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The Neoclassical Transport of Toroidal Momentum in Tokamaks

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Abstract

Allowing for the presence of an expected moderate concentration ($\sim 10\%$) of low energy ions in excess of the Maxwellian distribution causes a dramatic change in the neoclassical prediction for momentum transport, leading to agreement in magnitude with experiment.

It is well known that the experimentally observed rates of toroidal momentum transport in tokamaks[1,2] have not been explained by neoclassical theory. Taking the appropriate component of the ion pressure tensor to be given by $\pi_{\phi r} = -n_i m_i \mu_m \partial V_T/\partial r$, where ϕ is the toroidal angular coordinate and V_T the mean toroidal velocity on a magnetic surface, the neoclassical value found for the momentum diffusivity μ_m in the original work of Rosenbluth et al.[3] and confirmed more recently[4] was $0.1(r/R)^2\rho_{i\theta}^2\nu_{ii}$, where $\rho_{i\theta}$ is the ion Larmor radius in the poloidal field and ν_{ii} the standard ion collision frequency. This value of μ_m is about two orders of magnitude too small to explain the experimental results. (An attempt to explain the enhanced transport based on classical gyroviscosity[5] has been refuted[6] and has not been accepted by the theoretical community.) In contrast to this substantial discrepancy the much larger value for the neoclassical ion thermal diffusivity ($\chi_{i\mathrm{NC}}$) had the right magnitude to explain most experimental results throughout the 1970's and early 80's. More recently, with increased auxiliary ion heating and improved diagnostic measurements, experiments have given values of χ_i well above $\chi_{i\mathrm{NC}}$. The ratio $\chi_i/\chi_{i\mathrm{NC}}$ increases with minor radius and can be as large as 10 to 15 in the outer regions.[7,8] To explain the discrepancies in both μ_m and χ_i anomalous transport caused by electrostatic turbulence has generally been proposed[8,9] with the "η_i mode" the popular candidate for causing the turbulence.[10] There is experimental evidence for such turbulence at high densities in TEXT[11] and there is qualitative evidence[8,9] that ion energy containment is improved when $\eta_i\,(\equiv d\ln T_i/d\ln n_i)$ is reduced, but there is no definitive proof that such turbulence is the main cause of the enhanced ion transport.

An alternative explanation for the enhancement of χ_i over the standard neoclassical value is the increased neoclassical heat conduction which will be caused by an enhanced non-Maxwellian tail to ion velocity distribution f_{i0}.[12] Such non-Maxwellian tails are caused firstly by the ion heating process. With neutral beam heating, for example, the dominant collisions for the tail ions will be with the slowing down beam ions and with electrons; the reciprocal of the resultant negative slope for $\ln f_{i0}$ can be substantially larger than the temperature of the bulk ions.[13] Secondly, in the modern tokamak experiments with long pulse times and long heating pulses, once the slowing down beam ions have scattered sufficiently in pitch angle to populate the trapped region of velocity space, these ions will contribute to the enhanced ion heat conduction. Lastly, the heat conduction process contributes to the non-Maxwellian tail. The large poloidal Larmor radii of these energetic ions causes them to diffuse outward radially too rapidly to thermalize with the bulk ions.[12] The relative enhancement of the distribution tail increases with minor radius and hence the enhancement of χ_i will increase as observed experimentally. Such non-Maxwellian tails were observed experimentally in PDX,[14] ATC[15], and DIII[16], but for most experiments the presence or absence of such tails is unknown. Allowing for electron as well as ion collisions these tails can easily enhance the neoclassical heat conduction by an order of magnitude.[13]

But even if this increased neoclassical heat conduction due to the non-Maxwellian distribution tail is the prime cause of the anomalous χ_i, there still remained the abysmal failure of neoclassical theory to explain momentum transport. This has now changed. In a recent paper[17] the author has argued that an additional non-Maxwellian part of the ion velocity distribution will occur at low energies. Due to the combined effects of a large collision frequency for pitch angle scattering (because of the impurities present) and electrostatic diffusion, low energy ions diffuse inward too rapidly to thermalize with the outward diffusing energetic ions and a near singularity will occur in f_{i0} at low energies. It was shown that the presence of a moderate concentration of such excess low energy ions ($\sim 10\%$) was necessary to explain the density asymmetry measurements in PDX,[18] where conventional theory fails to explain even simple momentum balance within a magnetic surface. In addition, it was shown that the presence of these excess low energy ions leads to simple neoclassical explanations for many hydrogen and impurity ion transport phenomena. In particular, it was pointed out that there is a dramatic increase in the neoclassical prediction for the momentum diffusivity μ_m. The theory leading to that conclusion is presented here.

The neoclassical contribution to $\pi_{\phi r}$ for an ion species j is

$$\pi_{j\phi r} = -\int \frac{d\theta}{2\pi} \int m_j v_\| \frac{\left(\mu B + v_\|^2\right)}{\Omega_j R} \sin\theta \tilde{f}_{js} d^3v, \tag{1}$$

where Ω_j is the cyclotron frequency and $\tilde{f}_{js}$ is the part of the species distribution function which varies as $\sin\theta$ and is odd in $v_\|$ being given by

$$v_\| \frac{B_\theta}{B} \frac{\partial \tilde{f}_{js}}{r\partial\theta} = \tilde{C}_{jc} \equiv \sum_k \left[C_{jk}\left(\tilde{f}_{jc}, f_{k0}\right) + C_{jk}\left(f_{j0}, \tilde{f}_{kc}\right)\right]. \tag{2}$$

Here C_{jk} is the collision operator for collisions with species k and $\tilde{f}_{jc}$ is the part of f_j which varies as $\cos\theta$ and is even in $v_\|$. In particular the part of $\tilde{f}_{jc}$ which is proportional to $V_T'\,(\equiv \partial V_T/\partial r)$ for $V_T \ll v_{T_j}$ is

$$\tilde{f}_{jc} = -\frac{\left(\mu B + v_\|^2\right)}{\Omega_{j\theta}} \frac{m_j V_T'}{T_j} f_{j0} \frac{r}{R} \cos\theta, \tag{3}$$

where $\Omega_{j\theta}$ is the cyclotron frequency for species j in the poloidal magnetic field. The contributions to $\pi_{\phi r}$ due to the parts of $\tilde{f}_{jc}$ not proportional to V_T' are given at the end of this letter.

It is first noted that Eq. (1) can formally be written in the form

$$\pi_{j\phi r} = -\frac{\left(q_{j\|s} + q_{j\|\|s}\right)}{2\Omega_j R}, \tag{4}$$

where $q_{j\|s}$ is the amplitude of the $\sin\theta$ component of the total heat flow for species j parallel to $\mathbf{B}$ and $q_{j\|\|s}$ is the corresponding quantity for the flow of parallel energy. In the original treatment of Rosenbluth et al.[3] a pure hydrogen plasma was considered and only ion-ion collisions were included. As a result, since the collision operator in Eq. (2) then conserves energy, there can be no $\sin\theta$ heat flow to first order in the poloidal Larmor radius. Only $q_{i\|\|s}$ will contribute to $\pi_{\phi r}$; it is balanced by a reverse flow of perpendicular energy. The resultant value was found to be very small

$$\pi_{\phi r}\Big|_{ii} = -0.1 n_i \left(\frac{r}{R}\right)^2 \nu_{ii} \rho_{i\theta}^2 m_i V_T'. \tag{5}$$

If one allows for collisions with an impurity species, $q_{j||s}$ can be nonzero. $q_{z||s}$ is found to be negative and $q_{i||s}$ positive. If the $\cos\theta$ component of the collisional energy transfer from Z-ions to hydrogen ions is denoted by $\tilde{Q}_{zi}\cos\theta$, energy balance requires

$$\begin{aligned} \frac{B_\theta}{B}\frac{\partial}{r\partial\theta}\left(q_{z||s}\sin\theta\right) &= -\tilde{Q}_{zi}\cos\theta \\ &= -\frac{B_\theta}{B}\frac{\partial}{r\partial\theta}\left(q_{i||s}\sin\theta\right) \end{aligned} \tag{6}$$

and from Eqs. (4) and (6) the value of $\pi_{\phi r}$ due the two $q_{j||s}$ heat flows is

$$\pi_{\phi r} = \pi_{z\phi r} + \pi_{i\phi r} = \frac{r\tilde{Q}_{zi}}{2R\Omega_{i\theta}}\left[\frac{m_z}{Zm_i} - 1\right]. \tag{7}$$

The $q_{j||||s}$ terms give an extra small contribution. However, the net increase in momentum transport is not large enough to explain the experimental results if $T_z \simeq T_i$. The heat transfer caused by $C_{zi}\left(\tilde{f}_{zc}, f_{i0}\right)$ is reduced partly by the term $C_{zi}\left(f_{z0}, \tilde{f}_{ic}\right)$ and the square bracket term in Eq. (7) gives a further approximate 50% reduction. This picture changes when allowance is made for an excess of low energy hydrogen ions.

The hydrogen ion distribution function will be assumed to have the form $f_{i0} = f_H + f_C$ where f_H is Maxwellian (n_H, T_H) and f_C represents the excess low energy ions. The velocity dependence of f_C is unknown but will be assumed Maxwellian here for simplicity, with density n_C and temperature T_C. In fact, f_C will probably be more peaked towards low energies than a Maxwellian,[17] in which case a lower concentration n_C would suffice to give the observed momentum transport since it is the ions with $v \lesssim v_{TZ}$ which cause the dominant drag part of the collision operator C_{ZC}. Steady state conditions are assumed with the heating of the n_C ions due to the temperature difference balanced by the term $(3\Gamma_C/2)\,\partial T_C/\partial r$ as discussed in Ref. 17.

The contribution to $\pi_{\phi r}$ due to the n_C ions will be small, since n_C is highly nonuniform on a magnetic surface. $n_C \sim \exp\left(e\hat{\Phi}_0\cos\theta/T_C\right)$, where $e\hat{\Phi}_0 = m_iV_T^2(r/R) - e\tilde{\Phi}_0$ and $\tilde{\Phi}_0$ is the amplitude of the $\cos\theta$ part of the electrostatic potential. To explain the PDX results it was necessary to assume $e\hat{\Phi}_0/T_C \gtrsim 1.5$. As a result any $\sin\theta$ flows for the n_C ions will be small and, in particular, $\tilde{Q}_{CZ}$ will be balanced mainly by the $\cos\theta$ part of the divergence of their radial heat flow. There remains the Z-ion contribution to $\pi_{\phi r}$. Since the factor $\left(\mu B + v_\parallel^2\right) = [(4/3)P_0 + (2/3)P_2]\,(v^2/2)$, where P_n is the nth Legendre polynomial in $\xi \equiv v_\parallel/v$, after substituting from Eq. (2) into Eq. (1), one obtains

$$\pi_{\phi r}\Big|_{ZC} = -\int \frac{d\theta}{2\pi}\int d^3v \frac{m_z v^2 r}{2\Omega_{Z\theta} R}\cos\theta\left[\frac{4}{3}P_0\tilde{C}_{ZC}\left(P_0\right) + \frac{2}{3}P_2\tilde{C}_{ZC}\left(P_2\right)\right]. \tag{8}$$

Since $(4/9)\int P_2^2 d\xi$ is one twentieth of $(16/9)\int P_0^2 d\xi$, the P_2 terms are neglected. This is justified since the pitch angle scattering parts of $\tilde{C}_{ZC}$, which only appear in the P_2 term integrals, give smaller contributions than the energy scattering parts. Now only the energy part of C_{ZC} is required which has the form

$$C_{ZC} = -\frac{\gamma}{v^2}\frac{\partial}{\partial v}v^2\left(\frac{m_i}{m_z}f_z\frac{\partial H_C}{\partial v} - \frac{1}{2}\frac{\partial^2 G_C}{\partial v^2}\frac{\partial f_z}{\partial v}\right), \tag{9}$$

where $\gamma = 4\pi Z^2 e^4 \ln\Lambda/m_i^2$ and H_C, G_C are the Rosenbluth potentials[19] for the n_C ions. The required functions of H_C, G_C are[20]

$$\begin{aligned}\frac{\partial H_C}{\partial v} &= -\frac{4\pi}{v^2}\int_0^v f_C v^2 dv,\\ \frac{1}{2}\frac{\partial^2 G_C}{\partial v^2} &= \frac{4\pi}{3v^3}\int_0^v f_C v^4 dv + \frac{4\pi}{3}\int_v^\infty f_C v\, dv,\end{aligned} \tag{10}$$

where $f_C = f_{C0}\left(1 - \frac{2v^2 m_i V_T' r\cos\theta}{3\Omega_{i\theta}T_C R}\right)$ with only the P_0 part of $\tilde{f}_C$ having been retained. f_z in Eq. (8) will have the corresponding form.

After substituting from Eqs. (9) and (10) into Eq. (8), linearizing the collision operator with respect to the small $\cos\theta$ terms, integration yields

$$\begin{aligned}\pi_{\phi r}\Big|_{ZC} &= -\frac{10}{3}n_C\left(\frac{r}{R}\right)^2 \nu_{CZ}\rho_{Z\theta}^2 m_i V_T'\\ &\times\left\{\frac{\left[1+\frac{2}{5}\left(\frac{v_{T_Z}}{v_{T_C}}\right)^2 - \frac{3}{5}\frac{T_C}{T_Z}\right] - \frac{Zm_i}{m_z}\left[\frac{T_C}{T_Z}\left(\frac{2}{5}+\frac{v_{T_Z}^2}{v_{T_C}^2}\right) - \frac{3}{5}\left(\frac{v_{T_Z}}{v_{T_C}}\right)^2\right]}{\left[1+\left(\frac{v_{T_Z}}{v_{T_C}}\right)^2\right]^{5/2}}\right\},\end{aligned} \tag{11}$$

where $\nu_{CZ} = 4(2\pi)^{1/2} n_Z Z^2 e^4 \ln\Lambda / 3 m_i^{1/2} T_C^{3/2}$ and $\rho_{Z\theta}^2 = 2 m_Z T_Z / Z^2 e^2 B_\theta^2$. If T_C satisfies the inequalities

$$\left(\frac{m_i}{m_z}\right) \ll \frac{T_C}{T_Z} \ll 1, \tag{12}$$

then the quantity in the curly brackets in Eq. (11) is approximately unity and the ratio of Eq. (11) to Eq. (5) is

$$\frac{\pi_{\phi r}\,|_{ZC}}{\pi_{\phi r}\,|_{ii}} = 33\sqrt{2}\left(\frac{n_C}{n_i}\right)\left(\frac{T_H}{T_C}\right)^{3/2}\left(\frac{n_z Z^2}{n_i}\right)\left(\frac{m_z}{Z^2 m_i}\right). \tag{13}$$

Even for small (n_c/n_i) this ratio can easily be of order 10^2 if (T_H/T_C) is large, which is the required increase to give order of magnitude agreement with the experimental results. If v_{T_Z}/v_{T_C} is not small the ratio will still be large. If for example $v_{T_C} = v_{T_Z}$ the curly bracket terms in Eq. (11) reduce the numerical factor in Eq. (13) by a factor of 4 but then $(T_H/T_C)^{3/2}$ is very large; $(T_H/T_C)^{3/2} \simeq (m_z/m_i)^{3/2}$ for $T_H \simeq T_Z$.

If one includes the other parts of $\tilde{f}_{jc}$ which were omitted from Eq. (3) (see for example Ref. 21) and at the same time allows for $V_T \sim v_{T_Z}$, the extra contributions to momentum transport are, assuming the inequalities of Eq. (12) hold,

$$\begin{aligned}\pi_{\phi r} \;\simeq\; -n_C\left(\frac{r}{R}\right)^2 \nu_{CZ}\rho_{Z\theta}^2 m_i \Bigg\{ 2V_T\frac{p_z'}{p_z} + \left(\frac{m_z V_T^2 - \frac{R}{r} Ze\tilde{\Phi}_0}{T_Z}\right) V_T' \\ -\Omega_{Z\theta}\left[\left(\frac{m_z V_T^2 - \frac{R}{r} Ze\tilde{\Phi}_0}{T_Z}\right) + \frac{R\tilde{n}_C}{r n_C}\right]\Bigg\},\end{aligned} \tag{14}$$

where p_z is the impurity species' pressure and $\tilde{n}_C/n_C$ is the $\cos\theta$ component of $\exp\left(e\hat{\Phi}_0 \cos\theta / T_C\right)$ divided by its average. The first two terms in the curly brackets will add to the expression in Eq. (11) since they transport momentum outwards with the sign of V_T. The third term transports positive momentum outwards and this term will lead to improved momentum confinement with counter neutral beam injection, as is observed experimentally.[22] This term could also explain the observation that once substantial counter rotation has been generated it can be maintained with fewer beam lines plus more gas puffing.[22]

It is concluded that the expected presence of a moderate concentration of excess low energy hydrogen ions ($\sim 10\%$) will increase the neoclassical momentum diffusivity by two orders of magnitude giving order of magnitude agreement with experimental values.

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