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DRIFT WAVE TURBULENCE AND ANOMALOUS TRANSPORT

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I. Introduction

The properties of a magnetized plasma such as density and temperature are typically nonuniform in space. The presence of the spatial gradients across the magnetic field leads to the so-called diamagnetic drift currents of the particles in the direction mutually perpendicular to the gradient of the plasma and the direction of the magnetic field. In such nonuniform, magnetized plasmas there arise collective oscillations propagating along the diamagnetic currents called drift waves. Drift waves give rise to a motion of the charged particles across the magnetic field. The excursion of the particle across the magnetic field can be many times greater than its gyroradius. Through the particle dynamics in the collective modes of the system a net plasma transport occurs along the macroscopic plasma gradient. As reviewed in this chapter, the mechanisms for the transport are varied, depending on the regime of the magnetized plasma and the details of the drift modes. Quite generally, however, the effect of the plasma oscillations on the macroscopic scale is to produce a collective dissipation in the form of anomalous diffusion of particles and thermal energy along the gradients of the plasma density and temperature. The collective transport is independent of the binary particle collisions and may exceed the collisional transport by several orders of magnitude. For this reason, the transport is called anomalous, although it is the general mechanism of transport in high temperature plasmas and obeys the general thermodynamic laws of transport processes.

I. Introduction

The properties of a magnetized plasma such as density and temperature are typically nonuniform in space. The presence of the spatial gradients across the magnetic field leads to the so-called diamagnetic drift currents in the direction mutually perpendicular to the gradient of the plasma and the direction of the magnetic field. In such nonuniform, magnetized plasmas there arise collective oscillations propagating along the diamagnetic currents called drift waves. Drift waves give rise to a motion of the charged particles across the magnetic field. The excursion of the particle across the magnetic field can be many times greater than its gyroradius. Through the particle dynamics in the collective modes of the system a net plasma transport occurs along the macroscopic plasma gradient.

As reviewed in this chapter, the mechanisms for the transport are varied, depending on the regime of the magnetized plasma and the details of the drift modes. Quite generally, however, the macroscopic effect of the plasma oscillations is to produce a collective dissipation in the form of anomalous diffusion of particles and thermal energy along the gradients of the plasma density and temperature. The collective transport is independent of the binary particle collisions and may exceed the collisional transport by several orders of magnitude. For this reason, the transport is called anomalous, even though it is the general mechanism of transport in high temperature plasmas and obeys the general thermodynamic laws of transport processes.

The subject of linear drift wave instabilities arising from macroscopic inhomogeneities in magnetized plasma is reviewed by Mikhailovskii in Chapter III.4 of the Handbook. The drift wave instabilites occur as low frequency oscillations propagating along the diamagnetic particle currents required to support the cross-field plasma gradients as first explained by Rudakov and Sagdeev (1961). The oscillation frequencies are well below the ion gyrofrequency when the plasma gradients are weak on the scale of the average ion gyroradius. The electric field vector in the wave is nearly perpendicular to the magnetic field. The collective electric field produces $\underset{\sim}{E} \times \underset{\sim}{B}$ drifts of the charged particle guiding centers across the magnetic field which leads to the principal mechanism for the anomalous transport. The small electric field component aligned along the ambient magnetic field produces an oscillating plasma current which, in turn, generates a small oscillating magnetic field component perpendicular to the ambient magnetic field. The motion of the particles along the microscopically fluctuating magnetic field leads to the second mechanism for anomalous tranport.

The plasma transport at low plasma pressure is dominated by the collective electric field, since the magnetic fluctuation due to oscillating currents is a negligible perturbation to the ambient magnetic field. The electrostatic character of these low frequency electric fields introduces a substantial simplification to the self-consistent field problem. The theory of anomalous transport in the low plasma-to-magnetic pressure regime is developed in Secs. 2 and 3. At low plasma pressure there are laboratory experiments which measure the anomalous flux and correlate the flux to the amplitudes and phase relationships in the drift waves. In the low pressure regime there are also nonlinear theories for the self-consistent electrostatic waves. In simple geometries with discrete wavenumber spectra a small amplitude expansion of the nonlinear wave equations leads to formulas for the amplitude and the spectrum of the nonlinear oscillations. Single nonlinear waves are reviewed in Sections 2.3 to 2.5, and the drift wave soliton is described in Sec. 2.6. For typical system parameters the most important mode coupling process in nonlinear drift waves arises from the $\underset{\sim}{E} \times \underset{\sim}{B}$ convection of the hydrodynamic fields. The convective mode coupling produces a conservative, long range coupling in the wavenumber spectrum. In the presence of particle-wave resonances the conservative hydrodynamic coupling leads to stochastic fluctuations in the electric field in the presence of three or more interacting modes. Well past the threshold for instability the system is described by a broad spectrum of weakly correlated oscillations that form a turbulent plasma state. The nonlinear theories and the fluctuation measurements of drift wave turbulence are summarized in Sec. 3.

The anomalous transport depends on both the amplitude and the spectral properties of the collective electric field. The simplest calculations for transport solve the guiding center equation of motion for test particles in the low frequency drift modes. For simple models of the wave spectrum, the particle motion may be described by two-dimensional, area-preserving maps. The particle trajectories are known to change from confined orbits to stochastic orbits at the critical wave amplitude that causes neighboring particle-wave resonances to overlap (Chirikov, 1979). Once the resonance overlap condition is satisfied, the stochastic trajectories are well described statistically by quasilinear theory, which is reviewed by Sagdeev in Chapter IV. In the quasilinear regime the net plasma transport is proportional to the sum over all particles and all waves with the strength of each contribution weighted by the correlation of the particles with each spectral component of the fluctuating fields. The calculation of the correlation time of the particle-wave interaction requires the use of renormalized particle propagators in regimes of fully developed drift wave turbulence (Krommes in Chapter 5.4) and (Horton and Choi, 1979). After closure of the microscopic turbulence theory by a suitable truncation of high order correlations a closed system of equations for the spectral distribution function and the renormalized response function are obtained in Sec. 3.2. The macroscopic transport formulas for the turbulent plasma are analyzed for thermodynamic consistency, for ambipolarity, as an anomalous energy transfer mechanism, and for entropy production due to electrostatic fluctuations in Sec. 2.2 and due to electromagnetic fluctuations in Sec. 4.1.

In plasmas where the thermal energy density of the particles is a significant fraction of the magnetic field energy density, the collective drift wave oscillations produce substantial magnetic fluctuations due to the oscillating particle currents calculated in Sec. 4.1. The polarization of the perpendicular and the parallel components of the magnetic fluctuations are reviewed in Sec. 4.2. The component of the fluctuating magnetic field perpendicular to the average magnetic field vector permits the rapid parallel motion of the particles to contribute to the cross-field transport. As a result, the magnetic fluctuations are more important as a mechanism for anomalous electron transport due to the high speed of the electron motion along the magnetic field. In addition, the magnetic fluctuations due to drift modes lead to the concept of the microscopic destruction of the magnetic surfaces, as discussed in Sec. 4.3, and the occurrence of a component of the anomalous electron thermal transport due to the small scale magnetic turbulence as analyzed in Secs. 4.3 through 4.6. A central aspect of this theory is the calculation of the correlation time, or length, of the particle motion within the stochastic magnetic field.

Presently, there are no controlled plasma experiments measuring the anomalous transport produced by electromagnetic drift wave fluctuations. There are measurements, however, of significant electric and magnetic fluctuations in the drift wave frequency range in both tokamaks and tandem mirrors that clearly have the potential to dominate the radial transport of plasma. The problem of establishing under what circumstance the theoretical formulas reviewed in this chapter can explain the measured transport rates remains a multi-faceted problem for future studies. Finally, it is to be noted that due to the non-linear and self-consistent field nature of the problem, a definitive mathematical theory of anomalous transport has not been, and most likely, cannot be given. Instead, different approximate physical theories that explain certain features of an observed plasma behavior must provide the basis for an understanding of the phenomena of anomalous transport in plasmas.

2. Anomalous Transport Due to Drift Modes at Low Plasma Pressure

The collective modes of oscillation in a plasma confined by a uniform magnetic field give rise to the drift wave instability. The linear theory of drift wave instabilities is developed in Chapter III.4 by Mikhailovskii. The drift wave instabilities are driven by the radial gradients of the plasma density and temperature. In spatial structure the unstable drift waves are highly elongated parallel to the magnetic field. The perpendicular wavelengths $2\pi/k_\perp$ are shorter than the density and temperature gradient scale lengths r_n and r_T defined by

$$\frac{1}{r_n} = -\frac{1}{n}\frac{dn}{dr} \quad \text{and} \quad \frac{1}{r_T} = -\frac{1}{T}\frac{dT}{dr}\,, \tag{1}$$

but may be either longer or shorter than the ion gyroradius $\rho_i = c(m_i T_i)^{1/2}/e_i B$. The parallel wavelength $2\pi/k_\parallel$ scales with the gradient scale lengths r_n and r_T, and the longest parallel wavelengths may exceed fifty to one hundred times the smaller of r_n or r_T. For plasma gradients that are weak on the scale of the ion gyroradius ($r_n >> \rho_i$) the drift wave oscillations occur at frequencies $\omega_{\underset{\sim}{k}}$ that are well below the ion cyclotron frequency $\omega_{ci} = e_i B/m_i c$. Depending on the details of the system parameter, the oscillation frequencies are characterized by the diamagnetic drift frequencies ω_{nj} and ω_{*j} defined by

$$\omega_{nj} = k_y \frac{cT_j}{e_j B n_j} \frac{dn_j}{dr} \quad \text{and} \quad \omega_{*j} = k_y \frac{c}{e_j B n_j} \frac{dp_j}{dr} \tag{2}$$

in a right-handed coordinate system with the axial magnetic field $B\hat{z}$. The diamagnetic drift frequency $\omega_{*j}=\underset{\sim}{k}\cdot\underset{\sim}{v}_{dj}$ arises from the diamagnetic drift velocity $\underset{\sim}{v}_{dj}=cB\times\nabla p_j/e_j n_j B^2$ due to the pressure gradient of the charged particle species j in the non-uniform plasma [Chapter I.2]. It is conventional to define the ratio of r_n to r_T as $\eta=\nabla \ell nT/\nabla \ell nn=r_n/r_T$ and hence $\omega_{*j}=\omega_{nj}(1+\eta_j)$.

The most important of the drift waves is perhaps the electron drift wave with the long wavelength ($k_\perp \rho_i < 1$) frequency $\omega = \omega_{\underset{\sim}{k}}$ given by

$$\omega_k = \omega_{ne}\left[\frac{1-k_\perp^2\rho_i^2(1+\eta_i)}{1+k_\perp^2\rho^2}\right] . \tag{3}$$

Here, $\rho = c(m_i T_e)^{\frac{1}{2}}/e_i B$ describes the wave dispersion due to the ion inertia that produces the polarization current, and $\rho_i = c(m_i T_i)^{\frac{1}{2}}/e_i B$ gives the wave dispersion due to the finite ion gyroradius effects. The electron drift wave in Eq. (3) occurs when the electrons adjust adiabatically in the slowly varying electric field of the wave, as in an ion-acoustic wave. Due to the diamagnetic drift, however, the drift wave decouples from the ion-acoustic wave by propagating nearly perpendicular to the magnetic field such that $k_\parallel c_s < \omega_k < k_\parallel v_e$. At finite plasma pressure $\beta = 8\pi p/B^2$ the drift wave decouples from the Alfvén wave by propagating in the parallel phase velocity interval $c_s < \omega_k/k_\parallel < v_A$ where $v_A = B/(4\pi n_i m_i)^{\frac{1}{2}}$

is the Alfvén speed and $c_s = (T_e/m_i)^{1/2}$ is the ion-acoustic speed. At higher plasma pressure where $\beta = 8\pi p/B^2 > 1/10$ to 1/5 the unstable domain of the electron drift wave disappears except for some exceptional values of the temperature to density gradient ratio η_j found by Mikhailovskii and Fridman (1967).

Since the oscillation frequencies are well below the ion cyclotron frequency the principal motion of the electrons and the ions across the magnetic field is given by the $\underset{\sim}{E} \times \underset{\sim}{B}$ guiding center drift velocity

$$\underset{\sim}{v}_E = \frac{c\underset{\sim}{E} \times \underset{\sim}{B}}{B^2} = \frac{c\hat{z} \times \nabla\varphi(\underset{\sim}{x}t)}{B} \tag{4}$$

where the last formula applies for electrostatic $E(\underset{\sim}{x}t) = -\nabla\varphi(\underset{\sim}{x}t)$ motions. The leading order correction to the velocity (4) is proportional to ω_k/ω_{ci} arising from the ion polarization drift. The oscillating $\underset{\sim}{E} \times \underset{\sim}{B}$ drifts of the plasma particles give rise to the local fluctuating flux of particles $n_j\underset{\sim}{v}_E$ and thermal energy $p_j\underset{\sim}{v}_E$, where $n_j(\underset{\sim}{x}t)$ and $p_j(\underset{\sim}{x}t)$ are the density and pressure moments of the particle distribution function $f_j(\underset{\sim}{x}\underset{\sim}{v}t)$. To the first order in the amplitude of the oscillations these fluxes produce no net transport. At the second order in the amplitude there is, however, in general a net particle flux $\underset{\sim}{\Gamma}_j$ and a net thermal flux $\underset{\sim}{Q}_j$ produced by the drift wave oscillations. The fluxes are called anomalous since they occur independently, and often greatly exceed the fluxes produced by the Coulomb collisions between the charged particles, as given in Chapter I.6

by Hinton.

The fluctuating thermodynamic fields $n_j(\underset{\sim}{x}t)$, $p_j(\underset{\sim}{x}t)$, the electric field $\underset{\sim}{E}(\underset{\sim}{x}t)$, and the electrostatic potential $\varphi(\underset{\sim}{x}t)$ in a turbulent plasma are most appropriately represented in terms of their Fourier components $k = \underset{\sim}{k}\omega$. We write the fluctuating fields as

$$n_j(\underset{\sim}{x}t) = \sum_k n_j(k)\exp(i\underset{\sim}{k} \cdot \underset{\sim}{x} - i\omega t) \tag{5}$$

$$p_j(\underset{\sim}{x}t) = \sum_k p_j(k)\exp(i\underset{\sim}{k} \cdot \underset{\sim}{x} - i\omega t) \tag{6}$$

$$\varphi(\underset{\sim}{x}t) = \sum_k \varphi(k)\exp(i\underset{\sim}{k} \cdot \underset{\sim}{x} - i\omega t), \tag{7}$$

and likewise for the other fields such as $\underset{\sim}{E}(\underset{\sim}{x}t)$, $\delta\underset{\sim}{B}(\underset{\sim}{x}t)$ and $\underset{\sim}{v}_E(\underset{\sim}{x}t)$. Reality of the fields requires that $n_j(-k) = n_j^*(k)$. The k summation in Eqs. (5)-(6) is an abbreviated notation for the sum overall wavenumbers $\underset{\sim}{k}$ and the integration $\int_{-\infty}^{+\infty} d\omega/2\pi$ overall frequencies.

Although many correlation functions occur in plasma turbulence theory, a particularly important function is the $\underset{\sim}{k}\omega$ spectrum of the two-point potential correlation function. It is conventional to write $I(\underset{\sim}{k}\omega)$, or equivalently I_k, for the potential fluctuation spectrum where

$$\begin{aligned}\langle \varphi(\underset{\sim}{R},T)\ \varphi(\underset{\sim}{R}+\underset{\sim}{r},\ T+t)\rangle &= \sum_k I(\underset{\sim}{k}\omega)\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\omega t) \\ &= \sum_{\underset{\sim}{k}} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\ I_{\underset{\sim}{k}}(\omega)\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\omega t)\end{aligned} \tag{8}$$

which is equivalent to the statement that

$$\left\langle \varphi(\underset{\sim}{k}'\omega')\varphi(\underset{\sim}{k}\omega) \right\rangle = I_{\underset{\sim}{k}}(\omega)\,\delta_{\underset{\sim}{k}+\underset{\sim}{k}'}\,2\pi\delta(\omega+\omega') . \tag{9}$$

The space-time averages in Eqs. (8) and (9) are defined over the fluctuations on the drift-wave space-time scales which are well separated from the macroscopic space scales r_n, r_T and the anomalous transport time scale $\tau_D \simeq r_n^2/D$. Here D is the typical maximum anomalous drift wave diffusion rate estimated by $D \simeq (\rho/r_n)(cT/eB)$. The correlation functions defined by averaging over an intermediate space-time volume retain a parametric dependence given by $I_k(\underset{\sim}{R}T)$ on the macroscopic space-time scales $\underset{\sim}{R}T$ of the transport processes.

2.1 Particle and Thermal Flux Due to Electrostatic Drift Waves

General formulas for the anomalous flows in terms of the particle and field fluctuation spectra are obtained by extracting the average part of the local fluxes. Computing the space-time average of $n_j v_E$ and $\frac{3}{2}\, p_j v_E$, we obtain the anomalous flux formulas

$$\Gamma_j = -n_j \left(\frac{cT_e}{e\ B}\right) \sum_k k_y \mathrm{Im}\left[\frac{e\varphi^*(k)}{T_e}\ \frac{n_j(k)}{n_j}\right] \tag{10}$$

and

$$Q_j = -\frac{3}{2} p_j \frac{cT_e}{eB} \sum_k k_y \mathrm{Im}\left[\frac{e\varphi^*(k)}{T_e} \frac{p_j(k)}{p_j}\right] \tag{11}$$

where an appropriate normalization is introduced in the amplitude of fluctuating fields. The coefficient $D_B = cT_e/eB$ is sixteen times the so-called Bohm diffusion coefficient. In early confinement work, the Bohm diffusion coefficient $(1/16)(cT_e/eB)$ was used as a reference diffusion rate, although now no particular theoretical or experimental importance is attached to this formula.

In writing Eqs. (2), (10) and (11) the standard coordinate system in drift wave studies is adopted. Namely, $\hat{z}$ is taken in the direction of the local magnetic field $\underset{\sim}{B}$, $\hat{x}$ is taken in the direction of $\nabla n(r) \parallel \nabla T(r)$, and $\hat{y}$ is the direction mutually perpendicular to $\underset{\sim}{B}$ and $\nabla n(r)$, i.e. $\hat{y} = \hat{z} \times \hat{x}$. The components of $\underset{\sim}{\Gamma}_j$ and $\underset{\sim}{Q}_j$ in Eqs. (10) and (11) are the $\hat{x}$ components. Subsequently, we shall find that in general this is the only nonvanishing component.

The formulas for the particle flux Γ_j and the thermal flux Q_j show that only the part of the density or pressure fluctuation that is out-of-phase with the potential fluctuation makes a contribution to the net flow. Figure 1 shows how this dependence on the phase difference arises. At a given instant of time t_o and position z_o along the magnetic field, the contours of constant potential

$\varphi(x,y,z_o,t_o)$ = const are the stream lines for the guiding center drift velocity across the magnetic field, Eq. (4). As the field oscillates at the frequency ω, the guiding centers move along a fraction of the stream line given by $\underset{\sim}{v}_E/(-i\omega)$. At a sufficiently large amplitude of the electric field, $\underset{\sim}{k} \cdot \underset{\sim}{v}_E \sim \omega$, the guiding center circumnavigates the entire circumference in one oscillation period.

Since the flow is incompressible $\nabla \cdot \underset{\sim}{v}_E = 0$, the initially nonuniform particle fields $n_j(r)$ and $p_j(r)$ are convected along the stream lines by the oscillating $\underset{\sim}{E} \times \underset{\sim}{B}$ drift. When the variations, $\delta n_j(\underset{\sim}{x}t)$ and $\delta p_j(\underset{\sim}{x}t)$, of the particle fields are in phase with the potential field $\varphi(\underset{\sim}{x}t)$, as shown in Fig. 1c by the concentric level contours of φ and δn_j, the flux in the *outward* direction exactly compensates that in the *inward* direction. On the other hand, if the maximum of the density variation is shifted in phase by $\psi_{\delta n,\varphi}$ with respect to the maximum of the potential fluctuation, as shown in Fig. 1d, the fluxes no longer balance since in one direction the flow occurs with a higher density than in the other direction. For example, the electron drift wave, which by Eq. (3) propagates in the direction $\underset{\sim}{v}_{de} = (cT_e/eBr_n)\hat{y}$, has an excess particle flux in the direction $-\nabla n_e(r)$ when the density oscillation leads the potential oscillation in phase. The ion temperature gradient driven drift wave propagates in the direction $\underset{\sim}{v}_{di} = -(cT_i/eBr_T)\hat{y}$ and has an excess thermal flux in the direction $-\nabla p_i(r)$ when the potential oscillation leads the ion pressure fluctuation.

The general determination of the phase differences in the saturated nonlinear state is a difficult problem. In practice, however, one particle species develops the dominant nonlinear dynamics producing saturation of the linear growth. The other species remain well described by the quasilinear equations, which then determine the phase relations. In the electron drift wave, for example, it is usual that the ion dynamics becomes nonlinear, and the phase relations are computed from the quasilinear electron response.[1] Thus, when it is established that the species j responds quasilinearly, the fluctuating thermodynamic fields $n_j(k)$, $p_j(k)$, etc. are calculated directly from the low frequency ($\omega \ll \omega_{cj}$) fluctuating distribution function $\delta f_j(k,\underset{\sim}{v})$ given by (Mikhailovskii, Chapter III.4, Eqs. (29)-(36))

$$\delta f_j(k,\underset{\sim}{v}) = \frac{e_j}{m_j}\left[\frac{\partial f_j}{\partial \varepsilon} + \left(\omega \frac{\partial f_j}{\partial \varepsilon} + \frac{k_y}{\omega_{cj}} \frac{\partial f_j}{\partial x}\right) J_o^2\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) g_k^j(\underset{\sim}{v})\right] \varphi(k) \qquad (12)$$

[1]This situation is understood from the fact that the electron-wave correlation time $1/\Delta k_\parallel v_e$ is short compared to the ion-wave correlation time $1/\Delta\omega_k$. Consequently, as the amplitude of the electric field increases, the first interaction to be strongly nonlinear is that of the ions with the waves.

where $f_j(x,\varepsilon,t)$ is the background distribution function that varies on the macroscopic scales of transport r_n and r_n^2/D and where $\varepsilon = \frac{1}{2} v^2$ is the kinetic energy per unit of mass. In Eq. (12) the time history integral of the fluctuating field over the trajectory of the particle is $g_k^j(\underset{\sim}{v})$. In the simplest case of a collisionless nonuniform plasma slab $g_k^j(\underset{\sim}{v}) = (\omega - k_\| v_\| + io)^{-1}$, however, many different forms of $g_k^j(\underset{\sim}{v})$ are required for practical problems. In much of the following analysis, we only need two general properties of the particle propagator which follow from the conditions of reality and causality of the fields, namely

$$g_{-k}(\underset{\sim}{v}) = -g_k^*(\underset{\sim}{v}) \tag{13}$$

and

$$-\mathrm{Im} g_k(\underset{\sim}{v}) \geq 0. \tag{14}$$

These conditions on $g_k(\underset{\sim}{v})$ are satisfied by the propagators in complicated geometries and in the presence of either collisional or turbulent broadening of the wave-particle resonance.

From the fluctuating distribution (12) and property (13) it is straightforward to compute the fluctuating thermodynamic fields $n_j(k)$, $p_j(k)$ and the anomalous flux in Eq. (10) or Eq. (11) using, for example,

$$\text{Im}[\varphi^*(k)n_j(k)] = |\varphi(k)|^2 \int d\underset{\sim}{v} \left(\frac{e_j\omega}{m_j}\frac{\partial f_j}{\partial \varepsilon} + \frac{ck_y}{B}\frac{\partial f_j}{\partial x}\right) J_o^2\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) \text{Im}g_k^j(\underset{\sim}{v}) . \tag{15}$$

We also define the dimensionless dissipative contribution $\delta_j(k)$ by

$$\frac{e_j}{T_j}\delta_j(k) = \int d\underset{\sim}{v} \left(\frac{ck_y}{B}\frac{\partial f_j}{\partial x} + \frac{e_j\omega}{m_j}\frac{\partial f_j}{\partial \varepsilon}\right) J_o^2 \text{Im}g_k^j(\underset{\sim}{v}) .$$

The phase shift produced in the electron response in a drift-wave fluctuation has an important limiting form. Since the electron transit time over a parallel wavelength is short compared to the oscillation period, the general form of the density fluctuation is

$$n_e(k) = n_e\left[1 - i\delta_e(k)\right]\frac{e\varphi(k)}{T_e} \tag{16}$$

where the first correction to the in-phase response is of order $(\omega/k_\parallel v_e)^2 \ll 1$. A similar equation applies to the pressure fluctuation $p_e(k)$. With the quasilinear $n_e(k) - \varphi(k)$ relationship known, formula (10) for the anomalous flux becomes

$$\Gamma = n_e\left(\frac{cT_e}{eB}\right)\sum_k k_y\delta_e(k)\left|\frac{e\varphi(k)}{T_e}\right|^2 . \tag{17}$$

A similar formula for Q_e is readily derived using Eq. (11) and the pressure moment of Eq. (12). The phase shift shown in Fig. 1 is related to $\delta_e(k)$ by $\psi_{\delta n,\phi} = \sin^{-1}[\delta_e/(1 + \delta_e^2)^{\frac{1}{2}}]$.

In Eqs. (16) and (17) the function $\delta_e(k)$ is determined by the dissipative part of the electron propagator, Im $g_k^e(\underset{\sim}{v})$. When the electron mean-free-path $\lambda_e = v_e/\nu_e$ is long compared to the parallel wavelength, the resonant part of the electron propagator is Im $g_k^e(\underset{\sim}{v}) \cong -\pi\delta(\omega - k_\parallel v_\parallel)$. On the other hand, when the electron mean-free-path is shorter than the parallel wavelength, the electrons diffuse in space with $D_\parallel = \nu_e \lambda_e^2 = v_e^2/\nu_e$, rather than free streaming over the wavelength $2\pi/k_\parallel$. The electron dissipation is now determined by Im g_k^e = Im $(\omega - k_\parallel v_\parallel + i\hat{C}_e)^{-1}$ where $i\hat{C}_e$ is the Coulomb collision operator.

For the Lorentz approximation to the collision operator $i\hat{C}_e$ the result is approximately that $\mathrm{Im}\langle g_k^e \rangle_\Omega = \mathrm{Im}(\omega + ik_\parallel^2 v^2/\nu_e)^{-1}$ for $\nu_e > k_\parallel v_e$ and $\mathrm{Im}\,\langle g_k^e \rangle_\Omega = \pi/(2k_\parallel v)$ for $\nu_e < k_\parallel v_e$. Here, $\langle g_k^e(\underset{\sim}{v})\rangle_\Omega$ is the pitch-angle Ω averaged response function and $\nu_e = 2\pi n_e e^4 Z_{eff} \ln \Lambda/m_e^2 v_e^3$. Koch and Horton (1975) give the general form of the electron response with the Lorentz collision operator. Other results for Im $g_k^e(\underset{\sim}{v})$ including ∇B and curvature guiding center drifts and particle trapping by the magnetic mirror effect must be included for confinement problems as found in Horton (1976a) and Tang (1978). From these results it is straightforward to compute the electron velocity integrals which determine $\delta_e(k)$. For the values of Im $\langle g_k^e \rangle$ given in the preceeding, the out-of-phase component is approximately:

$$\delta_e(k) = \begin{cases} \left(\frac{\pi}{2}\right)^{1/2} \dfrac{[\omega_{*e}(1 - \frac{1}{2}\eta_e) - \omega]}{|k_\| | v_e} & \text{for } k_\| v_e > \nu_e \\ \dfrac{\nu_e}{k_\|^2 v_e^2}\left[\omega_{*e}(1 - \frac{3}{2}\eta_e) - \omega\right] & \text{for } k_\| v_e < \nu_e \end{cases} \tag{18}$$

in the case of a local Maxwell-Boltzmann electron distribution $f_e(x,\varepsilon) = F_e^M[\varepsilon, n_e(x), T_e(x)]$. Here, the temperature-to-density gradient ratio η_e is given by $\eta_e = \partial_x \ell n T_e / \partial_x \ell n n_e$.

A reduced description of drift-wave phenomena that is more tractable for nonlinear problems than the Vlasov equation is the two-component fluid description. Introducing the first order cross-field flows $\underset{\sim}{v}_E = c\underset{\sim}{E}\times\underset{\sim}{B}/B^2$, $\underset{\sim}{v}_{dj} = c\underset{\sim}{B}\times\nabla p_j/e_j n_j B^2$ and $\underset{\sim}{q}_j = (5p_j/2m_j\omega_{cj})(\underset{\sim}{B}\times\nabla T_j/B)$ due to plasma gradients and the electric field, the fluid conservation equations are

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j \underset{\sim}{v}_j) = 0$$

$$m_j n_j\left(\frac{\partial \underset{\sim}{v}_j}{\partial t} + \underset{\sim}{v}_j\cdot\nabla\underset{\sim}{v}_j\right) + \nabla\cdot\underset{\approx}{\Pi}_j = e_j n_j\left(\underset{\sim}{E} + \frac{\underset{\sim}{v}_j}{c}\times\underset{\sim}{B}\right) - \nabla p_j + \underset{\sim}{R}_j$$

$$\frac{3}{2} n_j\left(\frac{\partial T_j}{\partial t} + \underset{\sim}{v}_j\cdot\nabla T_j\right) + p_j\nabla\cdot\underset{\sim}{v}_j = -\nabla\cdot\underset{\sim}{q}_j - \underset{\approx}{\Pi}_j : \nabla\underset{\sim}{v}_j + Q_j \tag{19}$$

where the traceless part of the pressure tensor $\underset{\approx}{\Pi}$ is due to the finite-gyroradius transport of momentum given by

$$\Pi_{xx} = -\Pi_{yy} = -\frac{p}{2\omega_c}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) - \frac{1}{5\omega_c}\left(\frac{\partial q_x}{\partial y} + \frac{\partial q_y}{\partial x}\right)$$

$$\Pi_{xy} = \Pi_{yx} = \frac{p}{2\omega_c}\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y}\right) - \frac{1}{5\omega_c}\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y}\right)$$

$$\underset{\sim}{\Pi}_{\perp,z} = \frac{p}{\omega_c}\hat{b}\times\nabla v_{\parallel} .$$

For drift waves the electron inertia is negligible and the ion inertial acceleration is calculated perturbatively (Hinton and Horton, 1971) through

$$\underset{\sim}{v}_i^{(3)} = \frac{c}{e_i n_i B}\hat{b}\times\left[m_i n_i \frac{d\underset{\sim}{v}_i}{dt} + \nabla\cdot\underset{\sim}{\Pi}_i(\underset{\sim}{v}_i)\right]$$

and

$$\nabla\cdot\left(n_i \underset{\sim}{v}_i^{(3)}\right) = \nabla_\perp\cdot\left[\frac{n_i}{\omega_{ci}}\left(\partial_t + (\underset{\sim}{v}_E + \underset{\sim}{v}_{di})\cdot\nabla\right)\frac{c\underset{\sim}{E}_\perp}{B}\right] \qquad (20)$$

where cancellations occur in the divergence of the inertial acceleration and the finite-gyroradius stress tensor to yield Eq. (20).

A similar cancellation occurs in the thermal balance equation with $\frac{3}{2}n\underset{\sim}{v}_\perp\cdot\nabla T + p\nabla\cdot\underset{\sim}{v}_\perp + \underset{\sim}{\nabla}\cdot\underset{\sim}{q} = \frac{3}{2}\, n\underset{\sim}{v}_E\cdot\nabla T$ so that the ion-thermal balance reduces to

$$\frac{3}{2}\, n_j\left(\frac{\partial T_j}{\partial t} + \underset{\sim}{v}_E\cdot\nabla T_j\right) + p_j\nabla_{\parallel}v_{\parallel} = 0.$$

The electrons behave adiabatically with $k_{\|} v_e >> \omega$ which implies an infinite parallel thermal conductivity, $\underset{\sim}{B} \cdot \nabla T_e = 0$, in the collisionless plasma. For electrostatic motion the constraint $\nabla_{\|} T_e = 0$ implies $\delta T_e \simeq 0$, and for electromagnetic modes the constraint becomes $ik_{\|} \delta T_e + b_x \partial T_e / \partial x = 0$ where $b_x = \delta B_x / B_0$ as analyzed in Sec.4.

Linearization of the fluid equations for small amplitude oscillations in the inhomogeneous plasma is straightforward, although tedious, and yields the long wavelength $(k_{\perp} \rho_i < 1)$ form of the dispersion relation given in Eq. (3). Numerous nonlinear studies are based on the fluid description of drift waves.

A particularly simple regime occurs in the non-isothermal plasma with $T_e >> T_i$ where ion thermal motion is entirely negligible. The fluctuating component of the ion continuity equation gives that

$i\omega n_i(k) = \underset{\sim}{v}_E \cdot \nabla n_i(r) + \nabla \cdot (n_i \underset{\sim}{v}_p) = i n_i (\omega_{*e} - k_{\perp}^2 \rho^2 \omega)(e\varphi(k)/T_e)$

where $\underset{\sim}{v}_p = (c^2 m_i / e_i B^2)(d\underset{\sim}{E}_{\perp}/dt)$. Now demanding quasi-neutrality $n_i(k) = n_e(k)$ in the fluctuating state with Eq. (16) for $n_e(k)$ determines the complex frequency $\omega = \omega_{\underset{\sim}{k}} + i\gamma_{\underset{\sim}{k}} = \omega_{ne}/[1 + k_{\perp}^2 \rho^2 - i\delta_e]$ with ρ, ω_{ne} and δ_e defined previously in this section. From this derivation it is evident that the energy density W_k in the drift wave consists of the electrostatic potential energy of the floating electrons $\frac{1}{2} n_e e^2 \varphi_k^2 / T_e$ and the kinetic energy $\frac{1}{2} n_i m_i v_E^2(k)$ of the $\underset{\sim}{E} \times \underset{\sim}{B}$ motion of the ions.

Combining the two contributions, the wave energy density W_k expressed as a fraction of nT_e is

$$\frac{W(k)}{nT_e} = \frac{1}{2}[1 + k_\perp^2\rho^2]\left|\frac{e\varphi(k)}{T_e}\right|^2 , \tag{21}$$

a result used in the quasilinear and nonlinear analysis. The total fluctuation energy density is

$$W = \sum_k W(k) = \frac{e^2 n_e}{2T_e}\int d\underset{\sim}{r}\,[\varphi^2 + (\nabla_\perp\varphi)^2]$$

and is a small fraction of the thermal energy density.

2.2 Thermodynamics of Anomalous Transport

The anomalous transport of particles and energy due to drift wave fluctuations leads to a positive definite functional for the rate of internal entropy production as shown by Horton (1980). The entropy production functional is important for understanding the thermodynamics and the stability of the transport system. In thermodynamic stability theory, (Glandsdorff and Prigogine, 1974), the positive definite entropy production functional serves as the Lyapounov function that determines the stability of the steady state to thermal excursions.

In the general case, the particle and thermal fluxes are driven by complicated combinations of the density and temperature gradients, especially in the presence of significant ion temperature gradients. Coppi (1978) gives examples in tokamak geometry where the particle flux is inward, that is up the density gradient, while the thermal flux is outward or down the temperature gradient. The infinite plasma slab with reversed temperature and density gradients as produced, for example, in the skin current phase of heating contains complicated particle and thermal flows in opposite directions as analyzed by Liu et al. (1972). In these situations it is necessary to show that the particle and thermal flows satisfy the second law of thermodynamics. Any other behavior would lead to physical contradictions. The physical basis for the anomalous entropy production is the long range particle-particle interactions that occur through the collective electric field.

The transport equations for the macroscopic system are obtained by taking space-time averages of the microscopic transport equations. The anomalous particle flux $\underset{\sim}{\Gamma}_j$ and the anomalous thermal flux $\underset{\sim}{q}_j$ are written as vectors with nonvanishing components in the direction of the macroscopic gradients. The charged particle sources from atomic reactions are $S_j(\underset{\sim}{r},t)$ and the thermal sources from atomic reactions, auxiliary heating and radiative cooling are $P_j(\underset{\sim}{r},t)$. The macroscopic transport equation are

$$\frac{\partial n_j}{\partial t} + \nabla \cdot \underset{\sim}{\Gamma}_j = S_j(\underset{\sim}{r},t) \tag{22}$$

and

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_j T_j\right) + \nabla \cdot \underset{\approx}{Q}_j = P_j - \dot{W}_j , \tag{23}$$

where the macroscopic flows are given by

$$\underset{\sim}{\Gamma}_j = \langle n_j c \underset{\sim}{E} \times \underset{\sim}{B}/B^2 \rangle , \tag{24}$$

$$Q_j = \left\langle \frac{3}{2} n_j T_j \, c\underset{\sim}{E} \times \underset{\sim}{B}/B^2 \right\rangle = \frac{3}{2} T_j \underset{\sim}{\Gamma}_j + \underset{\sim}{q}_j \tag{25}$$

and

$$\dot{W}_j = - \left\langle \underset{\sim}{j}_j \cdot \underset{\sim}{E} \right\rangle = \left\langle \varphi \frac{\partial \rho_j}{\partial t} \right\rangle \quad . \tag{26}$$

Here, $\underset{\sim}{q}_j$ is the thermal flux relative to the convective energy flux $\frac{3}{2} T_j \underset{\sim}{\Gamma}_j$. The flow of energy between the fluctuation fields and the thermal energy of the jth species is given by $\dot{W}_j$. The collective electric field is electrostatic, $\underset{\sim}{E} = -\nabla\varphi(\underset{\sim}{r},t)$. Section 4.1 treats the case of electromagnetic fluctuations.

Drift wave fluctuations are charge neutral to the order $k_\perp^2 \lambda_{De}^2$ in both the linear and nonlinear states. From the condition of quasineutrality, $\rho_Q = \sum_j e_j n_j = 0$, it follows that the particle flux is ambipolar and that the net power flow vanishes, that is

$$\sum_j e_j \Gamma_j = 0 \quad \text{and} \quad \sum_j \dot{W}_j = 0 \quad . \tag{27}$$

In order that the thermodynamic fields in $n_j(\underset{\sim}{r},t)$, $T_j(\underset{\sim}{r},t)$ provide a well defined contracted description of the plasma we must restrict consideration to regimes where the Coulomb collisions between the charge particles are sufficiently frequent to maintain local Maxwell-Boltzmann velocity distributions $F_j^M(v^2)$ on the transport time scale. Let us define this regime by $\nu_e > \nu_*$ where ν_* is the critical collision frequency determined by the analysis of the background distribution function in Sec. 2.7. This critical collision frequency is a function of the fluctuation level. A consequence of the Maxwell-Boltzmann distribution $F_j^M(v^2)$ is that the characteristic microscopic diamagnetic frequency is given by

$$\omega_{*j}(v^2) = \frac{cT_j}{e_j B} \underset{\sim}{k} \times \hat{b} \cdot \left[\vec{X}_1 + \vec{X}_2\left(\frac{m_j v^2}{2T_j} - \frac{3}{2}\right)\right] \tag{28}$$

where

$$\underset{\sim}{X}_{j1} = \frac{\nabla n_j}{n_j} \quad \text{and} \quad \underset{\sim}{X}_{j2} = \frac{\nabla T_j}{T_j} \, . \tag{29}$$

We assume that $\vec{X}_1$ and $\vec{X}_2$ are parallel and define their ratio as $\eta_j = X_{j2}/X_{j1}$.

The microscopic fluctuating particle distributions must be obtained from the kinetic equation. In general, the fluctuation amplitudes are sufficiently large that the summation of high order secular contributions to the given fluctuation component $\delta f_{jk}(\underset{\sim}{v})$ is required to describe the particle-fluctuation interactions as shown by Horton and Choi (1979). Introducing the general particle propagator $g_k^j(\underset{\sim}{v})$, the fluctuating distribution is given by

$$f_{jk}(\underset{\sim}{v}) = -\frac{e_j F_j^M(v^2)}{T_j}\left\{1 - [\omega - \omega_{*j}(v^2)]\, g_k^j(\underset{\sim}{v})\right\}\varphi(k). \tag{30}$$

For example, the renormalized propagator in slab geometry is

$$g_k^j(\underset{\sim}{v}) = \frac{J_0^2(k_\perp v_\perp/\omega_{cj})}{\omega - k_\| v_\| - k_y v_{Dj} + i\nu_k}$$

where $v_{Dj} = (m_j c v_\perp^2/2e_j B)(d\ln B/dx)$ is the guiding center drift and $i\nu_k$ the resonance broadening frequency. Transport analysis with the particle propagator g_k^j specified for toroidal geometry, including a Lorentz collision operator broadening of the wave-particle resonances, is given in Horton and Estes (1979). The general properties of $g_k^j(\underset{\sim}{v})$ that follow from the conditions of reality and causality of the collective fields, $g_k^j(\underset{\sim}{v}) = -g_k^{j*}(\underset{\sim}{v})$ and $-\mathrm{Im}\, g_k^j \geqslant 0$, are given in Eqs. (13) and (14).

From the formulas (10), (11) and (24)-(30) we obtain the formulas for the particle flux and the thermal flux

$$\underset{\sim}{\Gamma}_j = -n_j\int dk\int d\underset{\sim}{v}\, I_j(k) F_j^M(v^2)\frac{cT_j}{e_j B}\frac{\underset{\sim}{k}\times\hat{b}}{B^2}[\omega - \omega_{*j}(v^2)]\,\mathrm{Im}\, g_k^j(\underset{\sim}{v}) \tag{31}$$

$$\underset{\sim}{q}_j = -n_j T_j\int dk\int d\underset{\sim}{v}\; I_j(k) F_j^M(v^2)\frac{cT_j}{e_j B}\frac{\underset{\sim}{k}\times\hat{b}}{B^2}\left(\frac{m_j v^2}{2T_j} - \frac{3}{2}\right)$$

$$[\omega - \omega_{*j}(v^2)]\,\mathrm{Im}\, g_k^j(\underset{\sim}{v}) \tag{32}$$

and the energy transfer flux

$$\dot{W}_j = n_j T_j \int dk \int d\underset{\sim}{v} I_j(k) F_j^M(v^2)\,\omega[\omega - \omega_{*j}(v^2)]\,\mathrm{Im}\,g_k^j(\underset{\sim}{v}) . \tag{33}$$

In Eqs. (31)-(33) the normalized spectral distribution $I_j(k)$ is defined $\langle e_j^2 \varphi^2(\underset{\sim}{r},t)/T_j^2 \rangle = \int dk I_j(k)$. Formulas (31)-(33) state that the anomalous flows are given by a sum over all particles and all fluctuations weighted by the strength of the particle-fluctuation interaction given by $-\mathrm{Im}\, g^j_{\underset{\sim}{k}\omega}(\underset{\sim}{v})$.

The anomalous flows $\underset{\sim}{\Gamma}_j$, $\underset{\sim}{q}_j$ and $\dot{W}_j$ are driven by the three forces $X_{1j} = \nabla \ln n_j$, $X_{2j} = \nabla \ln T_j$ and $X_{3j} = T_j^{-1} - T_w^{-1}$. Here, T_w is the effective temperature of the fluctuation field energy density $W = n_w T_w$ where n_w is the number of fluctuation modes per unit volume. In the regimes of interest the fluctuation temperature T_w greatly exceeds the particle temperature T_j so that the driving force $T_j^{-1} - T_w^{-1}$ reduces to $X_3 = 1/T_j$.

Each component of the entropy change, such as that produced by the flow of particles along the density gradient, may be either positive or negative. The total entropy production, $\sigma = -\sum_\alpha \underset{\sim}{J}_\alpha \cdot \underset{\sim}{X}_\alpha$, however, is positive definite. Combining the entropy production caused by the particle flow $\sigma_{\nabla n} = -\underset{\sim}{\Gamma} \cdot \underset{\sim}{X}_1$, due to the thermal conduction $\sigma_{\nabla T} = -\underset{\sim}{q} \cdot \underset{\sim}{X}_2/T$, and the energy transfer $\sigma_W = -\dot{W}_j X_3 = -\dot{W}_j/T_j$, it is straightforward to show that the total rate of internal entropy production is given by

$$\sigma_j = -\int dk \int d\underset{\sim}{v}\; I(k) F_j^M(v^2) \, \mathrm{Im}\; g_k^j(\underset{\sim}{v})$$

$$\left\{ e_j \omega X_3 - \frac{c}{B} \underset{\sim}{k} \times \hat{b} \cdot \left[\underset{\sim}{X}_1 + \underset{\sim}{X}_2 \left(\frac{m_j v^2}{2} X_3 - \frac{3}{2} \right) \right] \right\}^2 \geqslant 0 \; . \qquad (34)$$

A minimum entropy production principle follows by observing that for a frozen fluctuation spectrum $I(k)$ the entropy production functional $\sigma_j(X_2)$ acts as a potential for the flows. Namely, from Eq. (17) it follows that

$$\begin{aligned} \underset{\sim}{J}_1 &= \underset{\sim}{\Gamma} = -\frac{1}{2} \frac{\partial \sigma}{\partial \underset{\sim}{X}_1} \\ \underset{\sim}{J}_2 &= \underset{\sim}{q}/T = -\frac{1}{2} \frac{\partial \sigma}{\partial \underset{\sim}{X}_2} \\ J_3 &= \dot{W} + \dot{\underset{\approx}{Q}} \cdot \underset{\sim}{X}_2 = -\frac{1}{2} \frac{\partial \sigma}{\partial X_3} \; . \end{aligned} \qquad (35)$$

From these canonical forms for the flows, $J_\alpha = -\frac{1}{2}\, \partial\sigma/\partial X_\alpha$, it follows that the state of absolute minimum $\sigma(X_\alpha)$ is one with no macroscopic flow.

In terms of the generalized flows J_α the transport equations (22) and (23) become

$$\frac{\partial n_j}{\partial t} = -\nabla \cdot \underset{\sim}{J}_1 + S_j \qquad (36)$$

$$\frac{3}{2} n_j \frac{\partial T_j}{\partial t} = -T_j \nabla \cdot \underset{\sim}{J}_2 - J_3 + P_j - \frac{3}{2} T_j S_j . \quad (37)$$

In thermodynamic stability theory (Glansdorff and Prigogine, 1974), the stability of the transport system is analyzed by using the positive definite property of the entropy production σ to construct the Lyapounov functional

$$\mathcal{P}(t) = \frac{1}{2} \int d\underset{\sim}{r} \sigma(X_\alpha) \ . \quad (38)$$

The time rate of change of $\mathcal{P}(t)$ due to a perturbation of the steady state follows from the transport equations (36) and (37) and is given by

$$\frac{d\mathcal{P}}{dt} = -\int d\underset{\sim}{r} \left[\frac{1}{n}\left(\frac{\partial n}{\partial t}\right)^2 + \frac{3}{2} \frac{n}{T^2}\left(\frac{\partial T}{\partial t}\right)^2 \right] + \left(\frac{d\mathcal{P}}{dt}\right)_{ext} \quad (39)$$

where

$$\left(\frac{d\mathcal{P}}{dt}\right)_{ext} = \int d\underset{\sim}{r} \left[S \frac{\partial}{\partial t} \ln\left(\frac{n}{T^{3/2}}\right) - \frac{P}{T} \frac{\partial}{\partial t} \ln T \right] .$$

Since $\mathcal{P}(t)$ is a positive-definite form, the condition for asymptotic stability is that $dP/dt < 0$. Equation (30) states that thermal instability can occur only through the influence of the sources and sinks. Sufficient conditions for thermal stability on the parametric dependence of the particle source $S(n,T)$ and the thermal source $P(n,T)$ can be derived from Eq. (39).

2.3 Experiments on the Anomalous Transport Produced by Drift Waves

Basic plasma physics experiments in quiescent steady state plasmas show the presence of drift wave oscillations. The oscillations are identified as drift waves through studies of the parametric variation of the frequency and the wavelength as functions of the physical parameters in the linear dispersion relation. Two studies of particular interest with regard to the anomalous transport are the collisional drift wave experiments of Hendel, et al. (1967), (1968) and the collisionless drift wave experiments of Brossier et al. (1973), (1978). In both experiments the measured density to potential phase shifts are shown to agree with those given by the quasilinear electron response.

The collisional drift wave experiments are in a Q-machine with $\eta_e = d \ln T_e / d \ln n_e \cong 0$. The electron-wave dissipation is determined by the resistivity and the electron thermal conductivity along the magnetic field. The dissipation is given by Eq. (18) to within a constant factor that depends on the coefficients in the resistivity and thermal conductivity transport formulas.

Figure 2a shows the mean density profile in the stable state $n_{os}(r)$ and in the presence of the drift wave $n_{ow}(r)$. The amplitudes of the density and potential oscillations are shown in Fig. 2b along with the phase angle $\psi_{\delta n,\varphi}$ by which the density wave leads the potential wave in Fig. 2c. Figure 3 shows the particle flux $\Gamma(r) = F_{wave} = \langle n_e v_x \rangle$ computed from the amplitude and phase in Fig. 2. The particle flux is approximately one order of magnitude larger than the collisional flux F_{e-i} also shown in Fig. 3. Here $F_{e-i,s}$ and $F_{e-i,w}$ are the collisional fluxes computed from the density profile with the stable profile $n_{os}(r)$ and the mean profile $n_{ow}(r)$ with a wave present, respectively. The anomalous particle flux shown in Fig. 3a, along with the anomalous diffusion coefficient D(r) defined by

$$\Gamma(r) = -D(r)\,\frac{dn(r)}{dr}$$

and shown in Fig. 3b,

is necessary and sufficient to explain particle balance in the experiment.

Collisionless drift wave experiments by Brossier, et al. (1973), (1978) in a quiescent steady state hydrogen plasma have established the nature of the anomalous particle and electron thermal flux in the collisionless regime. In these experiments the electron temperature profile $T_e(r)$ is varied over the range $\eta_e = d \ln T_e / d \ln n_e = 0.2$ to 1.0. The resulting change in the density-to-potential phase shift, given approximately by Eq. (18), dictates the reduction of the growth rate and the particle transport as η_e increases. The measured wave amplitude and phase shift decreases with increasing η_e as predicted. The measured density oscillation and electron temperature oscillation are shown in Fig. 4, and the phase shifts $\psi_{\delta n,\varphi}$ and $\psi_{\delta T,\delta n}$ are shown in Fig. 5. Both the density and the electron temperature oscillations lead the potential oscillation by 30° to 50°. Calculation of the linear electron response with the self-consistent value of $\omega/k_{\parallel} v_e$ explains these two measured phase shifts.

The anomalous flux in this experiment is three orders of magnitude greater than the collisional flux. The anomalous particle flux and the anomalous thermal flux obtained from the amplitudes and phases of the oscillations account for particle and energy balance in the experiment. The particle and energy confinement times τ_p, τ_E are obtained from the decay rates of n_e and T_e after switch-off of the plasma and heating sources. The observed decay rates are consistent with the anomalous particle flux Γ and thermal flux q_e.

2.4 Finite Amplitude Drift Waves in Weakly Unstable Systems

For system parameters that are not far past the threshold condition for the onset of instability, there is a straightforward procedure to calculate the amplitude of the saturated modes. The method is based on the Landau picture for the transition to turbulence in which at a small degree of excess over the threshold only a few interacting modes need to be taken into account. In this regime the nonlinear partial differential equations for the fields $\varphi(\underset{\sim}{x}t)$, $\delta n_j(\underset{\sim}{x}t)$ and $\delta T_j(\underset{\sim}{x}t)$ are solved by a perturbation expansion in the small parameter Δ that measures the excess from threshold of the system. Expanding in powers of $\Delta^{\frac{1}{2}}$ both the fields $\Psi_\alpha = \Delta^{\frac{1}{2}}\Psi_\alpha^{(1)} + \Delta\Psi_\alpha^{(2)} + \Delta^{3/2}\Psi_\alpha^{(3)}$ and the nonlinear operators $\mathcal{O}_\alpha = \mathcal{O}_\alpha^{(0)} + \Delta\mathcal{O}_\alpha^{(2)}$, the resulting system of equations determines the harmonic production and the change in the background distributions at second order, and at third order, the nonlinear dispersion relation $\varepsilon_{\underset{\sim}{k}}^{n\ell}(\omega_{\underset{\sim}{k}} + \delta\omega_{\underset{\sim}{k}}^{n\ell}, |\Psi^{(1)}|^2, \Delta) = 0$. The vanishing of the complex nonlinear dispersion relation determines the amplitude of oscillation $|\Psi^{(1)}|^2 = f(\Delta)$ and the nonlinear frequency shift $\delta\omega_{\underset{\sim}{k}}^{n\ell} = g(\Delta)$. This calculation for the amplitude of the drift waves has been carried out for several problems. In the case of the collisional drift wave in the experiments of Hendel, et al. (1967), (1968) the principle mechanism for the amplitude limit is well understood.

2.5 Amplitude Limit for the Collisional Drift Wave

In the experiments of Hendel, et al. (1968) the critical parameter that is varied to destabilize the system is the magnetic field, $\Delta = (B-B_c)/B_c$. The threshold condition is determined by the balance of the growth rate $\gamma_k^e \cong \nu_e\omega_{*e}(k_\perp^2\rho^2)/k_\parallel^2 v_e^2$ arising from the cross-field ion inertia compared with the damping $\gamma_k^i \cong -\mu_o \nu_i (k_\perp \rho_i)^4$ from the ion viscosity where $\mu_0 \simeq 0.3$. In the Q-machine experiment the electron temperature gradient is negligible, $\eta_e = 0$. For $B_1 > B > B_c$ the growth rate exceeds the damping for a single low order azimuthal mode m_o, ω_o. At second order in the wave amplitude the harmonic $2\ m_o$, $2\ \omega_o$, which is not a normal mode due to the wave dispersion, absorbs energy from the fundamental by the nonlinear coupling to the strong viscous damping at $2m_o$. There is also a small change in the background density profile as shown experimentally in Fig. 2 a, but this change in the density is subdominant to the harmonic damping. The calculations are given by Hinton and Horton (1971), Monticello and Simon (1974) and Nishikawa et al. (1978). The results yield formulas for the amplitude a_k, the frequency $\omega_{\underset{\sim}{k}}^{n\ell}$ of the three dimensional drift mode $\tilde{n}_e/n_e = a_{\underset{\sim}{k}} \sin(k_x x)\sin(k_z z)\cos(k_y y - \omega_{\underset{\sim}{k}}^{n\ell} t)$ and the anomalous flux. The calculations yield amplitudes, frequencies and fluxes that are consistent with the experiments. This agreement in the amplitude is in contrast to the excessively large amplitudes obtained in a simpler one dimensional mode coupling theory by Stix (1969) in which limit cycles are found for drift waves of the form $\tilde{n}_e/n_e = F(k_y y + k_z z - \omega t)$.

2.6 Drift Wave Solitons

There are simple, nondissipative models for coherent large amplitude periodic traveling drift waves and localized drift wave solitons. These models must take the electron response as adiabatic $n_e = N_o \exp(e\varphi/T_e)$, and thus give no net transport.

The one dimensional drift wave equation contains the hydrodynamic type of wave steepening term $\eta_e \varphi \partial\varphi/\partial y$ as pointed out by Tasso (1967). Including the finite ion inertia produces wave dispersion $\omega_k = \omega_{*e}(1 - k_y^2\rho^2)$ which allows a soliton drift wave solution to develop as shown by Oraevskii et al. (1969). Petviashivili (1977) gives the one dimensional drift wave soliton as

$$\Phi = \frac{e\varphi(xt)}{T_e} = - \frac{3uA^2}{\eta_e v_{de}} \operatorname{sech}^2\left[\frac{A}{2\rho}(y + \kappa z - ut)\right] \tag{40}$$

where A is the real constant related to azimuthal speed u through $A^2 = (u^2 - uv_{de} - \kappa^2 c_s^2)/u^2$.

The drift wave soliton (40) is a solution of the model equations

$$\begin{aligned}(1 - \rho^2\nabla_\perp^2)\frac{\partial\Phi}{\partial t} + v_{de}\frac{\partial\Phi}{\partial y} - \eta_e v_{de}\Phi\frac{\partial\Phi}{\partial y} + \frac{\partial v_\parallel}{\partial z} &= 0 \\ \frac{\partial v_\parallel}{\partial t} &= - c_s^2\frac{\partial\Phi}{\partial z} \ .\end{aligned} \tag{41}$$

Petviashivili shows that the one-dimensional drift wave soliton is unstable to a finite k_x filamentation instability. More recently two-dimensional solitons and their interactions are investigated by Manakov et al. (1977) and Petviashivili (1978).

2.7 Quasilinear Limit of the Anomalous Flux

The anomalous flux depends on the particle distributions and the fluctuations spectrum. In the presence of a low level of drift wave turbulence the collision frequency, although low compared to the wave frequency, is still sufficient to maintain the local Maxwell-Boltzmann velocity distribution. The critical collision frequency $\nu_e = \nu_*$ above which the velocity distribution is collisionally dominated is determined by the balance of the wave induced parallel velocity diffusion given by $D_\parallel = \pi(e/m_e)^2 \sum_{\underset{\sim}{k}} |E_\parallel(k)|^2 \delta(\omega_{\underset{\sim}{k}} - k_\parallel v_\parallel)$ and the collisional diffusion due to $\nu_e = 2\pi n_e e^4 Z_{eff} \ln \Lambda / m_e^2 v_e^3$. In analyzing the $v_\parallel$-diffusion, it must be noted that the resonant electrons which determine the growth rate have $v_\perp \sim v_e = (2T_e/m_e)^{1/2}$ and $|v_\parallel| \leq v_c \equiv \max(\omega_{\underset{\sim}{k}}/k_\parallel) < v_e$. The resonant electrons are scattered in pitch angle out of this sector of velocity space at the rate $\nu_{eff} = \nu_e (v_e/v_c)^2$. When the competition between the quasilinear diffusion and the collisional restoration of the Maxwell-Boltzmann is expressed in terms of the fractional turbulence level W/nT_e given in Eq. (21) and the maximum parallel velocity v_c, the critical collision frequency is

$$\nu_* = \frac{c_s}{r_n}\left(\frac{v_e}{v_c}\right)^2\left(\frac{W}{nT_e}\right) \tag{42}$$

as shown by Galeev and Rudakov (1963) in the first quasi-linear analysis of drift wave turbulence.

In the regime $\nu_e > \nu_*$ the growth rate $\gamma_{\underset{\sim}{k}}$ and transport are determined by the local Maxwell-Boltzmann distribution as analyzed in Secs. 2.1 and 2.2. At lower collision frequencies $\nu_e < \nu_*$ the background electron distribution $f_e(x, v_\perp, v_\parallel, t)$ changes form under the influence of the turbulence.

Quasilinear theory, as developed by Galeev and Rudakov (1963) and Sagdeev and Galeev (1969), for the change in the background distribution function $f_j(x, v_\perp, v_\parallel, t)$ gives

$$\frac{\partial f_j}{\partial t} = \pi \left(\frac{e_j}{m_j}\right)^2 \sum_{\underset{\sim}{k}} \left(k_\parallel \frac{\partial}{\partial v_\parallel} + \frac{k_y}{\omega_{cj}} \frac{\partial}{\partial x}\right) |J_0 \varphi_{\underset{\sim}{k}}|^2 \delta(\omega_k - k_\parallel v_\parallel) \left(k_\parallel \frac{\partial f_j}{\partial v_\parallel} + \frac{k_y}{\omega_{cj}} \frac{\partial f_j}{\partial x}\right) \quad (43)$$

where

$$\frac{d|\varphi_{\underset{\sim}{k}}|^2}{dt} = 2\gamma_{\underset{\sim}{k}} |\varphi_{\underset{\sim}{k}}|^2$$

with $J_0 = J_0(k_\perp v_\perp / \omega_{cj})$. The dispersion relation for the unstable part of the spectrum in the resonant velocity interval $c_s \lesssim v_\parallel = \omega_{\underset{\sim}{k}}/k_\parallel \leqslant v_c$ yields

$$\gamma_{\underset{\sim}{k}}(t) = \frac{\pi \omega_k^2 T_e}{m_e |k_\parallel| \omega_{*e} \Gamma_o} \left(k_\parallel \frac{\partial F_e}{\partial v_\parallel} + \frac{k_y}{\omega_{ce}} \frac{\partial F_e}{\partial x}\right) \quad (44)$$

where

$$F_e(x,v_\parallel,t) = \int d\underset{\sim}{v}\,\delta(v_\parallel - \omega_{\underset{\sim}{k}}/k_\parallel) f_e(x,v_\perp,v_\parallel,t). \tag{45}$$

Here, $\omega_{\underset{\sim}{k}} = \omega_{*e}\Gamma_o/[1 + \tau(1 - \Gamma_o)]$ where $\Gamma_o = \langle J_0^2\rangle = I_o(b)\exp(-b)$, $b = (k_\perp\rho_i)^2$ and $\tau = T_e/T_i$. The quasilinear equations (43) - (45) apply when the width of the parallel phase velocity spectrum $\Delta(\omega_{\underset{\sim}{k}}/k_\parallel) \simeq v_c$ is greater than the mean trapping velocity $v_{tr} = v_e(W/nT_e)^{\frac{1}{4}}$, which is well satisfied when $W/nT_e \ll (v_c/v_e)^4$.

The maximum parallel phase velocity in the spectrum depends on the particular system. In the infinite slab without shear $v_c = v_A = v_e(m_e/\beta m_i)^{\frac{1}{2}} \cong c_s/\beta^{\frac{1}{2}}$ for $\beta > m_e/m_i$ and $v_c \cong v_e/2$ for $\beta < m_e/m_i$. In the sheared slab $v_e = c_s(L_s/r_n)^{\frac{1}{2}}$ for $\beta < r_n/L_s$ and $v_c = c_s/\beta^{\frac{1}{2}}$ for $\beta > r_n/L_s$ as shown by Gladd and Horton (1973). For a finite length L_o, shearless system, $v_c = \max(\omega_{\underset{\sim}{k}})L_o = c_s(L_o/r_n) < v_e$.

As the turbulence level develops the electrons diffuse in parallel velocity more rapidly than in radial position. The $v_\parallel$-slope of the distribution evolves to the "plateau" where

$$\frac{\omega_k}{v_\parallel}\frac{\partial f_e^{pl}}{\partial v_\parallel} + \frac{k_y}{\omega_{ce}}\frac{\partial f_e^{pl}}{\partial x} = 0 \quad \text{for} \quad |v_\parallel| \leqslant v_c \,. \tag{46}$$

For $\omega_{\underset{\sim}{k}} = \omega_{*e}$ and $d\,T_e/dx = 0$ the marginal distribution f_e^{pl} is the Maxwell-Boltzmann distribution $F_e^M(v)$. With $\omega_{\underset{\sim}{k}} < \omega_{*e}$ the plateau distribution develops a steeper $v_\parallel$-slope that arises from the cooling in parallel kinetic energy of the electrons that emit the waves. The cooling of the $v_\parallel$-distribution is a prominent feature in the particle simulations of Lee et al. (1978).

The extent to which the $v_\parallel$-distribution is distorted from $F_e^M(v_\parallel)$ by the waves increases strongly with $k_\perp \rho_i$. From the nonlinear transformation of the wave spectrum analyzed in Sec. 3 it is found that energy in the short waves $k_\perp \rho_i = (m_i \beta/m_e)^{\frac{1}{2}} > 1$ is strongly transformed to the long wave region $k_\perp \rho_i \lesssim 1$. Here, we restrict consideration to spectra $|\varphi_{\underset{\sim}{k}}|^2$ that are peaked at $\bar{k}_\perp$ such that $\bar{k}_\perp \rho_i \leqslant 1$. For such wave spectra the quasilinear plateau obtained from Eqs. (46) is

$$f_e^{pl} = F_e^M(v_\perp)\left(\frac{m_e}{2\pi T_e}\right)^{\frac{1}{2}}\left\{1 - \frac{m_e v_\parallel^2}{2T_e} - \frac{m_e(v_\parallel^2 - v_c^2)}{2T_e}\left[(\bar{k}_\perp \rho)^2 + \eta_e\left(\frac{m_e v^2}{2T_e} - \frac{3}{2}\right)\right]\right\} \tag{47}$$

for $|v_\parallel| \leqslant v_c$ and $f_e^{pl} = F_e^M(v)$ for $|v_\parallel| > v_e$. The parallel kinetic energy released in the evolution from F_e^M to f_e^{pl} is

$$\int \frac{1}{2} m_e v_\parallel^2 (F_e^M - f_e^{pl})\,dv \simeq n_e\left(\frac{v_c}{v_e}\right)\left(\frac{m_e v_c^2}{2T_e}\right)^2\left(\frac{\gamma_{\bar{k}}}{\omega_{\bar{k}}}\right) \tag{48}$$

where $n_r = n_e(v_c/v_e)$ is the density of resonant electrons with parallel kinetic energy $\frac{1}{2} m_e v_c^2$. In the plateau the turbulent wave energy density is given by $W^{pl}/nT_e \cong (v_c/v_e)^5(\gamma_{\overline{k}}/\omega_{\overline{k}})$. The diffusion to the plateau occurs at constant $\xi = x - \overline{k}_y v_\parallel^2/2\omega_{ce}\omega_{\overline{k}}$ which bounds the displacement of the resonant electrons by

$$\delta x = r_n\left(\frac{m_e v_c^2}{2T_e}\right) \tag{49}$$

as shown by Galeev and Rudakov (1963). To the extent that the plateau is achieved the anomalous diffusion ceases in the collisionless plasma.

Clearly, it is now essential to take into account the fact that the particle collisions act to disrupt the plateau. Since the resonant electrons lie in the velocity space sector restricted by $|v_\parallel| < v_c$ and $v_\perp \sim v_e$, particle collisions act predominantly to pitch-angle scatter the electrons as described by the Lorentz operator

$$Cf_e = \nu_e(v)\partial_\lambda(1 - \lambda^2)\partial_\lambda f_e \tag{50}$$

where $\lambda = v_\parallel/v = \cos\theta$ and $\nu_e(v) = \nu_e\hat{\nu}(v/v_e)$ is the energy dependent collision frequency. Introducing the quasilinear phase space flux

$$J_e(x,v_\perp,v_\parallel,t) = \pi(\frac{e}{m_e})^2 \sum_{\underset{\sim}{k}} k_\parallel |\varphi_{\underset{\sim}{k}}|^2 \delta(\omega_{\underset{\sim}{k}} - k_\parallel v_\parallel)$$

$$[k_\parallel \frac{\partial f_e}{\partial v_\parallel} + \frac{k_y}{\omega_{ce}} \frac{\partial f_e}{\partial x}] , \tag{51}$$

the quasilinear or turbulent collision operator for fluctuation spectra with $\bar{k}_\perp \rho_i \leqslant 1$ is

$$\hat{D}_{q\ell} f_e = \frac{\partial J_e}{\partial v_\parallel} + \frac{v_\parallel}{\omega_{ce} v_{de}} \frac{\partial J_e}{\partial x} \tag{52}$$

and the complete kinetic equation is

$$\frac{\partial f_e}{\partial t} = \hat{D}_{q\ell} f_e + \hat{C} f_e . \tag{53}$$

The $v_\parallel$-divergence in Eq. (52) dominates the x-divergence by the factor $(v_e/v_c)^2$ so that f_e evolves rapidly to the balance

$$\frac{\partial J_e(f_e)}{\partial v_\parallel} = - \nu_e(v) \partial_\lambda (1 - \lambda^2) \partial_\lambda f_e$$

$$\cong - v^2 \nu_e(v) \frac{\partial^2}{\partial v_\parallel^2} [f_e - F_e^M(v)] . \tag{54}$$

The radial diffusion is determined by the subdominant terms in the kinetic equation (53) from which the density and the energy moments give the transport equations (22) and (23) with the formulas

$$\Gamma_e = -\int d\underset{\sim}{v} \frac{v_\parallel}{\omega_{ce} v_{de}} J_e(f_e) \tag{55}$$

$$Q_e = -\int d\underset{\sim}{v} \frac{v_{\|}}{\omega_{ce} v_{de}} \frac{1}{2} m_e v^2 J_e(f_e) \tag{56}$$

The transfer of energy from the electrons to the drift wave is

$$\dot{W}_e = \int d\underset{\sim}{v}\, m_e v_{\|} J_e(f_e), \tag{57}$$

as first given by Gladd and Horton (1973).

The two mechanisms determining f_e in Eq. (53) compare as ν_* to ν_e. For $\nu_e > \nu_*$ the distribution is the Maxwell-Boltzmann with a ν_*/ν_e correction. For $\nu_* > \nu_e$ the distribution is

$$f_e = f_e^{pl} + f_e^{(1)}$$

where

$$J_e(f_e) = \hat{J}_e f_e^{(1)} = -v^2 \nu_e(v) \frac{\partial}{\partial v_{\|}} [f_e^{pl} - F_e^M(v)] \;. \tag{58}$$

Galeev and Sagdeev (1973) point out that by taking the $\underset{\sim}{v}_{\perp}$ integral of Eq. (58), using Eq. (46) to write

$$\partial f^{pl}/\partial v_{\|} = (\bar{k}_y/\bar{k}_{\|}\omega_{ce})(\partial f^{pl}/\partial x) \cong (\bar{k}_y/\bar{k}_{\|}\omega_{ce})(\partial F_e^M/\partial x),$$

and using Eqs. (44) and (45), the balance expressed by Eq. (58) determines the relation between $\gamma_{\underset{\sim}{\bar{k}}}(f_e)$ and $\gamma_{\underset{\sim}{\bar{k}}}(F_e^M) = \gamma_{\underset{\sim}{\bar{k}}}^M$. To within a factor of order unity the relationship is expressed in terms of ν_*/ν_e as

$$\gamma_k = \frac{\gamma^M_{\underset{\sim}{k}}}{1 + \nu_*/\nu_e} \quad .$$

Although Eq. (58) does not determine $f_e^{(1)}$ and $|\varphi_{\underset{\sim}{k}}|^2$ individually, it does determine the combination, namely J_e, required to evaluate the anomalous flows. From Eqs. (47) and (58) the first order flux is shown to be

$$J_e^{(1)} = \frac{2\ \nu_e(v)}{\pi^{\frac{1}{2}}} \left(\frac{v_{\|} v^2}{v_e^3}\right) F_e^M [(\bar{k}_\perp \rho)^2 + \eta_e (\frac{m_e v^2}{2T_e} - \frac{3}{2})] \tag{59}$$

for $|v_{\|}| \leqslant v_c$ and $J_e = 0$ elsewhere.

The anomalous flows calculated from the first order phase space flux $J_e^{(1)}$ using Eqs. (55), (56) and (57) are

$$\Gamma_e = - \frac{4\nu_e n_e v_c^3}{3\omega_{ce} v_{de} v_e} [C_0 (\bar{k}_\perp \rho)^2 - C_1 \eta_e] \tag{60}$$

$$q_e = - \frac{4\nu_e n_e T_e v_c^3}{3\omega_{ce} v_{de} v_e} [-C_1 (\bar{k}_\perp \rho)^2 + C_2 \eta_e] \tag{61}$$

$$\dot{W}_e = \frac{4\nu_e n_e T_e}{3} \left(\frac{v_c}{v_e}\right)^3 [C_0 (\bar{k}_\perp \rho)^2 - C_1 \eta_e] \tag{62}$$

where q_e is the thermal conduction $q_e = Q_e - \frac{3}{2} T_e \Gamma_e$ and

$$C_n = \frac{1}{\pi^{\frac{1}{2}}} \int d\underset{\sim}{v}_\perp F_e^M (v_\perp) \left(\frac{v_\perp}{v_e}\right)^2 \hat{\nu}(v_\perp) \left(\frac{3}{2} - \frac{m_e v_\perp^2}{2T_e}\right)^n \quad . \tag{63}$$

For $Z_{eff} \gg 1$ the velocity dependence of the collision frequency is $\hat{\nu}(u) \cong 1/u^3$ and the integrals reduce to $C_0 = C_1 = 1$ and $C_2 = 2/3$. At the high collision frequency limit of formulas (60) - (62) where $\nu_e = \nu_*$ the flux becomes $\Gamma \sim n_e c_s (W/nT_e)(\bar{k}_y \rho)\delta_{\bar{k}}$ where $\delta_{\bar{k}} \sim (v_c/v_e)[(\bar{k}_\perp \rho)^2 - \eta_e]$ so that the formulas join continuously with those in Sec. 2.1 and 2.2 where $\nu_e > \nu_*$.

The anomalous flows written in terms of effective transport coefficients are

$$\Gamma_e = -D \frac{dn_e}{dx}$$

$$q_e = -\chi_e \frac{dT_e}{dx} \qquad (64)$$

$$\dot{W}_e = \nu_{eff} n_e T_e$$

where

$$D = D_0[C_0(\bar{k}_\perp \rho)^2 - C_1 \eta_e] \qquad (65)$$

$$\chi_e = n_e D_0 [C_2 - C_1 (\bar{k}_\perp \rho)^2/\eta_e] \qquad (66)$$

with

$$D_0 = \frac{4}{3} \nu_e r_n^2 (\frac{v_c}{v_e})^3. \qquad (67)$$

The integrals C_0 and C_2 are positive definite, whereas C_1 is indefinite.

In the regime of reversed temperature and density gradients where $\eta_e < 0$ the particles flow down the density gradient, and the thermal energy flows down the temperature gradient. The net energy flux $Q_e = \frac{3}{2} T_e \Gamma + q_e$ nearly vanishes due to the cancelling contributions, as noted by Liu et al. (1972).

The turbulent diffusion is characterized by two regimes. For $\nu_e < \nu_*$ the quasilinear diffusion from the turbulence actually limits the anomalous flux to a value determined ~~by the collisional destruction of the plateau.~~

Between collision periods the resonant electrons diffuse the distance δx given by Eq. (49). The resonant electrons of density $n_r = n_e(v_c/v_e)$ are scattered by collisions at the rate $\nu_e(v_e/v_c)^2$. Thus, the diffusion rate in this regime is

$$D_0 \cong (\frac{n_r}{n_e}) \nu_e (\frac{v_e}{v_c})^2 (\delta x)^2 = \nu_e r_n^2 (\frac{v_c}{v_e})^3$$

as pointed out by Galeev and Sagdeev (1973). For $\nu_e > \nu_*$ the collisions maintain the Maxwell-Boltzmann slope to the resonant electrons. Here, the diffusion of the resonant electrons occurs through the turbulent velocity $\langle v_E^2 \rangle$ with a correlation time $\gamma_{\bar{k}}^{(M)}/\omega_{\bar{k}}^{2}$ which leads to

$$D_1 = (\frac{n_r}{n_e})\langle v_E^2 \rangle \tau_c = \frac{\rho}{r_n} (\frac{cT_e}{eB}) \frac{v_c}{v_e} \frac{W}{W_f}$$

where W_f is the maximum turbulent energy density given in Sec. 3 from nonlinear saturation, $W_f = n_e T_e (\rho/r_n)^2$ and W is the wave energy density.

The two regimes are summarized in Fig. 6. For nonlinear saturation in the strong drift wave turbulence regime where $W = W_f = nT_e(\rho/r_n)^2$, the anomalous diffusion coefficient varies as

$$D = \begin{cases} \nu_e r_n^2 (\frac{v_c}{v_e})^3 & \text{for } \nu_e \leqslant \nu_* = \frac{c_s}{r_n}(\frac{\rho}{r_n})^2 (\frac{v_e}{v_c})^2 \\ \\ \frac{\rho}{r_n}(\frac{cT_e}{eB})(\frac{v_c}{v_e}) & \text{for } \nu_e \geqslant \nu_* = \frac{c_s}{r_n}(\frac{\rho}{r_n})^2 (\frac{v_e}{v_c})^2 \end{cases} \qquad (68)$$

where the different values of v_c/v_e in terms of the plasma pressure β and the shear $S = r_n/L_s$ are given in the paragraph following Eq. (45). From Eqs. (65) and (66) we observe that

$$\frac{\chi_e}{nD} = \frac{C_2 - C_1 (\bar{k}\rho)^2/\eta_e}{C_0 (\bar{k}\rho)^2 - C_1 \eta_e} \gtrsim 1 .$$

Depending on the system parameters the electron thermal conduction due to χ_e may be several times greater than the convective particle transport from drift waves.

3. Drift Wave Fluctuation Spectra

The rate of anomalous transport depends directly on the magnitude and the spectral distribution of the electric potential fluctuations $I(\underset{\sim}{k}\omega)$. The fluctuation spectrum is influenced by many complicated processes in the plasma and, consequently, cannot be given precisely except in certain exceptional cases. On the other hand, the general behavior of the spectrum has been extensively investigated with the result that certain features of the fluctuations are well-known. We now consider the physical processes that determine the potential fluctuation spectrum.

The drift wave dispersion relation yields a growth rate $\gamma(k_y, k_\perp, k_\parallel)$ for a broad range of wavenumbers once the system parameters are past their threshold values. The quasilinear decorrelation rate for the electron interactions is $\Delta k_\parallel v_e$ and for the ion interactions is $\Delta\omega_{\underset{\sim}{k}}$ where $\Delta\underset{\sim}{k}$ and $\Delta\omega_{\underset{\sim}{k}}$ are the widths of the unstable spectrum. The correlation times are such that $\Delta k_\parallel v_e \gg \Delta\omega_{\underset{\sim}{k}} \gtrsim \gamma_{\underset{\sim}{k}}$. For macroscopic gradients that are weak over the scale of the ion gyroradius the energy density due to the diamagnetic currents which produce the drift instability is a small fraction of the thermal energy density, $W_d = \frac{1}{2} n_i m_i (v_{di} - v_{de})^2 \approx (\rho/r_n)^2 nT_e$. The maximum energy W_f available to drive the collective field energy density W due to low frequency fluctuations with an average radial width $\langle \Delta x^2 \rangle \approx \langle k_x^2 \rangle^{-1}$ is bounded by the thermodynamic free energy of the system $W/nT_e \leqslant W_f/nT_e \cong (\Delta x/r_n)^2$, or $(\Delta x/r_T)^2$ for $r_T < r_n$, Fowler (1968). These circumstances indicate that the general method for the problem of nonlinear drift waves is the statistical theory of plasma turbulence which takes into account the large number of weakly interacting fluctuations.

The general turbulence theory consists of expanding the nonlinear equations in powers of W/nT. The truncated weak turbulence expansion becomes inadequate in the state of fully developed drift wave turbulence at the level where the $\underset{\sim}{E} \times \underset{\sim}{B}$ mixing of the plasma produces nonlinear decorrelations $\langle (\underset{\sim}{k} \cdot \underset{\sim}{v}_E)^2 \rangle^{\frac{1}{2}}$ at a rate comparable to the quasilinear decorrelations rates. At the turbulence level given by the free energy bound $W \lesssim W_f$ the random Doppler shifts from the $\underset{\sim}{E} \times \underset{\sim}{B}$ motion are comparable to ω_{*e}. As a result in fully developed drift wave turbulence it is necessary to sum high order secular contributions in the interaction propagators. These renormalized drift wave turbulence theories are developed in Kadomtsev (1963), (1965), Dupree (1967), Galeev (1967) and Horton and Choi (1979). The renormalized propagators describe the decorrelation of the collisionless resonance $\omega \cong \underset{\sim}{k} \cdot \underset{\sim}{v}$ from the stochastic $\underset{\sim}{E} \times \underset{\sim}{B}$ drifts produced by the turbulent collective electric field.

3.1 Qualitative Picture of Drift Wave Turbulence

Before proceeding to the systematic nonlinear theory of the drift wave instability let us follow the early analysis by Oraevskii and Sagdeev (1963) that uses weak turbulence theory to give a qualitative picture of drift wave turbulence and the associated transport. First, it is observed that the linearly unstable region of $\underset{\sim}{k}$-space is limited to domain I in Fig. 7. The larger $k_\| r_n$ region in domain II is a region of strong collisionless ion damping. The smaller $k_\| r_n$ region in domain III is a region of collisionless electron damping. Above a critical wave amplitude for the drift wave oscillations there is a strong coupling of the active k-scales to the damped k-scales to balance the input of wave energy $2\gamma_{\underset{\sim}{k}} W_{\underset{\sim}{k}}$.

The strongest wave input occurs for $k_\perp \rho_i \gtrsim 1$ and $c_s \ll \omega_k/k_\| \lesssim v_A$ where

$$\omega_{\underset{\sim}{k}} + i\gamma_{\underset{\sim}{k}} \cong \frac{\omega_{*e}}{k_\perp \rho_i}\left(1 + i\,\frac{\omega_{*e}}{|k_\||v_e}\right) \tag{69}$$

with $\gamma_{\underset{\sim}{k}}$ driven by the square of the density gradient and reaching the maximum $\gamma_k \cong c_s/r_n$ when $\overline{k}_\| r_n \cong \frac{1}{2}\,(\beta/2)^{\frac{1}{2}}$ and $k_\perp \rho_i \sim (m_i \beta/m_e)^{\frac{1}{2}}$. An elementary estimate of the critical amplitude for nonlinear mode coupling in the ion dynamics to balance the linear growth γ_k is

$$\gamma_k \delta f_{ik}(\underset{\sim}{v}) \sim \frac{e}{m_i} E_\perp(k') \frac{\partial}{\partial v_\perp} \delta f_{ik-k'}(\underset{\sim}{v})$$

which occurs when $E_\perp(k) = -ik_\perp \varphi(k)$ is of order

$$\frac{e\varphi(k)}{T} \sim \frac{\gamma_{max}}{k_\perp v_i} \sim \frac{1}{k_\perp r_n} \quad . \tag{70}$$

From the phase shift formula $\delta_e(k) \sim \omega_{*e}/k_\parallel v_e \sim (k_y \rho)(m_e/m_i\beta)^{1/2}$, the approximate spectrum in Eq. (70) and the anomalous flux formula (17) the $k_\perp$-dependence of the spectral integral cancels, and the resulting diffusion coefficient is

$$D = \frac{\rho}{r_n} \left(\frac{cT_e}{eB}\right) \left(\frac{m_e}{m_i \beta_e}\right)^{1/2} \tag{71}$$

for $\beta_e > m_e/m_i$ and simply

$$D = \left(\frac{\rho}{r_n}\right) \left(\frac{cT_e}{eB}\right) \left(\frac{v_c}{v_e}\right)$$

for $\beta < m_e/m_i$. The more precise evaluations of the spectrum and diffusion given in Sec. 3.4 below show that formula (71) over estimates the diffusion by the factor $(m_i\beta/m_e)^{1/2}$.

3.2 Systematic Theory of Drift Wave Turbulence

The quantitative description of the nonlinear dynamics begins with a computation of the nonlinear charge densities from a small amplitude expansion of the Vlasov equation. The expansion is straightforward, as given by Sagdeev and Galeev (1969), and yields the nonlinear equation for φ_k

$$\varepsilon_k \varphi_k + \sum_{\substack{k_1+k_2\\=k}} \varepsilon^{(2)}_{k_1,k_2} \varphi_{k_1} \varphi_{k_2} + \sum_{\substack{k_1+k_2+k_3\\=k}} \varepsilon^{(3)}_{k_1,k_2,k_3} \varphi_{k_1} \varphi_{k_2} \varphi_{k_3} + \ldots = 0 \tag{72}$$

where k stands for the Fourier space variables $\underset{\sim}{k}\omega$.

For the background particle distribution $f_j(\epsilon, X)$ where $\epsilon = \frac{1}{2} v^2$, $X = x + v_y/\Omega$, with negligible velocity space anisotropy, the linear and nonlinear dielectric functions are

$$\varepsilon_k = 1 - \sum_j \frac{4\pi e_j^2}{k^2 m_j} \int d\underset{\sim}{v} \left[\frac{\partial f_j}{\partial \epsilon} - \frac{J_0^2\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right)\left(\omega \frac{\partial f_j}{\partial \epsilon} + \frac{k_y}{\omega_{cj}} \frac{\partial f_j}{\partial x}\right)}{\omega - k_\parallel v_\parallel + i\nu} \right] \tag{73}$$

$$k^2 \varepsilon^{(2)}_{k',k''} = -i \sum_j \frac{2\pi e_j^3 (\underset{\sim}{k}' \times \underset{\sim}{k}'' \cdot \hat{z})}{m_j^2 \omega_{cj}} \int d\underset{\sim}{v} \frac{J_0\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) J_0\left(\frac{k_\perp' v_\perp}{\omega_{cj}}\right) J_0\left(\frac{k_\perp'' v_\perp}{\omega_{cj}}\right)}{\omega - k_\parallel v_\parallel + i\nu}$$

$k' + k'' = k$

$$\times \left[\frac{\omega' \frac{\partial f_j}{\partial \epsilon} + \frac{k_y'}{\omega_{cj}} \frac{\partial f_j}{\partial x}}{\omega' - k_\parallel' v_\parallel + i\nu} - \frac{\omega'' \frac{\partial f_i}{\partial \epsilon} + \frac{k_y''}{\omega_{cj}} \frac{\partial f_j}{\partial x}}{\omega'' - k_\parallel'' v_\parallel + i\nu} \right] \tag{74}$$

$$k^2 \varepsilon^{(3)}_{k',k'',k'''} = \sum_{k'+k''+k'''=k} \frac{2\pi e_j^4 (\mathbf{k}' \times \mathbf{k}'' \cdot \hat{z})[\mathbf{k}''' \times (\mathbf{k}' + \mathbf{k}'') \cdot \hat{z}]}{m_j^3 \omega_{cj}^2}$$

$$\int d\mathbf{v} \, \frac{J_0\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) J_0\left(\frac{k'_\perp v_\perp}{\omega_{cj}}\right) J_0\left(\frac{k''_\perp v_\perp}{\omega_{cj}}\right) J_0\left(\frac{k'''_\perp v_\perp}{\omega_{cj}}\right)}{(\omega - k_\parallel v_\parallel + i\nu)(\omega' + \omega'' - (k'_\parallel + k''_\parallel) v_\parallel + i\nu)}$$

$$\times \left[\frac{\omega' \frac{\partial f_j}{\partial \epsilon} + \frac{k'_y}{\omega_{cj}} \frac{\partial f_j}{\partial x}}{\omega' - k'_\parallel v_\parallel + i\nu} - \frac{\omega'' \frac{\partial f_j}{\partial \epsilon} + \frac{k''_y}{\omega_{cj}} \frac{\partial f_j}{\partial x}}{\omega'' - k''_\parallel v_\parallel + i\nu} \right] . \tag{75}$$

In the collisionless weak turbulence regime the $i\nu$ in the particle propagator is the positive infinitesimal arising from the time history integral which guarantees causal, analytic response functions. As the turbulence increases to the level where $\langle (\mathbf{k} \cdot \mathbf{v}_E)^2 \rangle \cong \omega_{\mathbf{k}}^2$ there is a nonlinear or finite amplitude decorrelation of the wave-particle resonance. The propagator then contains the non-Markovian turbulent collision operator $i\nu \rightarrow i\nu_{\mathbf{k}\omega}$ given by renormalized turbulence theory. Horton and Choi (1979) obtain the formula for $i\nu_{\mathbf{k}\omega}$ by a selective summation of high order secular contributions to the free particle propagator which gives

$$i\nu_{\mathbf{k}\omega}(v_\parallel, v_\perp) = -\frac{c^2}{B^2} \sum_{k'} \frac{(\mathbf{k} \times \mathbf{k}' \cdot \hat{z})^2 |\varphi_{k'}|^2 J_0^2\left(\frac{k'_\perp v_\perp}{\omega_{cj}}\right)}{\omega - \omega' - (k_\parallel - k'_\parallel) v_\parallel + i\nu_{k-k'}} . \tag{76}$$

In the region where $\mathbf{k}\omega$ is small compared to the peak $\overline{\mathbf{k}\omega}$ of the fluctuation spectrum, the turbulent collision operator reduces to $i\nu_{\underset{\sim}{k}\omega} \rightarrow \underset{\sim}{k} \cdot \underset{\approx}{D}(\underset{\sim}{v}) \cdot \underset{\sim}{k}$ where $\underset{\approx}{D}(\underset{\sim}{v})$ is the Markovian turbulent diffusion coefficient

$$\underset{\approx}{D}(\underset{\sim}{v}) = -\frac{c^2}{B^2}\sum_{k'} \frac{(\underset{\sim}{k}' \times \hat{z})(\underset{\sim}{k}' \times \hat{z})|\varphi_{k'}|^2 J_0^2\left(\frac{k_\perp' v_\perp}{\omega_{cj}}\right)}{\omega' - k_\parallel' v_\parallel + i\, \underset{\sim}{k}' \cdot \underset{\approx}{D}(\underset{\sim}{v}) \cdot \underset{\sim}{k}'} \tag{77}$$

given by Dupree (1967). Kadomtsev (1963) introduces still a different turbulent broadening operator $i\nu_{\underset{\sim}{k}\omega} \rightarrow \eta_{\underset{\sim}{k}\omega}(v_\parallel)$ given in Sec. 3.4. Kadomtsev (1963) points out that the renormalization of the propagator does not appreciably alter the value of the nonlinear transfer rates in the mode coupling equations. Rather, the role of the renormalization is to describe the finite correlation time $1/\nu_{\underset{\sim}{k}\omega}(\underset{\sim}{v})$ that limits the resonance period of the particle $\underset{\sim}{v}$ with the fluctuation $\underset{\sim}{k}\omega$. Although the turbulent energy density W is quite small compared with the thermal energy density, the shortening of the interaction time with increasing field amplitudes is an important nonlinear process in collisionless drift wave turbulence.

As explained earlier, in the theories of drift wave turbulence it is argued that the nonlinear interactions produce sufficient mixing of the modes that a statistical description of the $\varphi_{\underset{\sim}{k}\omega}$ fields is justified (Terry and Horton, 1981). The degree of randomness of the fields is assumed sufficient to allow the fourth and higher order cumulants to be neglected. The third order correlation $\langle\varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\rangle$ is computed by perturbation theroy. The equation for the spectral distribution I_k, Eq. (9), of the two point correlation function of the potential fluctuations that follows from the statistical description of the mode couplings in Eq. (72) is

$$\varepsilon_k I_k - I_k \sum_{k'} I_{k'} \left[\frac{4\varepsilon^{(2)}_{k',k-k'}\varepsilon^{(2)}_{-k',k}}{\varepsilon_{k-k'}} - 2\varepsilon^{(3)}_{k,k',-k'} \right]$$

$$= \sum_{k'} \frac{2|\varepsilon^{(2)}(k',k-k')|^2}{\varepsilon^*_k} I_{k'} I_{k-k'} . \qquad (78)$$

This well known result is derived, for example, in Kadomtsev (1965) and in Sagdeev and Galeev (1969). We now proceed to the principal results that follow from Eqs. (72) - (78) for drift wave turbulence. The results given here are obtained in different limiting approximations to this nonlinear system of equations since no general solutions are known.

First two general limiting forms of Eq. (78) are to be noted. In the theory of weak turbulence Eq. (78) is reduced to a balance equation for the $\underset{\sim}{k}$-spectrum by observing that for sufficiently small amplitudes the lowest order solution is $I(\underset{\sim}{k}\omega) \cong I(\underset{\sim}{k},t)\delta(\omega-\omega_{\underset{\sim}{k}})$, where the time dependence of $I(\underset{\sim}{k},t)$ describes the slow variation induced by the nonlinear couplings. The imaginary part of the Eq. (78) determines $dI(\underset{\sim}{k},t)/dt$ and when calculated reveals two physical processes. The processes are the three wave couplings and the nonlinear collisionless damping, often called induced wave scattering, [Chapter IV.2].

The second point of view of Eq.(78) is that first advocated by Kadomtsev (1963, 1965). Returning to the problem of the truncation of the correlations that is made in going from Eq. (72) to Eq. (78), we may view Eq. (78) as containing the leading terms in a small W/nT expansion for two coupled equations for the renormalized fluctuation propagator $\varepsilon_k^{n\ell}$ and the spectral distribution I_k. In this framework these equations are the plasma physics analog of the Direct Interaction Approximation of fluid turbulence introduced by Kraichnan (1965) as pointed out by Kadomtsev (1965). The relevance of this point of view becomes clear when the equations are analyzed in the fluid limit in Sec. 3.3. Following Kadomtsev (1965) and Tsytovich (1977) the coupled system of equations for the spectral distribution I_k and the renormalized response function $\varepsilon_k^{n\ell}$ are

$$\varepsilon_k^{n\ell} I_k = \frac{2}{\varepsilon_k^{n\ell *}} \sum_{\substack{k'+k'' \\ =k}} |\varepsilon^{(2)}_{k',k''}|^2 \, I_{k'} I_{k''} \tag{79}$$

and

$$\varepsilon_k^{n\ell} = \varepsilon_k - \sum_{k'} I_{k'} \left[\frac{4\varepsilon^{(2)}_{k',k-k'}\,\varepsilon^{(2)}_{-k',k}}{\varepsilon^{n\ell}_{k-k'}} - 2\varepsilon^{(3)}_{k,k',-k'} \right] . \tag{80}$$

In Navier-Stokes turbulence where $\varepsilon^{(3)}_{k_1,k_2,k_3} = 0$, the second equation is often written for the nonlinear response function $R_k = 1/\varepsilon_k^{n\ell}$.

$$\left[\varepsilon_k - 4\sum_{k'}\varepsilon^{(2)}_{k',k-k'}R_{k-k'}I_{k'}\varepsilon^{(2)}_{-k',k}\right]R_k = 1 \tag{81}$$

which is the fluctuation analog of the renormalized propagator equation for $g_k(\underset{\sim}{v})$. It is evident the Eq. (78) is the first iteration of the DIA equation (80) in which the approximation $R_{k''} \cong 1/\varepsilon_{k''}$ is made in the interaction integral.

3.3 Fluid Limit to Drift Wave Turbulence

Some particular results for the behavior of the nonlinear transfer and the form of the fluctuation spectrum are known in the fluid limit of drift waves.

The fluid limit for the drift wave problem is obtained directly from the nonlinear Vlasov response functions in the limit that $k_\perp \rho_i \ll 1$ and $k_\parallel v_i \ll \omega \ll k_\parallel v_e$. Alternatively, the problem may be formulated in terms of hydrodynamic equations derived in the drift ordering as given in Eqs. (19)-(21).

The hydrodynamic formulation is particularly appropriate when there are sufficient collisions to justify the use of collisional transport theory to determine the plasma flows parallel to the magnetic field, such as $\eta j_\parallel = E_\parallel + \nabla_\parallel p_e/en_e$ for parallel electron momentum balance. In the collisionless regime, the hydrodynamic approximation to the nonlinear coupling remains a useful approach which can be partially justified. In the collisionless description the electron resonances that produce the phase shift and the growth rate are retained with kinetic theory formulas.

In the fluid approximation to the nonlinear dielectric functions in Eqs. (73) - (75) we write that $k^2\lambda^2_{De}\varepsilon_k^{n\ell} \to \varepsilon_k^{n\ell}$ to remove the Debye length dependence of the theory in the long wavelength regime, $k^2\lambda^2_{De} \ll 1$, where the quasi-neutral approximation is well satisfied. The fluid limit of the Vlasov response functions gives

$$\varepsilon_k = 1 - i\delta_k^e - \frac{\omega_*}{\omega} + k_\perp^2\rho^2\left(1 - \frac{\omega_{*pi}}{\omega}\right) - \frac{k_\parallel^2 c_s^2}{\omega^2}\left(1 - \frac{\omega_{*pi}}{\omega}\right) \tag{82}$$

from Eq. (73) and

$$\begin{aligned}
\varepsilon^{(2)}_{k',k''} = \frac{i}{2}\frac{cT_e}{eB}\frac{\underset{\sim}{k}' \times \underset{\sim}{k}'' \cdot \hat{z}}{\omega' + \omega''} \Bigg\{ & \left[\frac{\omega_*(k'')}{\omega''} - \frac{\omega_*(k')}{\omega'}\right]_A \\
& + \frac{T_e}{m_i}\left[\frac{k_\parallel k_\parallel''}{\omega\omega''} + \frac{k_\parallel''^2}{\omega'^2} - \frac{k_\parallel k_\parallel'}{\omega\omega''} - \frac{k_\parallel'^2}{\omega'^2}\right]_B \\
& - \frac{\omega_{*pi}(k'')}{\omega''}\frac{T_e}{m_i}\left[\frac{k_\parallel^2}{\omega^2} + \frac{k_\parallel k_\parallel''}{\omega\omega''} + \frac{k_\parallel''^2}{\omega''^2}\right]_C \\
& + \frac{\omega_{*pi}(k')}{\omega'}\left[\frac{k_\parallel^2}{\omega^2} + \frac{k_\parallel k_\parallel'}{\omega\omega'} + \frac{k_\parallel'^2}{\omega'^2}\right]_C \Bigg\}
\end{aligned} \tag{83}$$

from Eq. (74). The third order dielectric function $\varepsilon^{(3)}(k_1,k_2,k_3)$ is also readily obtained from the small $k_\perp\rho_i$ and $k_\parallel v_i/\omega$ expansion of Eq. (75).

The subscripts A, B and C in Eq. (83) label the origin of that nonlinearity in the hydrodynamic equations. In drift wave fluctuations only, the ion behavior is hydrodynamic. The coupling terms denoted by A arise from the $\mathbf{E} \times \mathbf{B}$ convective nonlinearity and the nonlinear polarization drift. The terms denoted by B arise from the $\mathbf{E} \times \mathbf{B}$ convection of the parallel ion velocity and the terms C from the $\mathbf{E} \times \mathbf{B}$ convection of the ion pressure.

The A terms proportional to $(\omega_*''/\omega'' - \omega_*'/\omega')$ typically dominates for the $(k_\perp \rho)^2$ and η_e driven electron drift wave. The A term arises from the $\mathbf{E} \times \mathbf{B}$ convective nonlinearity $\nabla \cdot (n^{(1)} \mathbf{v}_E^{(1)}) \propto \mathbf{k}' \times \mathbf{k}'' \cdot \hat{z}[\omega_*''/\omega' - k_\perp''^2 \rho^2] \varphi_{k'} \varphi_{k''}$ and from the polarization drift nonlinearity in which $\nabla \cdot (n^{(0)} \mathbf{v}_E \cdot \nabla \mathbf{v}_E) \propto \mathbf{k}' \times \mathbf{k}_\perp'' \cdot \hat{z}(k_\perp''^2 \rho^2) \varphi_{k'} \varphi_{k''}$ where in both terms the $k_\perp^2$ contribution is due to the polarization drift. These polarization drift contributions cancel in obtaining the final term A in Eq. (83).

The simplest model for drift wave turbulence follows from considering only the $\mathbf{E}\times\mathbf{B}$ and polarization drift nonlinearities in the hydrodynamic equations corresponding to the A terms in Eq. (83). Within this model there is an instructive analogy between drift waves and two-dimensional fluid turbulence. Consider the limit in which $i\delta_k^e$, $k_\parallel^2 c_s^2/\omega^2$ and ω_{*pi}/ω are negligible in Eqs. (82) and (83) and return to Eq. (72) inverted from ω-space to t-space. The quadratic mode coupling equation that results is

$$\frac{d\varphi_{\underset{\sim}{k}}}{dt} = -i\omega_{\underset{\sim}{k}}\varphi_{\underset{\sim}{k}} + \frac{cT_e}{2eB}\sum_{\substack{\underset{\sim}{k}'+\underset{\sim}{k}''\\=\underset{\sim}{k}}} \frac{\underset{\sim}{k}'\times\underset{\sim}{k}''\cdot\hat{z}}{1+k_\perp^2\rho^2}[(k_\perp''\rho)^2 - (k_\perp'\rho)^2]\varphi_{\underset{\sim}{k}'}\varphi_{\underset{\sim}{k}''} \qquad (84)$$

where $\omega_k = \omega_{*e}/(1+k_\perp{}^2\rho^2)$. Equation (84) is derived by Hasegawa and Mima (1977 and 1978) from the pressureless ion hydrodynamic equations and the adiabatic electron approximation. The mode coupling element in Eq. (84) is a strongly increasing function of $k_\perp\rho$ and vanishes unless $\underset{\sim}{k}_\perp'$ and $\underset{\sim}{k}_\perp''$ are in different direction.

In terms of the space-time dependence of the electrostatic potential $\varphi(\underset{\sim}{r},t)$ the mode-coupling equation (84) is equivalent to the two-dimensional partial differential equation

$$(1-\rho^2\nabla_\perp^2)\,\frac{\partial\varphi(x,y,t)}{\partial t} = -v_{de}\frac{\partial\varphi}{\partial y} + \frac{cT_e}{eB}[\varphi,\rho^2\nabla_\perp^2\varphi] \qquad (85)$$

where the Poisson bracket $[\varphi,f]$ expresses the convective derivative

$$\underset{\sim}{v}_E\cdot\nabla f = -\frac{c}{B}\frac{\partial\varphi}{\partial y}\frac{\partial f}{\partial x} + \frac{c}{B}\frac{\partial\varphi}{\partial x}\frac{\partial f}{\partial y} \equiv \frac{c}{B}[\varphi,f] \quad .$$

Equation (85) can also be derived from $\nabla\cdot\underset{\sim}{j}_\perp + \nabla_\parallel j_\parallel = 0$, using the ion polarization current for $\underset{\sim}{j}_\perp$ and the collisionless electron conductivity $\sigma_\parallel = -in_e e^2(\omega-\omega_{ne})/k_\parallel^2 v_e^2$ for the parallel electron current. The analogy with two-dimensional incompressible, inviscid neutral fluid dynamics is evident when the adiabatic shielding of the electrons and the drift velocity v_{de} are neglected (equivalent to dropping $j_\parallel$). In this limit, Eq. (85)

is equivalent to the neutral fluid equations $\partial_t \omega = -[\psi,\omega]$ where $\psi(x,y,t)$ is the stream function and $\omega = \nabla_\perp^2 \psi$ is the vorticity of the incompressible flow.

Futhermore, exactly the same equation as (85) is obtained for two-dimensional atmospheric flows on a rotating planet (Charney, 1948). In the atmospheric problem the gradient of the vertical component of Coriolis force $f(x)$ with longitude produces the wave frequency $\omega_{\underset{\sim}{k}} = \underset{\sim}{k} \cdot \hat{z} \times \nabla \ell n f/(1 + k_\perp^2 \rho_g^2)$ with $\hat{z}$ the vertical coordinate. The dispersion distance is set by the Rossby radius $\rho_g = (gH_o)^{\frac{1}{2}}/f$ where g is the acceleration of gravity and H_o the vertical scale height for the atmosphere. There are observations and computer simulations of Rossby waves in the atmosphere and oceanographic literature, (Rhines, 1975; Williams, 1978). From this literature it is also known that there are large amplitude two-dimensional solitary wave solutions of the form $\varphi(x,y,t) = \varphi[\sqrt{x^2 + (y - ct)^2}]$ where $\tilde{\varphi}(r) \sim \exp(-kr)$ for $r \to \infty$ (Flierl, et al., 1980). The two-dimensional drift wave or Rossby wave solitons propagate with a velocity $c \geqslant v_{de}$ or in the direction opposite to v_{de}.

The convective nonlinearity from the $\underset{\sim}{E} \times \underset{\sim}{B}$ convection of the plasma is an important characteristic of nonlinear drift-wave theory. The quadratic coupling $[\varphi, f]$ describing the nonlinear transport is a conservative process that gives rise to long-range coupling of the modes in $\underset{\sim}{k}$ space proportional to $\underset{\sim}{k}_1 \times \underset{\sim}{k}_2 \cdot \hat{z}$ (the area of the interaction triangle $\underset{\sim}{k} = \underset{\sim}{k}_1 + \underset{\sim}{k}_2$ in $\underset{\sim}{k}$ space). The conservation laws for the interactions follow from the properties

$$\int f[g,h]\, d\underset{\sim}{x} = \int g[h,f]\, d\underset{\sim}{x} = \int h[f,g]\, d\underset{\sim}{x}$$

and

$$[g,h] = 0 \Leftrightarrow h = h(g).$$

Since [g,h] is also the Jacobian of the coordinate transformation from x,y to g(x,y),h(x,y) its vanishing gives the generalization to non-sinusoidal functions of the condition shown in Fig. 1. For example, in the drift wave model (85) of Hasegawa and Mima, it is evident that the conservative properties of the mode coupling guarantee that the normalized energy density, Eq. (25) $W = \frac{1}{2}\int[\varphi^2 + (\nabla_\perp\varphi)^2]\,d\underset{\sim}{x}$ and the potential entrophy $U = \frac{1}{2}\int[(\nabla_\perp\varphi)^2 + (\nabla_\perp^2\varphi)^2]\,d\underset{\sim}{x}$ are constants of the motion.

When a large number of modes are present the dynamics is strongly mixing in the $\varphi_{\underset{\sim}{k}}(t)$ phase space. This hypothesis is tested for the two-dimensional inviscid initial value problem in a study by Fyfe and Montgomery (1979). In an example with a truncated, isotropic $\underset{\sim}{k}_\perp$ spectrum with $1 \leq k_\perp\rho \leq 15$ the numerical integration for $\varphi_{\underset{\sim}{k}}(t)$ is shown to agree with the equilibrium statistical theory that dictates that the observed spectrum is the most probable phase space distribution consistent with the constants of the motion. For a large number of modes the most probable phase space distribution consistent with the constant energy W and enstrophy U is $\exp(-\alpha W - \beta U)$ where $W = \frac{1}{2}\sum_k (1 + k_\perp^2)|\varphi_k|^2$ and

$U = \frac{1}{2}\Sigma k_\perp^2(1 + k_\perp^2)|\varphi_{k_\perp}|^2$. Thus, the statistical equilibrium distribution $|\varphi_k|^2$ for the truncated inviscid system is

$$\langle|\varphi_{k_\perp}|^2\rangle = \frac{1}{(1 + k_\perp^2)(\alpha + \beta k_\perp^2)}$$

with the constants α and β determined by W and U, Fyfe and Montgomery (1979). This equipartion distribution is divergent as $k_{max} \to \infty$ with $W \sim \frac{1}{2}\ln(k_{max})$ and $U \sim \frac{1}{\beta} k_{max}^2$ and does not represent the physically observed nonequilibrium spectra.

The observed turbulent spectra, as reviewed by Rose and Sulem (1978), vary more nearly as $W_k \sim k_\perp^{-4}$ or $\langle|\varphi_k|^2\rangle \propto 1/k_\perp^6$ for $k_\perp\rho > 1$. Such a distribution can be understood as the broadest spectrum consistent with a logarithmically convergent value for the enstrophy in the $k_{max} \to \infty$ limit. A simple derivation of this $1/k_\perp^4$ spectrum is given by Hasegawa, et al. (1979) from a wave-wave decay analysis of Eq. (84). The decay process proceeds with the intermediate wavenumber mode $k_2(k_1 < k_2 < k_3)$ decaying by giving the fraction $(k_3^2 - k_2^2)/(k_3^2 - k_1^2)$ of its energy to mode k_1 and the fraction $(k_2^2 - k_1^2)/(k_3^2 - k_1^2)$ of its energy to mode $\underset{\sim}{k}_3$. A Monte-Carlo simulation of this decay process shows that the spectrum becomes anisotropic at long wavelengths, $k_\perp^2\rho^2 < 1$, where energy preferentially flows to $k_y \ll k_x$ at small $k_\perp$. This anisotrophy is consistent with the conservation of the action density in the long wavelength region.

The anisotropic feature of the k-spectrum is supported by observational and computational studies of Rossby waves (Rhines, 1975). It appears that most of the energy in the Rossby wave is in the longest wavelength modes of the system, and furthermore, the energy appears preferentially in the $k_y \cong 0$ modes and at a finite k_x. This energy distribution produces large zonal flows in the y-direction with a periodic variation in x.

In weakly ionized plasmas drift wave turbulence is limited by the $\underset{\sim}{E} \times \underset{\sim}{B}$ mode coupling as given by Sudan and Keskinen (1977, 1979) and Keskinen (1981) using DIA theory. The theoretical results are compared with computer simulations and ionospheric measurements.

The drift wave problem differs in some important aspects from the neutral fluid problem. One important difference is the presence of dissipation throughout broad regions of $\underset{\sim}{k}\omega$ space due to the particle-wave interactions. The most important dissipation is that from the electron-wave interactions $i\delta_k^e$ which leads to the excitation of the instability and the n-φ phase shift that produces the anomalous plasma transport. The drift wave mode coupling equation based on the ion hydrodynamic description taking into account the dissipative phase shift $i\delta_k^e$ is developed by Horton (1976b).

In the presence of the phase shift the $\underset{\sim}{E} \times \underset{\sim}{B}$ nonlinearity dominates the finite ion inertia nonlinearity in Eq. (84) in the long wavelength part of the spectrum where $(k_\perp \rho)^2 < \delta_e(k_\perp)$. The mode coupling element is now

$[n,\varphi] \sim i\underset{\sim}{k}' \times \underset{\sim}{k}'' \cdot \hat{z}(\delta_{k''} - \delta_{k'})\varphi_{\underset{\sim}{k}'}\varphi_{\underset{\sim}{k}''}$ which also conserves the wave energy $W = \frac{1}{2}\int[\varphi^2 + (\nabla_\perp\varphi)^2]d\underset{\sim}{x}$ in the mode coupling. Due to the nonlinearity, wave energy is coupled from regions of growth $\gamma_k/\omega_k \cong \delta_k^e > 0$ to regions of collisionless damping. From the spectral equation for $dI(\underset{\sim}{k},t)/dt$ given by Horton (1976), the balance of power input to the spectrum $2\omega_k\delta_k^e W_k$ occurs with nonlinear transfer to regions of absorption when the spectrum is given by

$$\frac{e^2\left\langle|\varphi_{\underset{\sim}{k}}|^2\right\rangle}{T_e^2} = \frac{1}{\pi}\frac{\delta_e(\underset{\sim}{k}_\perp)}{\langle k_x^2 r_n^2\rangle(k_y\rho)} \tag{86}$$

where $\delta_e(\underset{\sim}{k}_\perp) = -\mathrm{Im}\ \varepsilon_{\underset{\sim}{k}}(\omega_{\underset{\sim}{k}}) > 0$. The $k_\perp$-spectrum in Eq. (86) reflects the transfer of wave energy from short wavelengths to long wavelengths where it is absorbed by collisionless ion damping when $\omega_{*e} \sim k_\parallel v_i \sim v_i/L$ where L is the effective parallel length of the system or the electron Landau damping when $(k_\perp\rho)^2 \ll \eta_e/2$.

The transfer of energy to long wavelengths shifts the peak of the spectrum from the fastest growing mode at the maximum of $k_y\delta_e(k)$ to the maximum of $\delta_e(k)/k_y$ according to formula (86). Other studies suggest that the maximum of the spectrum occurs near the long wavelength point of marginal stability $\mathrm{Im}\ \varepsilon_{\underset{\sim}{k}}(\omega_{\underset{\sim}{k}}) \approx 0$. The renormalized turbulence theory used to obtain Eq. (86) predicts broadening of the spectral lines at $\omega_{\underset{\sim}{k}}$ given by

$$\nu_k = \omega_k \langle k_x^2 r_n^2 \rangle (k_y \rho) \ \ I(k_y \rho) \tag{87}$$

where $\nu_k \lesssim \omega_k$ is assumed in the theory for $I(k_y)$. Further aspects of this theory are described by Tang (1978).

Microwave scattering experiments by Mazzucato (1976) and Goldston et al. (1978) measure the $k_\perp$ spectrum and the ω spectrum of drift wave fluctuations in a confinement experiment. The experiments are analyzed by Horton (1976b), and Horton and Estes (1979) using the spectral formula (86) derived from the hydrodynamic ion nonlinearity and the linear kinetic theory electron response for the system. The variable scattering angle defines the $\underset{\sim}{k}_\perp$ of the electron density fluctuation producing the scattered microwave signal. Fluctuations with $k_\perp \rho \sim 0.6$ have the frequency spectrum shown in Figure 8 where the maximum of the spectrum rotates with the electron diamagnetic velocity or somewhat faster due to an ion confining radial electric field. Figure 8 shows the change of rotational direction with the change $B_z \to -B_z$. The width of the fluctuation spectrum is comparable to ω_k as expected for the measured turbulence level. The frequency integrated wavenumber spectra obtained from different scattering angles is shown in Fig. 9. In addition the value for the spectrum given by formula (86) is also shown when evaluated for the experimental parameters. With this spectrum the anomalous transport given by Eq. (17) is substantial due to the phase shift in the region where the fluctuation amplitudes are significant. Analysis of the electron thermal transport in this and other experiments with microwave scattering show a significant correlation between the drift wave turbulence theory and anomalous electron thermal losses, Horton and Estes (1979).

3.4 Ion Temperature Gradient Drift Wave Turbulence

In the regime of strong ion temperature gradients $\eta_i = d\ell n T_i / d\ell n n_i \geqslant 2$ the dispersion relation (82) yields the ion temperature gradient-driven drift wave instability with $\omega_{\underset{\sim}{k}} \cong \frac{1}{2}|\omega_{*pi}|^{1/3}|k_\parallel c_s|^{2/3}$ and the growth rate

$$\gamma_{\underset{\sim}{k}} \cong \frac{\sqrt{3}}{2}|\omega_{*pi}|^{1/3}|k_\parallel c_s|^{2/3} . \tag{88}$$

The fastest growing mode in the unstable spectrum occurs at $k_\perp \rho_i \simeq (1 + \eta_i)^{-\frac{1}{2}} < 1$ and $k_\parallel r_n \simeq (1 + \eta_i)^{\frac{1}{2}}$ and has $\gamma_{max} \cong c_s/(r_n r_T)^{\frac{1}{2}}$. Coppi et al. (1967) show that the mode is not stabilized by shear $S = r_n/L_s$ and that the basic features of the instability are given by the two component fluid equations with adiabatic electrons $\delta n_e = n_e[e\varphi(\underset{\sim}{r},t)/T_e]$. The nonlinear saturation and transport has been studied in terms of the hydrodynamic equations. The ion temperature gradient-driven drift wave turbulence is quasineutral with $n_i(\underset{\sim}{r},t) = n_e(\underset{\sim}{r},t) = n_e(x)[1 + e\varphi(\underset{\sim}{r},t)/T_e]$ which allows the plasma density to be eliminated in terms of the electrostatic potential. The instability arises from the cubic polynomial in ω describing the small amplitude drift-ion acoustic waves in the presence of a nonuniform ion pressure gradient. The equations for $\partial_t\varphi$, $\partial_t v_{\| i}$ and $\partial_t p_i$ follow from $\nabla\cdot \underset{\sim}{j} = 0$, the ion parallel-momentum balance and the ion thermal balanace equations, respectively. The elementary estimates for the nonlinear limit of the amplitudes suggests scaling the amplitude of fields in terms of small gyroradius parameter $\alpha = \rho_i/r_n$. The unstable growth rate and wavenumber suggest scaling the space-time variables according to $\underset{\sim}{r}_\perp = \underset{\sim}{x}_\perp \rho$, $z = \zeta r_n$, and $t = \tau r_n/c_s$. In terms of these dimensionless variables the hydrodynamic equations are

$$(1 - \nabla_\perp^2)\,\frac{\partial\Phi}{\partial\tau} = -\frac{\partial\Phi}{\partial y} - \nabla_\| v + [\Phi,\ \nabla_\perp^2\Phi]$$

$$\frac{\partial v}{\partial\tau} = -\nabla_\|(\Phi + p) - [\Phi, v] - \alpha v\nabla_\| v$$

$$\frac{\partial p}{\partial\tau} = -[\Phi, p] - \alpha v\nabla_\| p - \alpha\gamma p\nabla_\| v\ , \qquad (89)$$

where $\nabla_\| = \partial_\zeta + S(x - x_o)\partial_y$ and

$$\frac{e\varphi(\underset{\sim}{r},t)}{T} = \frac{\rho}{r_n}\Phi\ ,\quad \frac{v_{i\|}}{c_s} = \frac{\rho}{r_n}v\ ,\qquad \frac{p_i}{n_i T_i} = \frac{\rho}{r_n}p\ . \qquad (90)$$

The system (89) is nondissipative with the total energy $W_f = W/(\alpha^2 nT)$

$$W_f = \int \left[\frac{1}{2}\Phi^2 + \frac{1}{2}(\nabla_\perp\Phi)^2 + \frac{1}{2}v^2 + \frac{p}{\alpha(\gamma - 1)}\right] d\underset{\sim}{x}$$

being a constant of the motion. Three-dimensional simulations of these reduced hydrodynamic equations were performed by Horton et al. (1979).

The mode-coupling theory of Sections 3.2 and 3.3 applied to Eq. (89) indicates that the terms labelled C in Eq. (83) are dominant. The numerical simulations show that the limit $\alpha \to 0$ can be taken in Eq. (89) The fluctuations saturate at the root-mean-square level $\Phi_o = \langle\Phi^2\rangle^{1/2} \sim (1 + \eta_i)^{1/2}$. The mode coupling is strong so that even with the initial data taken as a single small amplitude eigenmode, the saturated state contains a spectrum of $\underset{\sim}{k}$ modes.

In the saturated state there is a turbulent convection of ion thermal energy given by

$$Q_i = \langle v_{Ex} \delta p_i \rangle = -\chi_i n_i \frac{dT_i}{dr}$$

where

$$\chi_i = C_1 \frac{\rho_i}{r_n} \left(\frac{cT_i}{eB}\right) \left(\frac{L_s}{r_n}\right)^{1/2} (1 + \eta_i)^{1/2} \qquad (91)$$

with $C_1 \sim 0.25$ to 0.5 for $L_s/r_n < 100$. For weak shear the finite length of the system becomes important. In a toroidal system with low shear we find that $\chi_i = C_1(\rho_i/r_n)(cT_i/eB)q$ where q is the toroidal safety factor.

3.5 Kinetic Theory of Drift Wave Turbulence

In general the hydrodynamic equations do not adequately describe the particle and fluctuation dynamics that occurs in drift wave turbulence. In particular, the important role of fluctuations with $k_\perp \rho_i \sim 1$ and fluctuations with $\omega \sim k_\parallel v_i$ are not included in the nonlinear transfer processes described by the fluid approximation equations. The nonlinear analyses of Kadomtsev (1963), Galeev and Rudakov (1963), and Sagdeev and Galeev (1969) examine the influence of these kinetic processes in the theory of drift wave turbulence. These investigations consider that the turbulent fields are sufficiently random and weakly coupled to close the hierarchy of correlation functions by neglecting fourth and higher order cumulants. Galeev and coworkers assume further that it is sufficient to approximate the frequency spectrum by $I(\underset{\sim}{k}\omega) = I(\underset{\sim}{k},t)\delta(\omega - \omega_{\underset{\sim}{k}})$ and apply weak turbulence theory to reduce Eq. (78) directly to the weak turbulence theory spectral balance equation for $dI_{\underset{\sim}{k}}/dt$. In contrast, Kadomtsev (1963) introduces a renormalization of the equation for the $v_\perp$-integrated kinetic equation and derives a formula for the frequency spectrum. Both investigations, however, lead to similar conclusions. The investigations show that the role of the nonlinear ion dynamics dominates in the mode coupling equations and that this coupling leads to a transformation of the $k_\perp$-spectrum from short perpendicular wavelengths $k_\perp \rho \approx (m_i \beta / m_e)^{1/2}$ to long wavelengths where $k_\perp \rho_i \lesssim 1$. The important results from these investigations are summarized in this subsection.

Analysis of the weak turbulence spectral equations by Sagdeev and Galeev (1969) leads to the conclusion that the induced ion wave scattering is the dominant physical mechanism. The induced scattering rate is calculated from the imaginary part of the mode coupling elements in Eq. (78) with Eqs. (73)-(75) and is given by

$$\gamma^{n\ell}_{(k)} = \frac{\pi (cT_e/eB)^2}{1 + (T_e/T_i)(1 - \langle J_0^2 \rangle)} \sum_{k'} (\underset{\sim}{k} \times \underset{\sim}{k}' \cdot \hat{z})^2 \left| \frac{e\varphi_{\underset{\sim}{k}'}}{T_e} \right|^2 M(k_\perp, k_\perp')$$

$$\left(\frac{\omega_*' - \omega_*}{\omega_{\underset{\sim}{k}}} \right) \frac{\pi^{1/2}}{|k_\parallel| v_i} \exp \left[- \frac{(\omega_{\underset{\sim}{k}} - \omega_{\underset{\sim}{k}'})^2}{(k_\parallel - k_\parallel')^2 v_i^2} \right] \quad (92)$$

where $M(k_\perp, k_\perp')$ is the positive definite symmetric matrix

$$M(k_\perp, k_\perp') = \left\langle J_0^2\left(\frac{k_\perp v_\perp}{\Omega}\right) J_0^2\left(\frac{k_\perp' v_\perp}{\Omega}\right)\right\rangle - \frac{\left\langle J_0\left(\frac{k_\perp v_\perp}{\Omega}\right) J_0\left(\frac{k_\perp' v_\perp}{\Omega}\right) J_0\left(\frac{k_\perp'' v_\perp}{\Omega}\right)\right\rangle^2}{\left\langle J_0^2\left(\frac{k_\perp'' v_\perp}{\Omega}\right)\right\rangle} \qquad (93)$$

where $\underset{\sim}{k}'' = \underset{\sim}{k} - \underset{\sim}{k}'$. The total mode balance equation is

$$\frac{d|\varphi_{\underset{\sim}{k}}|^2}{dt} = [2\gamma^{\ell}(\underset{\sim}{k}) + 2\gamma^{n\ell}(\underset{\sim}{k})]|\varphi_{\underset{\sim}{k}}|^2. \qquad (94)$$

For large $k_\perp \rho_i$ the drift wave frequency $\omega_{\underset{\sim}{k}} = \omega_{*e} e^{-b} I_o(b)/[1 + T_e/T_i(1 - e^{-b} I_o)] \cong (c_s/r_n)/2\sqrt{2}\pi$ is nearly independent of $k_\perp$ so that three wave decay processes are strongly forbidden. For large $k_\perp \rho$ the matrix $M(k_\perp, k_\perp') \sim 1/(k_\perp \rho)(k_\perp' \rho)$ and the induced scattering from $k_\perp'' \rho \sim 1$ is strong. In particular $\gamma^{n\ell}(k) \sim -|\omega_{*e}(k)|\sum_{k'} |e\varphi_{k'}/T_e|^2 k'^2_x r_n^2/|k_y' \rho|$ which overcomes the maximum growth rate $\gamma_{max} \cong (c_s/r_n)(m_e/m_i \beta)^{1/2}$ when $W/nT_e \cong (m_e/m_i \beta)^{1/2}/\langle k_x^2 r_n^2\rangle$. Consequently, as in the fluid model, the kinetic ion non-linearity shifts the spectrum to smaller $k_\perp \rho_i$. For $k_\perp' \rho_i \sim k_\perp \rho_i < 1$, however, induced ion scattering becomes weak due to the cancellation that occurs between the bare ion scattering and that from the screening cloud as shown ~~shown by the reduction~~ $M(k_\perp, k_\perp') \cong (k_\perp \rho)^2 (k_\perp' \rho)^2$. Thus, Sagdeev and Galeev (1969) conclude that the spectrum is peaked at $k_\perp = \bar{k}_\perp$ with $\bar{k}_\perp \rho_i \lesssim 1$.

A constraint imposed by the mode balance equation is given for the $\underset{\sim}{k}$-integrated spectrum. The authors introduce $\delta\omega_{\underset{\sim}{k}} = \omega_{\underset{\sim}{k}} - \omega_{*e}(k)$ and obtain from the balance equation

$$\sum_{\underset{\sim}{k}} \left|\frac{e\varphi_{\underset{\sim}{k}}}{T_e}\right|^2 k_x^2 \cong \frac{\gamma_{\bar{\underset{\sim}{k}}}}{|\delta\omega_{\bar{k}}|\, r_n^2} \tag{95}$$

where $\gamma_{\bar{k}}/\delta\omega_{\bar{k}}$ is evaluated at the peak of the spectrum.

Using this result (95) and the formula (18) with $\delta_e(\bar{k}) \cong (m_e/m_i\beta)^{1/2}$ for the linear n_e-φ phase difference, the anomalous particle flux is computed as

$$\Gamma = - \frac{m_e}{m_i\beta} \frac{\rho}{r_n} \frac{cT_e}{eB} \frac{dn_e}{dx} \tag{96}$$

which is smaller by the additional power of δ_e in Eq. (95) than the estimate (71) of Oraevskii and Sagdeev (1963).

Galeev and Rudakov (1963) emphasize that for Eq. (96) to be valid the collision frequency must be greater than $\nu_* \cong \omega_{ci}(\rho/r_n)^3(m_e/m_i\beta)$ to maintain the slope of the resonant electron velocity distribution close to the Maxwell-Boltzmann value. For $\nu_e < \nu_*$ Eq. (96) remains valid with the value $\gamma_k(f_e)$ computed from the quasilinearly flattened distribution given in Sec. 2.7 from which it is evident that the final diffusion coefficient is obtained from Eq. (96) by dividing by the factor $1 + \nu_*/\nu_e$.

Kadomtsev (1963) obtains a different nonlinear transfer equation by introducing a renormalized particle-wave resonance $g_k(v_\parallel) = [\omega - k_\parallel v_\parallel + \eta_k(v_\parallel)]^{-1}$ obtained from the $v_\perp$-integrated nonlinear kinetic equation. From the fact that $\omega \ll \Omega_i$ and $|\rho_i \nabla n/n| \ll 1$ the $v_\perp$-dependence of the nonlinear fluctuating distribution is obtained in terms of $J_0(k_\perp v_\perp/\Omega)$. The turbulent collision operator obtained by Kadomtsev is

$$\eta_{\underset{\sim}{k}\omega}(v_\parallel) = \left(\frac{c}{B}\right)^2 \sum_{\underset{\sim}{k}'} \frac{(\underset{\sim}{k} \times \underset{\sim}{k}' \cdot \hat{z})^2 \left\langle J_0\left(\frac{k_\perp v_\perp}{\Omega}\right) J_0\left(\frac{k'_\perp v_\perp}{\Omega}\right) J_0\left(\frac{k''_\perp v_\perp}{\Omega}\right)\right\rangle^2 I(\underset{\sim}{k}')}{\left\langle J_0^2\left(\frac{k_\perp v_\perp}{\Omega}\right)\right\rangle \left\langle J_0^2\left(\frac{k''_\perp v_\perp}{\Omega}\right)\right\rangle (\omega'' - k''_\parallel v_\parallel + i\nu)} \tag{97}$$

where $\underset{\sim}{k}'' = \underset{\sim}{k} - \underset{\sim}{k}'$ and $\omega'' = \omega - \omega'$. The operator $\eta_{\underset{\sim}{k}\omega}(v_\parallel)$ reduces to the $v_\perp$-average of the $i\nu_{\underset{\sim}{k}\omega}(v_\parallel, v_\perp)$ operator in Eq. (76) when $\underset{\sim}{k}\omega \ll \underset{\sim}{k}'\omega'$. Physically, the operator $\eta_{\underset{\sim}{k}\omega}$ describes the decorrelations produced by the kinetic nonlinear $\underset{\sim}{k} \cdot \underset{\sim}{v}_E$ effects in the fluctuation $\underset{\sim}{k\omega}$ interacting with particles with parallel velocity $v_\parallel$. The broadening operator takes into account the averaging by the fast cyclotron motion of the large gyroradius particle orbit over the coupled fluctuations $k_\perp$, $k'_\perp$, $k''_\perp$.

Kadomtsev (1963) also includes the electromagnetic polarization of the wave which reduces the parallel electric field and hence the growth rate when the Alfvén frequency $k_\parallel v_A$ becomes comparable to the drift frequency ω_{*e}. The polarization effect is evaluated with the linear dispersion relation, and its principal effect is to determine the parallel wavenumber at which the spectrum is peaked. For $m_e/m_i < \beta < (m_e/m_i)^{1/3}$ Kadomtsev finds that the $k_{\parallel}$-spectrum is peaked at

$$k_{\parallel} r_n \simeq \frac{1}{2} \beta^{1/2} \quad . \tag{98}$$

The spectral equations given by Kadomtsev, which are too complicated to give here, are of the form given in Eqs. (79) and (80). This pair of coupled equations for the nonlinear response function $\varepsilon_k^{n\ell}$ and the spectral distribution I_k is the plasma physics analog of the Direct Interaction Approximation of fluid turbulence. Properties such as energy conservation of the renormalized dielectric function are given by Dupree and Tetreault (1978).

From analysis of the frequency integrated mode balance equation Kadomtsev concludes that the quasilinear spectrum is peaked at $k_{\perp}\rho_i \lesssim 1$. Since this is a region where the Bessel function matrix elements take on a complicated form Kadomtsev notes that it is not feasible to give a formula for the spectrum. Instead a constraint for the $\underset{\sim}{k}$-integrated spectrum is derived from the balance equation which leads to

$$\frac{e^2}{T_e^2} \int k_{\perp}^2 I(\underset{\sim}{k}) d\underset{\sim}{k} = \frac{A}{r_n^2} \left(\frac{m_e}{m_i \beta}\right)^{1/2} \tag{99}$$

where the estimate $A \sim 1/10$ is given. Furthermore, it is argued that at the peak of the spectrum the δn_e-φ phase shift remains as given by the linear electron response since the ion nonlinearities produce the saturation. Using the linear phase shift $\delta_e(k) \cong (\pi/2)^{1/2}\,\omega_{*e}/|k_\parallel|v_e$ which at the peak of the spectrum gives $\delta_e(\bar{k}) \cong (m_e/m_i\beta)^{1/2}$, the anomalous particle flux from formula (17) is

$$\Gamma = -\frac{Am_e}{m_i\beta}\,\frac{\rho}{r_n}\,\frac{cT_e}{eB}\,\frac{dn}{dx} \tag{100}$$

which is the same formula obtained by Sagdeev and Galeev, except for the smaller coefficient. For $\beta > (m_e/m_i)^{1/3}$ it is the ion ∇B collisionless resonance $\omega_k \cong \omega^i_{\nabla B}(E) \cong \frac{1}{2}\,\beta\omega_{*e}(E/T)$ that changes the linear theory and reduces the flux from that given in Eq. (100).

With knowledge of the approximate $\underset{\sim}{k}$-spectrum the ω-spectrum is obtained from the $\varepsilon_k^{n\ell} I_k$ balance equation (79). The fact that $\varepsilon_k^{n\ell} = 0$ describes the decay of a fluctuation $\underset{\sim}{k}\omega$ in the turbulent plasma, while the right-hand-side of the equation for I_k gives the production of fluctuations at $\underset{\sim}{k}\omega$ due to the nonlinear transformations. For the turbulence level obtained in Eq. (99) Kadomtsev estimates that the shift in the natural frequencies $\omega_{\underset{\sim}{k}}$ are small and that the line shape is Lorentzian with a width given by

$$\gamma_k^{n\ell} = -\,\mathrm{Im}\tilde{\varepsilon}_k^{n\ell}/(\partial\varepsilon_{\underset{\sim}{k}}/\partial\omega_{\underset{\sim}{k}}) \cong \left|\gamma^{\ell}_{\underset{\sim}{k}}\right| \lesssim \omega_{\underset{\sim}{k}} \;.$$

With the $\underset{\sim}{k}$-spectral distribution obtained from the preceding analysis the equation for the correlation function I_k is analysed for driven low frequency fluctuations as explained by Tsytovich (1977, Chapter 2.7-2.8). For the nondecay dispersion relation of drift waves in the region $k_\perp \rho \stackrel{\sim}{<} 1$ where the primary spectrum is most intense, the correlation equation (79) yields a strong production of low frequency long wavelength fluctuations $I^{LF}_{\underset{\sim}{k}\omega}$. For these low frequency fluctuations there is no particular relation between $\underset{\sim}{k}$ and ω. The low frequency spectrum is given by

$$I^{LF}_{\underset{\sim}{k}\omega} = \frac{2\pi}{|\varepsilon^{n\ell}_{k}|^2} \int d\underset{\sim}{k}_1 \, \delta\left[\omega - \underset{\sim}{k} \cdot \frac{\partial\omega(\underset{\sim}{k}_1)}{\partial \underset{\sim}{k}_1}\right] \left|\varepsilon^{(2)}_{\underset{\sim}{k}_1,\underset{\sim}{k}-\underset{\sim}{k}_1}(\omega_{\underset{\sim}{k}_1}, \omega_{\underset{\sim}{k}-\underset{\sim}{k}_1})\right|^2 [I(\underset{\sim}{k}_1)]^2 . \tag{101}$$

Equation (101) predicts a broad band filling of the low frequency spectrum when the fluctuation amplitudes are at the level given by formula (86) or formula (99). This low frequency filling of the spectrum can be a driving mechanism for very low frequency motions of the plasma such as convective cells.

3.6 Anomalous Transport Due to Convective Cells

Particle simulations and theoretical studies by Dawson and collaborators show that zero frequency collective modes with $k_\perp \neq 0$ known as convective cells are an important mechanism for anomalous transport across the confining magnetic field. Particle simulation studies by Okuda et al. (1975) and Chu et al. (1975) compare the particle diffusion D and the convective cell life $\Gamma_{k_\perp}$ with the formulas used in the analysis given here. Agreement between theory and simulation is reported by Dawson and collaborators for the anomalous diffusion produced by the weakly damped two dimensional convective cells.

At long wavelengths $k_\perp \rho_i \ll 1$ the convective cell appears in the retarded branch of the shear Alfvén wave dispersion relation for the inhomogeneous plasma

$$k_\perp^2 \rho^2 [(\omega + ik_\perp^2 \mu)(\omega - \omega_{*pi}) - k_\parallel^2 v_A^2] = 0. \tag{102}$$

As $k_\parallel v_A/\omega_{*pi} \to 0$ the two plasma modes are the ion flute mode $\omega_1 \cong \omega_{*pi} + k_\parallel^2 v_A^2/\omega_{*pi}$ and the convective cell

$$\omega_2 \cong -k_\parallel^2 v_A^2/\omega_{*pi} - ik_\perp^2 \mu \tag{103}$$

where $\omega_{*pi} = (k_y c/e_i n_i B)(dp_i/dx)$. In the long wavelength regime the collective mode with finite $k_\perp$ and $\omega_k^c = \omega_2(k_\perp, k_\parallel) \cong 0$ is a large scale hydrodynamic ($E_\parallel \cong 0$) roll whose damping is determined by the ion collisional viscosity $\mu = 0.3\nu_i \rho_i^2$. The cross-field $E \times B$ flows produced by these zero frequency modes, in the frame with $E_r = 0$, are different from that of drift waves in that the convection shown in Fig. 1 is now steady and closed. For drift waves the convective flow changes direction after transport over the distance $\underset{\sim}{v}_E/\omega$. The hydrodynamic modes appear at the thermal fluctuation level determined by the weak damping until driven up in amplitude by some mechanism.

Sagdeev and collaborators (1978) show that a natural mechanism exists for driving the convective cells through the parametric decay of the drift waves. Their theory is motivated by three dimensional particle simulations of drift wave instabilities by Cheng and Okuda (1977) and (1978). The simulations show that in the first stage of saturation the wave spectrum is dominated by drift modes with $k_{\|} v_e > \omega_{\underset{\sim}{k}}$, but at a later stage a significant fraction of the fluctuation energy is in a $k_{\|} = 0$, low $k_{\perp}$ mode of the spectrum.

Sagdeev et al. (1978) observe that since the drift wave frequency $\omega_k^d = \omega(k_{\perp}^2, k_y)$ has a broad maximum for $k_{\perp} \rho_i \lesssim 1$, there are many unstable drift modes which beat to produce zero-frequency, zero-parallel wavenumber fluctuations. Formulating the problem following the well known methods of Langmuir-ion acoustic wave interactions, the coupled drift wave $\delta n_e^d(\underset{\sim}{x}t)/n_e = e\varphi^d(\underset{\sim}{x}t)/T_e$ and convective cell $\delta n_e^c(\underset{\sim}{x}t)/n_e \gg |e\varphi^c/T_e|$ equations are derived and reduced to

$$[(1 - \rho^2\nabla_\perp^2)\frac{\partial}{\partial t} + v_{de}\frac{\partial}{\partial y}]\ \delta n^d = -\frac{cT_e}{eBn_0}[\delta n^d, \delta n^c] \tag{104}$$

$$(\frac{\partial}{\partial t} - \mu\nabla_\perp^2)\nabla_\perp^2\delta n^c = \frac{cT_e^2}{eBT_i n_0} < [\delta n^d, \nabla_\perp^2 \delta n^d] > \tag{105}$$

where the average <....> is over the z and t dependence of the drift wave nonlinearity which projects out the $k_\parallel = 0$, $\omega = 0$ components of the interaction term.

Eqs. (104) and (105) determine the stability to parametric decay [see Chapter IV.4] of a large amplitude drift wave $\delta n_{k_0}^d\ exp(ik_0 \cdot x - i\omega_{k_0}^d t)$ into a small amplitude drift wave $\delta n_{k_1}^d(t)\ exp(ik_1 \cdot x - i\omega_{k_1}^d t)$ and a convective cell $\delta n_{k_2}^c(t)\ exp(i\ k_2 \cdot x)$ where $k_0 = k_1 + k_2$, $k_{\parallel 0} = k_{\parallel 1}$, and $\omega_{k_0}^d = \omega_{k_1}^d$. When $k_0^2 > k_1^2$ the parametric decay occurs at the rate

$$\gamma_k^{n\ell} = \frac{cT_e}{eB}\ |k_1 \times k_0| \left[\frac{T_e}{T_i}\ \frac{(k_0^2 - k_1^2)}{k_2^2}\ \left|\frac{\delta n_{k_0}}{n_e}\right|^2\right]^{1/2} . \tag{106}$$

The anomalous diffusion coefficient $D_\perp$ produced by the two dimensional convective cells is investigated by Dawson and collaborators as the zero-frequency limit of the Dupree (1967) diffusion coefficient. This diffusion coefficient follows from the $k_\parallel = 0$, $k_\perp \rho_i = 0$ limit of formula (77) and is given by

$$D_{\perp} = -\frac{c^2}{B^2} \operatorname{Im} \sum_{k_{\perp}} \frac{|E_{\perp}^{c}(k)|^2}{\omega_k + i\Gamma_k} \tag{107}$$

with

$$\Gamma_k = k_{\perp}^2 D_{\perp} \ , \tag{108}$$

which for $k_{\perp}^2 D_{\perp} \gg \omega_k^c$ reduces to

$$D_{\perp} \simeq \frac{cT_e}{eB} \left\langle \left| \frac{\delta n^c(k_{\perp})}{n_e} \right|^2 \right\rangle^{1/2} . \tag{109}$$

Sagdeev and collaborators (1978) propose that a steady turbulent state is reached in which the energy from the gradients in the nonuniform electron distribution which drive the drift waves at the rate $2\gamma_k^d W_k^d$ is balanced by the decay into convective cells. Power balance in the convective cells is reached when the cell damping rate $\gamma_k^c = -k_{\perp}^2 D_{\perp}$ balances the rate of drift wave pumping which for a turbulent spectrum is given by

$$\gamma_k^{n\ell} = \frac{\omega_k^d}{(k_{\perp}\rho)^2} \frac{T_e}{T_i} \left(\frac{r_n}{\Delta x_c^c}\right)^2 < \left|\frac{\delta n^d}{n_e}\right|^2 > \tag{110}$$

where Δx^c is the average radial width of the convective cells. This balance determines the drift wave turbulence level as

$$<|\frac{\delta n^d}{n_e}|^2>^{1/2} = \frac{-\gamma_k^c}{\omega_{\bar{k}}^d}\frac{T_i}{T_e}(\bar{k}_\perp\rho)^2(\frac{\Delta x_c}{r_n})^2 \quad (111)$$

where $\bar{k}$, $\omega_{\bar{k}}^d$ are mean wavenumbers and frequencies in the drift wave spectrum. The cell size Δx_c is of order $(\rho_i r_n)^{1/2}$ when the shear is zero and of order $\rho_i(\beta L_s/r_n)^{1/2}$ in the presence of shear, $\underset{\sim}{B} = B_0(\hat{z} + \hat{y}x/L_s)$.

Power balance through the entire fluctuation spectrum requires

$$2\gamma^d W^d \cong 2\gamma^{n\ell} W^c \quad (112)$$

with $W^c \cong \sum_k (k_\perp\rho_i)^2|\delta n^c(k_\perp)/n_e|^2$ and leads to the amplitude of the convective cell motion

$$|\frac{\delta n^c}{n_e}|^2 \cong \frac{\gamma_{\bar{k}}^d}{\omega_{\bar{k}}^d}(\frac{\Delta x^c}{r_n})^2 \quad . \quad (113)$$

From Eq. (109) the diffusion due to convective cells at this amplitude is

$$D = (\frac{cT_e}{eB})\left(\frac{\gamma_{\bar{k}}^d}{\omega_{\bar{k}}^d}\right)^{1/2}(\bar{k}_\perp\rho_i)\,|\frac{\Delta x^c}{r_n}| \quad (114)$$

$$\cong \begin{cases} (\frac{cT_e}{eB})\left(\frac{\gamma_{\bar{k}}^d}{\omega_{\bar{k}}^d}\right)^{1/2}\frac{\rho_i}{r_n}(\frac{\beta L_s}{r_n})^{1/2} & \text{for } \beta\frac{L_s}{r_n} < \frac{r_n}{\rho_i} \\ (\frac{cT_e}{eB})\left(\frac{\gamma_{\bar{k}}^d}{\omega_{\bar{k}}^d}\right)^{1/2}(\frac{\rho_i}{r_n})^{1/2} & \text{for } \beta\frac{L_s}{r_n} > \frac{r_n}{\rho_i}\ . \end{cases} \quad (115)$$

For $\gamma^d_{\underset{\sim}{k}}/\omega^d_{\underset{\sim}{k}} \cong (m_e/m_i\beta)^{1/2}$ the diffusion reduces to

$$D = (\rho_i/r_n)(cT_e/eB)(m_e\beta/m_i)^{1/4}(L_s/r_n)^{1/2} . \tag{116}$$

This secondary diffusion from convective cells driven by the drift wave spectrum can dominate the primary diffusion given in Eqs. (96) and (100).

For the doubly periodic system with integer $k_y r = m$, $k_z R = \ell$ and with shear Bakai (1979) notes that the drift modes must be strongly overlapping in order to produce the convective cells. His consideration of this geometrical aspect of the problem also leads to the possibility of convective cells and predicts a Gaussian $\underset{\approx}{k}_\perp$-fluctuation spectrum.

4. Anomalous Transport from Electromagnetic Fluctuations

The electrostatic drift wave turbulence of Secs. 2 and 3 is the dominant mechanism for anomalous transport in plasmas where thermal pressure p is sufficiently small compared to the magnetic pressure, $\beta = 8\pi p/B^2 \ll 1$. On the other hand, in moderate to high pressure plasmas the self-consistent collective modes are typically strongly magnetic in character, and additional formulas taking into account the electromagnetic nature of the fluctuations are required to describe anomalous transport. With higher plasma pressure there are a variety of low frequency plasma instabilities that are related to the spatial gradients of the particle distribution and lead to anomalous transport. In particular, the temperature gradient driven tearing modes and the current filamentation instabilities produce electromagnetic turbulence and associated transport. The direct $\underset{\sim}{E} \times \underset{\sim}{B}$ transport from the electron drift wave fluctuations becomes less important than the electromagnetic transport as the value of beta for the plasma increases. The competing roles of the anomalous diffusion related to different instabilities and transport mechanisms at moderate to high plasma beta remains an area of active research. Some principal results are given here without attempting to make a definitive assessment of their relative importance.

At high plasma pressure the anomalous transport must be considered as a function of the full electromagnetic fluctuation spectrum specified by six correlation functions, $\langle E_\alpha(\underset{\sim}{r}'t')E_\beta(\underset{\sim}{r}t)\rangle$, rather than the single electrostatic potential correlation function $\langle\Phi(r't')\Phi(\underset{\sim}{r}t)\rangle$ and its associated spectral distribution $I(\underset{\sim}{k}\omega)$ introduced in Sec. 2, Eq. (8). Two methods are widely used to represent the electromagnetic fluctuations. One method is to retain the electrostatic potential $\Phi(\underset{\sim}{r}t)$, and the associated correlation function, and to introduce the two additional vector potential fields $A_\parallel(\underset{\sim}{r}t)$ and $a(\underset{\sim}{r}t)$.

The parallel vector potential $A_\parallel(\underset{\sim}{r}t)$ describes electromagnetic perturbations in which the perpendicular component of the electric field remains irrotational, $\nabla \times \underset{\sim}{E}_\perp = 0$, and consequently, there is no compression in the magnetic perturbation, $\delta B_\parallel = 0$. The fluctuating $A_\parallel(\underset{\sim}{r}t)$ therefore produces a magnetic field $\delta\underset{\sim}{B}_\perp = \nabla \times (A_\parallel\hat{z}) = \nabla A_\parallel \times \hat{z}$ and an inductive parallel electric field $E_\parallel^T = -\partial A_\parallel/c\partial t$. The second component of the vector potential $a(\underset{\sim}{r}t)$ is perpendicular to $\underset{\sim}{B}_0$ and produces the rotational electric field $\underset{\sim}{E}_\perp = \hat{z} \times \nabla_\perp a$ and the compressional magnetic field $\delta B_\parallel = (ic/\omega)\nabla_\perp^2 a$.

The problem of drift instabilities is formulated in terms of these potentials by Rosenbluth and Sloan (1971) and by Berk and Dominguez (1977). Berk and Dominguez give the complete form of the low frequency conductivity tensor for high beta system with background particle distributions of the form $f_j(\varepsilon,x)$.

The second commonly used representation for the electromagnetic fluctuations is obtained by choosing $E_y(r,t)$, $B_x(\underset{\sim}{r}t)$ and $\delta B_\parallel(\underset{\sim}{r},t)$ as the independent field components and using the homogeneous Maxwell equations to derive the other three fields E_x, $E_\parallel$ and B_y. In $\underset{\sim}{k}\omega$ space these relations are

$$
\begin{aligned}
E_x(k) &= \frac{k_x}{k_y} E_y(k) - \frac{\omega}{k_y c} \delta B_\parallel(k) \\
E_\parallel(k) &= \frac{k_\parallel}{k_y} E_y(k) + \frac{\omega}{k_y c} B_x(k) \qquad (117) \\
B_y(k) &= -\frac{k_\parallel}{k_y} \delta B_\parallel(k) - \frac{k_x}{k_y} B_x(k)
\end{aligned}
$$

This representation of the fields is used for drift modes by Catto, et al. (1974) and Tange, et al. (1979), and it is used here.

4.1. Electromagnetic Formulas for Particle and Thermal Transport

The anomalous flux of particles, momentum, and thermal energy are given by the pair correlations between fluctuating thermodynamic particle variables and electromagnetic field components, such as $\langle \tilde{n}_j \tilde{E}_y \rangle$. That only such pair correlations are required follows from the fact that the interaction term in the Vlasov equation, $(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B}) \cdot \partial f/\partial \underset{\sim}{v}$, is bilinear in the fields and particles. It follows from this bilinear feature that general expressions for the anomalous fluxes correct to first order in the field correlation functions are obtained from linear relations between the fluctuating thermodynamic variables and the electromagnetic fields. The fluctuating thermodynamic variables are calculated from the fluctuating particle distributions.

For a background distribution function of the form $f_j(\varepsilon, X)$, where $\varepsilon = \frac{1}{2} v^2$ is the energy per unit of mass and $X = x + (v_y - v_{Dj})/\omega_{cj}$ is the guiding center coordinate in the nonuniform plasma, the calculation of the fluctuating particle distributions follows from integrating the perturbing force $[\underset{\sim}{E}(\underset{\sim}{r}'t') + \underset{\sim}{v}' \times \underset{\sim}{B}(\underset{\sim}{r}'t)] \cdot \partial f_j/\partial \underset{\sim}{v}'$ along the particle orbits. Including the ∇B-guiding center drift of the particles, the calculation [Catto, et al. (1974) and Berk and Dominguez (1977)] of the orbit integral yields

$$\delta f_{jk}(\underset{\sim}{v}) = \frac{i e_j E_y(k)}{m_j k_y} \frac{\partial f_j}{\partial \varepsilon} + \left(\frac{e_j \omega}{m_j k_y} \frac{\partial f_j}{\partial \varepsilon} + \frac{c}{B} \frac{\partial f_j}{\partial x} \right) g_k^j(v_\parallel, v_\perp)$$

$$\times \quad \exp(i\underset{\sim}{k} \times \underset{\sim}{v} \cdot \hat{z}/\omega_{cj}) \left[J_0 E_y(k) + \frac{v_\parallel}{c} J_0 B_x(k) - \frac{i v_\perp}{c} J_1 \delta B_\parallel(k) \right]$$

(118)

where the particle propagator is

$$g^j_{\underset{\sim}{k}\omega}(v_\parallel, v_\perp) = \frac{1}{\omega - k_y v_{Dj} - k_\parallel v_\parallel + i\nu_k} \tag{119}$$

as introduced in Sec. 3.2. The Bessel functions J_0 and J_1 in Eq. (118) have arguments $k_\perp v_\perp/\omega_{cj}$ where $\omega_{cj} = e_j B/m_j c$. The guiding center drift velocity is $\underset{\sim}{v} = v_{Dj}\hat{y} + v_\parallel \hat{z}$ where $v_{Dj} = \frac{1}{2}(v_\perp^2/\omega_{cj})(d \ln B/dx)$. In the limit of electrostatic turbulence Eq. (118) reduces to formula (12) for $\delta f_{jk}(\underset{\sim}{v})$ used in Secs. 2 and 3. The same discussion concerning the role of $i\nu_k$ in the particle propagator given in Sec. 2.1 and 3.2 pertains here, but the electromagnetic formula for $i\nu_{\underset{\sim}{k}\omega}$ requires a generalization of the previous formulas (76) and (97) or the Markovian limit (77).

The fluctuating particle density and currents,

$$\tilde{n}_{jk} = \int d\underset{\sim}{v}\; \delta f_{jk}(\underset{\sim}{v}), \qquad \underset{\sim}{\Gamma}_{jk} = \int d\underset{\sim}{v}\; \underset{\sim}{v}\; \delta f_{jk}(\underset{\sim}{v}), \tag{120}$$

and the fluctuating thermal energy density,

$$\frac{3}{2}(n_j\tilde{T}_j)_k = \int d\underset{\sim}{v}\ \frac{1}{2}\ m_j v^2 \delta f_{jk}(\underset{\sim}{v}), \tag{121}$$

are readily computed from Eq. (118) for $\delta f_{jk}(\underset{\sim}{v})$.

In particular, the fluctuating thermodynamic variables required for the transport theory are

$$\tilde{n}_j(k) = \frac{i e_j E_y(k)}{m_j k_y}\int d\underset{\sim}{v}\ \frac{\partial f_j}{\partial \varepsilon} + \int d\underset{\sim}{v}\left(\frac{e_j}{m_j k_y}\ \frac{\partial f_j}{\partial \varepsilon} + \frac{c}{B}\ \frac{\partial f_j}{\partial x}\right)$$

$$\times\ g_k^j(v) J_0\left(\frac{k_\perp v_\perp}{\Omega_j}\right) F_y^j(k, v_\perp, v_\parallel) \tag{122}$$

$$\tilde{\Gamma}_x(k) = \int d\underset{\sim}{v}\left(\frac{e_j\omega}{m_j k_y}\ \frac{\partial f_j}{\partial \varepsilon} + \frac{c}{B}\ \frac{\partial f_j}{\partial x}\right) g_k^j(\underset{\sim}{v})\, i v_\perp J_1\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) F_y^j(k, v_\perp, v_\parallel) \tag{123}$$

$$\tilde{\Gamma}_\parallel(k) = \int d\underset{\sim}{v}\left(\frac{e_j}{m_j k_y}\ \frac{\partial f_j}{\partial \varepsilon} + \frac{c}{B}\ \frac{\partial f_j}{\partial x}\right) g_k^j(\underset{\sim}{v})\, v_\parallel J_0\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) F_y^j(k, v_\perp, v_\parallel) \tag{124}$$

where F_y^j is the gyro-orbit average of the y component of the Lorentz force

$$F_y^j(k, v_\perp, v_\parallel) = J_0\left(\frac{k_\perp v_\perp}{\Omega_j}\right) E_y(k) + \frac{v_\parallel}{c} J_0\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) B_x(k) -$$

$$-\ \frac{i v_\perp}{c}\ J_1\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) \delta B_\parallel(k). \tag{125}$$

The fluctuating thermal energy density is given by

$$\frac{3}{2}(\tilde{n_j T_j})_k = -\frac{i3e_j n_j E_y(k)}{2k_y}$$

$$+\int d\underset{\sim}{v}\left(\frac{e_j\omega}{m_j k_y}\frac{\partial f_j}{\partial \varepsilon}+\frac{c}{B}\frac{\partial f_j}{\partial x}\right)\left(\frac{m_j v^2}{2}\right) g_k^j(\underset{\sim}{v}) J_0 F_y^j(k, v_\perp, v_\parallel). \tag{126}$$

In Eq. (122) the local contribution reduces according to $\int d\underset{\sim}{v}\ \partial F_j/\partial\varepsilon = -\frac{1}{2}n_j\langle 1/v^2\rangle$, which for a Maxwell-Boltzmann distribution is $-n_j/T_j$. A similar integration is performed in the first term in Eq. (126).

For the case of a Maxwell-Boltzmann distribution with inhomogeneous density $n_j(x)$, temperature $T_j(x)$ and parallel drift velocity $u_\parallel(x)$ the velocity integrals in Eqs. (122) - (126) are performed in terms of modified Bessel functions $I_n(b)\exp(-b)$ with $b = k_\perp^2 T_j/m_j\Omega_j^2$ and the plasma dispersion function $Z[(\omega - k_\parallel u_\parallel)/|k_\parallel|v_j]$ with $v_j = (2T_j(x)/m_j)^{1/2}$. The formulas that result for the local Maxwell-Boltzmann are given by Tange, et al. (1979).

In the general case of electromagnetic fluctuations the complete set of anomalous transport processes are too complicated to present here. For general expressions the reader is referred to Tange, et al. (1979). The formulas are derived under the same conditions discussed in Sec. 2. To summarize, we assume that (1) the crossfield transport is produced by the macroscopic gradients, (2) the macroscopic inhomogeneity is weak in terms of the mean gyroradius, $\rho_j \nabla n/n \ll 1$, $\rho_j \nabla T/T \ll 1$ and (3) the space-time scales for transport $\partial \ln n_j/\partial t$, $\nabla \cdot \underset{\sim}{\Gamma}_j/n_j$ are well separated from the space-time scales of fluctuations, $1/\Delta\omega$ and $1/\Delta k$. When the last condition is satisfied the averages, such as $\langle n_j E_y \rangle$, are well-defined when taken on a space-time scale small compared to the macroscopic variations and large compared to those characteristic of the fluctuations.

The basic result for the anomalous particle flux is obtained from the space-time average of particle momentum balance in the direction mutually perpendicular to the ambient magnetic field and the macroscopic gradient.

After assessing the order of the averaged inertia terms and the divergence of the momentum-stress tensor as sub-dominant to the y-component of the Lorentz force in the momentum balance equation for particle species j, the basic result

$$\langle (\tilde{n}_j \underset{\sim}{\tilde{E}}_y + \frac{1}{c} \tilde{\Gamma}_j \quad \underset{\sim}{\tilde{B}}) \rangle - \frac{1}{c} \Gamma_{xj} B = - \frac{m_j}{e_j} \int v_y C_j(f,f) d\underset{\sim}{v} \tag{127}$$

follows where $m_j \int v_y C_j d\underset{\sim}{v}$ is the collisional change in the momentum. The well-known (Hinton, Chapter I.6) contribution of the particle-particle collisions is retained here to show clearly when the turbulent transport dominates. According to Eq. (127), there is a net loss of y-momentum in each species by the effective turbulent friction of those particles with the collective fluctuations and particle-particle collisions. This loss of momentum is balanced by a steady radial transport Γ_{xj} of that species across the ambient magnetic field. For example, electron-ion collisions contribute to the momentum loss of electrons through $m_e \int v_y C_{ei} d\underset{\sim}{v} \cong -m_e(\bar{v}_{ye} - \bar{v}_{yi})/\tau_e$ where the equilibrium diamagnetic drift is $\bar{v}_{ye} - \bar{v}_{yi} = -(c/eBn_e)(dp/dx)$ and $\tau_e^{-1} = \nu_e = 4(2\pi)^{1/2} n_e e^4 Z_{eff} \ln\Lambda / 3m_e^{1/2} T_e^{3/2}$. The collisional increment to the transport is then

$$\Gamma_{coll} = -\nu_e \frac{m_e c^2}{e^2 B^2}(T_e + T_i)\frac{dn}{dx} . \qquad (128)$$

Throughout the present Chapter the amplitude of the turbulent fluctuations is assumed sufficient to produce an "anomalous" or collisionless transport which dominates that due to the direct particle collisions.

The anomalous particle transport in the direction of the macroscopic gradient is

$$\Gamma_j = \frac{c}{B} \langle (\tilde{n}_j \tilde{E}_y + \frac{1}{c}\,\tilde{\Gamma}_{j\parallel}\tilde{B}_x - \frac{1}{c}\,\tilde{\Gamma}_{jx}\delta\tilde{B}_\parallel) \rangle$$

where the density fluctuation $\tilde{n}_j$ and the fluctuating currents $\tilde{\Gamma}_x$, $\tilde{\Gamma}_\parallel$ are given in terms of the first order field fluctuations by Eqs. (122), (123) and (124). Calculating the space-time averages by neglecting any weak correlation between different $\underset{\sim}{k}\omega$ modes, the anomalous electromagnetic particle flux reduces to

$$\Gamma_j = \frac{c}{B} \int d\underset{\sim}{k}\, d\omega \int d\underset{\sim}{v} \left(\frac{e_j \omega}{m_j k_y} \frac{\partial F_j}{\partial \varepsilon} + \frac{c}{B} \frac{\partial F_j}{\partial x} \right) \mathrm{Im}\, g_k^j(v_\parallel, v_\perp)$$

$$\times \left| J_0 E_y(k) + \frac{v_\parallel}{c} J_0 B_x(k) - \frac{i v_\perp}{c} J_1 \delta B_\parallel(k) \right|^2. \tag{129}$$

The dominant processes in the thermal balance equation are the generalized $c\underset{\sim}{E} \times \underset{\sim}{B}/B^2$ convective transport, the parallel motion of the particles along the bending magnetic field described by $\underset{\sim}{B}+\delta \underset{\sim}{B}_\perp$, and the $<\underset{\sim}{j} \cdot \underset{\sim}{E}>$ power exchange between the particles and the fluctuations. Equation (129) may be viewed as a sum over the energy spectrum of a set of independent cross-field transport processes each having its own strength according to $I_{\underset{\sim}{k}\omega}$, $F_j(\varepsilon, X)$ and $\mathrm{Im}\, g_k^j(\underset{\sim}{v})$. From this view it is evident that depending on whether the dominant contributions to Γ_j arise from particles that are subthermal or superthermal there is a net energy flux Q_j that is smaller than or greater than $3/2 T_j \Gamma_j$ where $3/2 T_j$ is the mean particle energy $<\frac{1}{2} m_j v^2>$. The total energy flux Q_j is given by

$$Q_j = \frac{c}{B} \int d\underset{\sim}{k}\, d\omega \int d\underset{\sim}{v} \left(\frac{e_j \omega}{m_j k_y} \frac{\partial F_j}{\partial \varepsilon} + \frac{c}{B} \frac{\partial F_j}{\partial x} \right) \left(\frac{1}{2} m_j v^2 \right) \mathrm{Im}\, g_k^j(\underset{\sim}{v})$$

$$\left| J_0 E_y(k) + \frac{v_\parallel}{c} J_0 B_x(k) - \frac{i v_\perp}{c} J_1 \delta B_\parallel(k) \right|^2 \tag{130}$$

and the conduction q_j of energy relative to the convective transport $\frac{3}{2} T_j \Gamma_j$ is defined by $q_j = Q_j - \frac{3}{2} T_j \Gamma_j$. Finally, resonant transfer of energy between the particles and the fluctuations is given by

$$\dot{W}_j = -\langle \underset{\sim}{j}_j \cdot \underset{\sim}{E} \rangle = \frac{e_j^2}{m_j} \int d\underset{\sim}{k} d\omega \int d\underset{\sim}{v} \left(\frac{\partial F_j}{\partial \varepsilon} + \frac{k_y}{\omega \Omega_j} \frac{\partial F_j}{\partial x} \right) \frac{\omega^2}{k_y^2} \text{ Im } g_k^j(v)$$

$$\left| J_0 E_y(k) + \frac{v_\parallel}{c} J_0 B_x(k) - \frac{i v_\perp}{c} J_1 \delta B_\parallel (k) \right|^2 . \qquad (131)$$

The particle and thermal balance equations are the same as given in Eqs. (22) and (23) of Sec. 2.2.

Formulas (129)-(131) show that the anomalous flux is proportional to the product of the fluctuation spectrum at $\underset{\sim}{k}\omega$ times the gradient of particle distribution at (x,v) weighted by the strength of the fluctuation-particle interaction given by Im $g_k^j(\underset{\sim}{v})$. The Bessel function $J_0(k_\perp v_\perp/\Omega_j)$ gives the gyroradius average of $E_y(\underset{\sim}{r}')$ and $B_x(\underset{\sim}{r}')$ along the particle orbit whereas the Bessel function $J_1(k_\perp v_\perp/\Omega_j)$ gives the average of $\delta B_\parallel$ over the area of the gyro-orbit or, equivalently the line integral $\oint \underset{\sim}{E}_\perp(\underset{\sim}{r}) \cdot d\underset{\sim}{l}$ around the orbit.

For a given magnetic fluctuation spectrum arising from an external mechanism, the anomalous particle flux from Eq. (129) is not ambipolar. Typically, the flucutating parallel electron current dominates reducing the formula to

$$\Gamma_e \simeq - \frac{\langle \tilde{B}_x \tilde{j}_{\parallel e} \rangle}{eB} = \int dk \int d\underset{\sim}{v} v_{\parallel}^2 \mathrm{Im} g_k^e (\underset{\sim}{v}) \left| \frac{B_x(k)}{B} \right|^2 \frac{\partial F_e}{\partial x} .$$

The electron loss leads to a net positive charge of the plasma calculated in Section 4.3. In contrast, when the electromagnetic turbulence arises from self-consistent flucutations satisfying the Vlasov-Maxwell equations, the particle flux averaged over the fluctuations is ambipolar. From Eq. (129), we have

$$\sum_j e_j \Gamma_j = \frac{[\langle \tilde{j}_{\parallel} \tilde{B}_x \rangle - \langle \tilde{j}_x \delta \tilde{B}_{\parallel} \rangle]}{B} = - \frac{c}{4\pi B} [\langle \tilde{B}_x \partial_y \tilde{B}_x \rangle + \langle \delta B_{\parallel} \partial_y \delta B_{\parallel} \rangle]$$

$$= 0$$

where we use Ampere's law and change neutrality $\tilde{\rho}_Q = \sum_j e_j \tilde{n}_j = 0$. The translational invariance of the equilibrium implies that both the stability $\gamma_{\underset{\sim}{k}}$ and the anomalous fluxes Γ_j, Q_j, $\dot{W}_j$ are independent of a Galilean transformation by $v_y \to v_y + V$, whereas the fluctuation frequencies and the radial electric field change according to $\omega \to \omega - k_y V$ and $E_x \to E_x + (V/c)B$. In cylindrical geometry these transformation properties are valid to the extent that the centrifugal acceleration V_θ^2/r remains neglible, $\Delta\omega_c = k_\theta (cm_i V_\theta^2/erB) < \Delta\omega_k$.

For a Maxwell-Boltzmann background distribution $f_0 = F_j^M(\varepsilon,x)$ with a nonuniform density and temperature the characteristic frequency

$$\omega_{*j}(\varepsilon) = k_y \frac{cT_j}{e_j B} \frac{1}{n_j} \frac{dn_j}{dr}[1 + \eta_j(\varepsilon/T_j - \frac{3}{2})]$$

measures the effect of the nonuniformity. In this case the formulas for Γ_j, $q_j - \frac{3}{2} T_j \Gamma_j$ and $\dot{W}_j$ simplify to

$$\Gamma_j = \frac{e_j c}{B} \int d\underset{\sim}{k}\, d\omega \int d\underset{\sim}{v}\, F^M(\varepsilon) \frac{[\omega - \omega_{*j}(\varepsilon)]}{k_y} \text{Im } g^j(\underset{\sim}{v}) |F_y^j(k,v_\perp,v_\parallel)|^2, \quad (132)$$

$$q_j = \frac{ce_j T_j}{B} \int d\underset{\sim}{k}\, d\omega \int d\underset{\sim}{v}\, F_j^M(\varepsilon) (\frac{\varepsilon}{T_j} - \frac{3}{2}) \frac{[\omega - \omega_{*j}(\varepsilon)]}{k_y} \text{Im } g^j(\underset{\sim}{v}) |F^j(k,v_\perp,v_\parallel)|^2 \quad (133)$$

and

$$\dot{W}_j = \frac{e_j^2}{T_j} \int d\underset{\sim}{k} d\omega \int d\underset{\sim}{v}\, F_j^M(\varepsilon) \frac{\omega}{k_y} [\omega - \omega_{*j}(\varepsilon)] \text{Im } g_k^j(\underset{\sim}{v}) |F_y^j(k,v_\perp,v_\parallel)|^2 . \quad (134)$$

The condition for the background distribution to be close to a Maxwellian-Boltzmann is found by following Sec. 2.7.

Now we reconsider for the high pressure plasma the entropy production functional σ_j introduced in Sec. 2.2. For the Maxwell-Boltzmann background distribution the entropy density is $s_j(\underset{\sim}{r}t) = -n_j \ln(n_j/T_j^{3/2})$ and the local rate of change is computed from the transport Eqs. (22) and (23). Apart from the external particle and thermal sources introduced in Sec. 2.2, the rate of entropy production is given by

$$\sigma_j = -\sum J_\alpha X_\alpha = -\frac{\Gamma_j}{n_j}\frac{dn_j}{dx} - \frac{q_j}{T_j^2}\frac{dT_j}{dx} - \frac{\dot{W}_j}{T_j} .$$

Upon using Eqs. (132)-(134) the entropy production reduces to

$$\sigma_j = -\frac{e_j^2}{T_j}\int dk \int d\underset{\sim}{v}\, F_j^M(\varepsilon)\, \frac{[\omega - \omega_{*j}(\varepsilon)]^2}{k_y^2} |F_y^j(k,\underset{\sim}{v})|^2 \mathrm{Im}\, g_k^j(\underset{\sim}{v}) . \quad (135)$$

Now we consider the transport in problems with a substantial level of magnetic fluctuations.

4.2 Polarization Relations for Low Frequency Electromagnetic Fluctuations

At moderate plasma pressure, $\beta < 1$, there are general polarization features of the low frequency, drift wave and tearing mode type of fluctuations that simplify the transport analysis. In contrast, for $\beta \gtrsim 1$, all three electric field components are coupled in a complicated manner; however, the drift-type modes are then unstable only under rather special conditions, such as the $\eta = d\ln T/d\ln = -\frac{1}{2}$ mode found by Mikhailovskii and Fridman (1967).

In moderate pressure plasma the highly elongated fluctuations $k_\perp \gg k_\parallel$ of the drift wave and tearing mode type decouple from the fast compressional motion with $\omega^2 \ll k^2 v_A^2$. The polarization of the slow convective modes is determined by the total pressure balance across the magnetic field

$$B\delta B_\parallel(k) + 4\pi\delta p_\perp(k) = 0$$

that is maintained on the scale $k_\perp^{-1}$ during the slow oscillations. From this relation and the formula for $\delta p_\perp(k)$ the magnitude of the magnetic compression follows as

$$\frac{\delta B_\parallel(k)}{B} = \frac{1}{2}\beta\left[\frac{ieE_y(k)}{k_y T}\right] \cong \frac{1}{2}\beta\left[\frac{e\varphi(k)}{T}\right] \tag{136}$$

which is valid for modes rotating in either the ion or the electron diamagnetic direction. The change in the magnetic field strength $\delta B_\parallel(k)$ ensures that there is no compression of the plasma $\nabla \cdot \underset{\sim}{v}_E = 0$ in the convection of the plasma across the finite beta equilibrium with $B^2(r) + 8\pi p_\perp(r) = \text{const.}$ In addition to the diamagnetic drifts v_{dj} defined in Sec. 2, the guiding centers drift with the velocity $\underset{\sim}{v}_{Dj} = (m_j c v_\perp^2/2e_j B)(d\ln B/dx)\hat{y}$ which also influences the transport. In the absence of strong magnetic field curvature we observe that $\langle v_{Dj}\rangle = (cp_{\perp j}/e_j B^2)(dB/dx) = -\frac{1}{2}\beta_j v_{dj}(1+\eta_j)$.

The perpendicular magnetic perturbation $\delta B_\perp(k)$ considerably exceeds $\delta B_\parallel(k)$ and takes on the magnitude and phase required to short out the parallel electric field, $E_\parallel(k) = -ik_\parallel\varphi(k) - i(\omega/c)A_\parallel(k) \approx 0$. The important case where the inductive part of the parallel electric field is not able to cancel the parallel electric field occurs when the $\underset{\sim}{k}\omega$ components are such as to incur an anomalously low parallel conductivity $\sigma_\parallel(\underset{\sim}{k},\omega)$. The fluctuations for which the parallel conductivity is low are $\omega \sim \omega_{*e}$ and $k_\parallel v_e \gtrsim \omega$ since from the electron current $\Gamma_\parallel(k)$,

$$\sigma_\parallel(k) \cong \frac{i m_e n_e e^2}{2k_\parallel^2 T_e}(\omega - \omega_{*e})Z'\left(\frac{\omega}{|k_\parallel|v_e}\right) , \tag{137}$$

from Eq. (124). For fluctuations in this domain it follows from $\nabla \times \delta \underset{\sim}{B}_\perp = 4\pi\delta j_\parallel/c$ that the perpendicular magnetic perturbation is

$$\frac{\delta B_\perp(k)}{B} \cong i\left(\frac{\beta}{2}\right)^{1/2}\left(\frac{\omega_{*e}}{k_\parallel v_A}\right)k_\perp\rho_i\left[\frac{e\varphi(k)}{T_e}\right] \tag{138}$$

where $E_y(k) \cong -ik_y\varphi(k)$ is used.

In characterizing the moderate β drift-tearing mode fluctuations it must be observed that the field line bending from $\delta B_\perp(k)$ couples the parallel transport into a radial motion. As an example, we observe that in the absence of line bending the electron temperature fluctuation determined for $\nu_e > k_\| v_e$ from the thermal balance equation with $q_{\| e} = -\chi_{\| e} \nabla_\| T_e$ where $\chi_{\| e} = n_e T_e / m_e \nu_e$ yields

$$\frac{\delta T_e(k)}{T_e} = \left(\frac{i\nu_e}{k_\|^2 v_e^2}\right) [\omega - \omega_{*e}(1 - \frac{3}{2}\eta_e)] [\frac{e\varphi(k)}{T_e}] = \left[\frac{\delta T_e(k)}{T_e}\right]_{conv} \qquad (139)$$

from the balance of convection and parallel diffusion. In the presence of line bending, however, the thermal transport along the field produces the fluctuation

$$\frac{\delta T_e(k)}{T_e} = \frac{-1}{ik_\|} \frac{B_x(k)}{B} \frac{1}{T_e} \frac{dT_e}{dx} + \left[\frac{T_e(k)}{T_e}\right]_{conv} \qquad (140)$$

from the balance equation. For $B_x(k)/B \sim k_\| r_n [e\varphi(k)/T_e]$ the temperature fluctuation from the parallel transport exceeds that due to convection. At the magnetic amplitude $\delta B_\perp(k)$ where this occurs the parallel inductive field $(\omega/c) A_\|(k)$ due to $\delta B_\perp(k) = ik_y A_\|(k)$ is such that

$$E_\|^{(A)} = \frac{i\omega}{c} A_\|(k) \simeq \left(\frac{\omega}{\omega_{*e}}\right) [-ik_\| \varphi(k)].$$

The opposite parity in $k_\|$ of $A_\|(k)$ and $\varphi(k)$ is a general feature of the low frequency fluctuations.

The polarization relations for the low frequency drift and tearing mode fluctuations at $\beta < 1$ allow the particle and energy flux formulas to be simplified. First consider the contribution of $\delta B_{\parallel}(k)$ to the Lorentz force $F_y^j(k, v_\perp, v_\parallel)$ in Eq. (125). Using Eq. (136) to relate $E_y(k)$ and $\delta B_\parallel(k)$ we observe that $F_y^j \simeq E_y(k)\,[J_0 + i\beta_j J_1/(k_\perp \rho_j)] \simeq E_y(k) J_0(k_\perp v_\perp/\omega_{cj})$ for $\beta_j < 1$ and arbitrary $k_\perp \rho_j$. Now, consider the contribution to F_y^j due to $\delta B_\perp(k)$. For the $\delta B_\perp(k)$ given in Eq. (138) it follows that $F_y^j \cong E_y J_0(1 + v_\parallel/v_A)$ when $\omega_{*e} \sim k_\parallel v_A$. For the plasma pressure $\beta < 1$ the Alfvén speed is intermediate to the ion and electron thermal velocities, so that the magnetic force is small on the ions with $F_y^i \cong J_0 E_y$ and dominant on the electrons $F_y^e \cong (v_\parallel/c) B_x(k)$.

Taking into account these simplifications we have

$$\Gamma_i = \sum_k \left|\frac{cE_y(k)}{B}\right|^2 \int d\underset{\sim}{v}\left(\frac{\partial f_i}{\partial x} + \frac{\omega\Omega_i}{k_y}\frac{\partial f_i}{\partial \varepsilon}\right) J_0^2\left(\frac{k_\perp v_\perp}{\Omega_i}\right) \mathrm{Im}\; g_k^i(\underset{\sim}{v}) \tag{141}$$

$$\Gamma_e = \sum_k \left|\frac{B_x(k)}{B}\right|^2 \int d\underset{\sim}{v}\left(\frac{\partial f_e}{\partial x} + \frac{\omega\Omega_e}{k_y}\frac{\partial f_e}{\partial \varepsilon}\right) v_\parallel^2 \mathrm{Im}\; g_k^e(\underset{\sim}{v}), \tag{142}$$

and the energy flux Q_j is obtained by including the kinetic energy $\frac{1}{2} m_j v^2$ under the velocity integral. For fluctuations with sufficiently low frequencies,

$$|\omega| \ll \frac{k_y \langle v^2 \rangle}{2\Omega_j} \left\langle \frac{d \ln f_j}{dx} \right\rangle ,$$

the anomalous flux is driven directly by $\partial f_j/\partial x$.

4.3. Qualitative Picture of Anomalous Transport Due to Magnetic Fluctuations

Low frequency fluctuations near a neutral sheet such as in the geomagnetic tail plasma, e.g. Galeev (1978), or in the laboratory near the mode rational surfaces of the helical magnetic fields used for confinement, e.g. Callen (1977), produce parallel inductive electric fields $E_{\|} = -\partial A_{\|}/c\partial t$ and perpendicular magnetic fields $\delta \underset{\sim}{B}_{\perp} = \nabla A_{\|} \times \hat{z}$. Such magnetic perturbations lead to anomalous transport. According to the quasilinear formula (142) the electron transport is proportional to $\langle |\delta B_x(k)|^2 \rangle$ and the interaction strength $\text{Im}\, g_k^e(\underset{\sim}{v})$. For time dependent magnetic fluctuations Callen describes the process as transport due to "magnetic flutter." Even in the case of static magnetic perturbations with $\langle |\delta B_{\perp}(k)|^2 \rangle = \langle |\delta B_{\perp}(\underset{\sim}{k})|^2 \rangle \delta(\omega)$, there is an enhanced transport due to the particle motion along the magnetic field line. The anomalous transport produced by the static magnetic field perturbations is described by Stix (1973) as transport due to "magnetic braiding."

Stix points out that a magnetic perturbation $\delta \underset{\sim}{B}_{\perp}(\underset{\sim}{x})$ that is resonant with a helical magnetic field produces a chain of magnetic islands of width Δ_k about the resonant surface where

$\underset{\sim}{B}\cdot\nabla = ik_{\parallel}B = 0$. Writing $\underset{\sim}{B} = B_y(x)\hat{y} + B_0\hat{z}$ for the local magnetic field and $L_s^{-1} = dB_y/B\,dx$ for the local shear, the width Δ_k of the magnetic island is given by

$$\Delta_k = \left(\frac{2L_s}{k_y}\frac{\delta B_x}{B_0}\right)^{1/2} . \tag{143}$$

When two or more of these chains of islands occur as shown in Fig. 10 the magnetic field becomes ergodic first along the separatrix and then throughout the region between the islands with increasing amplitude of the magnetic perturbation. Owing to their high speed the electrons follow the braided field lines to produce a transport across the ambient helical field. The magnetic perturbations arise from a variety of reasons such as filamentation of an Ohmic heating current, electromagnetic fields from auxiliary heating methods or collective instabilities in the plasma.

The low frequency drift waves and tearing modes which arise from the gradient and magnetic energy density in non-uniform plasmas produce small scale magnetic perturbations. For example, Callen (1977) suggests that the drift wave fluctuations measured by Mazzucato (1976) have sufficient amplitude to produce a substantial anomalous thermal transport due to their magnetic component.

For finite beta drift wave turbulence, the case emphasized by Callen (1977), $\delta B_x(x)$ vanishes at the resonant surface, rises to a maximum of δB_{xA} at $k_\parallel v_A \simeq \omega_{*e}$, and decreases as $\delta B_{xA}(x_A/x)$ for $k_\parallel v_A > \omega_{*e}$. Here, x_A is the distance from the resonant surface $k_\parallel = 0$ to the region where $k_\parallel v_A = k_y v_A x_A/L_s = \omega_{*e}$ which is $x_A = \rho\beta^{1/2}(L_s/r_n)$. For a drift wave island width $\Delta_k^{dw} > x_A$, Callen finds

$$\Delta_k^{dw} \simeq \left(\frac{L_s x_A}{k_y}\frac{\delta B_{xA}}{B_0}\right)^{1/3} ,$$

and the distance L_k along the magnetic field to circumnavigate the magnetic island

$$L_k = \frac{L_s}{k_y \Delta_k} \gg L_s .$$

Since L_k is greater than L_s the drift wave frequency $\omega_{\underset{\sim}{k}}$ resonates with the electron transit frequency around the island $\omega_k \cong v_e/L_k$. Including the buildup of an ambipolar potential $\Phi(x)$ to equalize the ion and electron loss rates, Callen (1977) estimates that the electron diffusion and ambipolar potential are given by

$$\Gamma_e \simeq - \frac{n_e}{2}\sum_k \Delta_k^2(\nu_e + \gamma_k)\left[\frac{d\ln n_e}{dx} - \frac{e}{T_e}\frac{d\Phi}{dx} - \frac{\omega e B}{c k_y T_e}\right] \simeq 0 \qquad (144)$$

$$\frac{e}{T_e}\frac{d\Phi}{dx} = \left(1 - \frac{\omega_{\bar{k}}}{\omega_{*e}}\right)\frac{d\ln n_e}{dx} \approx \frac{d\ln n_e}{dx}$$

or essentially electrostatic confinement of the electrons. The electron thermal transport

$$Q_e = -\chi_e \frac{dT_e}{dx}$$

is estimated by

$$\chi_e = \frac{3n_e}{4} \sum_k \Delta_k^2(\nu_e + \gamma_k) \tag{145}$$

where ν_e is the electron collision frequency and $1/\gamma_k$ is the correlation time or the life time of the magnetic island Δ_k. Equation (145) is a nonlinear generalization of the quasilinear formula in which the island width

$$\Delta_k \propto \begin{cases} \left[\dfrac{\delta B_{xA}}{B}\right]^{1/3} & \text{for drift waves} \\ \\ \left[\dfrac{\delta B_x(0)}{B}\right]^{1/2} & \text{for tearing modes} \end{cases} \tag{146}$$

replaces the linear radial excursion $v_\parallel B_x(k)/B\Delta(\omega - \underset{\sim}{k} \cdot \underset{\sim}{v}) = [B_x(k)/B]v_\parallel \tau_c$ that occurs in the quasilinear theory, Eq. (142).

The magnitude of the anomalous transport produced by magnetic braiding depends sensitively on the correlation time of the electron with the magnetic perturbation $\delta \underset{\sim}{B}_\perp$, as well as the obvious amplitude dependence. Rosenbluth and Rechester (1978) introduce the methods of stochastic trajectories to obtain formulas that describe the finite correlation time τ_c or length L_c along the field line.

First the statistical properties of the magnetic field in the presence of the stochastic perturbations $\delta\underset{\sim}{B}_\perp = \sum_k \delta B_k \exp(i\underset{\sim}{k} \cdot \underset{\sim}{x})$ are established. Rechester and Rosenbluth introduce the stochasticity parameter

$$s = \frac{1}{2}(\Delta_{k_1} + \Delta_{k_2})/|(r_{k_1} - r_{k_2})| \tag{147}$$

that compares the total width of two magnetic islands with their radial separation, Fig. 10. When this parameter exceeds unity the magnetic surfaces are destroyed between the chains of islands. In this region where the field lines wander ergodically, a small circle of radius ℓ_0 is mapped by following the field line into a complex, elongated shape that has the same area as the initial circle since $\nabla \cdot \underset{\sim}{B} = 0$. The mapping of the circle by the magnetic field is shown in Fig. 11. The stretching of a typical side of this area is given by

$$\ell(z) = \ell_0 \exp(z/L_c) \tag{148}$$

where $L_c = \pi R/\ln(\pi s/2)$ which is called the stochastic instability of trajectories. To conserve area the width

$\delta(z)$ of the area must decrease according to $\delta(z) = \ell_0 \exp(-z/L_c)$ as shown in Fig. 11b. When the elongation $\ell(z)$ exceeds the cross field correlation distance δ different regions separated by δ evolve independently as shown in Fig. 11c. This condition, $\ell(z) = \delta = 1/\Delta k_\perp$, defines the correlation length

$$L_{c0} = L_c \ln\left(\frac{\delta}{\ell_0}\right) \tag{149}$$

for the area. For $L \gg L_{c0}$ the magnetic mapping is a random walk with the average squared displacement of the area described by

$$\langle \Delta r^2 \rangle = 2D_F L \tag{150}$$

where

$$D_F = \pi \sum_{\underset{\sim}{k}} \left| \frac{B_x(\underset{\sim}{k})}{B} \right|^2 \delta(k_\parallel) \tag{151}$$

is the field diffusion coefficient.

Now consider the motion of a small patch of electrons whose guiding centers follow the magnetic field lines. The small patch of electrons of radius r_e move along the field mapping into the complicated thin filaments of width $r_e \exp(-z/L_c)$ until they suffer a collision at the average distance $z = \lambda_e = v_e/\nu_e$. At the time of the collision the initial patch of electrons has spread the average squared perpendicular distance $\langle\Delta r^2\rangle = 2D_F\lambda$ provided $\lambda > L_{c0}$. After the collision the thin filaments expand to r_e, and subsequently the new subsections of radius r_e repeat the process. This gives rise to the thermal diffusion

$$\chi_e = \frac{1}{2}\langle\Delta r^2\rangle\nu_e = v_e D_F \qquad (152)$$

valid for $\lambda_e \gg L_c \sim 1/\Delta k$. Rechester and Rosenbluth (1978) note that this transport applies to the electron thermal energy since the particle transport must be limited by an ambipolar potential.

In the collisional regime $\lambda_e < L_c$ the electrons change direction along the field line many times before the reaching uncorrelated regions. During the parallel diffusion $\Delta z^2 = \chi_{\parallel e} t$ the patch of electrons expands across the magnetic field at the rate $(\chi_{\perp e} t)^{1/2}$ by collisional diffusion while the field line mapping contracts the width of the patch according to $\exp(-z/L_c)$. The balance between these processes occurs when the width is of order $L_c(\chi_{\perp e}/\chi_{\parallel e})^{1/2}$. A small area of this radius becomes uncorrelated when $L_c(\chi_{\perp e}/\chi_{\parallel e})^{1/2} \exp(-z/L_c) \approx 1/\Delta k_\perp$ which yields the collisional correlation length

$$L_{c\delta} = L_c \ln\left[\frac{\delta}{L_c}\left(\frac{\chi_{\parallel e}}{\chi_{\perp e}}\right)^{1/2}\right]. \qquad (153)$$

The associated time for parallel diffusion $t_\delta = L_{c\delta}^2/\chi_{\parallel e}$ is the time during which a particle may be thought of as orbiting along a single field line before collisionally scattered to a new uncorrelated field line. The random walk of the electron patch proceeds with the step time t and mean radial step size $<\Delta r^2> = D_F L_{c\delta}$. For this collisional regime the thermal transport is

$$\chi_e = \frac{<\Delta r^2>}{t_\delta} = D_F\left(\frac{\chi_{\parallel e}}{L_{c\delta}}\right) \tag{154}$$

which is smaller than the collisionless χ_e in formula (152) by the factor λ_e/L_c. To within the logarithmic factor the transport in Eq. (154) is simply $\chi_e = \chi_{\parallel e}\langle B_x^2/B^2\rangle$ which follows from the fluid description of the transport due to magnetic turbulence as shown in the next section.

Rechester and Rosenbluth (1978) suggest that finite beta drift waves and tearing modes with fluctuation scales $k_\parallel R \sim 1$ and $k_\perp \rho_i \sim 1$ have sufficient amplitude in confinement experiments to be well into the regime of strong stochasticity where the field diffusion is $D_F \cong R<\delta B_x^2>/B^2$ and χ_e follows from Eqs. (152) for $\lambda_e > L_c$ and (154) for $\lambda_e < L_c$.

4.4 Electron Thermal Transport from Magnetic Fluctuations in the Fluid Approximation

Kadomtsev and Pogutse (1978) address the problem of calculating the anomalous electron transport from stochastic mangetic perturbations within the MHD approximation for thermal conduction. The basic transport law is taken as

$$\nabla \cdot \mathbf{q} = 0 \tag{155}$$

with the collisional formula for the local thermal flux

$$\mathbf{q} = -\chi_{\parallel}\mathbf{h}(\mathbf{h} \cdot \nabla T_e) - \chi_{\perp}\nabla T_e \tag{156}$$

where $\chi_{\parallel}$ exceeds $\chi_{\perp}$ by several orders of magnitude. The collisional values are approximately $\chi_{\parallel} = nT_e/m_e\nu_e$ and and $\chi_{\perp} = nT_e\nu_e/m_e\omega_{ce}^2$ and give the ratio $\chi_{\perp}/\chi_{\parallel} = (\nu_e/\omega_{ce})^2 = \gamma^2$ where γ is assumed small. The vector $\mathbf{h}$ along the magnetic field is written as the sum

$$\mathbf{h} = \hat{e}_z + \mathbf{b} \tag{157}$$

of the constant vector in the direction of the ambient magnetic field and small stochastic vector $\mathbf{b} = \delta\mathbf{B}_{\perp}/B$. It is seen from Eq. (156) that for $\langle b^2\rangle\chi_{\parallel} \sim \chi_{\perp}$ the contribution from the parallel transport is comparable to that from the cross field transport.

The stochastic magnetic field $\mathbf{b}(\mathbf{r}_{\perp} z)$ has $\nabla_{\perp} \cdot \mathbf{b} = 0$ since $\mathbf{B}_0$ = const. The stochastic field $\mathbf{b}$ is characterized by (1) the small root-mean-square value $b_0 = \langle b^2\rangle^{1/2}$, (2) isotropic properties in the plane perpendicular to the ambient magnetic field, (3) the cross field correlation distance δ and (4) the parallel field correlation distance L_c. In this and subsequent sections the logarithmic factors that distinguish L_c, L_{co} and $L_{c\delta}$ are disregarded.

The equation for the transverse coordinates $\underset{\sim}{r}_\perp$ of a point following the field lines is

$$\underset{\sim}{r}_\perp(z) = \int_0^z \underset{\sim}{b}[r(z'),z']dz'. \tag{158}$$

In the limit of small b_0, the quasilinear approximation for $\langle r_\perp^2(z)\rangle$ becomes at large z

$$\langle r_\perp^2\rangle = 4D_F z \tag{159}$$

where

$$D_F = \frac{1}{4}\int_{-\infty}^{+\infty} \langle \underset{\sim}{b}(0,z)\underset{\sim}{b}(0,0)\rangle \, dz. \tag{160}$$

The parallel correlation length L_c is defined by

$$D_F = \frac{1}{4} b_0^2 L_c \tag{161}$$

with which the quasilinear diffusion may be written as $\langle r_\perp^2\rangle = b_0^2 L_c z$ for $z > L_0$. The quasilinear approximation of neglecting $\underset{\sim}{r}(z)$ in Eq. (160) is valid when $\langle r_\perp^2\rangle^{1/2} \simeq b_0 L_c \leqslant \delta$ where δ is the cross field correlation distance. Thus, the amplitude parameter R,

$$R = \frac{b_0 L_c}{\delta} , \tag{162}$$

defines the quasilinear regime.

To determine the field diffusion D_F in the regime $R > 1$, Kadomtsev and Pogutse (1978) use the statistical theory of plasma turbulence, Dupree (1967). With $N(\underset{\sim}{r},z)$ defined as the conserved density of magnetic field lines, the magnetic transport equation is

$$\frac{\partial N}{\partial z} + \underset{\sim}{b} \cdot \nabla N(\underset{\sim}{r}_\perp, z) = 0. \tag{163}$$

Splitting the density into its mean value $\bar{N}$ and its fluctuation N', the well known coupled turbulence equations

$$\frac{\partial \bar{N}}{\partial z} = -\nabla \cdot \langle \underset{\sim}{b} N' \rangle = D_F \nabla_\perp^2 \bar{N}$$

$$\frac{\partial N'}{\partial z} - D_F \nabla_\perp^2 N' = -\underset{\sim}{b} \cdot \nabla N' ,$$

are used to obtain

$$D_F = \frac{1}{2} \int \frac{b^2(\underset{\sim}{k})}{ik_z + k_\perp^2 D_F} \, d\underset{\sim}{k} \tag{164}$$

where the factor 1/2 results from the angle average of the isotropic spectrum. Here, the Fourier spectrum of the istropic magnetic correlation function is

$$b^2(\underset{\sim}{k}) = \frac{1}{(2\pi)^3} \int d\underset{\sim}{r} \langle \underset{\sim}{b}(0) \underset{\sim}{b}(\underset{\sim}{r}) \rangle \quad \exp\,(-i\underset{\sim}{k} \cdot \underset{\sim}{r}) .$$

The quasilinear formula is recovered from (164) when $\Delta k_z > k_\perp^2 D_F$ where $D_F = (\pi/2) \int d\underset{\sim}{k}\, b^2(\underset{\sim}{k}) \delta(k_z) = b_0^2 L_c$. For large amplitudes $R > 1$ the cross field decorrelation dominates, $k_\perp^2 D_F > \Delta k_z$, and Eq. (164) reduces to

$$D_F^2 = \frac{1}{2} \int \frac{b^2(\underset{\sim}{k})}{k_\perp^2}\, d\underset{\sim}{k} \cong b_0^2 \delta^2 . \tag{165}$$

Thus, the field diffusion varies according to

$$D_F = \begin{cases} b_0^2 L_c & \text{for} \quad R = b_0 L_c/\delta < 1 \\ b_0 \delta & \text{for} \quad R = b_0 L_c/\delta > 1 \end{cases} . \tag{166}$$

Returning to the solution of Eqs. (155) and (156), the temperature field is split into its mean $\bar{T}_e$ and fluctuating components T_e' where linearization gives

$$(k_z^2 + \gamma^2 k_\perp^2) T_e'(\underset{\sim}{k}) = i k_z b_x(\underset{\sim}{k}) \frac{d\bar{T}_e}{dx} . \tag{167}$$

The mean value of the radial component of the heat flux is

$$q_e = -\chi_{\parallel}\langle b_x^2\rangle \frac{d\bar{T}_e}{dx} - \chi_{\parallel}\langle b_x \frac{dT'_e}{dx}\rangle$$

$$= -\frac{\chi_{\parallel}}{2}\int \frac{\gamma^2 k_{\perp}^2 b^2(k)\,d\underset{\sim}{k}}{k_z^2 + \gamma^2 k_{\perp}^2}\frac{d\bar{T}_e}{dx}\,. \qquad (168)$$

When we restrict consideration to $b_0 > \gamma = \nu_e/\omega_{ce}$ and demand that $R < 1$, it follows that for $\Delta k_z > \gamma k_{\perp}$ the quasi-lienar approximation for the effective transport resulting from Eq. (168) is

$$\chi_e = (\chi_{\parallel}\chi_{\perp})^{1/2}\left(\frac{b_0^2 L_c}{\delta}\right)$$

$$= \left(\frac{cT_e}{eB}\right)\left(\frac{b_0^2 L_c}{\delta}\right) \qquad (169)$$

where $(\chi_{\parallel}\chi_{\perp})^{1/2} = D_B = cT_e/eB$ and $b_0 L_c \leqslant \delta$ is the limit of the quasilinear regime.

In the limit $\chi_{\perp} \to 0$ formula (169) breaks down because of the neglect of higher order terms in $\underset{\sim}{b}$. Kadomtsev and Pogutse (1978) give a theory for the nonlinear regime defined by $b_0 > \gamma/R$. They show that in this regime the thermal conductivity is

$$\chi_e = \chi_{\parallel}\left(\frac{b_0^2 L_c}{\delta}\right)^2 \quad \text{for} \quad R = b_0 L_c/\delta_0 < 1. \qquad (170)$$

In both parameter ranges of the ratio $\chi_{\parallel}/\chi_{\perp}$ the anomalous thermal conductivity is related to the field diffusion by

$$\chi_e = D_F \bar{v} \tag{171}$$

where $\bar{v} = \chi_{\parallel}/L^*$ with L^* the characteristic length over which the temperature perturbation is transported. Formula (171) with $D_F = b_0^2 L_c$ agrees with formulas (152) and (154) to within logarithmic factors of order unity and extends the results for χ_e outside the quasilinear regime.

In a plasma with high electrical conductivity it is natural to consider that $\delta B_\perp$ arises from displacements with $\xi \sim \delta$ which bend the field according to $\delta B \approx B_0 \delta / L_c$, according to Kadomtsev and Pogutse (1978). Such displacements put the magnetic turbulence in the $R \sim 1$ regime where χ_e is only weakly dependent on $\gamma = (\chi_\perp / \chi_\parallel)^{1/2}$. The value of χ_e is

$$\chi_e = \begin{cases} \chi_\parallel (\delta^2 / L_c^2) & \text{for} \quad \lambda_e < L_c \\ \\ v_e (\delta^2 / L_c) & \text{for} \quad \lambda_e > L_c \end{cases} \tag{172}$$

from formulas (170) and (171).

For the cross field correlation distance δ Kadomtsev and Pogutse (1978) propose that for low frequency fluctuations of the drift mode type the natural scale for the destruction and reconnection of the magnetic field lines is the collisionless skin depth $\delta = c/\omega_{pe}$. The argument is that at these low frequencies the magnetic surfaces are frozen into the electrons on this scale. With this assumption the formula (172) for the anomalous electron thermal conductivity becomes

$$\chi_e = \frac{c^2}{\omega_{pe}^2} \frac{v_e}{L_c} \quad \text{for} \quad \nu_e < v_e / L_c \tag{173}$$

where the parallel correlation length L_c is now the effective length of the system. In a toroidal confinement device with major radius R and inverse rotational transform or safety factor q the effective length is $L_c = qR$. In this case the formula agrees with the empirical formula proposed by Ohkawa (1978) to explain the parametric dependence observed in toroidal confinement experiments.

4.5 Saturation Levels of the Gradient Driven Tearing Modes and Finite Beta Drift Waves

The electron temperature gradient destabilizes short wavelength tearing modes also called microtearing modes. These gradient driven modes produce substantial $\delta \underset{\sim}{B}_{\perp}(k)$ about the resonant surfaces $\underset{\sim}{k} \cdot \underset{\sim}{B} = 0$ which lead to anomalous transport via the mechanism analyzed in Secs. (4.1)-(4.4). The traditional long wavelength tearing mode is driven by the magnetic energy produced by force free plasma currents, $\underset{\sim}{j} \parallel \underset{\sim}{B}$, as discussed in Chapter III.5, and, although transport is produced as a consequence of the magnetic activity, the mode is not inherent to the anomalous transport due to macroscopic plasma gradients.

In the collisional regime the electron temperature gradient driven tearing mode is analyzed by Hazeltine and Strauss (1976) in the quasilinear regime of Sec. 4.2. The turbulent diffusion of the temperature gradient across a single magnetic island produces saturation when the magnetic fluctuation reaches the level

$$b_o = \frac{<\delta B_\perp>^{1/2}}{B} \lesssim k_y \rho_e \left(\frac{\lambda_e}{r_T}\right)^{1/2} \tag{174}$$

where ρ_e is the electron gyroradius and r_T is the electron temperature gradient scale length.

A mode coupling analysis of the temperature gradient driven problem by Drake et al. (1980) in a semicollisional regime, $\omega_{*e} \sim \nu_e$, leads to the conclusion that the nonlinear transfer of mode energy from $\delta j_\| B_x$ balances the growth rate $\gamma_k \simeq (\omega^2_{*e}/\nu_e)\eta_e(1 + \frac{5}{2}\eta_e)$ (Gladd et al., 1980) when the fluctuation amplitude reaches the level

$$b_o = \frac{<\delta B_\perp^2>^{1/2}}{B_0} \cong \frac{\rho_e}{r_T} \tag{175}$$

In the amplitude regime where the magnetic island width $\Delta_k = (L_s b_0/k_y)^{1/2}$ is greater than the separation of the resonant surfaces, $k_y > (r_T L_s/r^2 \rho_e)^{1/3}$, the anomalous thermal transport is estimated by

$$\chi_e = \frac{v_e^2 \rho_e^2}{\nu_e r_T^2} = \frac{\rho_e}{r_T} \frac{cT_e}{eB} \frac{\lambda_e}{r_T} \tag{176}$$

which Drake et al. (1980) discuss in terms of recent experiments on magnetic turbulence.

The renormalized electron-wave interactions due to the magnetic fluctuations are investigated by Galeev (1978) for the short wavelength collisionless tearing modes. The fluctuations are dominated by perpendicular wavelength of order of the collisionless skin depth c/ω_{pe}. Galeev finds saturation of the modes at the turbulence level

$$\frac{\langle(\delta B_\perp)^2\rangle}{B^2} = \sum_{k} \left|\frac{\delta B_\perp(k)}{B}\right|^2 \cong \frac{\bar{k}_\parallel^2 c^2}{\omega_{pe}^2} \frac{1}{(1+T_i/T_e)} \tag{177}$$

where $\bar{k}_\parallel$ is the mean parallel wave number determined by the geometry and the shear. In view of the analysis in Sec. 4.4 this saturation is at the transition point $R = b_0 L_c/\delta \sim 1$ where $\delta \sim c/\omega_{pe}$ and $L_c \cong 1/\bar{k}_\parallel$. The saturation level (177) provides an example where the anomalous thermal conductivity in Eq. (173) is predicted.

For finite β drift waves, Molvig, et al. (1979) use the renormalized electron propagator and the stabilizing effect of magnetic shear to obtain saturation. The anomalous electron transport obtained by Molvig, et al. is

$$\chi_e = 0.1 \frac{m_e}{m_i \beta_e} \frac{\rho_i}{r_n} \frac{cT_i}{eB} \left[\frac{L_s}{r_n(1+T_i/T_e)}\right]^2$$

which exceeds the transport in Eq. (100) by the factor $(L_s/r_n)^2$, due partly to the higher fluctuation level. This model neglects the ion nonlinearities analyzed in Sec. 3.

4.6 Kinetic Theory of Transport due to Magnetic Fluctuations

More information about the character of the anomalous electron thermal conductivity due to magnetic turbulence is obtained from a kinetic description of the fluctuating particle distribution. In the analysis of the transport due to tearing modes, Galeev (1978) shows that the net particle and thermal flux across the ambient magnetic field is limited by the randomization or stochastization of the particle-fluctuation phase relation. For example, the cross-field electron diffusion $\langle \Delta x^2 \rangle = \chi_e \tau$ in the sheared magnetic field results in the decorrelation in time τ of the parallel phase relationship according to $\langle \exp(i\int^z k_\parallel dz') \rangle \cong \exp\left(-\frac{1}{6} k_\parallel'^2 v_\parallel^2 \chi_e \tau^3\right)$ where $k_\parallel' = \partial k_\parallel/\partial x = k_y/L_s$.

To describe both the turbulent phase stochastization and the collisional diffusion from electron-ion ν_{ei} collisions and electron-electron ν_{ee} collisions, Galeev and Zeleny (1979) introduce the renormalized kinetic equation with a Batnagar-Gross-Krook collision operator that conserves particles and parallel momentum. From this kinetic equation the fluctuating parallel electron current $j_\parallel(k)$ is calculated and used in Eq. (129) to determine the radial transport due to $\langle |\delta B_x(k)|^2 \rangle$. In the quasistatic, renormalized quasilinear regime defined by

$$\omega_k < \nu_{eff} < \nu_{ei}, \tag{178}$$

the formula for the anomalous thermal conductivity reduces to

$$\chi_e = \chi_{\parallel e} \sum_{\underset{\sim}{k}} \left| \frac{\delta B_x(\underset{\sim}{k})}{B} \right|^2 \frac{\nu_{eff}}{k_\parallel^2 \chi_{\parallel e} + \nu_{eff}} \tag{179}$$

where

$$\nu_{eff} = \lambda_e^2 (k_\parallel')^2 (\chi_{\perp e} + \chi_e) \tag{180}$$

with $\chi_{\parallel e} = \lambda_e^2 \nu_{ei}$ and $\chi_{\perp e} = \rho_e^2 \nu_{ei}$ being the classical parallel and perpendicular electron thermal conductivities. Here, $\lambda_e = v_e/\nu_{ei}$ is the electron mean-free-path and $\rho_e = v_e/\omega_{ce}$ the electron gyroradius.

The $k_\parallel$ width of the correlation function is $\Delta k_\parallel = (\nu_{eff}/\chi_{\parallel e})^{1/2}$ which defines the characteristic scale length L_e^* for stochastization of the electron-fluctuation phase

$$L_e^* = 1/\Delta k_\parallel = \lambda_e \left(\frac{\nu_{ei}}{\nu_{eff}}\right)^{1/2} . \tag{181}$$

When the $\delta B_x(k)$ spectrum is broad compared with $\Delta k_\parallel$ the conductivity formula reduces to

$$\chi_e = v_e \frac{\lambda_e}{L_e^*} D_F \tag{182}$$

where the magnetic field diffusion coefficient D_F is given in Eq. (151). At the collisionless limit of condition (178) where the mean-free-path exceeds the phase stochastization length the electrons diffuse at the same rate as the magnetic field

$$\chi_e = v_e D_F \tag{183}$$

as discussed in Sec. 4.3. For $\lambda_e < L_e^*$ the anomalous transport is reduced by λ_e/L_e^* but does not go directly to the collisional regime of Rechester and Rosenbluth (1978) given in Eq. (154). Between the collisional and collisionless regime there is a plateau or semi-collisional regime where the mean-free-path is long compared to the shear length $L_s \approx L_0$ but shorter than the phase stochastization length L_e^*. In this amplitude regime where χ_e exceeds $\chi_{\perp e}$ the length L_e^* varies inversely with $\chi_e^{1/2}$ according to Eqs. (180) and (181). Expressing L_e^* in terms of χ_e and solving Eq. (179) yields the plateau thermal conductivity

$$\chi_e = \chi_{\parallel e} b_0^4 \bar{k}_y^{-2} \lambda_e^2 \tag{184}$$

where b_0 is defined in Sec. 4.4 and $\bar{k}_y$ is a mean wavenumber in the spectrum. The plateau or semi-collisional formula (184) is valid for b_0 constrained by $\chi_e > \chi_{\perp e}$ and $\nu_{eff} < \nu_{ei}$, or equivalently $L_e^* >> \lambda_e$, which defines the domain

$$\frac{L_s^{1/2}}{\bar{k}_y \lambda_e^{3/2}} > b_0 > \left(\frac{\rho_e}{\bar{k}_y \lambda_e^2}\right)^{1/2} . \tag{185}$$

For b_0 less than the lower limit in Eq. (185) χ_e equals $\chi_{\perp e}$ plus a small increment from the magnetic turbulence. For higher fluctuation levels the mean-free-path exceeds the stochastization length and the collisionless formula (183) applies. At still larger amplitudes $b_0 > \delta/L_s$ the thermal conductivity becomes $\chi_e = v_e b_o \delta_x$.

These four regimes are shown in the left half of Fig. 12.

The limited region of radial correlation in the magnetic turbulence also influences the form of the anomalous thermal flux. Here, we define $\delta_x = 1/\bar{k}_x$ as the radial scale distance over which the magnetic fluctuations are correlated. There are two effects that arise from the finite radial correlation distance. First, there is a further loss of the phase coherence due to the perpendicular diffusion over δ_x which is described by $\nu_{eff} \to \nu_{eff} + k_x^2 \chi_e$; and secondly, there is an additional decorrelation from the parallel motion of the electron along the radially tilted field line which is described by the nonlinear $\underset{\sim}{k} \cdot \underset{\sim}{B} = k_{\|0} B + k_x \delta B_x$. With regard to the first effect, a comparison of $k_x^2 \chi_e$ with ν_{eff} in Eq. (180) shows that the radial decorrelation dominates when

$$\frac{L_s}{\lambda_e} > \frac{\bar{k}_y}{\bar{k}_x} = \bar{k}_y \delta_x . \tag{186}$$

From the second effect it is adequate to take into account that the mean value of the stochastic $k_\|$ is given by

$$\langle k_\|^2 \rangle = k_\|^2 + k_x^2 b_0^2 \tag{187}$$

from which it is evident that the nonlinear broadening of the parallel phase dominates when

$$b_0 > \frac{\max(k_\|)}{k_x} = \frac{\delta_x}{L_s} \quad . \tag{188}$$

Taking these two effects into account generalizes correlation function in Eq. (179) so that the quasistatic thermal conductivity becomes

$$\chi_e = \chi_{\| e} \sum_{\underset{\sim}{k}} \left| \frac{B_x(\underset{\sim}{k})}{B} \right|^2 \frac{\nu_{eff} + k_x^2 \chi_e}{(k_\|^2 + k_x^2 b_0^2)\chi_{\| e} + \nu_{eff} + k_x^2 \chi_e} \quad . \tag{189}$$

The approximation of replacing $k_\|^2$ by the average nonlinear $\langle k_\|^2 \rangle$ given by Eq. (187) appears adequate to recover the essential results of the nonlinear physics.

First, it is observed from Eq. (189) that in the quasilinear regime, $b_0 < \delta_x/L_s$, that the scale length for stochasization of the electron phase is shorter by $1/L_e^* = \Delta k_\| = [(\nu_{eff} + k_x^2 \chi_e)/\chi_{\| e}]^{1/2} \cong k_x(\chi_e/\chi_{\| e})^{1/2}$ when when condition (186) is satisfied. For $\lambda_e < L_e^*$ the plateau formula that follows from L_e^* is

$$\chi_e = \chi_{\| e}\left(\frac{b_0^4 L_s^2}{\delta_x^2}\right) \tag{190}$$

which agrees with Eq. (170) from Kadomtsev and Pogutse (1978) and connects the thermal conductivity to Eq. (184) when condition (186) is reversed.

The domain of applicability of Eq. (190) is shown in the middle-right hand section of Fig. 12. For the anomalous conductivity χ_e to exceed the collisional conductivity $\chi_{\perp e}$ it is necessary that $b_0 > (\rho_e \delta_x / \lambda_e L_s)^{1/2}$.

In the nonlinear regime $b_0 > \delta_x / L_s$ the stochastization of the parallel phase is unimportant according to Eq. (189) Clearly, there remain two sub-domains depending on the magnitude of the nonlinear broadening $k_x^2 b_0^2 \chi_{\parallel e}$. In the lower amplitude subdomain clearly Eq. (189) reduces simply to

$$\chi_e = \chi_{\parallel e} b_0^2 \tag{191}$$

where now the condition for applicability becomes

$$\frac{\delta_x}{L_s} < b_0 < \frac{\delta}{\lambda_e} .$$

When the nonlinear parallel decorrelation is substantial, the only consistent solution of Eq. (189) has $k_x^2 b_0^2 \chi_{\parallel e} \sim k_x^2 \chi_e$ and consequently $\chi_e \cong \chi_{\parallel e} b_0^2$, valid when the mean free path is less than the parallel correlation length. The parallel correlation length is given by $1/L_{\parallel} \sim k_x b_0 \sim b_0 / \delta_x$. Thus, the well known self-similar form $\chi_e \cong \chi_{\parallel e} b_0^2$ remains valid until

$$b_0 > \frac{\delta_x}{\lambda_e}$$

whereupon

$$\chi_e = v_e D_F \cong v_e b_0 \delta_x \qquad (192)$$

gives the nonlinear, collisionless thermal transport shown in the upper part of Fig. 12.

Finally, it should be noted that the self-similar result $\chi_e = \chi_{\parallel e} \langle \delta B_x^2 \rangle / B^2$ is universal in the sense that it is realized both for strong collisions and for a high level magnetic fluctuations. In the regime of strong collisions the result is obtained by Rechester and Rosenbluth (1978) in Eq. (154). As shown by the kinetic analysis in this section and summarized in Fig. 12, there are plateau or semicollisional regimes of practical importance where the transport follows different laws.

In conclusion, there are different mechanisms for anomalous transport depending on the regime of the plasma. At the present, there are no general rules for determining which of the formulas presented here govern transport in a specific plasma. As a guide, however, we may remark that formulas (60)-(68) are important for low plasma pressure $<$ and are supported by the experiments discussed in Sec. 2.3. The formulas (86) and (87) for the $\underset{\sim}{k}$ spectrum and the line width $\nu_{\underset{\sim}{k}}$ are supported by the microwave scattering experiments. The formula (91) for the ion thermal conduction is supported by computer simulations. For $\beta > m_e/m_i$ the formula (96), or equivalently (100), appears consistent with the empirical scaling law for tokamak confinement. In this regime, the magnetic turbulent transport formulas (172)-(173) yield a result rather similar to (96) and may also be a reasonable approximation to the empirical scaling law for thermal transport in low beta tokamaks. Finally, the transport produced by convective cells is given by formulas (115) and (116) which are also important results for the analysis of plasma transport.

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FIGURE CAPTIONS

Figure 1 (a) A segment of a drift wave fluctuation showing the variation of the electrostatic potential perpendicular to the magnetic field at a given instant of time. The contours of φ = const in the plane perpendicular to $B\hat{z}$ are the stream lines of the $\underset{\approx}{E} \times \underset{\approx}{B}$ particle motion.

(b) A segment of the correlated but phase shifted density variation.

(c) Top view of the potential and density contours in (a) and (b) in the case where the density and potential variation are in phase.

(d) Top view in the case where the potential and density variation are out of phase by $\psi_{\delta n, \varphi}$.

Figure 2. From the collisional drift wave experiment of Hendel, et al. (1968) showing (a) the plasma density profile $n_e(r)$, (b) the amplitudes of the density and potential waves, and (c) the phase shift $\psi_{\delta n, \varphi}$ by which the density wave leads the potential wave.

Figure 3. From the collisional drift wave experiment in Fig. 2, showing (a) the radial particle flux $\Gamma(r) = F_{wave}$ produced by the drift wave compared to the collisional flux $F_{e-i}(r)$ and (b) the anomalous diffusion coefficient $D(r)$ produced by the drift wave compared to the collisional diffusion D_{e-i}.

Figure 4. From the collisionless drift wave experiment of Brossier, et al. (1978) showing the equilibrium plasma density and the amplitudes of the density, electron temperature and the potential waves.

Figure 5. From the collisionless drift wave experiment in Fig. 4, showing the phase shift between the density and potential wave $\psi_{\delta n, \varphi}$ and the phase shift between the density and the temperature wave $\psi_{\delta n, \delta T}$.

Figure 6. The anomalous diffusion coefficient for collisionless drift waves as a function of collision frequency. For $\nu_e < \nu_*$ the quasi-linear change in the electron velocity distribution limits the anomalous flux.

Figure 7. Diagram showing the unstable region of wavenumber space that is coupled through nonlinear interactions to the larger damped regions of wavenumber space as discussed by Oraevskii and Sagdeev (1963).

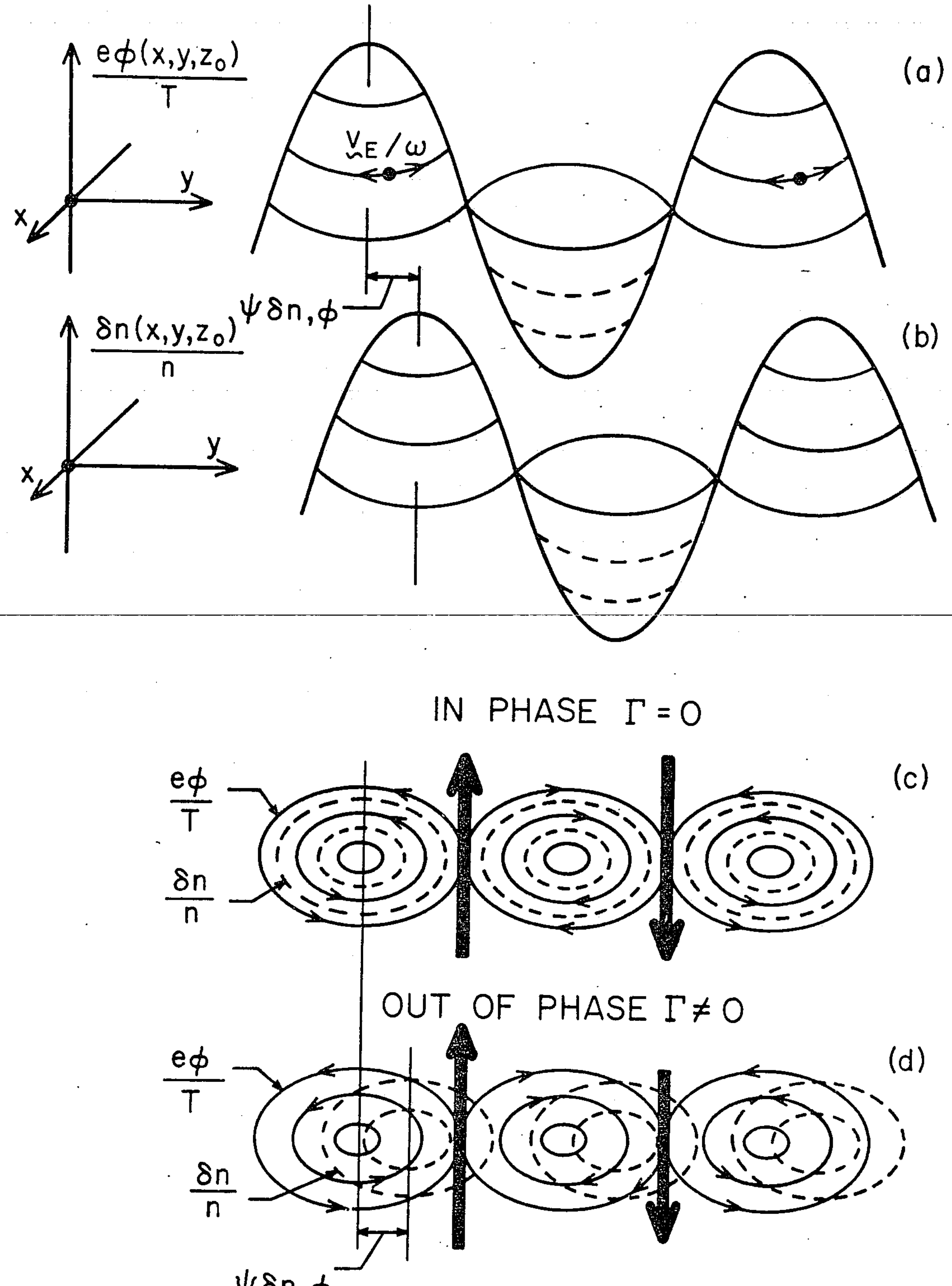
Figure 8. The frequency spectrum obtained from microwave scattering off the one centimeter wavelength, $k_\perp \rho_i \sim 0.6$, in the drift wave turbulence studies in the ATC experiment, Goldston, et al. (1978). From (a) to (b) the direction of rotation of the drift wave fluctuations is changed by reversing the direction of the magnetic field. The peak of the spectrum corresponds approximately to the drift wave frequency ω_k and the width of the spectrum to the Doppler broadening frequency ν_k.

Figure 9. The $k_\perp$-spectrum obtained from microwave scattering at six different angles in the ATC experiment by Mazzucato (1976). The spectral formula of Horton (1976) given Eq. (86) is shown evaluated for the experiment.

Figure 10. The magnetic island chains produced by two nearby resonant perturbations $k_1 \cdot B = 0$ and $k_2 \cdot B = 0$ in the sheared magnetic field. The $\delta B_\perp(k)$ amplitudes are small enough that overlapping has not yet occured $s \leqslant 1$. Adopted from Galeev (1978).

Figure 11. In the stochastic regime of strong island overlapping $s >> 1$, the small circle in (a) is mapped by the magnetic field to (b) showing the stochastic instability of trajectories. When the filament length in (b) exceeds the cross-field correlation distance the mapping evolves as shown in (c) Rechester and Rosenbluth (1978).

Figure 12. Regimes and formulas for the anomalous electron thermal conductivity χ_e due to quasistatic magnetic turbulence. Formulas taken from Rechester and Rosenbluth (1978), Kadomtsev and Pogutse (1978) and Galeev and Zeleny (1979).



eφ(x,y,z₀) / T
y
x
V_E / ω
(a)
ψδn,φ
δn(x,y,z₀) / n
y
x
(b)
IN PHASE Γ = 0
eφ / T
δn / n
(c)
OUT OF PHASE Γ ≠ 0
eφ / T
δn / n
(d)
ψδn,φ

FIGURE 1

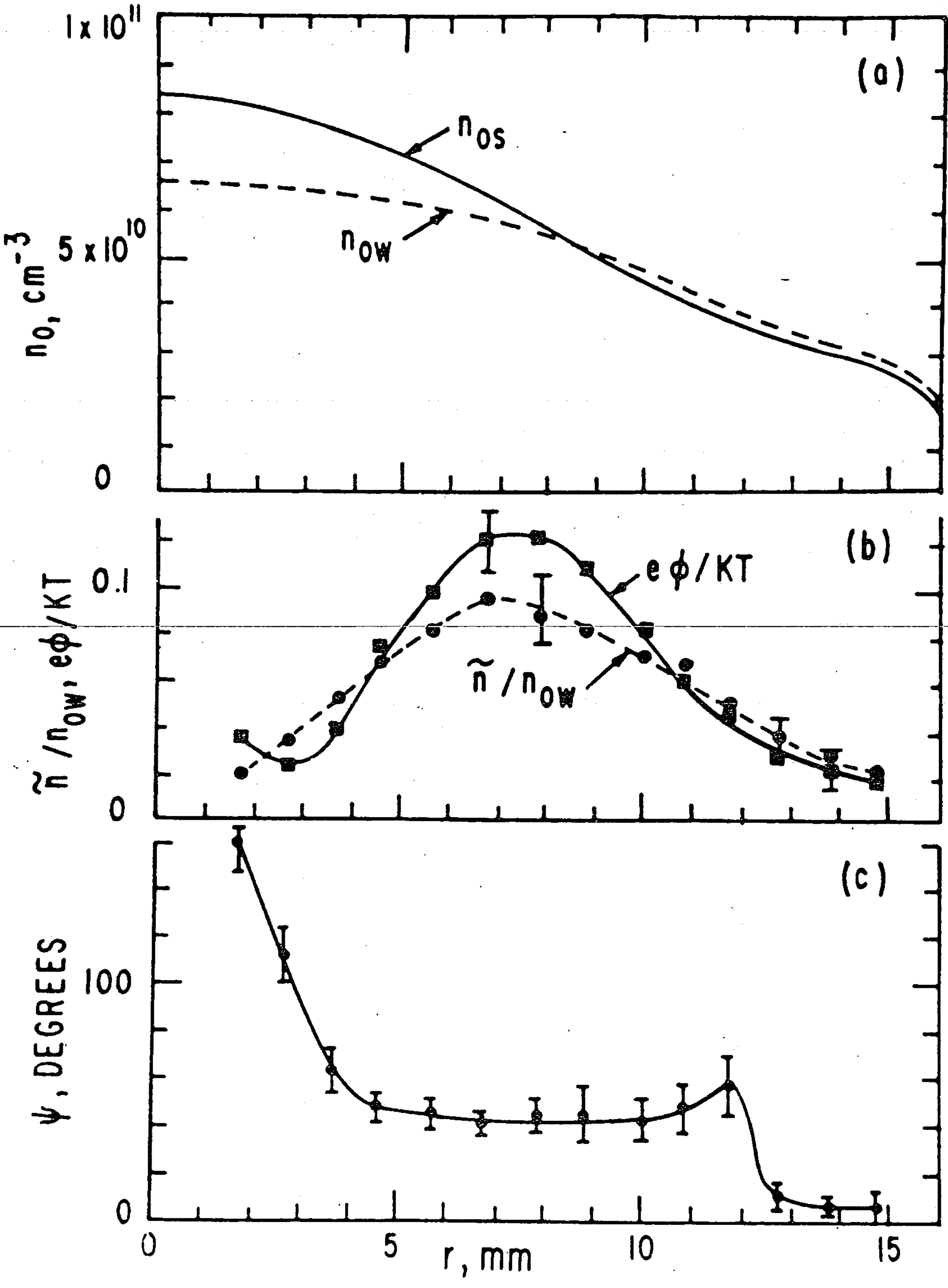
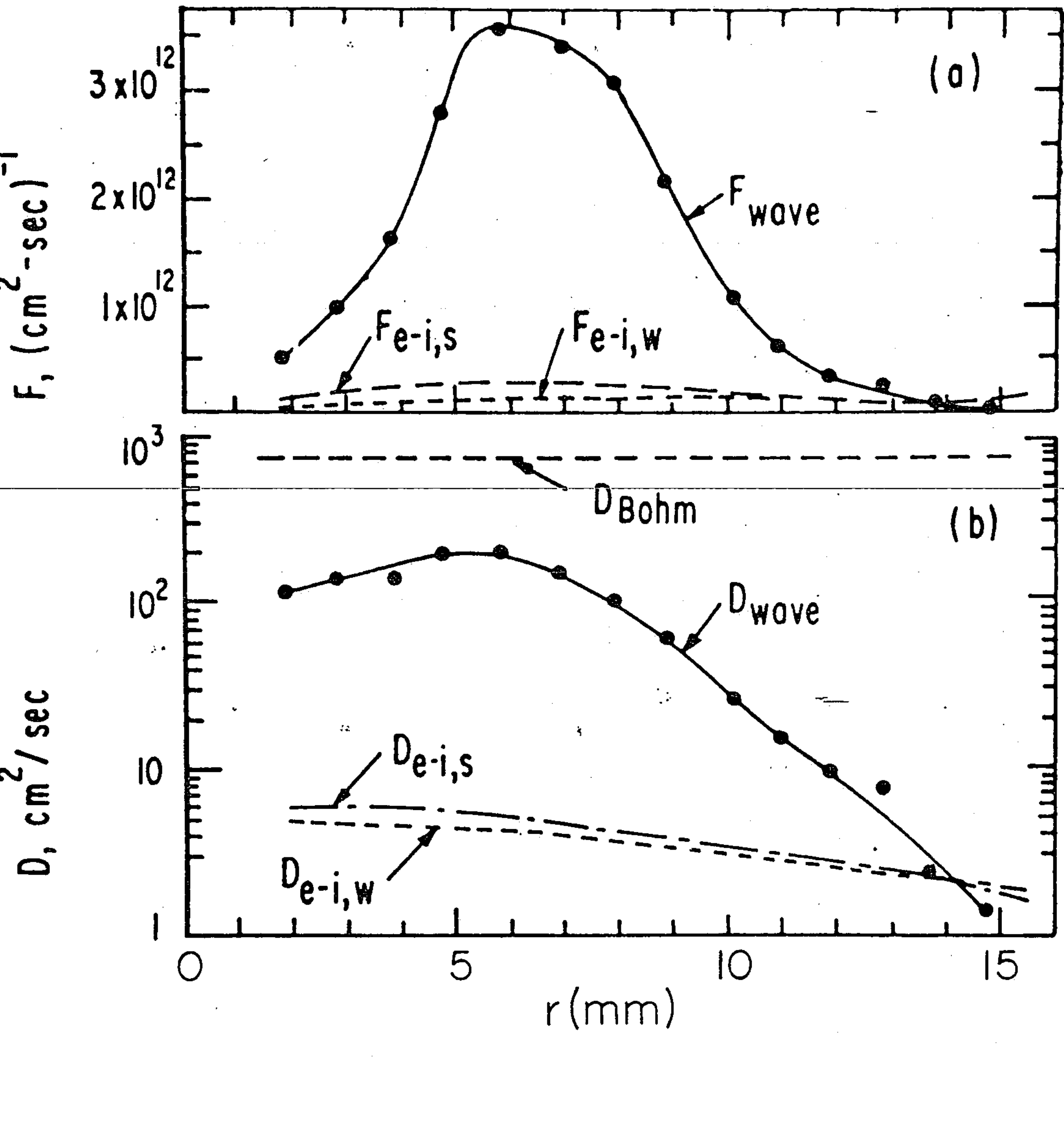

1 x 10^11
5 x 10^10
0
n_o, cm^{-3}
(a)
n_{os}
n_{ow}
0.1
0
$\tilde{n}/n_{ow}$, $e\phi/KT$
(b)
$e\phi/KT$
$\tilde{n}/n_{ow}$
100
0
ψ, DEGREES
(c)
0
5
10
15
r, mm

FIGURE 2



F, $(cm^2\text{-sec})^{-1}$
3×10^{12}
2×10^{12}
1×10^{12}
(a)
F_{wave}
$F_{e-i,s}$
$F_{e-i,w}$
D, cm^2/sec
10^3
10^2
10
1
D_{Bohm}
(b)
D_{wave}
$D_{e-i,s}$
$D_{e-i,w}$
0
5
10
15
r(mm)

FIGURE 3

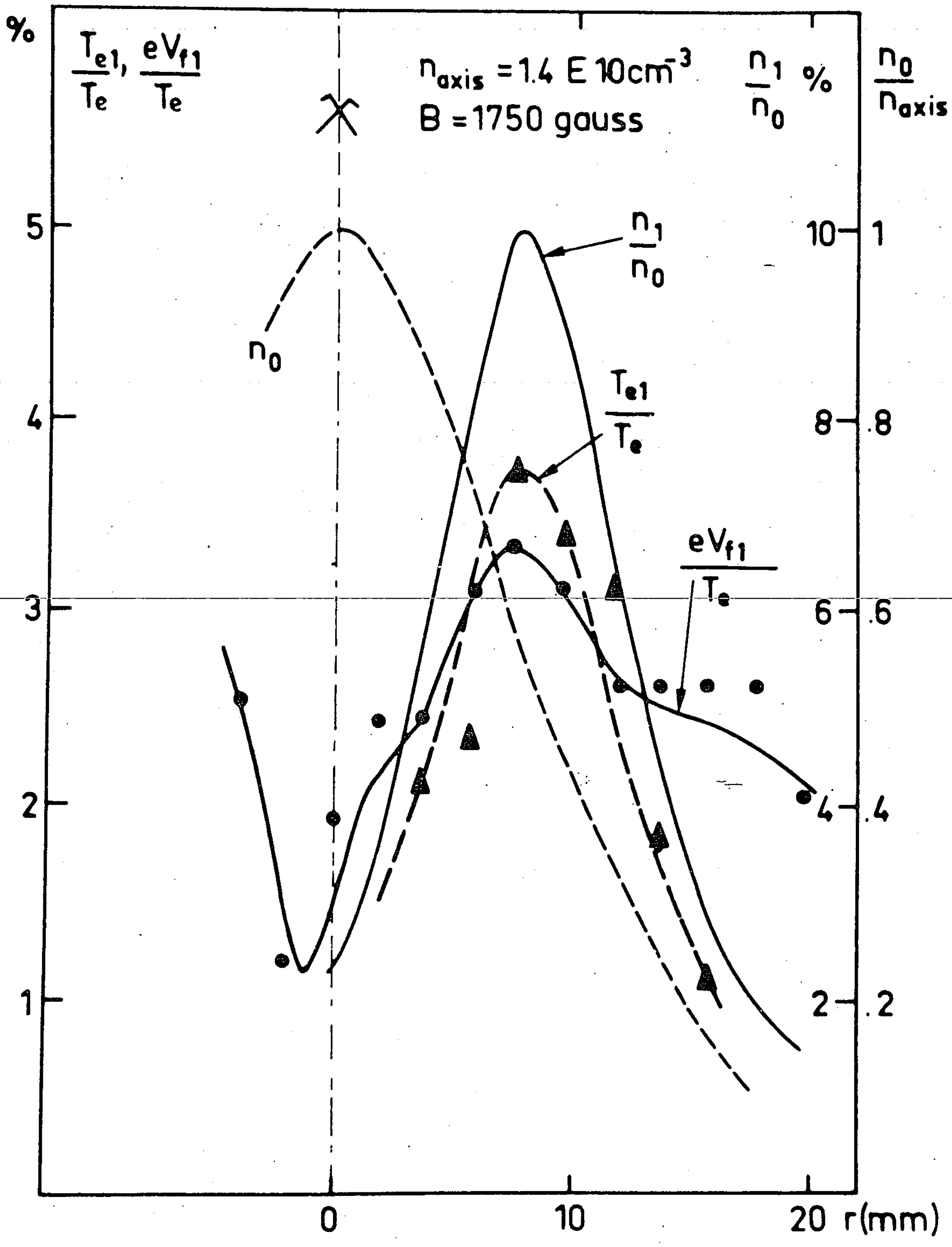

%
$\frac{T_{e1}}{T_e}$, $\frac{eV_{f1}}{T_e}$
$n_{axis} = 1.4$ E $10 cm^{-3}$
B = 1750 gauss
$\frac{n_1}{n_0}$ %
$\frac{n_0}{n_{axis}}$
n_0
$\frac{n_1}{n_0}$
$\frac{T_{e1}}{T_e}$
$\frac{eV_{f1}}{T_e}$
5
4
3
2
1
10
8
6
4
2
1
.8
.6
.4
.2
0
10
20
r(mm)

FIGURE 4

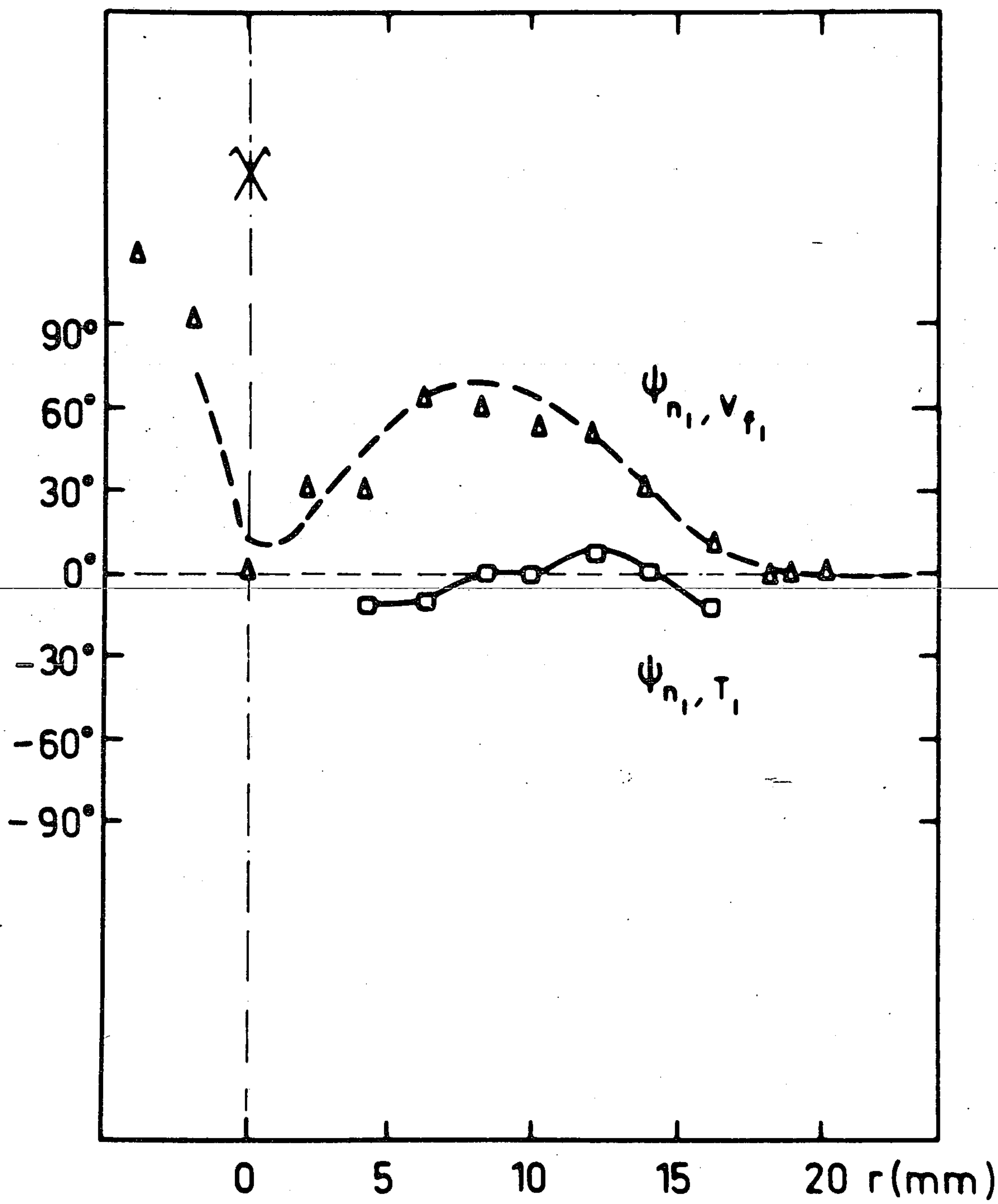


FIGURE 5

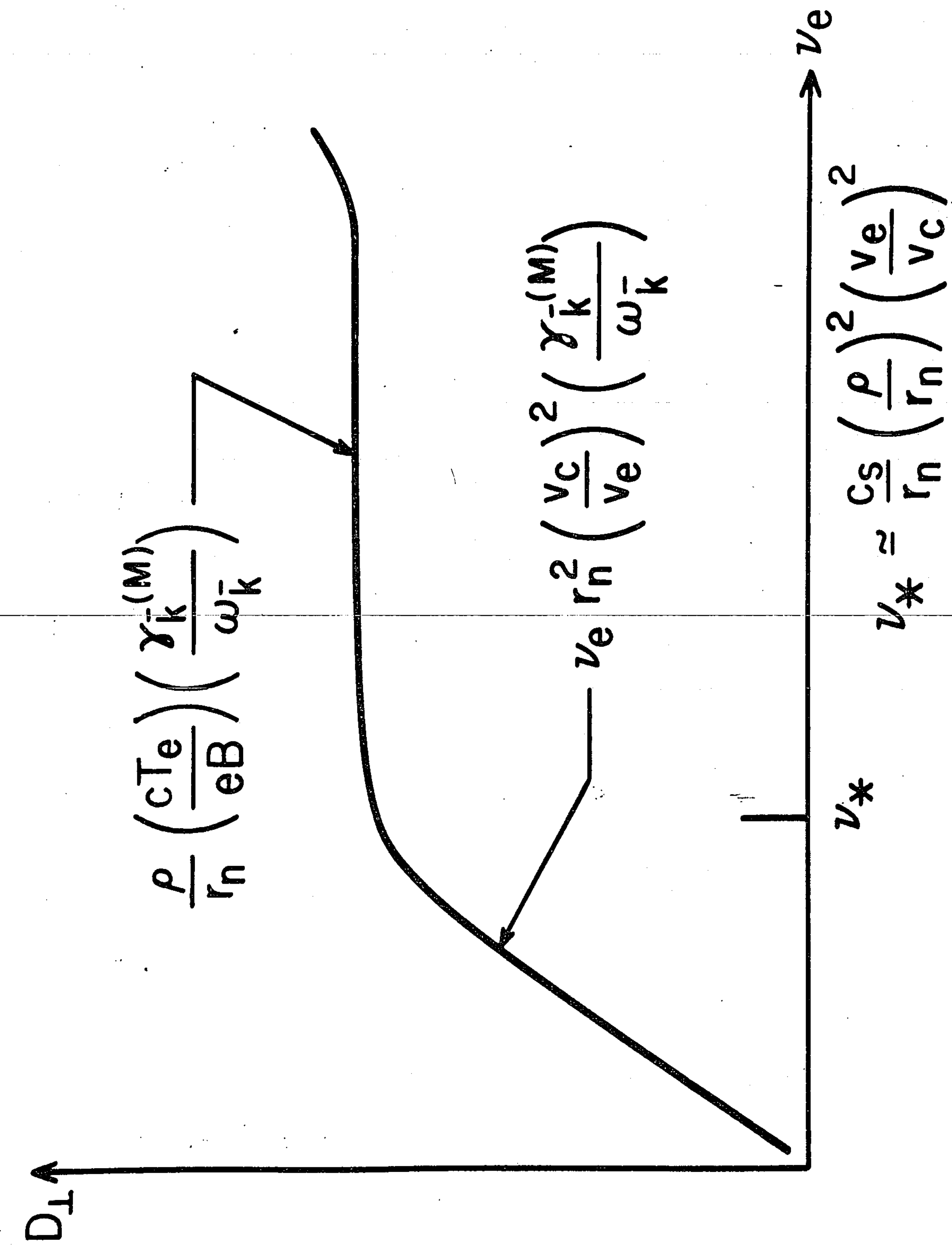

$D_\perp$
ν_e
ν_*
$\frac{\rho}{r_n}\left(\frac{cT_e}{eB}\right)\left(\frac{\gamma_k^{-(M)}}{\omega_k^-}\right)$
$\nu_e r_n^2\left(\frac{v_c}{v_e}\right)^2\left(\frac{\gamma_k^{-(M)}}{\omega_k^-}\right)$
$\nu_* \simeq \frac{c_s}{r_n}\left(\frac{\rho}{r_n}\right)^2\left(\frac{v_e}{v_c}\right)^2$

FIGURE 6

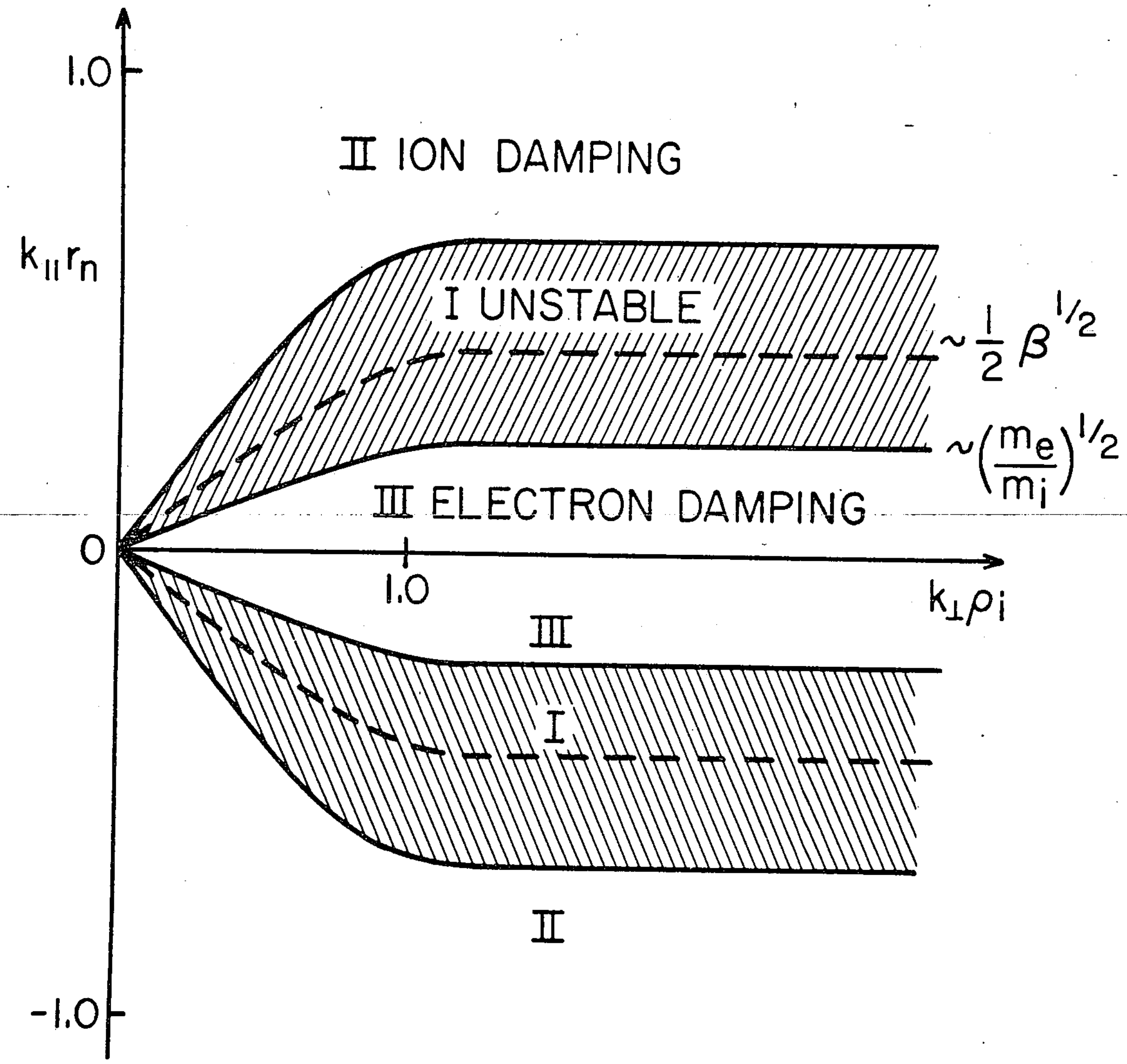

1.0
II ION DAMPING
$k_{\parallel} r_n$
I UNSTABLE
$\sim \frac{1}{2} \beta^{1/2}$
$\sim \left(\frac{m_e}{m_i}\right)^{1/2}$
III ELECTRON DAMPING
0
1.0
$k_{\perp} \rho_i$
III
I
II
-1.0

FIGURE 7

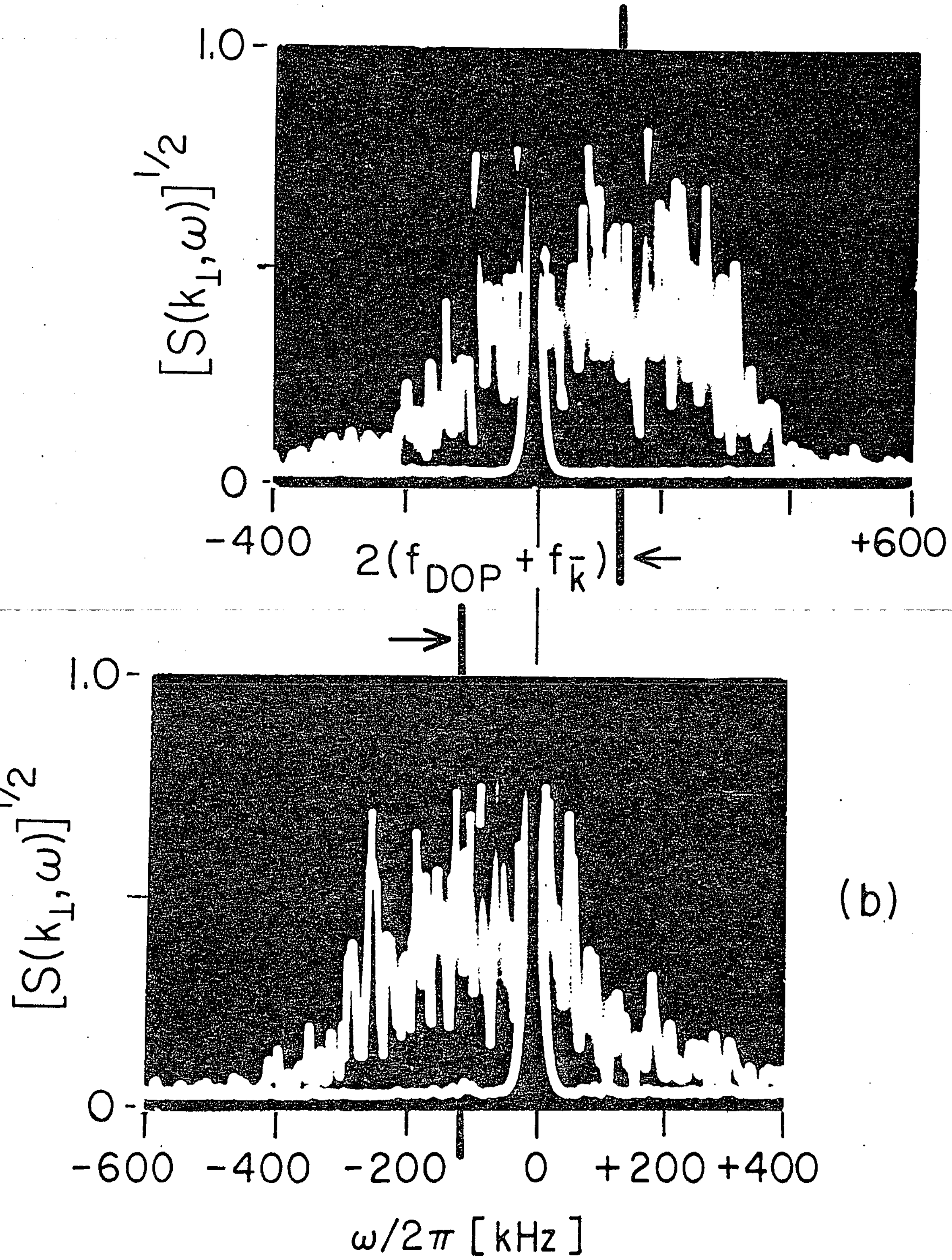
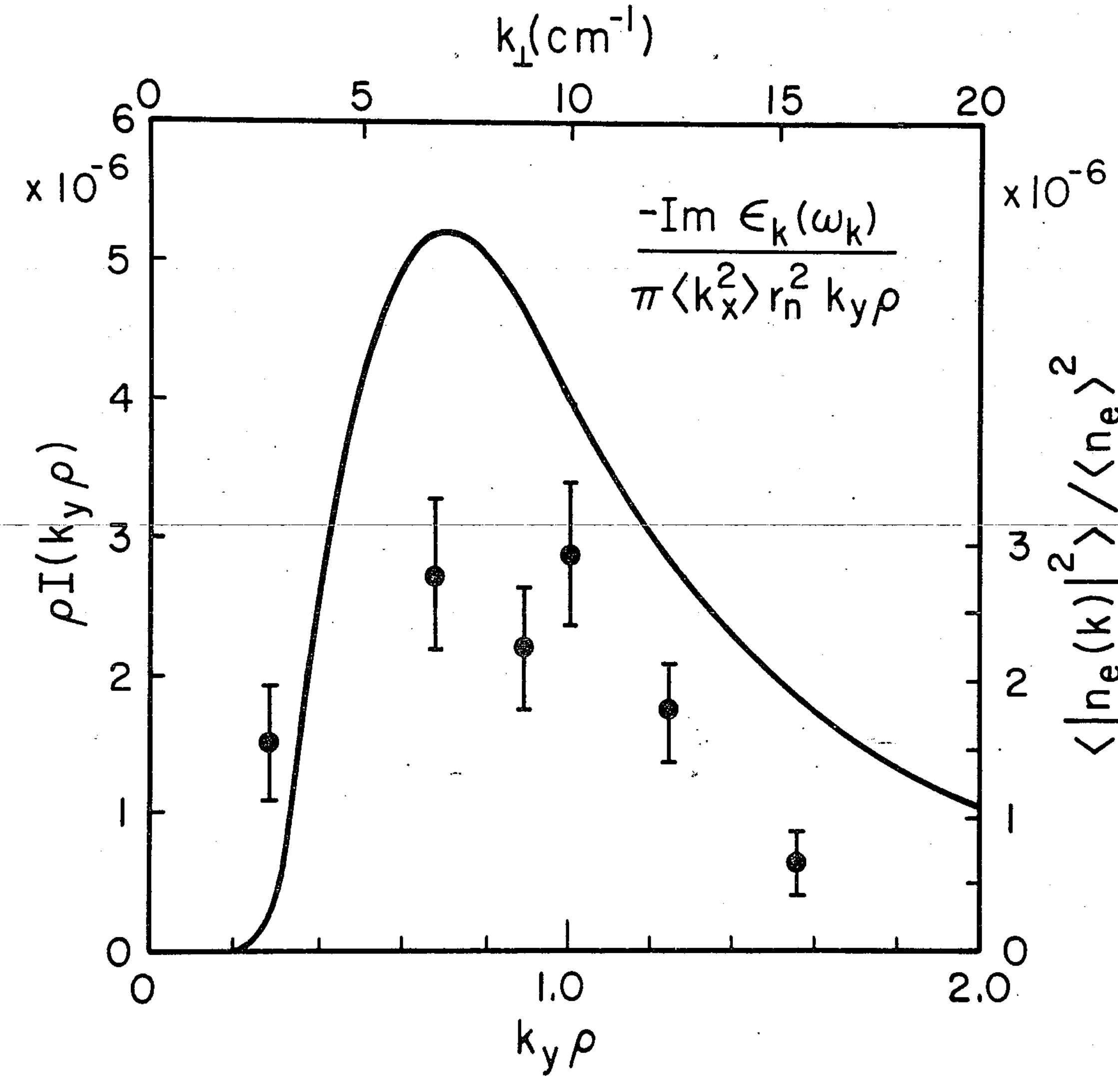

1.0
0
$[S(k_\perp,\omega)]^{1/2}$
-400
+600
$2(f_{DOP} + f_{\bar{k}})$
1.0
0
$[S(k_\perp,\omega)]^{1/2}$
(b)
-600 -400 -200 0 +200 +400
$\omega/2\pi$ [kHz]

FIGURE 8



$k_\perp(cm^{-1})$
0
5
10
15
20
6
x 10^{-6}
5
4
3
2
1
0
0
1.0
2.0
$k_y\rho$
$\rho I(k_y\rho)$
x 10^{-6}
3
2
1
0
$\langle|n_e(k)|^2\rangle/\langle n_e\rangle^2$
$\dfrac{-\mathrm{Im}\,\epsilon_k(\omega_k)}{\pi\langle k_x^2\rangle r_n^2 k_y\rho}$

FIGURE 9

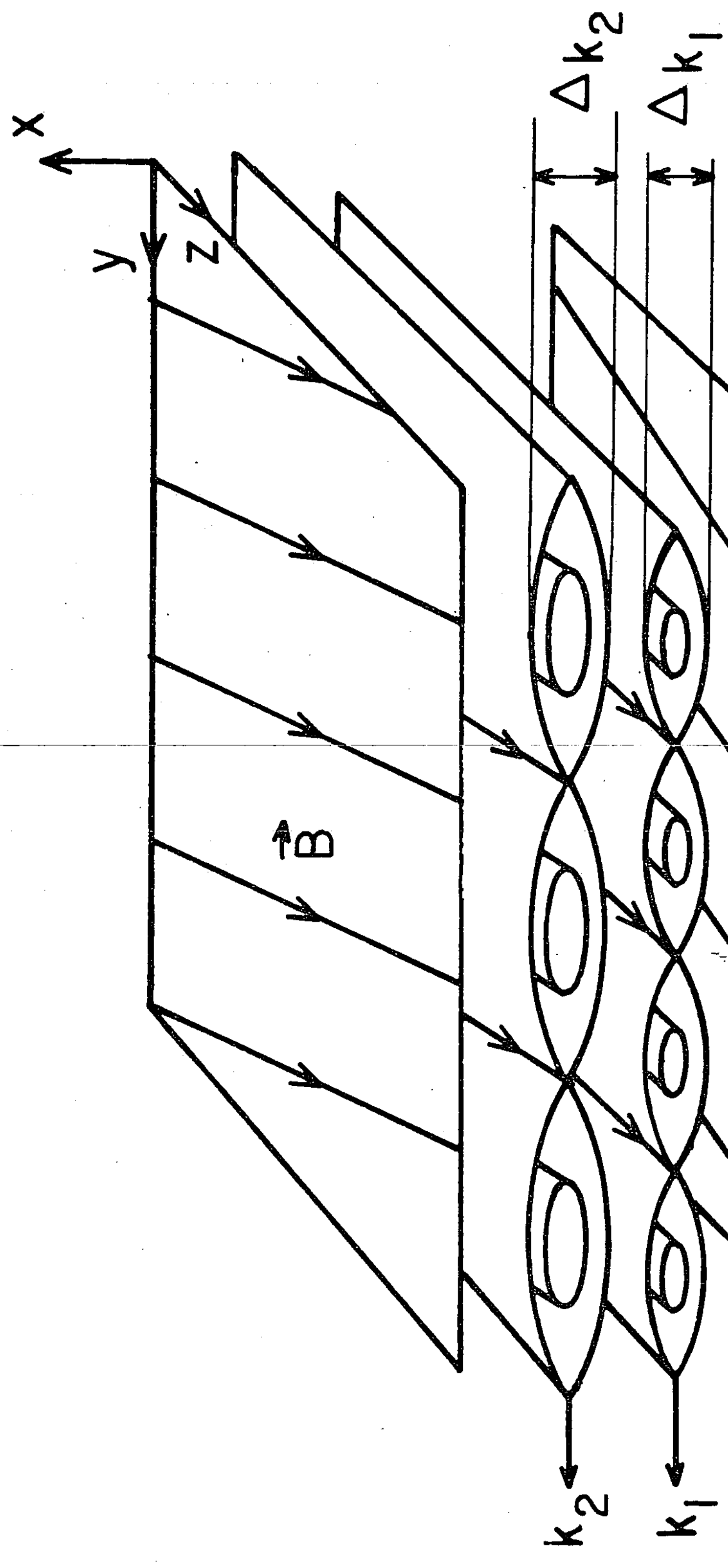


FIGURE 10

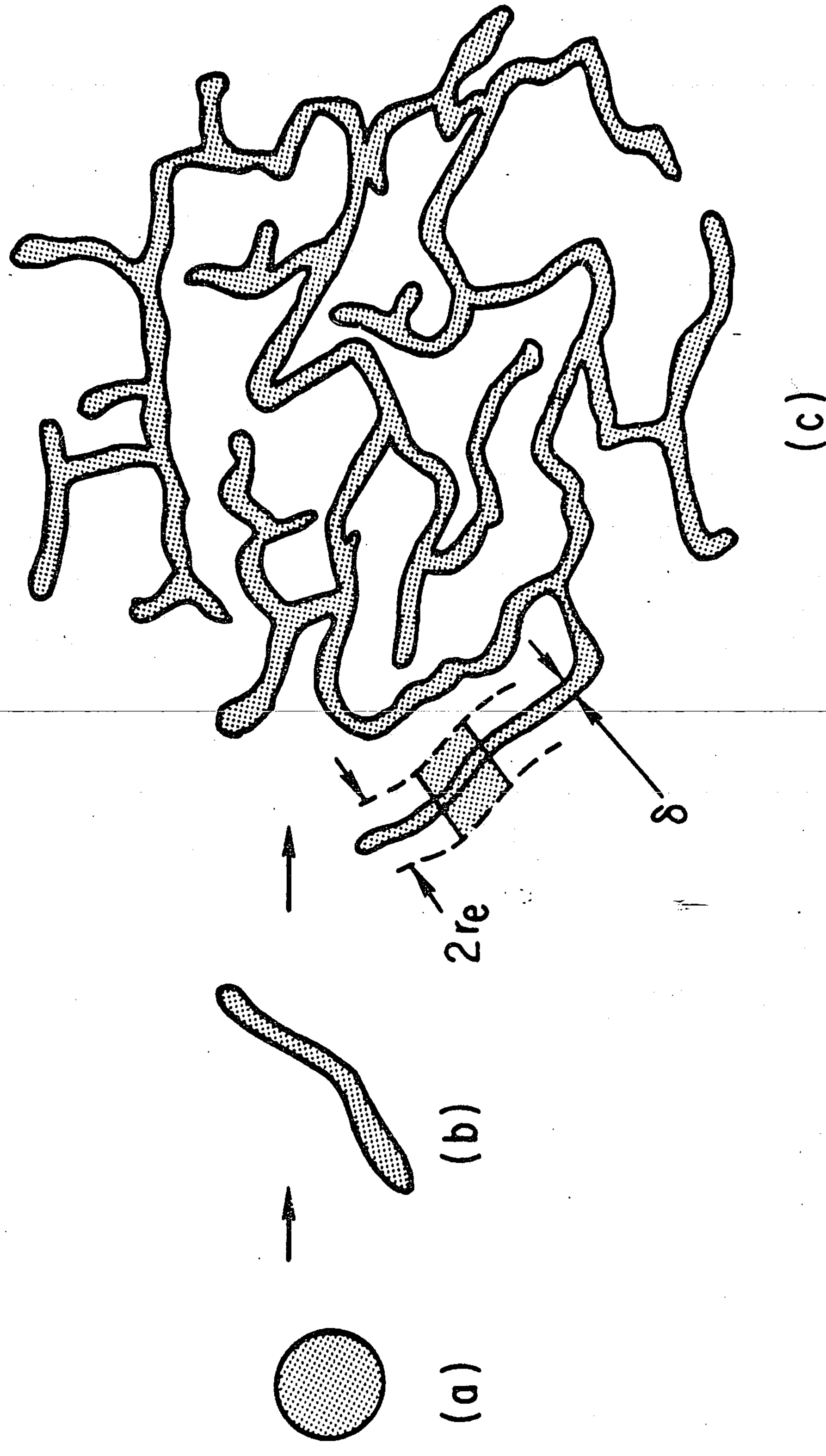


FIGURE 11

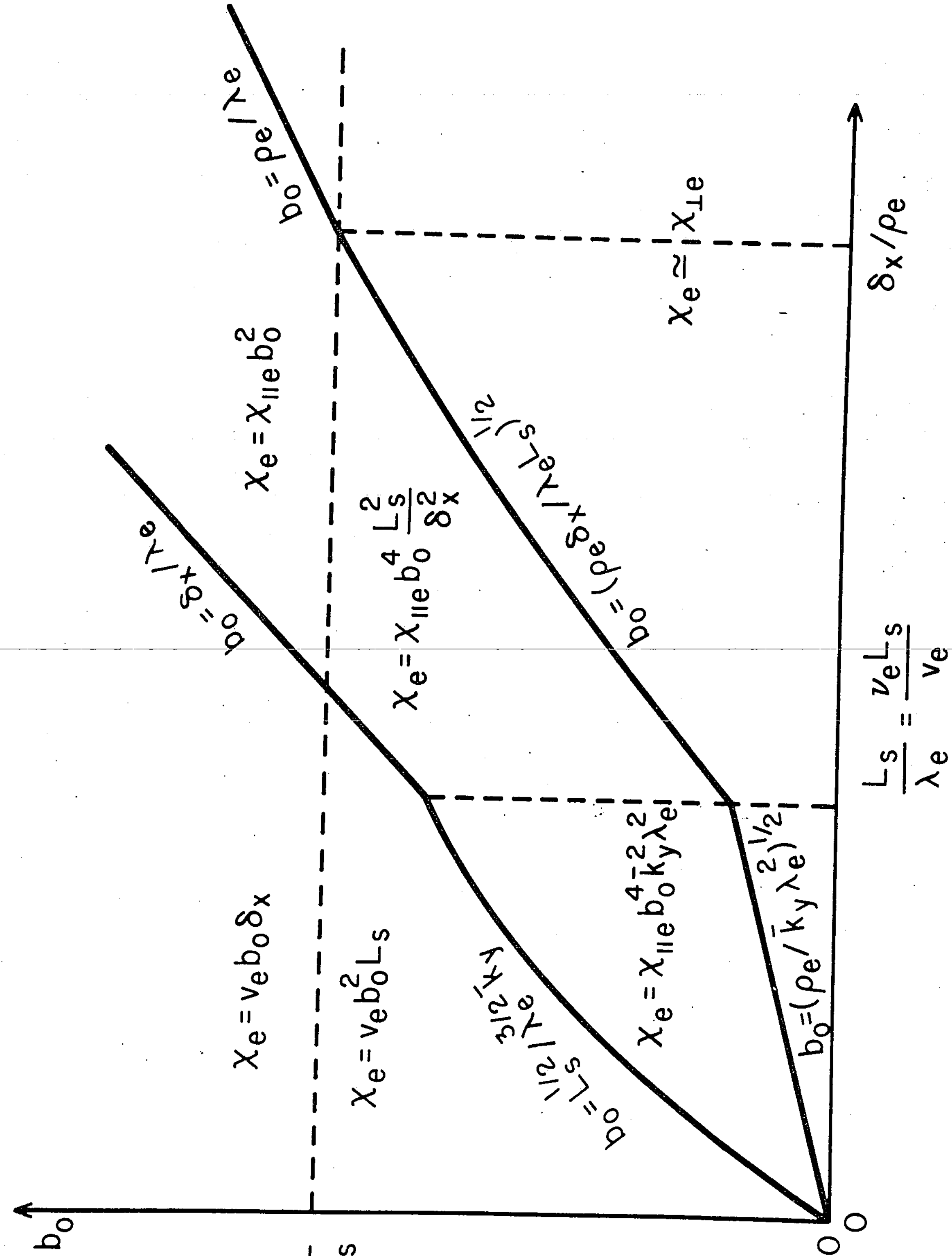

b_o
δ/L_s
0
0
δ_x/ρ_e
$\frac{L_s}{\lambda_e} = \frac{\nu_e L_s}{v_e}$
$b_o = \rho_e/\lambda_e$
$\chi_e \simeq \chi_{\perp e}$
$\chi_e = \chi_{\parallel e} b_o^2$
$b_o = (\rho_e \delta_x/\lambda_e L_s)^{1/2}$
$b_o = \delta_x/\lambda_e$
$\chi_e = \chi_{\parallel e} b_o^4 \frac{L_s^2}{\delta_x^2}$
$\chi_e = v_e b_o \delta_x$
$\chi_e = v_e b_o^2 L_s$
$b_o = L_s^{1/2}/\lambda_e^{3/2} \bar{k}_y$
$\chi_e = \chi_{\parallel e} b_o^4 \bar{k}_y^{-2} \lambda_e^2$
$b_o = (\rho_e/\bar{k}_y \lambda_e^2)^{1/2}$

FIGURE 12