Ion Temperature Gradient Driven Turbulence
in Tokamaks with Flat Density Profiles

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Abstract

The theory of ion temperature gradient driven turbulence in tokamaks is extended to the flat density regime. The values of ion and electron thermal flux, χ_i and χ_e, momentum diffusivity, χ_φ, and particle flux, Γ_r, are also calculated. These formulas extend previous calculations, which were restricted to the regime $L_n < \sqrt{L_s L_T}$ (Lee and Diamond, Phys. Fluids **29**, 3291 and Mattor and Diamond, Phys. Fluids **31**, 1180). This allows an assessment of the role of ion temperature gradient turbulence in H-modes, where the density gradient is often observed to be flattened in the plasma core.

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I. Introduction

The theory of ion temperature gradient driven turbulence (ITGDT) which develops from ion temperature gradient driven modes[1] ("η_i modes") has been reasonably successful in explaining several aspects of anomalous transport in tokamaks.[2,3] However, the standard theory is not applicable to the important case of H-mode discharges, where the core density profile is (approximately) flat, so that $L_n \to \infty$ and $\eta_i \to \infty$.[4,5] Indeed, the standard ITGDT model is valid only when $\omega < \omega_*$, which occurs in the regime $L_n^2 < L_s L_T$. In this regime only, density fluctuation evolution is slow compared to density evolution, $(\frac{\partial}{\partial t}(|e|\tilde{\phi}/T) \ll c\nabla_\theta\tilde{\phi}/BL_n)$, so that the total density flow is incompressible with radial convection balancing parallel compression $(\nabla \cdot (n\vec{v}) = \tilde{v}_r/L_n + \nabla_\parallel\tilde{v}_\parallel = 0)$. In contrast, the "flat density" ITGD mode is relevant when $\omega > \omega_*$, so that $L_n^2 > L_s L_s$. Here, density fluctuation evolution is faster than radial convection, so that $\frac{\partial}{\partial t}(|e|\tilde{\phi}T) + \nabla_\parallel\tilde{v}_\parallel \simeq 0$. The flat density "$\eta_i$ mode" is more properly termed a ∇T_i-driven mode, with the linear stability threshold given by $(L_s/L_{T_i})_{crit} \simeq 1.9\left(1 + \frac{T_i}{T_e}\right)\frac{(2l+1)}{L_s}$, in contrast to $\eta_{i,crit} \simeq 1$, as in the "standard case."[6] The $\nabla n_0 \to 0$ threshold of the toroidal ITGDT model has been studied as well.[7,8] However, comparitively little theoretical work on saturated ITGD *turbulence* in the flat density regime exists at this point.

In this paper, we discuss the theory of strong ITGD turbulence in the flat density regime, where $\nabla n_0 \to 0$. The renormalized ion fluid equations are solved to obtain $D_{\vec{k}}$, the diffusivity necessary for saturation of the mode $\vec{k}$. In the process, the "diffusivity as eigenvalue" calculation, developed previously,[9] is extended to the case vhere the turbulent fluctuations have a real frequency ω_r comparable to the decorrelation rate $\Delta\omega_{\vec{k}}$. The radial correlation lengths of turbulent temperature and density fluctuations are calculated as well. The ion thermal diffusivity χ_i, momentum diffusivity χ_φ, electron thermal diffusivity, χ_e, and particle flux Γ are determined, the last two assuming dissipative trapped electron regime coupling. It is found that $\chi_i = \chi_\varphi$, and that $Q_i = \chi_i n\nabla T_i > Q_e$, with different scalings. These transport coefficients, along with the linear instability threshold values of ∇T_i, and the corresponding coefficients for the weak turbulence regime[10] can then be utilized to asses the role of ITGDT in the flat density H-mode discharges.

The remainder of this paper is organized as follows. In Section II, the linear theory

of flat density ITGDT is reviewed briefly. Section III discusses the theory of saturated ITGDT when $\nabla n \to 0$. In Section IV, the fluctuation levels and trasport fluxes produced by ITGDT are calculated. Section V consists of a discussion and summary.

II. Basic Model and Linear Theory

The renormalized nonlinear equations below, Eqs. (1)-(3), are adapted directly from Ref. 1. They represent the ion continuity, ion parallel velocity, and ion pressure evolution equations:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{v}_{\perp i}) + \nabla_\parallel \left(n_i v_{\parallel i} \right) = 0$$

$$m_i n_i \left(\frac{\partial v_{\parallel i}}{\partial t} + \left(\vec{v}_E + \vec{v}_{\parallel i} \right) \cdot \nabla v_{\parallel i} \right) = -e n_i \nabla_\parallel \Phi - \nabla_\parallel P_i + \mu_\parallel \nabla_\parallel^2 v_{\parallel i}$$

$$\frac{\partial P_i}{\partial t} + \left(\vec{v}_E + \vec{v}_{\parallel i} \right) \cdot \nabla P_i + \Gamma P_i \nabla_\parallel v_{\parallel i} = 0,$$

where Φ is the electrostatic potential, and Γ is the ratio of specific heats. Electrons are assumed adiabatic, $\tilde{n}_e = n_0 \frac{e\tilde{\phi}}{T_e}$; and the quasineutrality condition, $n_i = n_e$, is assumed. The perpendicular dynamics are due to $\vec{E} \times \vec{B}$, ion diamagnetic, and polarization drifts. Temporal and spatial scales are normalized to units of Ω_i^{-1} (inverse gyrofrequency) and $\rho_s = c_s/\Omega_i$, where $c_s = \sqrt{T_e/m_i}$ is the sound speed. A sheared slab model of the magnetic field is used, with $\vec{B} = B_0 \left(\hat{z} + (x/L_s)\hat{y} \right)$, so that the parallel wave number is given by $k_\parallel = (x - x_s) k_y/L_s$ in the neighborhood of a rational surface, x_s. The dominant nonlinearities are $\vec{E} \times \vec{B}$ convection of parallel velocity, and pressure fluctuations, and are given in renormalized form on the right hand side of Eqs. (2)-(3), respectively. The vorticity nonlinearity in the continuity equation is small (of order $k_y^2/\langle k_y^2 \rangle$) for the low k_y regime considered here, which can be seen from a slight variation in the usual renormalization procedure in which the back reaction of $\tilde{\phi}$ to $\vec{E} \times \vec{B}$ vorticity diffusion is included.[9] A derivation of these equations may be found in Refs. 2 and 9, and the renormalization procedure is detailed in Refs. 9 and 11.

$$\frac{\partial}{\partial t} \left(1 - \nabla_\perp^2 \right) \tilde{\phi} + v_D \nabla_y \tilde{\phi} + v_D \left(\frac{1 + \eta_i}{\tau} \right) \nabla_\perp^2 \nabla_y \tilde{\phi} + \nabla_\parallel \tilde{v}_\parallel = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\tilde{v}_\| + \nabla_\|\tilde{\phi} + \nabla_\|\tilde{p} - \mu\nabla_\|^2\tilde{v}_\| = \frac{\partial}{\partial x}D_{\vec{k}}^{xx}\frac{\partial}{\partial x}\tilde{v}_{\|\vec{k}} - k_y^2 D_{\vec{k}}^{yy}\tilde{v}_{\|\vec{k}}, \tag{2}$$

$$\frac{\partial}{\partial t}\tilde{p} + v_D\left(\frac{1+\eta_i}{\tau}\right)\nabla_y\tilde{\phi} + \frac{\Gamma}{\tau}\nabla_\|\tilde{v}_\| = \frac{\partial}{\partial x}D_{\vec{k}}^{xx}\frac{\partial}{\partial x}\tilde{p}_{\vec{k}} - k_y^2 D_{\vec{k}}^{yy}\tilde{p}_{\vec{k}}, \tag{3}$$

where $\tilde{\phi} = e\tilde{\Phi}/T_e$, $\tilde{p} = \tilde{p}_i/\tau P_{i0}$, and

$$\eta_i = \frac{d(\ln T_i)}{d(\ln n_0)} \qquad v_D = -\frac{d(\ln n_0)}{dx} \qquad \mu = \frac{\mu_\|\Omega_i}{c_s^2} \qquad \tau = \frac{T_e}{T_i}.$$

The turbulent diffusion coefficients are given by

$$D_{\vec{k}}^{xx} \equiv \sum_{\vec{k}'}\frac{k_y'^2\left|\tilde{\phi}_{\vec{k}'}\right|^2}{-i\omega_{\vec{k}''} + \Delta\omega_{\vec{k}''}}, \tag{4}$$

$$D_{\vec{k}}^{yy} \equiv \sum_{\vec{k}'}\frac{\left|\partial\tilde{\phi}_{\vec{k}'}/\partial x'\right|^2}{-i\omega_{\vec{k}''} + \Delta\omega_{\vec{k}''}}. \tag{5}$$

The linear beat frequency, $\omega_{\vec{k}''}$, has been written explicitly in Eqs. (4) and (5) because in the flat density regime the real part of the frequency is comparable to the growth rate, as shown below.

In the flat density limit there are two changes in Eqs. (1)-(3). The first is that that $v_D(1 + \eta_i)$ is replaced by $-d(\ln T_i)/dx$, which does nothing to change the dynamics of the mode. Second is the vanishing of the drift term, $v_D\nabla_y\tilde{\phi}$, in Eq. (1). This term represents $\vec{E}\times\vec{B}$ convection along the density gradient, and is important in the lowest order dynamics of the (finite L_n) η_i-mode since it allows $\nabla_\|\tilde{v}_\|$ to be nonzero while maintaining incompressible mass flow (i. e., $\nabla\cdot(n\vec{v}) \simeq 0$). Without the density gradient, $\partial\tilde{n}/\partial t$ assumes this role, mass incompressibility no longer holds, and the theory must be reformulated.

Linearizing Eqs. (1)-(3), Fourier transforming in y, z, and t, with $k_\| = k_y x/L_s$, taking the drift term in Eq. (1) to zero, and solving for $\tilde{\phi}$, we obtain the following mode equation:

$$\frac{d^2\tilde{\phi}_{\vec{k}}}{dx^2} + Q(x,\omega)\tilde{\phi}_{\vec{k}} = 0, \tag{6}$$

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where the "potential" function is given by

$$Q(x,\omega) = \left\{ -k_y^2 - \frac{\omega}{\omega + \omega_*^T} + \frac{k_y^2 x^2}{L_s^2 \left(\omega^2 - \frac{\Gamma}{\tau} \frac{k_y^2 x^2}{L_s^2} \right)} \right\}, \qquad (7)$$

where $\omega_*^T = -k_y/\tau L_T$ (in dimensionless units).

Neglecting the term that varies as Γ (which gives corrections of order $\sqrt{L_T/L_s}$), then Eq. (6) is the usual Weber's equation, with solution given by the Hermite functions, and yields the following dispersion relation:

$$(1 + k_y^2) \left(\frac{\omega}{\omega_*^T} \right)^2 + \left(k_y^2 + i(2l+1)\frac{\tau L_T}{L_s} \right) \left(\frac{\omega}{\omega_*^T} \right) + i(2l+1)\frac{\tau L_T}{L_s} = 0, \qquad (8)$$

where l is a positive integer. For the regime $k_y^2 \lesssim \tau L_T/L_s$ ($\ll 1$) and $l < L_s/\tau L_T$, the roots are, approximately,

$$\omega \simeq \pm \left(\frac{1-i}{\sqrt{2}} \right) (2l+1)^{\frac{1}{2}} \left(\frac{\tau L_T}{L_s} \right)^{\frac{1}{2}} \omega_*^T, \qquad (9)$$

of which one root is unstable.[1,4] It is important to notice that here $Re(\omega) \simeq Im(\omega)$, whereas for the finite density gradient case, a purely growing portion of the spectrum exists at low k_y, where $\omega \simeq i\frac{L_n}{L_s}\frac{1+\eta i}{\tau}\omega_{*e}$. Physically this difference is due to the introduction of mass compression into the basic dynamics, an effect also present in the turbulent regime, which influences construction of the nonlinear theory.

The width of the mode may be found by taking the x^2 moment of the Hermite functions, and is, in the low l and low k_y-regime,

$$(\Delta x)^2 \simeq \frac{1+i}{\sqrt{2}} (2l+1)^{3/2} \left(\frac{L_s}{\tau L_T} \right)^{1/2}. \qquad (10)$$

Despite the imaginary component of Δx, the mode remains a bound state since $Im(\Delta x) < Re(\Delta x)$.

We have numerically compared the above fluid results with the more complete kinetic theory by using a shooting code with a potential derived from the ion gyrokinetic equation, similar to that used in Ref. 9. This analysis is tedious, and not presented here, but the

basic result is that fluid and kinetic theory agree at least as well in the flat density limit as in the usual finite L_n regime. The neglect of ion resonances requires that

$$\frac{k_{\parallel} v_i}{\omega} \simeq (2l+1)^{1/4} \left(\frac{L_T}{\tau L_s}\right)^{1/4} \ll 1,$$

or $\frac{\tau L_s}{L_T} \gg 2l+1$, which may be interpreted to mean that the temperature gradient must be well above threshold for a fluid theory to apply.

In Ref. 7 it was demonstrated that compression of the ion diamagnetic drift (where $\nabla \cdot \vec{v}_D \simeq [\hat{b} \times (\hat{b} \cdot \nabla)\hat{b}] \cdot \vec{k}_{\perp} \tilde{p} \simeq k_y \tilde{p}/R$, in undimensional units, and R is the major radius), which is neglected here, can have a stabilizing effect on the mode. However, this study also neglected the parallel sound wave dynamics, which is clearly the destabilization mechanism in the slab limit. A comparison of these terms in the present ordering shows:

$$\frac{\nabla_{\perp} \cdot \tilde{v}_D}{\nabla_{\parallel} \tilde{v}_{\parallel}} \simeq \left(\frac{L_s}{R}\right)^{\frac{1}{2}} = \frac{q}{\hat{s}}.$$

Thus, perpendicular compression is important in the weak shear limit where $q/\hat{s} > 1$ (*i.e.*, the toroidal limit), whereas for strong shear (the slab limit) it is replaced by parallel compression.

A mixing length estimate of the turbulent diffusion rate produces:

$$D_{\vec{k}}^{xx} \simeq \gamma_{\vec{k}} (\Delta x)_{\vec{k}}^2 \simeq (2l+1)^2 \frac{\omega_*^T}{2}. \tag{11}$$

The absence of L_s in this estimate is a result of the dual role of the shear, which both destabilizes the mode (which enhances D^{xx}), and localizes it (which decreases D^{xx}), with net cancellation. However, shear dependence will occur if the ion thermal transport is strong enough to hold the temperature gradient to the threshold value, given by $(L_s/L_T)_{crit} \simeq 1.9(1 + 1/\tau)(2l+1)$. The threshold effect also limits the radial eigenmode number to low values, since higher l have a higher threshold and thus might be expected not to be excited.

III. Nonlinear Theory

In this section, we seek to improve upon the mixing length estimate given above by solving for the renormalized diffusions as an *eigenvalue* of the system of differential equations.[9] This method has the capability of taking into account *all* of the terms in the renormalized equations. By comparison, the usual mixing length scheme, which employs such heuristic concepts as "asymptotic balance" to estimate scalings with various key terms, is inherently limited. The basic scheme here is to demand $Im(\omega) = 0$, at saturation, and solve an eigenvalue problem for D (henceforth D will denote $D_{\vec{k}}^{xx}$). Thus the calculation yields the level of turbulent energy diffusion necessary to shut off the growth. Here, this technique determines whether the renormalized nonlinear dynamics produce any modification over the mixing length estimate, as can occur in other cases.[12]

This approach was applied the case of finite density gradient η_i-turbulence. In that work, it was possible to drop all of the time derivative at saturation, since in the low k_y regime considered it represents growth only. However, in the flat density case $Re(\omega)$ assumes the important role of balancing parallel compression in the continuity equation. This compression (which vanishes as k_y) will persist in the saturated state, and cannot be balanced by any of the nonlinearities in Eq. (1) (which vanish as k_y^2 or faster, which is also a property of the unrenormalized $\vec{E} \times \vec{B}$ vorticity nonlinearity), in the $k_y \to 0$ limit. Thus, for large wavelength (the limit of interest in this one-point, transport theory), it is essential to retain $Re(\omega)$ to balance compression in the continuity equation.

Thus, the analytical method proceeds as follows: restricting consideration to the low k_y portion of the spectrum (which is responsible both for energy feed and for transport), we solve Eqs. (1)-(3) for $\tilde{\phi}$ retaining both ω_r *and* the renormalized diffusivities. The resulting differential equation is then manipulated into "Schrödinger" form, and a WKB approximation produces a complex dispersion relation for D. The other "eigenvalue" $Re(\omega)$, remains at its linear value, since any nonlinear frequency modifications are contained in $Im(D)$.

Proceeding along these lines, we Fourier transform Eqs. (1)-(3) in t, y, z, *and* x and solve for $\tilde{\phi}$, neglecting terms which vary as k_y^2 and higher. This results in the following

second order differential equation:

$$\left(\frac{1}{\omega + iDk_x^2}\frac{\partial}{\partial k_x}\right)^2 \psi + \frac{L_s^2}{k_y^2}\frac{\omega + (\omega_*^T + \omega)k_x^2}{\omega_*^T + \omega + iDk_x^2}\psi = 0 \tag{12}$$

where,

$$\psi = \frac{\omega_*^T + \omega + iDk_x^2}{\omega + iDk_x^2}\tilde{\phi}.$$

Applying the WKB phase quantization approximation to this equation produces:

$$i\left(\omega_*^T + \omega\right)^{\frac{1}{2}}\frac{L_s}{k_y}\int_{-k_T}^{k_T}\frac{\omega + iDk_x^2}{\left(\omega_*^T + \omega + iDk_x^2\right)^{\frac{1}{2}}}\left(k_T^2 - k_x^2\right)^{\frac{1}{2}}dk_x = \frac{\pi}{2}\left(2l + 1\right), \tag{13}$$

where,

$$k_T^2 = -\frac{\omega}{\omega_*^T + \omega}.$$

The choice of k_T as the turning point recovers all the characteristics of the linear mode in the $D \to 0$ limit, which demonstrates that this branch is the nonlinear extension of the linear theory. Expanding the denominator of the integrand to order k_x^2 and integrating, yields the following dispersion relation:

$$i\left(\omega_*^T + \omega\right) + \frac{D}{4}\left(1 - \frac{1}{2}\frac{\omega}{\omega_*^T + \omega}\right) = -\left(2l + 1\right)\frac{k_y}{L_s}\left(\frac{\omega_*^T + \omega}{\omega}\right)^2. \tag{14}$$

In the saturated state, the growth is shut off, so only the real part of ω remains, which is given by Eq. (9). Solution of Eq. (14) then yields:

$$D = 8\omega_*^T\frac{\left(1 - s_l^{\frac{1}{2}}\right)^3}{1 - s_l^{\frac{1}{2}}/2}\left(1 - \frac{i}{2}\frac{1}{1 - s_l^{\frac{1}{2}}}\right), \tag{15}$$

where $s_l = (l + 1/2)\frac{rL_T}{L_s}$, and $\frac{rL_T}{L_s} \ll 1$. The basic scaling, $D \sim \omega_*^T$, agrees with the mixing length estimate, Eq. (11). However, there is a discrepancy of $(2l + 1)^2$, missing in Eq. (15). Possibly, this is a consequence of Eq. (15) being calculated in k_x space (which leads to $D_{\rm eig} \simeq \Delta\omega_{\bar{k}''}/\left(\Delta k_x\right)^2$), while the linear estimate uses a mode width calculated in x-space (so that $D_{\rm ML} \simeq \Delta\omega_{\bar{k}''}\left(\Delta x\right)^2$). While it is true that $\Delta x \simeq 1/\Delta k_x$ for the $l = 0$ mode, one can easily show that for the higher l Hermite functions this must be generalized to $\Delta k \simeq (l + 1/2)/\Delta x$. This could account for the discrepancy between Eq. (11) and

Eq. (15). Since it is the mode width in x-space that determines the step length in the random walk, and hence the radial diffusion, we expect the mixing length estimate of Eq. (11) to yield the correct l scaling. However, in light of the ambiguity, we shall consider only the diffusivity of the $l = 0$ mode:

$$D_{l=0} = 2\omega_*^T.$$

Use of only the $l = 0$ mode is reasonable in light of threshold effects, which allow only the lowest radial eigenmodes to be unstable for realistic values of L_s/L_T. This may be used for rough comparison with the transport of the finite L_n case as described in Ref. 2 (which also considers only the $l = 0$ case).

Finally, it is useful to estimate the rms fluctuation levels at saturation. This is done using the approximation

$$D \simeq \sum_{\vec{k}'} \frac{k_y'^2 \left| \tilde{\phi}^2 \right|}{\Delta\omega_{\vec{k}''}} \sim \frac{\langle k_y^2 \rangle \langle \phi^2 \rangle}{\Delta\omega_{\vec{k}}}.$$

Estimating $\Delta\omega_{\vec{k}} \sim D/\Delta x^2$, and using Eq. (16) for D, we find

$$\langle \phi \rangle_{\mathrm{rms}} \sim D/\Delta x k_y \sim \left(\frac{\tau L_T}{L_s} \right)^{\frac{1}{4}} \frac{\rho_s}{\tau L_T}$$

in undimensional units. This also represents the level of density fluctuation, via adiabatic electron response. The saturated level of pressure fluctuation is estimated by balancing pressure diffusion with $E \times B$ mixing of the equilibrium pressure gradient (the second and fourth terms in Eq. 3), yielding:

$$\tilde{p} \sim \left(\frac{L_s}{\tau L_T} \right)^{\frac{1}{4}} \frac{\rho_s}{\tau L_T}.$$

Finally, since $\tilde{n} \ll \tilde{p}$, then $\tilde{p}$ is dominated by the contribution from temperature fluctuations, so that:

$$\tilde{T}/T_i \sim \left(\frac{L_s}{\tau L_T} \right)^{\frac{1}{4}} \frac{\rho_s}{L_T}.$$

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IV. Transport

Having obtained the saturation level of turbulent diffusivity, we next apply this knowledge to finding the saturation levels of ion and electron thermal conductivities, χ_i and χ_e, the momentum diffusivity, χ_φ and the particle convection velocity, V_r. The basic technique and formulas are given in Refs. 2 and 9, and since these formulas do not change in the $L_n \to \infty$ limit, here we apply them without rederivation.

The ion thermal flux is calculated from the correlation between ion pressure fluctuations and radial velocity fluctuations, which yields:

$$q_i = -\frac{T_i}{L_T} \left\langle D_{\vec{k}} \right\rangle,$$

with resulting ion thermal conductivity:

$$\chi_i = \left\langle D_{\vec{k}} \right\rangle \simeq 2 \frac{\langle k_y \rho_s \rangle_{\mathrm{rms}}}{\tau L_T} \rho_s^2 c_s. \tag{16}$$

Evaluation of $\langle k_y \rho_s \rangle_{\mathrm{rms}}$ would require solving a two-point spectrum equation, which is beyond the scope of the present study. For the purposes of an estimate, one may use $\langle k_y \rho_s \rangle_{\mathrm{rms}} \simeq 0.4$, which was calculated in Ref. 2 for the finite L_n limit. This is done in the summary provided in Table I.

Formal evaluation of the momentum diffusivity, χ_φ, was done in Ref. 1, which examined the effects of a shear flow on the ITGD mode in the limit $L_n < \sqrt{L_T L_s}$. The result was that the momentum diffusivity is equal to the ion thermal diffusivity (because of the sonic nature of the mode), and that both are enhanced by a small amount from the additional shear flow free energy source. Although here we have eschewed the detailed consideration of the shear flow, the analysis would proceed in an analogous fashion, yielding:

$$\chi_\varphi = \chi_i \simeq 2 \frac{\langle k_y \rho_s \rangle_{\mathrm{rms}}}{\tau L_T} \rho_s^2 c_s. \tag{??}$$

We neglect the shear flow enhancement factor, which is generally small.

The electron thermal conductivity (χ_e) is derived from the trapped electron response to the turbulent potential fluctuations in the dissipative trapped electron regime.[13] ($\omega_{*e} < \nu_{\mathrm{eff},e}$). Here, χ_e is estimated as:

$$\chi_e = 15\sqrt{2}\epsilon^{3/2} \frac{\rho_s^2 c_s^2}{\nu_e} \sum_{\vec{k}'} \left\langle k_y'^2 \left| \frac{e\tilde{\phi}_{\vec{k}'}}{T_e} \right|^2 \right\rangle.$$

Using the approximation $\sum_{\vec{k}'} k'^2_y \left| \frac{e\tilde{\phi}_{\vec{k}'}}{T_e} \right|^2 \simeq \langle \gamma_{\vec{k}'} D_{\vec{k}'} \rangle$, we find:

$$\chi_e \simeq 30 \frac{\epsilon^{3/2}}{\nu_e} \left(\frac{\tau L_T}{L_s} \right)^{\frac{1}{2}} \frac{\langle k^2_y \rho^2_s \rangle}{(\tau L_{T_i})^2} \rho^2_s c^2_s, \tag{17}$$

where ϵ is the inverse aspect ratio and ν_e is the electron collisionality.

For particle flux in the central region, the necessary phase shift between $\tilde{v}_{E_r}$ and $\tilde{n}_e$ (here adiabatic to lowest order) is also provided by dissipative trapped electron dynamics.[13] In the flat density limit, the flux is:

$$\Gamma_r = \langle \tilde{v}_r \tilde{n} \rangle \simeq \frac{3}{2} \frac{n_0 \epsilon^{3/2}}{\nu_e L_{T_e}} \sum_{\vec{k}'} \left\langle k'^2_y \left| \frac{e\tilde{\phi}_{\vec{k}'}}{T_e} \right|^2 \right\rangle$$

in the high-collisionality limit of the banana regime where $\nu_{\text{eff},e} \gg \omega, \overline{\omega}_{De}$. Redimensionalizing, and applying the same approximation that led to Eq. (17), we find that the particle convection velocity is given by:

$$V_r = \Gamma_r / n_0 \simeq 2.1 \frac{\epsilon^{3/2}}{\nu_e} \left(\frac{\tau L_T}{L_s} \right)^{\frac{1}{2}} \frac{\langle k^2_y \rho^2_s \rangle}{L_{T_e} (\tau L_{T_i})^2} \rho^3_s c^2_s. \tag{18}$$

This represents a purely outward particle flux, although for different collisionality regimes the flux can be inward.[14]

V. Discussion

This paper has developed the theory of ion temperature gradient driven turbulence in the presence of a flat density profile. The principal results of thes paper are:

1. For $\nabla n \to 0$, ITGDT has real frequency $\omega_r = \frac{1}{\sqrt{2}} \left(\frac{\tau L_T}{L_s} \right)^{\frac{1}{2}} \omega_*^T$, turbulent decorrelation rate $\Delta \omega_{\vec{k}} \simeq \omega_{\vec{k}}$, and radial correlation length $(\Delta x)_{\vec{k}} \simeq (L_s/\tau L_T)^{1/2}$.

2. The ion thermal diffusivity χ_i in this regime is given by $\chi_i = 2 \langle k_y \rho_s \rangle_{\text{rms}} \rho_s^2 c_s / \tau L_T$. This value is equal to the the momentum diffusivity, χ_φ.

3. For dissipative trapped electron response coupling ($\omega_{D,e} < \omega_r < \nu_{eff,e}$, the electron heat diffusivity and particle flux are given by, respectively,

$$\chi_e \simeq 30 \frac{\epsilon^{3/2}}{\nu_e} \left(\frac{\tau L_T}{L_s} \right)^{\frac{1}{2}} \frac{\langle k_y^2 \rho_s^2 \rangle}{(\tau L_{T_i})^2} \rho_s^2 c_s^2,$$

$$V_r = 2.1 \frac{\epsilon^{3/2}}{\nu_e} \left(\frac{\tau L_T}{L_s} \right)^{\frac{1}{2}} \frac{\langle k_y^2 \rho_s^2 \rangle}{L_{T_e} (\tau L_{T_i})^2} \rho_s^3 c_s^2.$$

4. Saturated flat density ITGDT supports fluctuations with

$$(\tilde{T}_i/T_0) \sim \left(\frac{L_s}{\tau L_T} \right)^{\frac{1}{4}} \frac{\rho_s}{L_T} > (\tilde{n}/n_0)_{\text{rms}} \left(\frac{\tau L_T}{L_s} \right)^{\frac{1}{4}} \frac{\rho_s}{\tau L_T}$$

These results are summarized in Table 1, along with their finite density gradient counterparts. Roughly speaking, either regime can be derived from the other by interchanging $(1 + \eta_i)/\tau$ with $\sqrt{L_s/\tau L_T}$.

The results of this study can be used, along with transport experimets and analyses, to assess the role of ITGDT in flat density discharges. However, it should be remembered that the results of this paper are valid only for strong turbulence, where $\Delta \omega \gtrsim \omega_r$. Thus, a complete theory of flat density ITGDT requires a model for weak turbulence, where $\Delta \omega \ll \omega_r$, which occurs near the instability threshold. Work on the weak turbulence theory of flat density ITGDT is in progress and will be discussed in a future publication.

Acknowledgements

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Physical Quantity	$L_n^2 < L_T L_s$	$L_n^2 > L_T L_s$
Threshold	$\frac{L_n}{L_T} \simeq 1$	$\frac{L_s}{L_T} \gtrsim 1.9\left(1 + \frac{T_i}{T_e}\right)$
ω_r	$O(L_n^2/L_s^2) \ll \gamma$	$\left(\frac{\tau L_T}{L_s}\right)^{\frac{1}{2}} \omega_*^T/\sqrt{2}$
$\Delta\omega$	$\left(\frac{1+\eta_i}{\tau}\right)\omega_{*e}$	$\left(\frac{\tau L_T}{L_s}\right)^{\frac{1}{2}}\omega_*^T$
χ_i	$1.3\left(\frac{1+\eta_i}{\tau}\right)^2 \rho_s^2 c_s/L_s$	$0.8\rho_s^2 c_s/\tau L_T$
χ_e	$10\frac{\epsilon^{3/2}}{\nu_e L_s^2}\left(\frac{1+\eta_i}{\tau}\right)^3 \rho_s^2 c_s^2$	$4.8\frac{\epsilon^{3/2}}{\nu_e}\left(\frac{\tau L_T}{L_s}\right)^{\frac{1}{2}}\frac{\rho_s^2 c_s^2}{(\tau L_{T_i})^2}$
χ_φ	$1.3\left(\frac{1+\eta_i}{\tau}\right)^2 \rho_s^2 c_s/L_s$	$0.8\rho_s^2 c_s/\tau L_T$
Γ_r	$\frac{0.5 n_0 \epsilon^{3/2}}{\nu_e L_n L_s^2}\left(1+\frac{3}{2}\eta_e\right)\left(\frac{1+\eta_i}{\tau}\right)^3 \rho_s^2 c_s$	$\frac{0.3 n_0 \epsilon^{3/2}}{\nu_e}\left(\frac{\tau L_T}{L_s}\right)^{\frac{1}{2}}\frac{\rho_s^3 c_s^2}{L_{T_e}(\tau L_{T_i})^2}$
$\tilde{T}_i/T_0$	$\left(\frac{1+\eta_i}{\tau}\right)^{3/2}\frac{\rho_s}{L_n}$	$\left(\frac{L_s}{\tau L_T}\right)^{\frac{1}{4}}\frac{\rho_s}{L_T}$
$\tilde{n}/n_0$	$\left(\frac{1+\eta_i}{\tau}\right)^{3/2}\frac{\rho_s}{L_s}$	$\left(\frac{\tau L_T}{L_s}\right)^{\frac{1}{4}}\frac{\rho_s}{\tau L_T}$

Table I: Summary of finite and flat density gradient results (slab limit only).