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Self-Consistent Collision Code for Beam Tracking in SSC

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Abstract The beam dynamics for an electron (or positron) storage ring and for the Superconducting Super Collider (SSC) at the collision point is crucial in determining the maximum luminosity and lifetime of the beam. For typical parameters of SSC the beam density is sufficiently high ($\approx 10^{15}$ cc) at the collision point and the interaction time sufficiently long ($\Gamma_{\max}\tau_{\text{coll}} \approx 0.1$, where $\Gamma_{\max} = \sqrt{2}\omega_{pi}/\sqrt{\gamma}$), so that effects of not only rigid body beam-beam interaction but also the softer beam-beam collective interaction should be assessed. A relatively simple model has been introduced to analyze the above problem in a self-consistent manner by the particle simulation method. Relativistic particles interact self-consistently with each other over the collision time τ_{coll} and then their transverse position (or betatron coordinates x, y) and beam angles (or aperature) θ_x, θ_y are transformed in a simplectic map of the magnets that lie between the collision point to the next. This is repeated over a necessary number of time steps. Results with parameters for colliders are presented.

INTRODUCTION

In typical collider storage rings most of the time particle collective effects are not significant. The densities are low and single particle motion dominates. However, at points where counterstreaming particle beams collide the densities are high enough ($\approx 10^{15}$ cc for the SSC) and the interaction time long enough for significant beam-beam collective effects to occur. It has been suggested[1] that self-field effects play a significant role in the stability behavior of transverse orbit or the expansion of the beam cross section in electron and positron colliding beams in a storage ring. The enhanced heating and filamentation brought about by collective effects could substantially reduce beam lifetimes in collider rings. It is the objective of this paper to examine the importance of collective effects in the heating and broadening of beam cross section by repeated interaction of counterstreaming proton-proton beams using particle simulation techniques (see, for example, Ref. 2).

MODELLING OF A COLLIDER RING

We model collective effects which occur in collider storage rings. In most collider machines the charged particles undergo lateral oscillations, betatron oscillations, about the ideal circular or racetrack shaped orbit. These betatron oscillations can be modelled by a simplectic map in phase space, x, p_x where x and p_x represent position and momentum perpendicular to the beam direction. The successive phase space position $(x, p_x) \rightarrow (x', p'_x)$ at the collision point is given by

$$\frac{x'}{\beta^*} = \frac{x}{\beta^*}\cos\theta + \frac{p_x}{p}\sin\theta$$

$$\frac{p'_x}{p} = -\frac{x}{\beta^*}\sin\theta + \frac{p_x}{p}\cos\theta \ . \qquad (1)$$

where β^* is a storage ring parameter which is the betatron oscillation wavelength at the interaction point, $p = \sqrt{p_x^2 + p_y^2 + p_z^2}$, and θ is the rotation angle. The rotation angle is related to a parameter called the tune Q where $\theta = 2\pi Q$.

We use a 1-2/2 dimensional (x, p_x, p_y, p_z) relativistic electromagnetic particle-in-cell (PIC) code where the electrostatic field is not computed to model beam-beam interactions in a collider ring. The interaction region is modelled by the particle simulation method,[2] since in this region the density of particles becomes high enough so that collective effects are important. Outside the interaction region the particles are assumed to execute single particle motion. Therefore, the rest of the storage ring can be modelled by simply (1) a rotation in phase space and (2) a reset of self-field quantities to zero upon re-entry to the collision point. A theoretical uncertainty and thus interest is to understand the cumulative effect of beam-beam interaction at collision points sandwiched by the successive Hamilton transports (1) & (2). The number of timesteps being determined by the time that the particles spend in the interaction region. Figure 1 shows the simulation geometry. The beams are drifting with high velocities in the y-direction, p_y, and are given finite thermal velocities in the x-direction, p_x, where $p_y \gg p_x$. The simulation box is in the x-direction with periodic boundary conditions in x.

BEAM-BEAM INTERACTION AND FILAMENTATION INSTABILITY

When collective effects are taken into account between two counterstreaming beams, one of the most important instabilities is the filamentation instability. It is a mode which propagates nearly perpendicular to the beam direction. The onset of this instability can lead to beam filamentation and heating. From linear theory the maximum growth rate for counterstreaming electron and positron beams is[1]:

$$\Gamma_{\max} = \frac{1}{2}\frac{\omega_{pe}}{\sqrt{\gamma}} \tag{2}$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$, and $\omega_{pe}^2 = 4\pi e\, n_e/m_e$. For counterstreaming proton-proton beams the maximum growth rate is:

$$\Gamma_{\max} = \sqrt{2}\omega_{pi}/\sqrt{\gamma} \tag{3}$$

Given typical interaction region lengths and growth times for the filamentation instability for colliding proton-proton beams one obtains $\Gamma_{\max}\tau_{\text{coll}} \approx 0.1$, where τ_{coll} is amount of time the beam spends in the interaction region, so repeated interactions should reveal the results of collective effects.

CODE TEST

To test the code a series of runs was performed. Figure 2 shows the measured growth rates of a homogeneous beam at early stages of the instability for various relativistic factor, γ, values. Good agreement is found between the simulation results and the theoretical values determined from the formula for counterstreaming proton beams.

Another check on the code was to compare results from the simulation with those of a tracking code.[3] In a tracking code collective effects are not taken into account, but hard beam-beam collisions are included by short impulses applied to the tracked particles. Results indicate good agreement for short run times before collective effects become significant.

PRELIMINARY RESULTS

Figure 3 shows phase space plots of counterstreaming proton-proton beams with $\gamma = 1600$ at initialization and after 50 rotations or interactions. The x and y axis are x/β^* and p_x/p respectively. The tune Q for this run is

0.285. As the figure shows the final phase space plot is not too different from the initial phase space plot. The beam has shifted to the left while the other beam has shifted to the right. This signals the initial phases of beam filamentation. The initial emittance of the beam was 3.32×10^{-3}. At the end of the run the emittance was 5.0×10^{-3}. In the tracking code with the same parameters the final emittance was 2.86×10^{-3}. This indicates that collective effects are causing beam expansion.

DISCUSSION

In summary we have modelled a collider storage ring using an electromagnetic PIC code. It has been found that collective effects due to beam-beam interactions contribute to beam expansion. Specifically, the filamentation instability has been observed. The growth rates at the linear stages of the instability agree with theory. Betatron oscillations are included by a rotation in phase space. Simulation runs with 50 rotations show that collective effects are causing expansion of the beam. Typical beam lifetimes in storage rings are on the order of 10^4 to 10^5 rotations with relativistic factor $\gamma \approx 10^4$. In future studies the code will be extended to much longer timescales, larger system sizes, and higher relativistic factors to model more realistically storage rings.

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Simulation Geometry

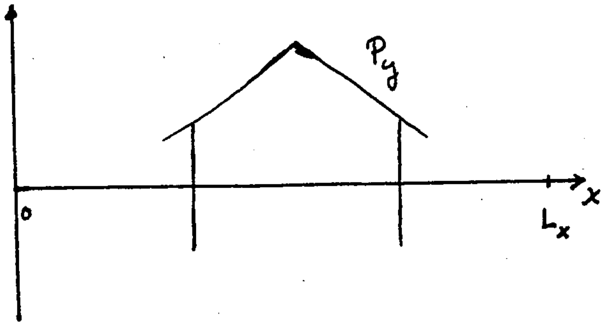

FIGURE 1: Model

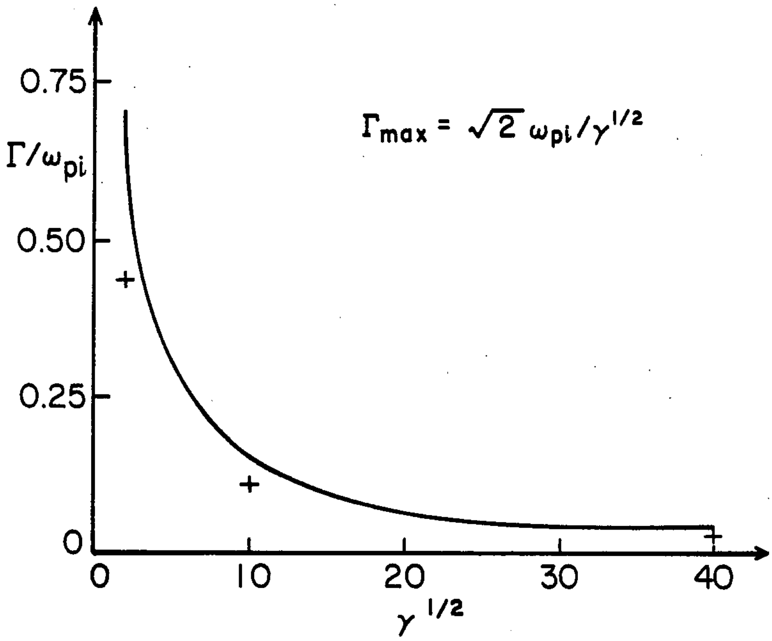

FIGURE 2: Growth rate of instability for counterstreaming electron beams or proton beams

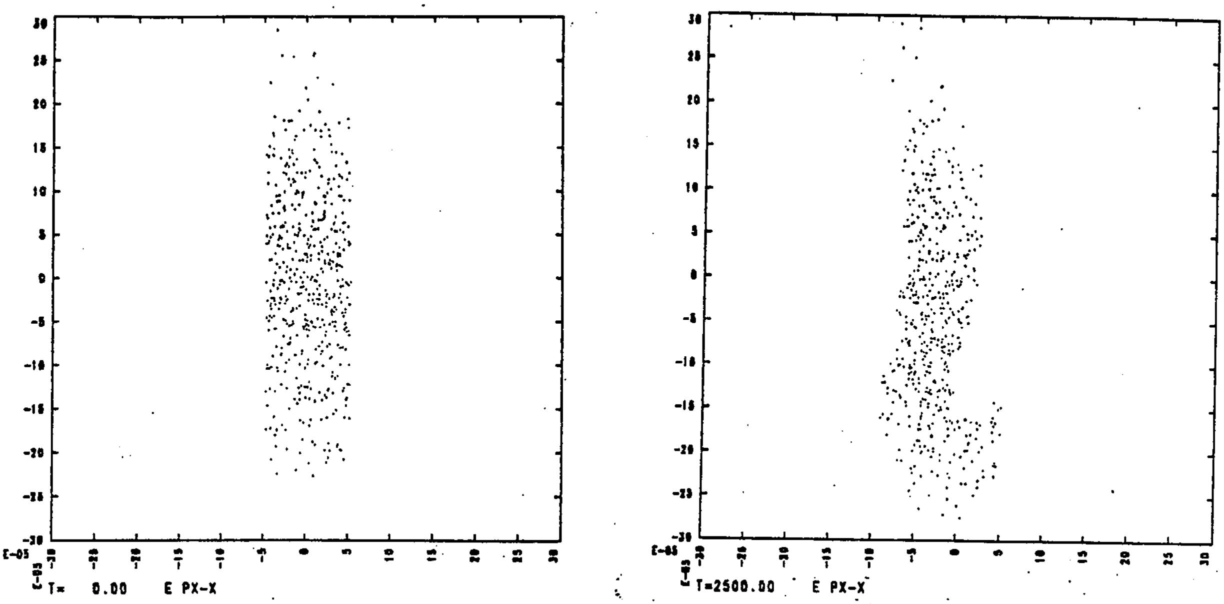
beginning

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FIGURE 3: Phase space plots

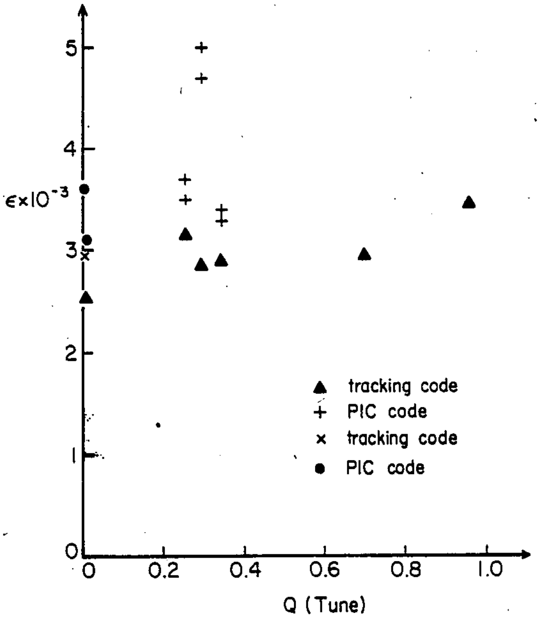

FIGURE 4: Emittance vs. tune