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Abstract The behavior of a short laser pulse with periodically peaked transverse intensity profile is important for the study of laser acceleration of particles. For a specific relation between the amplitudes and the separation of the peaks, this profile should remain undistorted while propagating in plasma. Carrying out numerical particle simulation runs in which a deviation from this relation is present, we have observed the system to exhibit a kind of bistability.

INTRODUCTION

Motivated by the need of laser beam transport with minimal loss in intensity over considerable distances by such new concepts as laser-plasma particle accelerators and laser ignited fusion, the asymptotic form of an optical beam travelling in a plasma is of great interest. In the case of a short intense laser pulse, when the ions can be taken to be immobile and the dominant forces on the electrons are the laser ponderomotive force and the electrostatic force, Kurki-Suonio, Morrison, and Tajima[1] found that the asymptotic solutions come in two kinds: one kind has a solitary-type transverse profile, and the other has a periodically peaked profile. Also an analytic solution for the solitary-type profile was obtained and tested with a recently developed particle simulation code[2] appropriate for transport of optical beams in plasmas. No further analysis of the multi-peaked profile was carried out. Here we present some preliminary numerical results on the multi-peaked profiles using the above mentioned particle simulation code.

ASYMPTOTIC SOLUTIONS

Expressing the electromagnetic fields in terms of the potentials, the following wave equation has been derived.[1]

$$2k_0 \frac{\partial \psi}{\partial z} - |\nabla\psi|^2 + \frac{1}{a}\nabla^2 a - \frac{1}{\lambda_c^2}\frac{N_e}{\sqrt{1+a^2}} + \frac{\omega_0^2}{c^2} - k_0^2 = 0$$

$$-k_0 \frac{\partial a^2}{\partial z} + (\nabla a^2)\cdot\nabla\psi + a^2\nabla^2\psi = 0 \ . \qquad (1)$$

where ψ and a are real normalized phase and amplitude fractions respectively, k_0 and ω_0 are the laser wavenumber and frequency respectively, and $\lambda_c = c/\omega_p$. The quantity N_e is the electron density including the ponderomotive perturbation,

$$N_e \equiv 1 + \frac{\delta n_e}{n_0} = 1 + \lambda_z^2 \left[\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right] \sqrt{1 + I_n} \ .$$

We look for an asymptotic intensity profile, independent of z, for the laser beam. Equation (1) is separable under the following ansatz:

$$a(r,z) = a(r) \ ,$$
$$\psi(r,z) = f(z) + g(r)$$

where we have allowed for phase modulation in z.

If we further assume a slab approximation ($r \rightarrow x$) we obtain an equation that was analyzed in detail in Ref. 1. There it was concluded that the bound physical solutions come in two kinds: One is a solitary solution, and the other is a multi-beamlet type. An exact analytical solution was obtained for the solitary solution, and the asymptotic nature of this solution was confirmed by a numerical particle simulation. Multi-peaked profiles were analyzed by exploiting the analogy of a particle in a classical potential. The energy-like integral is given by

$$\mathcal{E} = \frac{1}{2} g(a) \left(\frac{da}{dx} \right)^2 + V(a) \ , \tag{2}$$

where $g(a) = \frac{1}{1+a^2}$ is the metric of the system, and $V(a) = \frac{1}{2}\frac{C_4^2}{a^2} - \frac{1}{\lambda_c^2}\sqrt{1+a^2} - \frac{1}{2} C_1 a^2$ is the potential. An approximate form of a multi-peaked profile can be obtained by integrating Eq. (2) with respect to the amplitude a in the neighborhood of the minimum of the potential $V(a)$:

$$\lambda_s = \int_{a_1}^{a_2} \frac{da}{\sqrt{\mathcal{E} - V(a)}} \ . \tag{3}$$

Here a_1 and a_2 correspond to the same total energy, $\mathcal{E} = -\frac{1}{\lambda_c^2}\sqrt{1+a_i^2} - \frac{1}{2} C_1 a_i^2$, $i = 1, 2$. The solitary profile has $\mathcal{E} = \frac{-1}{\lambda_c^2}$ thus fixing a_1 at the origin. Equation (3) gives an approximate wavelength λ_s between the peaks in the profile corresponding to the specific amplitudes a_1 and a_2 (see Fig. 1).

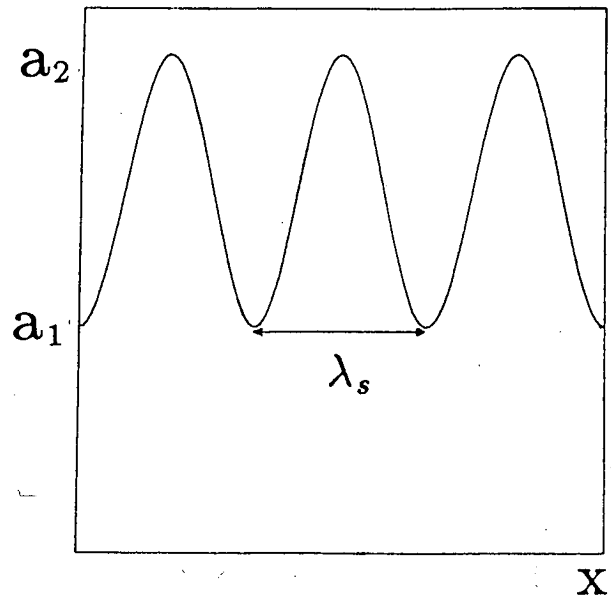

FIGURE 1: A periodically peaked amplitude profile

DYNAMICS OF A PERIODICALLY-PEAKED AMPLITUDE PROFILE

The code used is a time-averaged particle simulation code developed recently for modeling transport of optical beams in plasmas.[2] It uses periodic boundary conditions, the width of the simulation box is chosen to be $25.6\,\lambda_c$, and there are 100 electrons per grid cell. The number of grid points for the simulations discussed below is 256 and the time step was chosen at $dt = 0.1\,\omega_{pe}^{-1}$.

We ran several computed simulation runs with various parameter values for the multi-peaked amplitude profile. Since the exact form of the multi-peaked solution is not known, none of the runs corresponded to the exact solution for the asymptotic equation, and the profiles were not expected to remain undistorted. The locations of the peaks and troughs of the profile were seen to alternate so that for half of the time the peak would be located at the point where the trough was originally, and vice versa. Furthermore, the phase shift Q was observed to exhibit similar behavior but with a $\frac{\pi}{2}$-phase shift. In Fig. 2 this behavior of the field quantities is illustrated for a run with $\lambda_s = 5.12\,\lambda_c$, $a_1 = 0.02$, $a_2 = 0.05$.

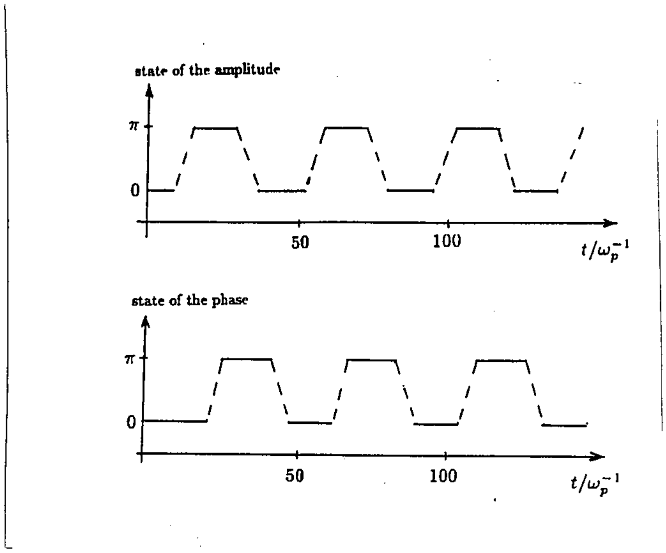

FIGURE 2: The flip-flopping of the states observed for field quantities. Zero corresponds to the original location of a peak, and π corresponds to the switched location

To gain insight on the observed process — and to make sure that what was seen was not a numerical artifact — we studied the relevant field equations at very early times when the process can be taken to be linear. Rewriting Eq. (1) in terms of $I = a^2$ and linearizing around an initial state given by $I = I_0(x)$, $\psi = 0$, we get

$$\frac{\partial \psi_1}{\partial z} = \frac{1}{2k_0} \left\{ \frac{1}{4I_0^2} \left(\frac{\partial I_0}{\partial x} \right)^2 - \frac{1}{2} \left(\frac{1}{I_0} - 1 + \frac{1}{2} I_0 \right) \frac{\partial^2 I_0}{\partial x^2} \right\} , \tag{4}$$

where we have assumed $I_1 \ll I_0 < 1$, and we have neglected all the terms involving I_1 compared to terms containing I_0 only on the right-hand side. Also, the dispersion relation $\omega_0^2 = \frac{\omega_{p0}^2}{\sqrt{1+I_0}} + c^2\, k_0^2$ was used. According to Eq. (4) the phase shift ψ should be driven by the gradients of the initial amplitude profile. In the simulation code the initial profile is given as

$$a_0(x) = \alpha - \beta \cos(k_s\, x) , \tag{5}$$

where $a = \frac{1}{2}(a_1 + a_2)$ and $\beta = \frac{1}{2}(a_2 - a_1)$. Therefore, at the locations where the amplitude peaks ($\cos(k_s\, x) = -1$), the phase shift should start according to

$$\frac{\partial \psi_1}{\partial z} \approx \frac{1}{2k_0} \frac{k_s^2\, \beta}{a_2} . \tag{6}$$

Accordingly, at the trough locations ($\cos(k_s\, x) = +1$), the phase shift should be given by

$$\frac{\partial \psi_1}{\partial z} \approx -\frac{1}{2k_0} \frac{k_s^2 \beta}{a_1} . \tag{7}$$

We ran a few cases varying the separation parameter $k_s = \frac{2\pi}{\lambda_s}$ for the amplitude profile but keeping the amplitude values fixed at $a_1 = 0.02$ and $a_2 = 0.05$. The simulation results together with the theoretical predictions are summarized in Fig. 3.

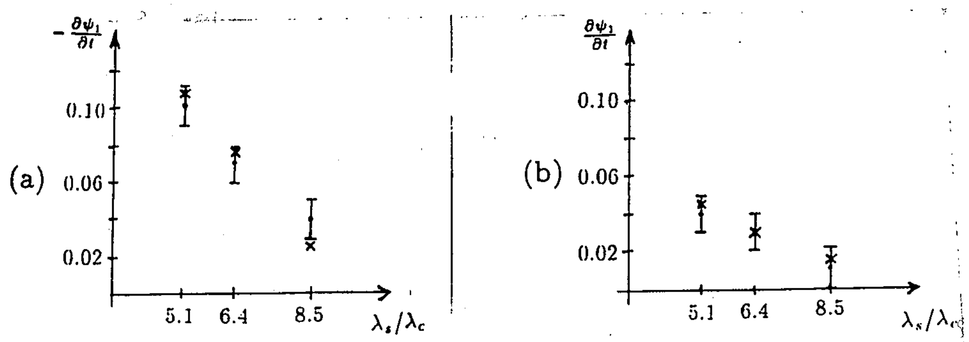

FIGURE 3: Simulation Results. The observed growth rate of the phase shift (dot) together with the theoretical value (cross). (a) At the location of a trough, and (b) at the location of a peak of the initial amplitude profile.

Figure 3(a) shows the behavior of the phase shift at the minimum amplitude location, and Fig. 3(b) shows corresponding result for the maximum amplitude location. The scaling in the simulation results is observed to follow that of the theory, and even the numerical values are surprisingly close considering the crudeness of the model.

Phenomenologically, what is taking place here seems to be the following: The optical beam has initially a flat phase front and a multi-humped amplitude profile as indicated in Fig. 4.

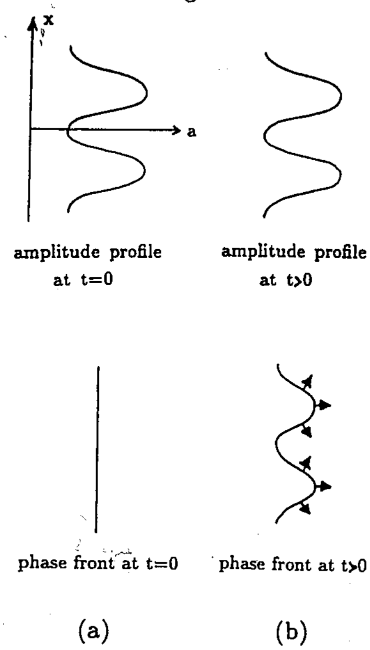

FIGURE 4: The interplay of the amplitude and phase. A multi-humped amplitude profile distorts an originally flat phase front. The curved phase front acts back on the amplitude causing periodical structure of self-focusing and defocusing regions.

The spatial gradients of the amplitude profile drive a deformation of the phase front in such a way that the phase front curvature will be reminiscent of the amplitude profile, i.e., a maximum on the phase front will form where the amplitude peaks etc. (see Fig. 4(b)). The curvature of the phase front will now drive the dynamics of the amplitude profile (as indicated by the arrows in Fig. 4(b)) so that the profile flattens out and eventually new peaks are formed at the locations of the former minima. The new amplitude peaks act back on the phase, and the cycle continues. The system thus flip-flops between two states exhibiting a kind of bistability or breathing.

As mentioned, these results are very preliminary and simplistic. The flip-flop behavior between two states that the amplitude exhibits could be of enormous importance to optical switching: the bistability could lead to an optical analog of an electronic transistor. Therefore, this phenomenon deserves a careful and detailed theoretical analysis.

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