of the projectile on a target that is separate from the synthesizer (along with his converter) and the

TEK's pellet synthesizer.<sup>8</sup> In Petrov's scheme multiple collisions of projectile particles with target particles are expected. With sufficiently large (or long) target this is accomplished. The target is a thin rod of Be or T with length of  $1 - 3\ell_{\rm mfp}$  in the direction of the beam injection with aspect ratio of 100. This guarantees that the portion of beam suffers only electromagnetic collisions through the target. This target is placed in front of the convertor which is in front of the pellet (synthesizer). The pellet is surrounded by the direct energy convertor. The recoverable portion of the beam is again recirculated as in the previous cases. If the target is Be,  $\mathcal{E}_{\rm em} \cong \frac{1}{2} \mathcal{E}_{\rm tot}$ ,  $\mathcal{E}_{\rm nuc} \cong \frac{1}{2} \mathcal{E}_{\rm tot}$ , as  $Z_{\rm Be} = 4$  and  $\mathcal{E}_{\rm em}({\rm Be}) = Z_{\rm Be} \mathcal{E}_{\rm em}({\rm H})$ . The energy cost of muon production  $E_{\mu}$  is thus reduced from  $\widetilde{E}_{0}$  to

$$E_{\mu} = \tilde{E}_{0} \left[ \left( \frac{\mathcal{E}_{\text{em}}}{\mathcal{E}_{\text{tot}}} \eta_{\text{dc}} + \frac{\mathcal{E}_{\text{nuc}}}{\mathcal{E}_{\text{tot}}} \eta_{\text{th}} \right) F + \eta_{\text{acc}} (1 - F) \right] , \qquad (41)$$

where F is the fraction of energy deposited in the target relative to  $\tilde{E}$ . For example, if the length of the target is  $3\ell_{\rm mfp}$ ,  $F=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}=0.85$ . The energy cost is  $E_{\mu}=0.38\tilde{E}_0$  and thus the reduction factor is  $\tilde{E}_0/E_{\mu}\sim 2.6$ , where F is assumed to be 0.8 and other parameters are those as used in the previous examples. The energy  $\tilde{E}_0$  according to some reports (Jändel et al.<sup>17</sup>) is as low as 2 GeV for multiple scattering. This however may have to be upward adjusted because our configuration for the target and direct energy convertor may or may not fit the assumptions for lowest  $\tilde{E}_0$  calculations. For this scheme to work, one needs enough energy by the projectile beam so that the target material evaporates and explodes. This sets the condition for the duty cycle of the operation.

# IV. Reactor Implications

We have proposed a method of direct conversion of the thermal energy of the target heated by the projectile beam in order to reduce the energy cost of muon production for the MCF reactor. The present method has been considered for employment in a few different configurations of reactor environment. In each of these applications our calculation shows a reduction of muon production energy cost thanks to the present method by a factor of roughly 2.5 from the value without direct conversion. This reduced muon energy cost imposes a less severe condition for energy extraction in the reactor.

This muon production is intrinsically a pulse operation and may be applicable to Petrov's hybrid fission-fusion reactor concept<sup>6</sup> as well as to Tajima-Eliezer-Kulsrud's pure fusion reactor concept.<sup>8</sup> In particular it plays an important role to ameliorate the energy cost associated with the coronal plasma heating necessary in the T-E-K concept. The energy needed to run the reactor has to be substantially less than the usable energy obtained by fusion reactions. The ratio  $\alpha$  of the muon production energy to the electric energy mode available by fusion is given by

$$\alpha^{-1} = \frac{\eta_a \, \widetilde{\eta}_{\rm th} \, k \, \mathcal{E}_f \, X_\mu}{E_\mu} \,, \tag{42}$$

where  $\mathcal{E}_f = 17.6 \,\mathrm{MeV}$ ,  $X_\mu$  is the number of muon catalysis per muon  $(\mu^-)$ , and k is the nuclear energy multiplication factor ranging 1 to  $\sim 6$  from pure fusion to hybrid. The effective thermal efficiency  $\tilde{\eta}_{\mathrm{th}}$  is given by

$$\widetilde{\eta}_{\rm th} = \frac{1}{5} \eta_{\rm dc} + \frac{4}{5} \mathcal{M} \eta_{\rm th} , \qquad (43)$$

where the fusion alpha particle energy has been assumed to be recovered by our present direction conversion method and the neutron energy by the conventional thermal conversion. If there is a neutron blanket, the multiplication factor  $\mathcal{M}$  can be larger than unity. The value of  $\tilde{\eta}_{th}$  is 0.44 if  $\eta_{th} = 0.35$ ,  $\eta_a = 0.9$  and  $\mathcal{M} = 1$ . For an (economically) attractive reactor  $\alpha$  is much less than unity. If we demand  $\alpha$  be 0.13, this sets a critical  $X_{\mu}$  at which the MCF becomes economically feasible (or economic breakeven), while the so-called scientific breakeven corresponds to  $\alpha = 1$ . The critical  $X_{\mu}$  is thus given by

$$X_{\mu}^{c} = \frac{E_{\mu}}{\alpha \eta_{a} \, \tilde{\eta}_{th} \, k \, \mathcal{E}_{f}} \,. \tag{44}$$

For example, the economically critical  $X^c_{\mu}(\alpha=0.13)\cong 1600$  for  $\eta_a=0.6$ ,  $\tilde{\eta}_{\rm th}=0.44$ , k=1 (pure fusion), and  $E_{\mu}\sim 1$  GeV, while the scientific breakeven  $X^c_{\mu}(\alpha=1)\sim 220$ . In Petrov's

concept it allows larger  $\eta_a$  although more quantity of the tritium inventory is needed and no possible employment of Kulsrud's idea.<sup>9</sup>

The pure fusion reactor based on MCF has been proposed by Tajima, Eliezer, and Kulsrud,<sup>8</sup> using Kulsrud's ICRH method<sup>9</sup> to reduce the muon sticking on alpha particles. Kulsrud's method required the solid DT rods surrounded the coronal plasma to establish pressure confinement, which in turn is inertially confined as an overall pellet. The slightly offset frequency<sup>9</sup> so designed applied to the corona is capable of accelerating  $(\alpha\mu)^+$  above the value of energy below which the stripping cross-section of  $(\alpha\mu)^+$  atom in dense DT matter becomes negligibly small. The original velocity  $v_0$  corresponding to 3.5 MeV spirals into a limit circle with the stabale velocity at  $v_s$  corresponding to  $\sim 0.1$  MeV. The stopping power of an alpha particle is

$$\frac{d\mathcal{E}}{dx} = 10^3 f \left(\frac{v_0}{v}\right)^3 \left(\frac{45 \text{ KeV}}{\text{cm}}\right) , \qquad (45)$$

where f is the filling factor of rods in the pellet and v is the instantaneous velocity of  $\alpha$  or  $(\alpha\mu)^+$  particle. The  $(\alpha\mu)^+$  atom obeys the equation of motion

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m}\mathbf{E}(t) + \frac{e}{m_c}\mathbf{v} \times \mathbf{B} - \alpha_0 \left(\frac{v_0}{v}\right)^3 \mathbf{v} , \qquad (46)$$

with

$$\mathbf{E} = \hat{x} E_0 \cos(\omega' t + \phi) + \hat{y} E_0 \sin(\omega' t + \phi) , \qquad (47)$$

where  $E_0$  is the microwave amplitude,  $\omega' = \Omega_{\alpha\mu} + \Delta\omega$  and  $\Omega_{\alpha\mu}$  and  $\Delta\omega$  are the  $(\alpha\mu)^+$  ion cyclotron frequency and the offset frequency,<sup>9</sup> respectively. A typical orbit of  $(\alpha\mu)^+$  that approaches the stable limit circle whose energy is above this value is generated by computer from Eq. (46) and is shown in Fig. 5 as an example.

The muon kinetic processes of sticking and stripping may be described as

$$\frac{\partial N_0}{\partial t} = -N_0 \,\lambda_c \,\omega_s + N_1 \,\frac{\partial P_1}{\partial t} + N_2 \,\frac{\partial P_s}{\partial t} - \lambda_\mu \,N_0 + S \,\,, \tag{48}$$

$$\frac{\partial N_1}{\partial t} = \lambda_c \,\omega_s \,N_0 - N_1 \,\frac{\partial P_1}{\partial t} - N_1 \,\frac{v_0 - v_s}{a_s} - \lambda_\mu \,N_1 \,\,, \tag{49}$$

$$\frac{\partial N_2}{\partial t} = \frac{v_0 - v_s}{a_s} N_1 - \lambda_\mu N_2 - \frac{\partial P_s}{\partial t} N_2 , \qquad (50)$$

where  $N_0, N_1$ , and  $N_2$  are muon numbers of free muons, stuck muons, and muons in the stable limit cycle, respectively,  $P_1$  and  $P_s$  are stripping probabilities from the respective state, S is the muon production source,  $\omega_s$  the sticking ratio, and  $\lambda_c$  and  $\lambda_\mu$  are the rate of muon creating and the muon decay. We consider the steady state  $\frac{\partial}{\partial t} = 0$ . Then

$$N_0 = \frac{S}{\lambda_{\mu} + \lambda_c \,\omega_s^0 (1 - R) \,\frac{\lambda_{\mu}}{\dot{P}_s + \lambda_{\mu}}} \,, \tag{51}$$

where  $\omega_s = \omega_s^0(1 - R)$ . On the other hand we have

$$\frac{\dot{P}_s}{\lambda_{\mu}} = \frac{1}{\lambda_{\mu}} \frac{\partial P_s}{\partial \mathcal{E}} \frac{d\mathcal{E}}{dt} = \frac{v}{\lambda_{\mu}} \frac{\partial P_s}{\partial \mathcal{E}} \frac{d\mathcal{E}}{dx} , \qquad (52)$$

where  $\partial P_s/\partial \mathcal{E} \sim 0.1 \,\mathrm{MeV^{-1}}, \, v \sim 2.2 \times 10^8 \,\mathrm{cm/sec}$  and

$$\frac{d\mathcal{E}}{dx} = e E \tag{53}$$

in the steady state. Thus  $\dot{P}_s/\lambda_\mu \simeq 4.8 \times E/(100 \, \mathrm{keV/cm})$ . The ICRH assisted MCF catalysis number per muon is now given as

$$X_{\mu} = \left(\frac{\lambda_{\mu}}{\lambda_{c}} + \Omega\right)^{-1} \,, \tag{54}$$

where

$$\Omega = \omega_s^0 (1 - R) \frac{1}{(\dot{P}_s/\lambda_\mu) + 1} , \qquad (55)$$

where  $\dot{P}_s/\lambda_{\mu}$  is given by Eq. (52). Assuming  $\lambda_{\mu}/\lambda_c \sim 1/2000$ ,  $X_{\mu} \sim 1530$  at  $E=300 \, \mathrm{keV/cm}$ ,  $X_{\mu}$  becomes  $10^3$  at  $E=100 \, \mathrm{keV/cm}$  for example. The necessary electric power of RF fields for this for our pellet is approximately  $10^6 \, \mathrm{W}$  at peak power at  $E=300 \, \mathrm{keV/cm}$  and  $\sim 10^3 \, \mathrm{W}$  on average power assuming the duty cycle of  $100/\mathrm{sec}$ . The value of  $X_{\mu}$  obtained in Eq. (54)

at  $E = 300 \,\mathrm{keV/cm}$  is in the neighborhood of the necessary value of  $X_{\mu}^{c}$  required in Eq. (44). One notes, however, that in deriving these equations a number of approximations and simplifications have been done and more research is necessary to determine the implication of these equations. Some of the processes which were neglected so far will be considered in the next section.

#### V. Discussion

The model considered in Sec. II for the direct conversion processes assumes a sharp clear-cut boundary between the DT material (or plasma) and the vacuum. In reality the boundary may be diffuse. Furthermore, this boundary may experience deceleration due to the rising magnetic pressure or other reasons and then experience the Rayleigh-Taylor instability. Under such circumstances the sharp and well-defined boundary assumed in Fig. 2 and in Sec. II is not realized any more. The boundary becomes milky and less efficient compression of the vacuum volume may result. The growth rate of the Rayleigh-Taylor instability is given by

$$\gamma \sim \sqrt{g/a} \sim \left(\frac{\Delta P}{a^2 \rho}\right)^{1/2} ,$$
 (56)

where  $\Delta P$  is the pressure difference over the boundary  $\Delta P = nT + B_1^2/8\pi - B_0^2/8\pi$ ,  $\rho$  the mass density, and a the thickness of the boundary. The safety condition for no mixing  $\gamma \tau_{\mu} \lesssim 1$  may be cast into

$$\tau_{\mu} \lesssim \frac{a}{\Delta c_s} \,,$$
(57)

where  $\Delta c_s \equiv (\Delta P/\rho)^{1/2}$  and the right-hand side is the differential sound time of the boundary. In the analysis in Secs. II–IV we have further neglected the possibility that the DT material during the adiabatic expansion can be cooled beyond the recombination temperature. If so, the adiabatic equation state that has been used for work calculation has to be replaced by an appropriate counterpart that takes this phase transition into account.

In a more realistic model of the TEK concept the coronal plasma is immersed between the DT fuel and the vacuum (Fig. 6). The boundary in Fig. 3 is now that of the corona and the vacuum in Fig. 6. If we ignore the rods, the efficiency of direct energy conversion by the corona is

$$\eta_{\rm dc} = 1 - \varepsilon^{\gamma - 1} \left[ 1 + \frac{\gamma + 1}{2} \left( \ell n \varepsilon \right) \right] ,$$
(58)

where we assumed that  $T_2 = T_r$  and  $\beta = \infty$ . When  $\beta < \infty$ ,  $\eta_{dc}$  is slightly larger than the value in Eq. (58).

The present method of direct energy conversion may help the TEK reactor concept.<sup>8</sup> In its operation it was necessary to heat the corona so that the pressure balance between the reacting DT fuel and the corona is maintained. This may be done by (i) the alpha particle heating of the corona, (ii) shock wave heating of the corona, (iii) the external heating of the corona. The first two seem to be unlikely to succeed. The third method calls for extra energy for heating the corona, which costs energy as much as the thermal energy of the rods times the ratio of the coronal volume to the DT fuel rods. This is clearly very costly. Further studies of feasibility of these are necessary.

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#### References

- 1. J.D. Jackson, 1957, Phys. Rev. 106, 330.
- 2. S.E. Jones et al., 1986, Phys. Rev. Lett. 56, 588.
- S.S. Gershtein and L.I. Ponomarev, Muon Physics, Vol. III, Eds. V.W. Hughes and C.S. Wu, (Academic Press, N.Y., 1975) p. 141.
- 4. L. Bracci and G. Fiorentini, 1982, Phys. Rep. 86, 169.
- 5. S. Eliezer, 1988, Laser and Particle Beams 6, 63.
- 6. Yu.V. Petrov, 1980, Nature 285, 466.
- 7. S. Eliezer, T. Tajima, and M.N. Rosenbluth, 1987, Nucl. Fusion 27, 527.
- 8. T. Tajima, S. Eliezer, and R.M. Kulsrud, Muon Catalyzed Fusion, AIP Conf. Proc. 181, Eds. S.E. Jones, J. Rafelski, and H. Monkhorst (AIP, New York, 1989) p. 423.
- 9. R.M. Kulsrud, Muon Catalyzed Fusion, AIP Conf. Proc. 181, Eds. S.E. Jones, J. Rafelski and H. Monkhorst (AIP, New York, 1989) p. 367.
- 10. Yu.V. Petrov and Yu.M. Shabelskii, 1979, Sov. J. Nucl. Phys. 30, 66.
- 11. L. Bracci and G. Fiorentini, 1982, Nature 297, 134.
- R.J. Noer, 1982, Appl. Phys. A28 (1); J.W. Wang and G.A. Loew, 1985, IEEE Trans.
   Nucl. Sci., NS-32, 2915.
- 13. G.I. Budkar and A. Skrinsky, 1978, Usp. Fiz. Nauk, 124, 561.
- 14. D. Möhl, G. Petrucci, L. Thormodahl and S. Van der Meer, 1980, Phys. Rep. 58, 73.
- 15. L.I. Menshikov and L.I. Ponomarev, 1987, JETP Lett. 46, 312.

- 16. B.C. Maglich and J. Norwood (Eds.), *Aneutronic Energy* (North Holland, Amsterdam, 1988).
- M. Jändel, M. Danos, and J. Rafelski, 1988, Phys. Rev. C. 37, 403; also, 1988, Muon Cat. Fus. 2-3.

#### Figure Captions

- 1. A schematic description of the configuration of the injected beam and the target immersed in the magnetic mirror.
- 2. A schematic external circuit for direct conversion of muon-catalyzed fusion energy through the compression of magnetic fields. (a) Before the reaction; (b) During and after the reaction.
- 3. Illustration of the states before (a) and after (b) the explosion.
- 4. The thermodynamical cycle for the present direct energy converter in the p-V diagram. The upper curve  $T_1$   $T_2$  is the isentrope, while the lower one  $T_r$   $T_r$  is the isotherm.
- 5. A typical orbit of  $(\alpha \mu)^+$  in the ICRH field. The mesoatom starts from the lower left corner in the  $v_x v_y$  plane and eventually ends up in the limit circle. (NB: This computer output has a scale in the  $v_y$  larger than that in the  $v_x$  so that a circle appears elongated in the y-direction).
- 6. A schematic picture of the system with a coronal plasma (compare with Fig. 3).

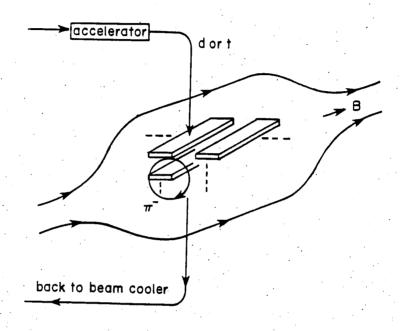
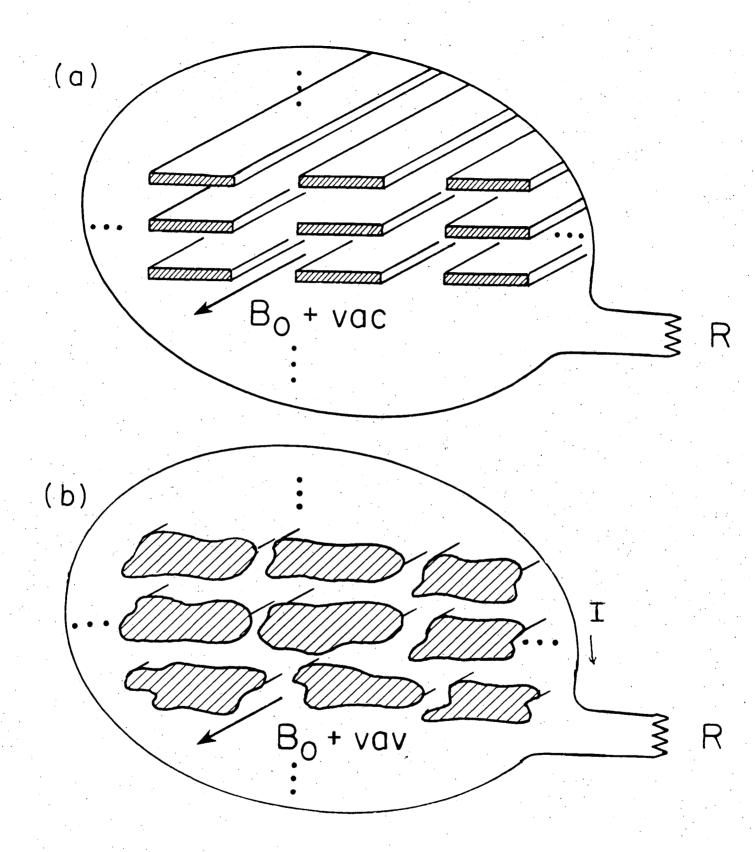


Fig. 1



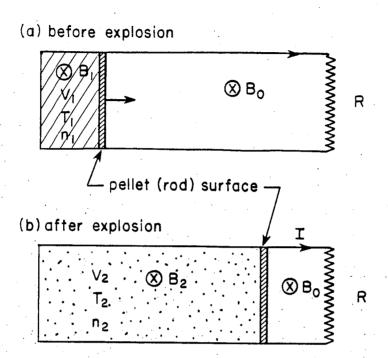


Fig. 3

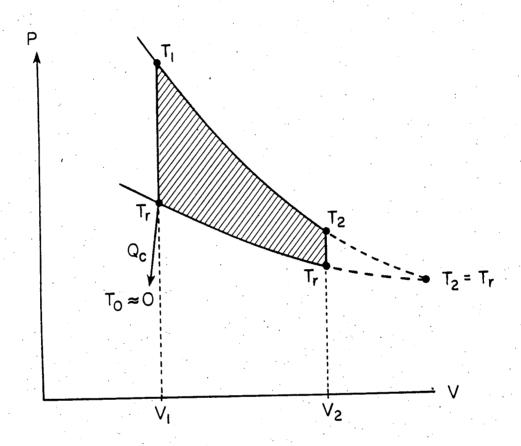


Fig. 4

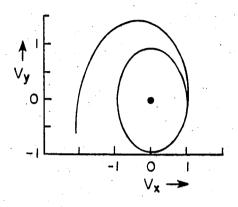


Fig. 5

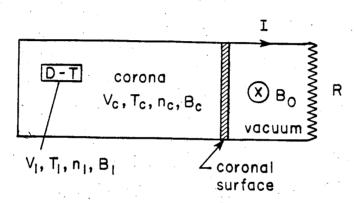


Fig. 6