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# The Rapid Inward Diffusion of Cold Ions in Tokamaks and Their Effect on Ion Transport

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**Abstract**

The observed increase with density of the density asymmetry caused by the centrifugal force of toroidal motion in the PDX tokamak [Plasma Physics and Controlled Nuclear Fusion Research, (IAEA, Vienna, 1981) Vol. 1, p. 665], which is contrary to conventional theory, is explained by the presence of an excess of low energy ions with 10–15% concentration. The prime source being recycling, it is shown that low energy ions undergo rapid inward diffusion (too rapid to thermalize with the outward diffusing energetic ions) due to the combined effects of large $\nu_{\mathrm{PA}}$, electrostatic diffusion, and negative $E_r$ and $\partial T_i/\partial r$. The presence of the low energy ions alters dramatically the predictions of neoclassical theory and many hydrogen and impurity ion transport phenomena now have simple explanations.

# I. Introduction

A basic assumption of both neoclassical and anomalous transport theory is that the velocity distribution function for each particle species is an approximate Maxwellian, which can be described by a temperature, and that perturbations caused by finite Larmor radius effects and the transport are small. This assumption clearly facilitates greatly both theory and the experimental description of the plasma. For standard neoclassical ion transport in a tokamak, if the second order drift-kinetic equation is averaged over velocity pitch angle and over a magnetic surface, the divergence of the radial diffusion is at most proportional to $(r/R)^{1/2}\nu_{\rm PA}\left(\rho_{i\theta}/L\right)^2$, while the divergence in velocity space is proportional to $\nu_{\rm ES}$. (Here $\rho_{i\theta}$ is the ion poloidal Larmor radius, $L$ the radial scale length, and $\nu_{\rm PA}$, $\nu_{\rm ES}$ the collision frequencies for pitch angle and energy scattering, respectively.) Thus, with similar magnitudes for $\nu_{\rm PA}$ and $\nu_{\rm ES}$, the perturbation to the distribution function caused by the transport is of order $(r/R)^{1/2}\left(\rho_{i\theta}/L\right)^2 f_0$, which is small since $\rho_{i\theta}/L$ is normally small. This assumption breaks down at high ion energies since $\rho_{i\theta}/L$ is then proportional to $\left(v^3/v_{T_i}^2\Omega_{i\theta}\right)(T_i'/T_i)$, which is greater than unity for sufficiently high $v$. The presence of a non-Maxwellian tail to $f_0$ has been observed experimentally[1,2,3] and explained theoretically.[4] (For most experiments the presence, or absence, of a non-Maxwellian tail is unknown.) Allowing for electron as well as ion collisions, the non-Maxwellian tail can enhance the neoclassical ion heat conduction by an order of magnitude.[5]

This paper is concerned with another breakdown of the above basic assumption which will occur at low ion energies. There are several effects which combine to cause an excess of low energy ions throughout the plasma. To facilitate the discussion of this excess the simplest model, which will be used in some of the calculations, is to assume that $f_0 = f_C + f_H$ where $f_C$ and $f_H$ are Maxwellians with densities $n_C$, $n_H$ and temperatures $T_C$, $T_H$. A more realistic model assumes $f_0$ is the Maxwellian $f_H$ for velocity magnitudes above a certain value $V_C$, but has a steeper negative slope $(\partial f/\partial v)$ below $V_C$. The excess density of low energy particles is then $n_C = \int_0^{V_C} (f_0 - f_H)\, d^3v$, but this excess density cannot be given a temperature. The processes which combine to generate $n_C$ are:

### (a) Recycling and gas puffing

Ionization of the neutral gas entering the plasma produces low energy ions near the wall with about 1 eV energy, whereas the local ion temperature ($T_H$) can be from 20 to 50 eV or more. A two component ion distribution is thus generated as a boundary condition near the wall.

### (b) $\nu_{\mathrm{PA}} \gg \nu_{\mathrm{ES}}$ for low ion energies

Allowing for the presence of impurities $\nu_{\mathrm{PA}}$ is large $\left(\sim n_Z Z^2/T_C^{3/2}\right)$, whereas $\nu_{\mathrm{ES}}$ is small $\left(\sim n_H/T_H^{3/2}\right)$. These effects are considered more precisely in Sec. III.

### (c) Electrostatic diffusion

Unlike the magnetic particle drift which causes normal neoclassical transport and is proportional to particle energy, the electric drift $(\mathbf{E} \times \mathbf{B}/B^2)$ is independent of particle energy. Under collisionless conditions this plasma motion is very different from a moving Maxwellian. With an electrostatic potential variation on a magnetic surface $\left[\tilde{\Phi}(\theta)\right]$, a near singularity in the Pfirsch-Schlüter friction results at low particle energies[6] and hence a corresponding near singularity in the electrostatic diffusion.[7]

For a stable pure hydrogen plasma $e\tilde{\Phi}/T \sim (\rho_{i\theta}/L)\,(r/R)$, but with impurities and poloidal rotation present the magnitude of $e\tilde{\Phi}/T$ can be much larger.[8] (The presence of electrostatic turbulence can be expected to enhance further the inward diffusion of low energy ions.)

### (d) The radial electric field

In ohmic discharges there is a strong negative radial electric field (with associated poloidal rotation) whose magnitude fits that expected for neoclassical ambipolarity. This electric field will be the dominant "force" driving the inward diffusion of low energy ions in such discharges. With substantial toroidal rotation (due to neutral beam injection) the effective radial electric field ($E_r - V_T B_\theta$) and the experimental poloidal rotation[9] are very small. (In neoclassical theory the strong poloidal nonuniformity in density inhibits poloidal rotation.) In such cases the effective potential is $\hat{\Phi}$ with $e\hat{\Phi} = m_i V_T^2 (r/R) \cos\theta - e\tilde{\Phi}$ and $e\hat{\Phi}/T_c$ will be large. Neglecting temperature gradient effects the dominant "force" driving inward diffusion will then be proportional to $(n'_Z/n_Z - n'_C/n_C)$ (see Sec. IV).

These effects combine to cause a strong inward diffusion of low energy ions which is too rapid to permit thermalization with the outward diffusing hot ions ($n_H$) as shown qualitatively in the theory presented in Sec. III. The presence of the excess low energy ions ($n_C$) leads to major changes in the predictions of neoclassical theory and to simple explanations of various hydrogen and impurity ion transport phenomena as discussed in Sec. IV.

The experimental results which convinced the author of the presence of an excess low energy component $n_C$ were the density asymmetry measurements made by Grek and Johnson on PDX.[10] The surprising but clear-cut result of these measurements is completely contradictory to the prediction of simple momentum balance within a magnetic surface if for each particle species $f_0$ is Maxwellian in lowest order; it presents strong proof that there is an excess low energy component $n_C$. This experimental result is therefore considered in some detail first in Sec. II.

## II. The PDX Density Asymmetry Measurements

Using the 56 channel Thomson scattering diagnostic Grek and Johnson[10] measured the electron density and temperature profiles on the horizontal midplane of PDX with neutral beam injection. The observed density profiles, reproduced in Fig. 1, show an in/out asymmetry as would be expected because of the centrifugal force of the toroidal mass motion. The surprising result is that as the plasma density is increased, all other experimental conditions being kept constant, the in/out density difference increases, despite the fact that the toroidal velocity was found to satisfy $V_T \sim n_e^{-1}$. Figure 2 is reproduced from a presentation by Goldston[11] and shows $\tilde{n}_e$ plotted against $\bar{n}_e$ for the minor radius $a/2$. Here $\tilde{n}_e = [n_e(\theta = 0) - n_e(\theta = \pi)]/2$ and $\bar{n}_e = [n_e(\theta = 0) + n_e(\theta = \pi)]/2$. Despite the scatter of the points there is a clear increase of $\tilde{n}_e$ with $\bar{n}_e$. The dotted straight line, added by this author, satisfies

$$\tilde{n}_e = 0.25 \left(\bar{n}_e - 2 \times 10^{13}\right) \text{cm}^{-3}. \tag{1}$$

In contrast to this result, conventional theory requires, firstly, there be little temperature variation on a magnetic surface and, secondly, from momentum balance parallel to $\mathbf{B}$,

$$\frac{\partial p}{r\partial\theta} = -\frac{n_e m_i V_T^2}{R}\sin\theta \qquad \text{or} \qquad \frac{\tilde{n}_e}{\bar{n}_e} = \frac{m_i V_T^2 r}{(T_i + T_e)\, R}. \tag{2}$$

Since the experiment showed both $T_i$ and $V_T$ varied inversely as $n_e$ but $T_e$ remained approximately constant, $\tilde{n}_e/\bar{n}_e$ should either vary as $n_e^{-1}$, for $T_i \gg T_e$ or vary as $n_e^{-2}$ if $T_e \gg T_i$ or something in between if $T_e \sim T_i$, all of which are in marked disagreement with experiment. Thus conventional theory cannot explain the simple momentum balance within a magnetic surface.

If instead one assumes that $f_0$ for the ions is the sum of two Maxwellians as described in the Introduction, i.e., $f_0 = f_C + f_H$ with $T_C \ll T_H$, an entirely different result is obtained. Since $\nu_{\text{PA}}$ is large for low energy ions, they will have the same toroidal mass motion as $f_H$ even though they have not thermalized. The poloidal variation of $n_C$ will be given by $n_C = N \exp\left(e\hat{\Phi}_0 \cos\theta / T_C\right)$ where $N$ is a constant and $e\hat{\Phi}_0 = m_i V_T^2 (r/R) - e\tilde{\Phi}_0$. Neglecting the much weaker poloidal variation of $n_H$ and using the above definitions of $\tilde{n}_e$, $\bar{n}_e$, then $\tilde{n}_e = \tilde{n}_i = N \sinh\left(e\hat{\Phi}_0/T_C\right)$ and $\bar{n}_e = n_H + N\cosh\left(e\hat{\Phi}_0/T_C\right)$. Eliminating $N$ from these two equations,

$$\tilde{n}_e = (\bar{n}_e - n_H)\tanh\left(\frac{e\hat{\Phi}_0}{T_C}\right)$$
$$\simeq \bar{n}_e - n_H \tag{3}$$

provided

$$\frac{e\hat{\Phi}_0}{T_C} \gtrsim 1.5 \tag{4}$$

for the range of densities in this PDX experiment. Thus, subject to the condition Eq. (4), $\tilde{n}_e$ will increase with density independently of the magnitude of $V_T$ in agreement with the experimental observation. Equations (1) and (3) will agree if $n_H = 0.75\bar{n}_e + 0.5 \times 10^{13}\text{cm}^{-3}$. Taking $e\Phi_0/T_C = 1.5$ the magnetic surface average of $n_C$ required to satisfy the experimental results varies from 10% to 15% of the total ion density as $\bar{n}_e$ varies from 4 to $7 \times 10^{13}\text{cm}^{-3}$. The percentage will decrease if $e\hat{\Phi}_0/T_C$ is larger.

A similar result to Eq. (3) is obtained if the more accurate model for $f_0$ is taken, namely $\partial f_0/\partial v$ more negative for $v < V_C$.

# III. Radial Diffusion of $n_C$

## A. The simple model

One example with the simple model of two Maxwellians is considered first. Treating the ohmic discharge case, where $E_r^* \equiv E_r - V_T B_\theta$ is the dominant force, and assuming $e\tilde{\Phi}(\theta)/T_C \gtrsim 1$ then the $n_C$ ions will nearly all be trapped and will have the mean velocity parallel to $\mathbf{B}$ of $E_r/B_\theta$. Since the passing ions, including the impurities, will have the mean velocity $V_T$ (approximately zero for ohmic discharges), the cold ions will experience the friction force $F_{CZ} = -n_C \nu_{CZ} m_i (E_r^*/B_\theta)$ and their mean radial diffusion is $\Gamma = -F_{CZ}/eB_\theta = n_C \nu_{CZ} E_r^*/B_\theta \Omega_{i\theta}$, where $\Omega_{i\theta}$ is the cyclotron frequency in the poloidal field. The cold ions will be heated collisionally primarily by the hot ions $n_H$, so that neglecting radial heat conduction for the cold ions, in the steady state the total time derivative $(\partial/\partial t + V_r \partial/\partial r)$ of $T_C$ is given by

$$\left(\frac{3}{2} n_C \frac{dT_C}{dt}\right) = \frac{3}{2} \Gamma_C \frac{\partial T_C}{\partial r} = 3 n_C \nu_{\text{CH}} \left(T_H - T_C\right), \tag{5}$$

there being no compressional heating with electrostatic diffusion. Neglecting $T_C$ with respect to $T_H$ on the right-hand side, Eq. (5) reduces to

$$\lambda \frac{\partial}{\partial r}\left(T_C^{-1/2}\right) = T_H^{-1/2}, \tag{6}$$

where $\lambda$ is the length $(n_z Z^2/n_H) \left|E_r^*\right| / B_\theta \Omega_{i\theta}$ and $E_r^*$ has been assumed negative. Taking the parabolic form, $T_H = T_{H0}\left[1 - (r/a)^2\right]$ the solution is

$$T_C = \frac{T_{ca}}{\left[1 - \frac{a}{\lambda}\left(T_{ca}/T_{H0}\right)^{1/2} \cos^{-1}(r/a)\right]^2}, \tag{7}$$

where $T_{ca}$ is the value for $T_C$ at $r = a$. This solution breaks down when $T_C$ becomes comparable with $T_H$ and $(T_H - T_C)$ will then decrease approximately exponentially in the negative $r$-direction with $e$-folding length $\lambda$. But from Eq. (7) it can be seen that for $\lambda/a \gg \left(T_{ca}/T_{H0}\right)^{1/2}$, then $T_C/T_H$ will remain small all the way to $r = 0$. Otherwise $T_C$ will remain small into the approximate radius given by

$$\cos^{-1}\left(\frac{r}{a}\right) = \frac{\lambda}{a}\left(\frac{T_{H0}}{T_{ca}}\right)^{1/2}.$$

A typical value of $-E_r/B_\theta$ for an ohmic discharge is $v_{T_i}/4$ and since for $Z_{\text{eff}} = 3$, $n_z Z^2/n_i = 16/5$, then $\lambda \approx \rho_{i\theta}$. This calculation has ignored anomalous transport caused for example by electrostatic turbulence. Such transport can be expected to increase the rate of inward diffusion for the cold ions.

## B. The averaged drift kinetic equation

The equation which would give the true variation of $f_0$ with energy and minor radius is the second order drift-kinetic equation, averaged over pitch angle in velocity space and poloidal angle in real space, which has the form[4]

$$-\frac{\gamma}{v^2}\frac{\partial}{\partial v} v^2 \left(f \frac{\partial H}{\partial v} - \frac{1}{2}\frac{\partial^2 G}{\partial v^2}\frac{\partial f}{\partial v}\right) = \frac{1}{r}\frac{\partial r \Gamma_v}{\partial r} - Q, \tag{8}$$

where $\gamma = 4\pi e^4 \ln\Lambda/m_i^2$, $4\pi v^2 \Gamma_v dv$ is the mean diffusion for ions in the spherical shell $4\pi v^2 dv$ and $H$, $G$ are the standard Rosenbluth potentials. $Q$ represents the ion heating term such as collisions with electrons or beam ions. For $f$ isotropic in pitch angle, the required functions of $H$ and $G$ reduce to

$$\begin{aligned} \frac{\partial H}{\partial v} &= -\frac{\hat{n}}{v^2}, \\ \frac{1}{2}\frac{\partial^2 G}{\partial v^2} &= \frac{\hat{n}\,\langle v'^2\rangle}{3v^3} + \frac{\hat{\hat{n}}}{3}\left\langle\frac{1}{v'}\right\rangle. \end{aligned} \tag{9}$$

Here $\hat{n} \equiv \int_0^v 4\pi v'^2 f(v')dv'$ is the number of ions in the sphere of radius $v$ in velocity space, $\hat{\hat{n}} = \int_v^\infty 4\pi v'^2 f(v')dv$ is the number outside the sphere, $\langle v'^2\rangle$ is the average of $v^2$ for the $\hat{n}$ ions and $\langle 1/v'\rangle$ is the average of $v^{-1}$ for the $\hat{\hat{n}}$ ions.

Not surprisingly, the author has not solved this integro-differential equation. Unlike the corresponding problem for the distribution tail at high energies,[4] there is no apparent valid approximation which will linearize and simplify Eq. (8) for low energies. Instead a series of points are made here which suggest that the effects listed in the Introduction will produce a near singularity in $f_0$ at low energies.

The effect of electrostatic diffusion is considered first. For neoclassical diffusion due to magnetic particle drift, $\Gamma_v$ has a maximum in $v$ at the boundary between the ions in the plateau regime and those in the banana regime, i.e., where $(\nu_{\rm PA}/\omega_b)\,(v_{T_i}/v)^4 = (r/R)^{3/2}$. (Here for simplicity $n_z Z^2$ has been assumed sufficiently large for $Z$-ion collisions to dominate in $\nu_{\rm PA}$ so that $\nu_{\rm PA} \sim v^{-3}$ for $v > v_{TZ}$.) In the case of electrostatic diffusion $\Gamma_v$ is proportional to $v^{-2}$ for the banana regime ions and $v^{-1}$ for the plateau regime ions. (More precisely for the part of the diffusion driven by the radial electric field, $\Gamma$ is proportional to $v^{-3}\,(\partial f_0/\partial v)$ and $v^{-2}\,(\partial f_0/\partial v)$ in the banana and plateau regimes.) The strong increase in $\Gamma_v$ with decreasing $v$ will continue below the high collision frequency end of the plateau regime range $(\nu_{\rm PA}/\omega_b)\,(v_{T_i}/v)^4 = 1$. As discussed in Ref. 6, the precise value of $v$ where the increase ceases depends on which of the three processes, energy scattering, poloidal rotation or $v = v_{TZ}$, comes into effect first with decreasing $v$.

It will be assumed that $f_0$ can be expanded in the form

$$f_0 = f_H + f_C, \tag{10}$$

where $f_H$ is Maxwellian and $f_C$ (not assumed Maxwellian) is small compared with $f_H$. The assumption that $f_C$ is small is not necessarily correct but is forced on the author to obtain an approximate solution. Even if $n_C/n_H$ is small $f_C$ can be comparable with $f_H$ for small $v$, since $f_C$ is localized at low energies. Linearizing the collision operator so that $C = C(f_C, f_H) + C(f_H, f_C)$, the first part contains products of the velocity gradients of $f_C$ and the magnitude of $f_H$ and the second part the reverse. The first part will therefore be larger for small $v$ and since only the solution for low energies is of interest, the second part is neglected. It is also assumed that $Q$ is negligible for low ion energies, which is certainly true for collisions with electrons and beam ions if $f_C/f_0$ is not too small, and this term is also omitted. Since $H$, $G$ now involve only the Maxwellian $f_H$, the expressions in Eq. (9) can be simplified further to give their standard form and Eq. (8), for $v/v_{TH}$ small, reduces to

$$\frac{\nu_H}{v^2}\frac{\partial}{\partial v}v^3\left(f_C + \frac{T_H}{mv}\frac{\partial f_C}{\partial v}\right) = \frac{1}{r}\frac{\partial}{\partial r}r\Gamma_v, \tag{11}$$

where $\nu_H = 16\pi^{1/2} n_H e^4 \ln\Lambda / 3 m_i^2 v_{TH}^3$.

To illustrate the energy dependence of $f_C$ for low energies only the parts of $\Gamma_v$ lowest order in $v$ are taken. Plateau regime conditions are assumed to apply for the range $x_m < x < 1$ where $x \equiv v/v_{TH}$ and $x_m = (\nu_{\mathrm{PA}}/\omega_b)^{1/4}$. Hence for this range the electrostatic part of $\Gamma_v$ is

$$\Gamma_v = -\frac{\pi^{1/2}}{4}\left(\frac{\tilde{\Phi}}{B_\theta}\right)^2 \frac{B_\theta}{rB}\left[\left(\frac{n'}{n} - \frac{3}{2}\frac{T_H'}{T_H} - \frac{eE_r}{T_H}\right)\frac{f_H}{v} + \frac{eE_r}{mv^2}\frac{\partial f_C}{\partial v}\right] \tag{12}$$

with the radial gradient of $f_C$ neglected in comparison with $(eE_r/mv)\,\partial f_C/\partial v$. The right-hand side of Eq. (11) is taken to be

$$\frac{1}{r}\frac{\partial}{\partial r}r\Gamma_v = -\nu_H A_1 \frac{f_H}{x} + \frac{T_H A_2}{mv^2}\frac{\partial f_C}{\partial x}, \tag{13}$$

where $A_1$, $A_2$ contain the unknown radial derivatives

$$-A_1 = \frac{\pi^{3/2} v_{TH}^2}{\nu_H n r} \frac{\partial}{\partial r} \frac{r\Gamma_v (f_H) v}{e^{-x^2}},$$

$$A_2 = \frac{2}{\nu_H v_T^3 r} \frac{\partial}{\partial r} \left( \frac{r\Gamma_v (f_C) v^2}{\partial f_C / \partial v} \right).$$

Once again only the terms lowest order in $v$ have been retained. With these assumptions Eq. (11) reduces to

$$\frac{\partial}{\partial x} \left( x^2 \frac{\partial f_C}{\partial x} + \left( 2x^3 - A_2 \right) f_C \right) = -2A_1 x f_H. \quad (14)$$

It will be assumed that $f_2$ is zero at approximately $x = 1$, since the terms in the divergence of $\Gamma_v$ higher order in $x$, including magnetic neoclassical diffusion, will be positive and $f_C$ can be expected to have a small negative value for $x > 1$. This is taken as one boundary condition and the second condition is given by

$$\left| \nu_H v^3 \left( \frac{T_H}{mv} \frac{\partial f_C}{\partial v} + f_C \right) \right|_{v = v_T x_m} = \int_0^{v_T x_m} dv \frac{v^2}{r} \frac{\partial r \Gamma_v}{\partial r}$$

$$\equiv -\Gamma_0 \frac{v_T^3}{2} \nu_H,$$

where the right-hand side represents the velocity integral of the divergence of the unknown inward diffusion for $v < v_T x_m$ which need not be small.[7] The solution to Eq. (14) is then

$$f_C = A_1 f_H e^{-A_2/x} \int_x^1 \left( \frac{e^{x^2 - x_m^2} - 1}{x^2} \right) e^{A_2/x} dx$$

$$+ \left( \Gamma_0 e^{-x^2} + A_2 f_C (x_m) e^{-x_m^2} \right) e^{-A_2/x} \int_x^1 \frac{e^{x^2 + \frac{A_2}{x}}}{x^2} dx \quad (15)$$

with

$$f_C (x_m) = \frac{\Gamma_0 e^{-x_m^2 - \frac{A_2}{x_m}} \int_{x_m}^1 \frac{e^{x^2 + \frac{A_2}{x}}}{x^2} dx + A_1 f_H (x_m) e^{-A_2/x_m} \int_{x_m}^1 \left( \frac{e^{x^2 - x_m^2} - 1}{x^2} \right) e^{A_2/x} dx}{1 - 2A_2 e^{-x_m^2 + \frac{A_2}{x_m}} \int_{x_m}^1 \frac{e^{x^2 + \frac{A_2}{x}}}{x^2} dx}. \quad (16)$$

The dominant term in $f_C$ is that proportional to $\Gamma_0$ and for small $A_2/x$ its approximate linear form is

$$f_C \simeq \Gamma_0 e^{-x^2} \left( \frac{1}{x} - 1 \right) \quad (17)$$

so that $f_C$ increases inversely as $x$. But the nonlinear terms in Eq. (15) and in particular the $f_C(x_m)$ term show that $f_C$ increases still more rapidly with decreasing $x$. An important feature of the solution is that the inward diffusion of low energy ions for $x < x_m$ requires a negative gradient to $f_C$ all the way to $x \simeq 1$ so that the excess ions will flow to where the divergence of $\Gamma_v$ changes sign.

The above result shows that even if $f_0$ is Maxwellian in lowest order, the electrostatic diffusion produces a perturbation given by $f_C$ which increases inversely with $v$. Since recycling and gas puffing produce an $f_0$ near the wall which already has an excess of low energy ions relative to a Maxwellian, the diffusion process will lead to an enhancement of this excess low energy component plus an increased $f_C$ all the way to $x \simeq 1$. By following the solution of Eq. (8) for a single time step it can be shown that $\partial f_0/\partial v$ becomes more negative at low energies as this part of $f_0$ diffuses inwards.

# IV. Changes in Ion Transport Caused by an Excess of Low Energy Ions

In this section the presence of an excess of low energy ions will be assumed ($f_0 = f_H + f_C$) and the effect of the excess low energy ions on transport will be considered. In fact a series of experimentally observed hydrogen and impurity ion transport phenomena now have simple explanations. In addition to the PDX density asymmetry discussed in Sec. II, these include:

(a) Pulsed gas puffing leading to rapid increase of central density but no increase in ion energy content.

(b) Improved energy containment when recycling is reduced—H-mode, pellet injection, supershots and improved ohmic containment in ASDEX.

(c) The absence of an effective electric field $E_r^*$ in PDX in conflict with that required to explain ion motion near the observed discontinuity in $\partial f_0/\partial v$.

(d) Anomalous transport of toroidal momentum.

(e) No neoclassical peaking of $n_z(r)$ with normal conditions.

(f) Large decrease in $n_z$ with pulsed gas puff.

(g) Increase of $n_z$ in center with pellet injection.

The explanation for these various phenomena is given briefly in what follows.

**Item (a).** In the pulsed gas puff experiment in PLT,[12] the rapid increase in central density by a factor of 2 could be modelled by empirical values for a diffusion coefficient $D$ and an inward convective velocity $V$. But when these parameters were used to model the ion energy balance, an increase in central ion temperature was predicted because of the compression associated with $V$ and increased electron collisional heating, whereas experimentally $T_i$ went down. A similar result has been obtained with TFTR.[13]

Since the increasing density involves electron diffusion as well as ion diffusion, this part of the diffusion must presumably be anomalous. If the turbulence is electrostatic the inward diffusion can be expected to involve low energy ions and electrons. In addition there will be the neoclassical type diffusion discussed in the previous section. The positive friction force ($\|\mathbf{B}$) experienced by the $n_C$ ions will be balanced by negative friction forces for the $Z$-ions (see below) and the hot ions $n_H$. (This is subsequently referred to as the ambipolarity effect.) The $n_H$ ions will therefore undergo increase diffusion outwards, meaning more ion heat conduction. Both effects, the inward velocity $V$ involving low energy ions and the increase heat conduction during the gas puff pulse, agree with the explanations invoked by the PLT team.[12]

**Item (b).** The various recent improvements in containment—H-mode, pellet injection, supershots[14] and improved ohmic containment in ASDEX[15] have all involved reduced recycling at the edge of the plasma. Since this will involve less inward diffusion of $n_C$ and from ambipolarity less outward diffusion of $n_H$, there is less ion heat conduction and better containment. In steady state $\Gamma_Z = 0$ and the positive friction experienced by $n_C$ is balanced by negative friction for $n_H$, the interaction being primarily through the impurities. Thus reducing recycling means less ion heat conduction.

In the case of pellet injection there is the added effect that the diffusion flow of cold ions will be reversed from the region of pellet evaporation outwards. This requires for ambipolarity reduced outward diffusion for the impurities (see below) and for $n_H$, the latter meaning less heat conduction.

**Item (c).** Using active charge exchange measurements on PDX Davis et al.[2] observed ion distributions with a discontinuity in slope at high ion energies. The non-Maxwellian tail could be explained due to neoclassical effects allowing for the large poloidal Larmor radius for these ions;[4] but to reconcile the flow of ions in phase space near the locus of the discontinuity it was necessary to invoke[4] a large negative $E_r^* = E_r - V_{\|} B_\theta$. However, the poloidal rotation of impurities ($V_{z\theta} \simeq E_r^*/B$) was found to be zero.[9]

With the presence of an excess of low energy ions $n_C$, reconciling the motion of ions near the singularity no longer needs a large $E_r$.

**Item (d).** The neoclassical transport of toroidal momentum is given by the pressure tensor component $P_{\|r}$ which can be written in the form[16]

$$\begin{aligned} P_{\|r} &= -\int \frac{d\theta}{2\pi} \int m_i h v_{\|} \frac{\left(\mu B + v_{\|}^2\right)}{\Omega_i R} \sin\theta f_i d^3 v \\ &= -\left(q_{i\|s} + q_{i\|\|\|s}\right) / 2\Omega_i R, \end{aligned} \tag{18}$$

where $q_{i\|s}$ is the part of the total ion heat flow parallel to $B$ which varies as $\sin\theta$ and $q_{i\|\|\|s}$ is the corresponding flow of parallel energy. For pure hydrogen and to order $(\rho_{i\theta}/L)\, V_T$, $q_{i\|s}$ must be zero. Only the component $q_{i\|\|\|s}$ is nonzero—it is balanced by a reverse flow of perpendicular energy—and the resultant value of $P_{\|r}$ was found to be very small[17]

$$P_{\|r}\Big|_{ii} = -0.1 n_i \left(\frac{r}{R}\right)^2 \nu_{ii} \rho_{i\theta}^2 m_i \frac{\partial V_T}{\partial r}, \tag{19}$$

where the $ii$ subscript denotes ion-ion collisions involved. The experimentally observed transport of momentum is about $10^2$ times larger than this value. If one allows for the presence of impurities, $q_{i\|s}$ can be nonzero but the increase in $P_{\|r}$ is insufficient if $T_i \simeq T_z$. For the model consider here $(f_H + f_C)$ the increase is very large. Assuming $f_C$ is Maxwellian for simplicity, with $T_C/T_Z$ small but greater than $m_i/m_z$, the value of $P_{\|r}$ due to the $Z$-ions is found to be

$$P_{\|r}\Big|_{zc} = -3n_C\left(\frac{r}{R}\right)^2 \nu_{cz}\rho_{z\theta}^2 m_i \frac{\partial V_T}{\partial r}. \tag{20}$$

The $\sin\theta$ part of $q_\|$ for the $n_C$ ions has been assumed small because of the strong nonuniformity of $n_C$. The $\cos\theta$ part of the heat received from the $Z$-ions will be conducted away radially and not poloidally. The ratio of the values in Eqs. (19) and (20) is

$$\frac{P_{\|r}\,|_{zc}}{P_{\|r}\,|_{ii}} = 30\left(\frac{n_C}{n_i}\right)\left(\frac{T_H}{T_C}\right)^{3/2}\left(\frac{n_z Z^2}{n_i}\right)\frac{m_z}{Z^2 m_i} \tag{21}$$

which can easily be $10^2$ even for small $n_C/n_i$.

**Item (e).** To illustrate the dramatic effect of the $n_C$ ions on impurity transport the case of $Z$-ions in the Pfirsch-Schlüter regime is considered. Treating only a single impurity species and neglecting for simplicity the term due to the ion temperature gradient, the impurity diffusion due to collisions with the $n_H$ ions is

$$\Gamma_Z\Big|_{ZH} = Z^{-1}n_H\nu_{HZ}\rho_{H\theta}^2\left(\frac{r}{R}\right)^2\left(\frac{n_H'}{n_H} - \frac{n_Z'}{Zn_Z}\right), \tag{22}$$

where a dash denotes the radial derivative. This formula led to the conclusion that in steady state the $Z$-ion density should be strongly peaked so that $n_z'/n_z = Zn_i'/n_i$. Allowing for the presence of $n_C$ the contribution to $\Gamma_Z$ due to $cz$ collisions has the form[6]

$$\begin{aligned}\Gamma_Z\Big|_{ZC} &= -Z^{-1}\Gamma_C\Big|_{CZ} = Z^{-1}\int\frac{d\theta}{2\pi}\left(\frac{\tilde{n}_C}{n_C} + 2\frac{r}{R}\cos\theta\right)\frac{\tilde{F}_{CZ}}{eB_\theta} \\ &\simeq \frac{Z^{-1}n_C\nu_{CZ}}{2\Omega_{i\theta}}\left(\frac{e\tilde{\Phi}}{T_C}\right)\left(\frac{\tilde{\Phi}}{B_\theta}\right)\left(\frac{n_C'}{n_C} - \frac{n_Z'}{n_Z}\right)\end{aligned} \tag{23}$$

which is the dominant electrostatic part since $r/R$ has been neglected with respect to $e\tilde{\Phi}/T_C$. For steady state, adding Eqs. (22) and (23), $\Gamma_Z = 0$ requires

$$\frac{n_Z'}{n_Z} = \frac{\alpha\frac{n_H'}{n_H} + \beta\frac{n_C'}{n_C}}{(\alpha/Z) + \beta}, \tag{24}$$

where $\alpha \equiv \left(n_H/T_H^{1/2}\right)(r/R)^2$, $\beta \equiv \left(n_C/T_C^{1/2}\right)\left(e\tilde{\Phi}/T_C\right)^2$. If $\beta \sim \alpha$ and $n_C'/n_C$ is small then $n_Z'/n_Z \sim n_H'/n_H$. That is the tendency for the $n_C$ ions to drive the impurities out balances the effect of $n_H$ and there no unusual peaking is predicted for $n_Z$.

**Item (f).** In the pulsed gas puff PLT experiment referred to above, spectroscopic measurements showed that the gas puff reduced the concentration of heavy impurities by an order of magnitude.[18]

Using the example conditions assumed to explain Item (e), the sudden increase in the concentration $n_C$ of cold ions diffusing inward involves a strong increase in the friction force $\tilde{F}_{CZ}$. Because of the non-steady state conditions this force will be balanced primarily by a negative friction force on $n_Z$, which means the impurities are driven outwards.

**Item (g).** With pellet injection a sudden and dramatic increase in impurity concentration is observed in the center of the plasma.[19] This is the reverse of Item (f). The copious source of low energy ions in the region of pellet evaporation will reverse the direction of diffusion for $n_C$. Now both $n_C$ and $n_H$ drive $n_Z$ inwards.

# V. Conclusions

In Sec. II it was shown that conventional theory, which assumes approximate Maxwellian distribution functions for each particle species, predicts a result for simple momentum balance within a magnetic surface which has the wrong sign to explain the PDX density asymmetry observations. In contrast, assuming a moderate concentration of low energy ions $n_C$ in excess of the Maxwellian distribution ($n_C/n_i$ from 10 to 15%) predicts the correct result.

In Sec. III arguments were presented to show that low energy ions will in fact undergo strong inward diffusion, too rapid to thermalize with the outward diffusing energetic ions. The prime source of low-energy ions (apart from pellet injection) is recycling. The rate of inward diffusion increases with decreasing ion energy because of the combined effects of the increasing collision frequency for pitch angle scattering, the negative radial electric field (for ohmic discharges) the negative temperature gradient and electrostatic diffusion. The velocity distribution for the excess low energy ions will have a near singularity at low energies.

The presence of the excess low energy ions leads to dramatic changes in the predictions for neoclassical theory. As shown in Sec. IV, there are simple (qualitative) explanations for a variety of hydrogen and impurity ion experimental results previously unexplained by neoclassical theory, and for toroidal momentum transport even a quantitative result.

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# Figure Captions

1. The variation of electron density with major radius in PDX with neutral beam heating. All parameters other than density were kept constant.

2. $\tilde{n}_e$ versus $\bar{n}_e$, where $\tilde{n}_e = [n_e(\theta = 0) - n_e(\theta = \pi)]/2$ and $\bar{n}_e = [n_e(\theta = 0) + n_e(\theta = \pi)]/2$.

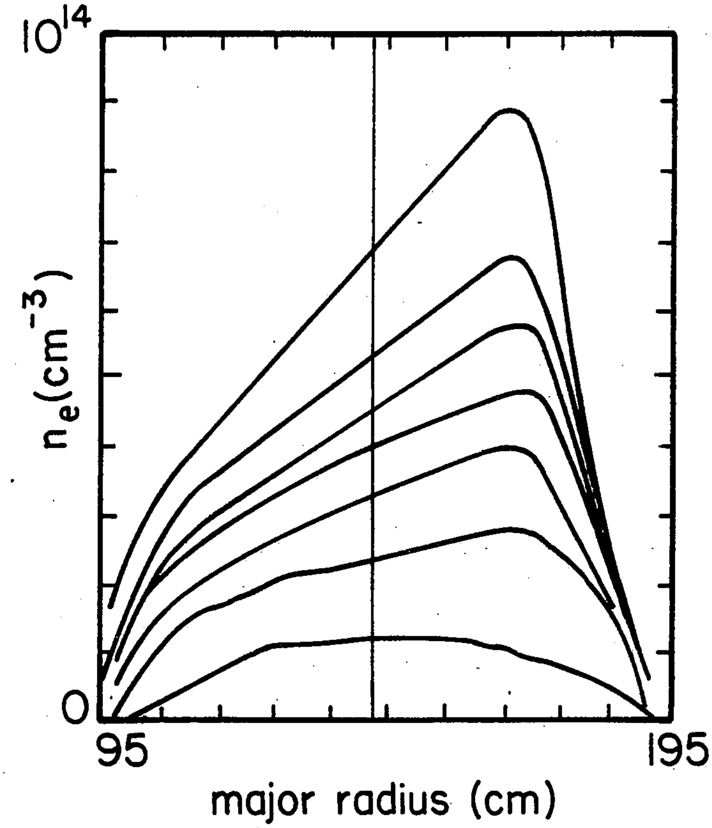


Figure 1

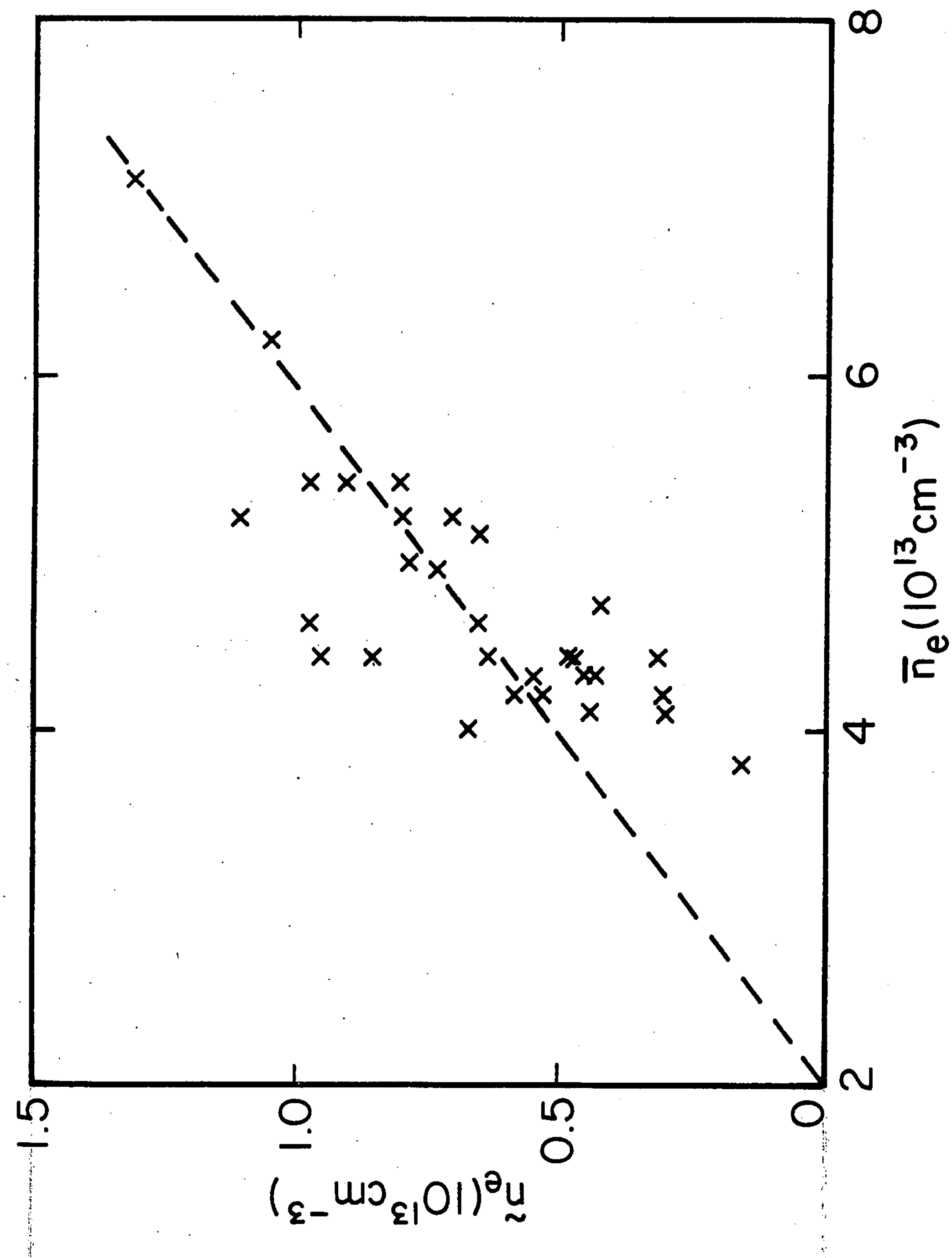


Figure 2