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Theory of Neoclassical Resistivity-Gradient-Driven Turbulence

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Abstract

It is shown that rippling instabilities can tap the density gradient expansion free energy source through the density depenence of the neoclassical resistivity. Linear analyses show that the region where neoclassical rippling modes are significantly excited extends from the edge of the plasma to the region where $\nu_{*e} \leq 1$. Since these modes are non-dispersive, diamagnetic effects are negligible in comparision to the nonlinear decorrelation rate at saturation. Thus, the relevant regime is the 'strong turbulence' regime. The turbulent radial diffusivities of the temperature and the density are obtained as eigenvalues of the renormalized eigenmode equations at steady state. The density gradient acts to enhance the level of turbulence, compared to that driven by the temperature gradient alone. The saturated turbulent state is characterized by: current decoupling, the breakdown of Boltzmann relation, a radial mode scale of density fluctuations exceeding that of temperature fluctuations, implying that density diffusivity exceeds temperature diffusivity, and that density fluctuation levels exceed temperature fluctuation levels. Magnetic fluctuation levels are negligible.

I. Introduction

Resistivity-gradient-driven turbulence[1] (RGDT) has been successful in explaining many characteristics[2] of tokamak edge plasmas: large fluctuation levels, radially increasing large diffusivities, and the breakdown of Boltzmann relation. Consideration of impurity density fluctuation dynamics[3,4] and radiation effects[5] has made RGDT an even more realistic tokamak edge turbulence model. Recently, it has been argued[6] that these attractive features are ill-founded from the conventional standpoint of linear stability. Indeed, RGDT evloves from unstable rippling modes,[7–9] and detailed linear analyses[6] indicate that the linear rippling modes tend to be quenched at moderate temperature due to stabilizing effects of large parallel thermal conduction and finite electron diamagnetism. However, both theoretical and computational analyses[1,4] have demonstrated that the nonlinear evolution of RGDT is characterized by the nonlinear broadening of an asymmetric mode structure, so that the resistivity and potential fluctuations move away from the rational surface and decouple from the current perturbation. This current decoupling drastically modifies the usual linear structure[7] of the perturbations, and renders the stabilzing influence of field line bending ineffective. Therefore, RGDT is nonlinearly robust. It is also crucial to note that the tokamak edge is rarely quiescent,[2] due to the presence of impurities, plasma-wall interaction, etc.. Therefore, the conventional picture of linear instability developing from a quiescent plasma at equilibrium is not necessarily appropriate. Hence, the tokamak edge fluctuations can only be understood by nonlinear theory. All afore-mentioned RGDT models[1,4,5] are based on a reduced, resistive MHD description[10] of a plasma, and consists of Ohm's law and the vorticity and resistivity evolutions. However, this model is valid only in the Pfirsch-Schlüter collisionality regime.[11]

Recently, neoclassical MHD equations[12] have been derived which are valid in the experimentally relevant banana-plateau collisionality regimes. The most significant modification in RGDT due to neoclassical effects is that the neoclassical resistivity[12,13] is now a function of the density as well as the temperature, while the classical Spitzer resistivity[14] depends on the temperature only. Therefore, rippling modes[3–6] can couple to the density gradient, and the density fluctuation dynamics becomes significant in determining the evolution of neoclassical rippling modes. Main purpose of this paper is to investigate neoclassical resistivity-gradient-driven turbulence (NRGDT) evolving from neoclassical rippling modes, to incorporate density fluctuation dynamics, and to extend the validity regime of RGDT further into high temperature, low collisionality regimes of tokamaks. The resulting particle diffusivity increases as temperature decreases, allowing a possibility of explaining the radially increasing profile of particle diffusivity from experimental measurements.[2,15] We note that drift wave instsabilities[16] are unable to account for this experimental result.

Linear stability analysis shows that without neoclassical coupling to the density gradient, the large parallel thermal conduction has a strong stabilizing effect for small scale modes, and that the growth rate decreases with the poloidal mode number, m, for large m. In a small-m limit, parallel conduction is less efficient, and the growth rate increases with m. Therefore, conventional rippling modes are most unstable (linearly) for moderate m-values. When the density gradient is included, the equilibration of density fluctuations along flux surfaces is affected by parallel stress tensor effects (due to collisions between trapped and untrapped particles). This has a much weaker stabilizing effect than thermal conduction. Therefore, the unstable region extends both in distance from the edge (i.e., into regions of higher temperature) and in m-values.

The radial asymmetry in the mode structure develops further during the nonlinear evolution, resulting in negligible current fluctuation in the region of interest. This leads to the decoupling of the vorticity evolution equation from the other basic equations, so that the system consists of Ohm's law and the temperature and density evolution equations. Nonlinear saturation is attained when parallel dissipations, enhanced by nonlinear mode broadening, balance linear instability sources. The mode broadening results from nonlinear mode couplings mediated via $E \times B$ convection. Convective nonlinearity is renormalized to yield spectrum-dependent turbulent diffusivities. The turbulent temperature and density diffusivities are obtained as eigenvalues of the stationary renormalized eigenmode equations at saturation. Their values are much larger than naive mixing length estimates, based upon linear growth rates and mode widths. The dominant effect of the inclusion of the density dynamics is the considerable increase in the level of turbulence, in comparision to the case with only the temperature gradient.

Levels of electrostatic potential, temperature, and density fluctuations are determined. The level of density fluctuation is higher than that of temperature fluctuation as a consequence of larger radial scale-length of density fluctuation, and is different from the level of electrostatic fluctuation. The magnetic fluctuation level is shown to be too low to induce any significant heat transport along perturbed magnetic field lines.

The remainder of this paper is organized as follows: In Sec. II, the theoretical model for NRGDT is presented. The structure of the neoclassical resistivity is investigated in detail. In Sec. III, the linear stability results are presented. In Sec. IV, the nonlinear saturation mechanism is identified and the level of turbulence is determined. A discussion of the theory in the context of ELMs in DIII-D H-mode[17] plasmas is also presented. Finally, Sec. V contains a summary and conclusions.

II. Neoclassical Resistivity-Gradient-Driven Turbulence Model

In this section, we present the basic neoclassical resistivity-gradient-driven model which is derived from the simplified neoclassical MHD equations.[12] The simplification is due to the omission of perpendicular flow. The neoclassical correction[12,13] to the Spitzer resistivity is most relevant to the evolution of NRGDT, and is discussed in detail.

The simplified neoclassical Ohm's law in the electrostatic approximation can be written as:

$$\tilde{E}_{\|} = -\nabla_{\|}\tilde{\phi} = \eta_{\rm nc}\tilde{J}_{\|} + \tilde{\eta}_{\rm nc}J_{\|} - \frac{1}{e}\nabla_{\|}\left(T_e\frac{\tilde{n}}{n_0} + \tilde{T}_e\right). \tag{1}$$

Here, $E_{\|}$ is the parallel electric field, ϕ is the electrostatic potential, $J_{\|}$ is the parallel current, T_e is the electron temperature, and n_0 is the density. Also, $\widetilde{(\cdots)}$ denotes the perturbed quantity and the $\eta_{\rm nc}$ is the neoclassical resistivity, the expression for which will be given later. The origin of Eq. (1) can be easily seen by considering the electron momentum balance equation,

$$m_e\frac{dv_{\|}}{dt} = -eE_{\|} - \frac{1}{n_0}\mathbf{b}\cdot\nabla P_e + \frac{1}{n_0}\mathbf{b}\cdot\nabla\cdot\Pi_{\|e} + e\eta_{\rm sp}J_{\|}. \tag{2}$$

Here, P_e is the electron pressure, m_e is the electron mass, $\Pi_{\|}$ is the parallel stress tensor, and $\mathbf{b} = \mathbf{B}/B$. Also, the Spitzer resistivity,[14] $\eta_{\rm sp}$ is given by

$$\eta_{\rm sp} = K_{\|}\frac{m_e}{n_0 e^2}\nu_e,$$

where $K_{\|} \simeq 0.51$ for $Z_{\rm eff} = 1$ plasma, and the electron collision rate, ν_e, is

$$\nu_e = \frac{4}{3}\sqrt{2\pi}\frac{n_0 e^4}{m_e^{1/2}T_e^{3/2}}\ln\Lambda.$$

The parallel viscous drag due to collisions with trapped particles is given approximately by:

$$\mathbf{b}\cdot\nabla\cdot\Pi_{\|e} \simeq m_e n_0 \mu_e v_{\|}. \tag{3}$$

Here,

$$\mu_e \simeq \frac{2.3\epsilon^{1/2}\nu_e}{1 + 1.02\nu_{*e}^{1/2} + 1.07\nu_{*e}}, \tag{4}$$

where ϵ is the inverse aspect ratio, $\nu_{*e} = \nu_e/\omega_{be}\epsilon^{3/2}$, $\omega_{be} = v_{th,e}/R_0 q$, R_0 is the major radius, and q is the safety factor. The poloidal component of the first order electron perpendicular flow, leading to the bootstrap current[12,13] is omitted for simplicity. From Eqs. (2) and (3), Ohm's law becomes

$$E_\| = \eta_{\rm sp}\left(1+\frac{4.51\epsilon^{1/2}}{1+1.02\nu_{*e}^{1/2}+1.07\nu_{*e}}\right)J_\| - \frac{1}{en_0}\mathbf{b}\cdot\nabla P_e. \tag{5}$$

Thus, the neoclassical resistivity[12,13] can be written as

$$\eta_{\rm nc} = \eta_{\rm sp}\left(1+\frac{4.51\epsilon^{1/2}}{1+1.02\nu_{*e}^{1/2}+1.07\nu_{*e}}\right).$$

Note that $\eta_{\rm nc}$ now depends on the density as well, through the density dependence of ν_{*e}, while the Spitzer resistivity is a function of T_e only, i.e., $\eta_{\rm sp}\propto T_e^{-3/2}$. Therefore, in the neoclassical theory, resistivity-gradient-driven turbulence[1] is modified by, and depends on density fluctuation dynamics as well as temperature fluctuation dynamics. Exploring the consequence of this novel feature is the motivation of the present work.

The structure of the neoclassical resistivity fluctuation can be written as

$$\frac{\tilde\eta_{\rm nc}}{\eta_{\rm nc}} = -C_t\frac{\tilde T_e}{T_e} - C_n\frac{\tilde n}{n_0}, \tag{6}$$

where

$$\begin{aligned} C_n &= -\frac{\partial \ln \eta_{\rm nc}}{\partial \ln n_0} \\ &= \frac{4.51\epsilon^{1/2}(0.51\nu_{*e}^{1/2}+1.07\nu_{*e})}{(1+1.02\nu_{*e}^{1/2}+1.07\nu_{*e})^2}\left(1+\frac{4.51\epsilon^{1/2}}{1+1.02\nu_{*e}^{1/2}+1.07\nu_{*e}}\right)^{-1}, \end{aligned} \tag{7}$$

$$\begin{aligned} C_t &= -\frac{\partial \ln \eta_{\rm nc}}{\partial \ln T_e} \\ &= \frac{3}{2} - 2C_n. \end{aligned} \tag{8}$$

Since C_n vanishes in both limits ($\nu_{*e}\to 0$ and $\nu_{*e}\to\infty$), the density gradient effect on rippling modes also vanishes for values of ν_{*e} far from the unity. For moderate values of ϵ, C_n has a maximum value of about 0.2 around $\nu_{*e}\sim 1$ and decreases slowly as ν_{*e} increases, as seen in Fig. 1. Also, note that C_n and C_t are always positive.

The vorticity evolution equation can be derived from the charge neutrality condition:

$$\rho\frac{d}{dt}\nabla_\perp^2\tilde\phi = \frac{B^2}{c^2}\nabla_\|\tilde J_\|, \tag{9}$$

where ρ is the mass density. We have neglected the neoclassical enhancement of the inertia (arising from an additional polarization drift) and the neoclassical cross viscosity[12,18] (arising from an anisotropy in the perturbed pressure) in Eq. (9). The neoclassical cross viscosity, along with the bootstrap current which is also neglected in Eq. (1), act as destabilizing sources for the neoclassical pressure-gradient-driven instabilities.[18,19] The neoclassical enhancement of the inertia as well as the ion diamagnetism acts to reduce the growth rate. However, in lowest order, these are irrelevant to the evolution of NRGDT due to current decoupling. The heating due to the bootstrap current is also neglected for simplicity.

The density equation is determined from the continuity equation:

$$\frac{d\tilde{n}}{dt} = -\frac{c}{B}\mathbf{b} \times \nabla\tilde{\phi} \cdot \nabla n_0 + \frac{1}{e}\nabla_{\parallel}\tilde{J}_{\parallel} - n_0\nabla_{\parallel}\tilde{u}_{\parallel}. \tag{10}$$

The center-of-mass velocity $\tilde{u}_{\parallel}$ evolves as

$$\frac{d\tilde{u}_{\parallel}}{dt} = -\frac{c_s^2}{n_0}\nabla_{\parallel}\tilde{n} - \frac{1}{M}\nabla_{\parallel}\tilde{T}_e - \mu_i\tilde{u}_{\parallel}. \tag{11}$$

Here, ion temperature fluctuations are neglected, M is the ion mass,

$$c_s^2 = \frac{1}{M}(T_e + T_i),$$
$$\mu_i = \frac{0.66\epsilon^{1/2}\nu_i}{1 + 1.03\nu_{*i}^{1/2} + 0.31\nu_{*i}},$$

where ν_i is the ion collision rate and $\nu_{*i} = \nu_{*e\rightarrow i}$. The term involving μ_i comes from the ion neoclassical parallel viscous force due to collisions with trapped ions. The first-order ion perpendicular flow component has been omitted, consistently with previous simplifications. In the regime where modes grow slower than the effective neoclassical collision time ($d/dt \ll \mu_i$), Eq. (10) simplifies to:

$$\frac{d\tilde{n}}{dt} - \chi_n\nabla_{\parallel}^2\tilde{n} - \frac{n_0}{\mu_i M}\nabla_{\parallel}^2\tilde{T}_e = -\frac{c}{B}\mathbf{b} \times \nabla\tilde{\phi} \cdot \nabla n_0 + \frac{1}{e}\nabla_{\parallel}\tilde{J}_{\parallel}, \tag{12}$$

where $\chi_n = c_s^2/\mu_i$. Note that χ_n, combined with the shear, cuts off sound wave propagation. Also, note that up to additional couplings to $\tilde{T}_e$ and $\tilde{J}_{\parallel}$ dynamics, density fluctutation dynamics is similar to impurity fluctuation dynamics,[3,4] with neoclassical viscosity replacing effective parallel diffusion χ_z. Therefore, results structurally similar to those of Ref. 4 are expected.

The temperature evolution equation is

$$\frac{d}{dt}\tilde{T}_e - \chi_{\|}\nabla_{\|}^2\tilde{T}_e = -\frac{c}{B}\mathbf{b}\times\nabla\tilde{\phi}\cdot\nabla T_e, \tag{13}$$

where $\chi_{\|}$ is the parallel electron thermal conductivity. The large value of $\chi_{\|}$ tends to localize classical rippling modes around the mode rational surface, and thus has a strong stabilizing effect. We have neglected impurity radiation cooling effects,[5] for simplicity.

Equations (1), (9), (12), and (13) constitute the basic NRGDT model to be studied analytically.

III. Linear Theory

We study the linear instability described by the basic NRGDT model by Fourier analyzing perturbations as:

$$\tilde{\phi} = \sum_{m,n} \Phi_{mn}(x) \exp[i(m\theta - n\frac{z}{R}) + \gamma t],$$

where m and n are poloidal and toroidal mode numbers, respectively, R is the major radius, z is the coordinate along the major axis, and $x = r - r_s$ is the distance from the singular surface, r_s, at which $q(r_s) = m/n$. Then, the eigenmode equation becomes

$$\begin{aligned}\frac{d^2}{dX^2}\Phi - \left[\frac{1}{4}X^2 - \frac{\delta_t X}{1+b_tX^2} - \frac{\delta_n X}{1+b_nX^2}\left(1 - \frac{d_tX^2}{1+b_tX^2}\right)\right]\Phi \\ - i\left[\frac{\alpha_t X^2}{1+b_tX^2} + \frac{\alpha_n X^2}{1+b_nX^2}\left(1 - \frac{d_tX^2}{1+b_tX^2}\right)\right]\Phi = 0. \qquad (14)\end{aligned}$$

Here,

$$\begin{aligned} X &= x/x_R, \\ x_R^4 &= \frac{1}{4}\eta_{\mathrm{nc}}\gamma\rho\left(\frac{cL_s}{Bk_\theta}\right)^2, \\ \delta_t &= \frac{cC_tE_\parallel L_s}{4B\gamma L_t x_R}, \\ b_t &= \frac{\chi_\parallel}{\gamma}\left(\frac{k_\theta x_R}{L_s}\right)^2, \\ \delta_n &= \frac{C_n\delta_t}{C_t\eta_e}, \\ d_t &= \frac{\eta_e T_e}{\gamma\mu_i M}\left(\frac{k_\theta x_R}{L_s}\right)^2, \\ b_n &= \frac{\chi_n}{\gamma}\left(\frac{k_\theta x_R}{L_s}\right)^2, \\ \alpha_t &= \frac{T_e c k_\theta}{4L_t e\gamma B}, \\ \alpha_n &= \alpha_t/\eta_e. \end{aligned}$$

We have used $\nabla_\perp^2 \simeq d^2/dx^2 \gg k_\theta^2$, $\nabla_\theta = ik_\theta = im/r_s$, and $\nabla_\parallel = ik_\parallel = -ik_\theta x/L_s$. Also, $L_s = Rq^2/rq'$ is the shear length, $L_t = -[d\ln T_e/dr]^{-1}$ is the temperature scale length, $L_n = -[d\ln n_0/dr]^{-1}$ is the density scale length, and $\eta_e = L_n/L_t$. The first term in Eq. (14) represents the inertia and the second represents field line bending which has a stabilizing effect. The third and the fourth represent drive due to the temperature gradient (δ_t) and the density gradient (δ_n), respectively, through temperature and density dependences of the neoclassical resistivity. The drive is limited to small regions by b_t and b_n. Also, the d_t factor represents the coupling of temperature evolution to density evolution. Finally, the imaginary part of Eq. (14) represents the electron diamagnetic effect, which gives rise to finite real frequency. It is crucial to note that the eigenmode equation is not symmetric in x, with the consequence that Φ is asymmetrically skewed about the rational surface. The resistivity fluctuation breaks the parity of Ohm's law to localize Φ at the high resistivity side around the rational surface. This is because the destabilizing part of $\tilde{\mathbf{J}} \times \mathbf{B}$ force (proportional to $\tilde{\eta}$) changes sign as the rational surface is crossed.[3,7]

Equation (14) can be solved appproximately[8,9] by using the WKB method, by ignoring the subdominant imaginary part. Eigenvalues are determined from the condition:

$$\int_0^{X_0} dX \left[\frac{\delta_t X}{1+b_t X^2} + \frac{\delta_n X}{1+b_n X^2}\left(1 - \frac{d_t X^2}{1+b_t X^2}\right) - \frac{X^2}{4}\right]^{1/2} = \frac{\pi}{2}(2l+1), \tag{15}$$

where l is an integer, and X_0 is a turning point, other than 0.

First, we consider the limit where $\delta_n = 0$, corresponding to the flat density profile, where the density coupling to the resistivity is neglected. In the limit of small electron parallel thermal conduction ($b_t = 0$), exact solutions can be found:

$$\Phi(X) = \Phi(0) \exp\left[-\frac{1}{4}(X - 2\delta_t)^2\right] H_l \left[2^{1/2}(X - 2\delta_t)\right], \tag{16}$$

where H_l is a Hermite polynomial. The eigenvalue condition is $2\delta_t^2 = 2l + 1$. Note that the center of modes are located $2\delta_t$ outside the rational surface. Most dominant modes are the localized $l = 0$ modes, for which

$$\gamma_t = \left(\frac{\pi}{4}\right)^{1/5} \left(\frac{R r^3 q k_\theta L_s C_t^2 J_\parallel^2}{c^2 B^2 L_t^2}\right)^{2/5} S_M^{-3/5} \tau_A^{-1}. \tag{17}$$

Here, $S_M = \tau_R/\tau_A$ is the local magnetic Reynolds number, $\tau_R = r^2/\eta c^2$ is the resistive time scale, $\tau_A = Rq/v_A$ is the poloidal Alfvén transit time, and $v_A = B/(4\pi\rho)^{1/2}$ is the Alfvén speed. In the limit of large $\chi_\parallel$ (b_t), the dominant mode has

$$\gamma_t = 0.78 \left(\frac{c E_\parallel C_t L_s}{B L_t \chi_\parallel}\right)^{4/3} \left(\frac{L_s}{k_\theta}\right)^{2/3} \frac{R^2 q^2}{r^2} S_M \tau_A^{-1}. \tag{18}$$

Except for the extreme edge region of tokamaks, the approximation given by Eq. (18) is more appropriate. The radial scale length, W, is also greatly reduced compared to the small $\chi_\parallel$ limit, and is given by

$$W_t \sim \left(\frac{cE_\parallel C_t}{BL_t\chi_\parallel k_\theta^2}\right)^{1/3} L_s. \tag{19}$$

The naive mixing length 'principle', based on linear mode widths and growth rates, thus predicts a thermal diffusivity:

$$D_t^{\mathrm{ML}} \sim \left(\frac{cE_\parallel C_t L_s Rq}{BL_t\chi_\parallel k_\theta r}\right)^2 S_M \frac{L_s^2}{\tau_A}, \tag{20}$$

which is small, due to strong dependence on $\chi_\parallel$. This is qualitatively different from the nonlinear prediction of D^t, as we will see later in Sec. IV. This is because the character of the nonlinear evolution of NRGDT differs significantly from its linear antecedent, because of current decoupling.

It is interesting to note that the growth rate for a small b_t scales as $\gamma_t \propto m^{2/5}$. The nonlinear saturation mechanism requires that the energy cascades to small scale modes where it is eventually dissipated. Therefore, additional sinks at small scales would be necessary to achieve nonlinearly saturated state in a small b_t limit. However, for a large b_t, $\gamma_t \propto m^{-2/3}$, and nonlinear saturation is attained without additional sinks.

Next, we consider the isothermal limit ($\delta_t = 0$), while retaining the density dependence of the resistivity ($\delta_n \neq 0$). The mathematical structure is same as when $\delta_n = 0$, since $b_t \gg d_t$. When b_n is small,

$$\gamma_n = \left(\frac{\pi}{4}\right)^{1/5} \left(\frac{Rr^3 q k_\theta L_s C_n^2 J_\parallel^2}{c^2 B^2 L_n^2}\right)^{2/5} S_M^{-3/5} \tau_A^{-1}. \tag{21}$$

When $b_n \to \infty$,

$$\gamma_n = 0.78 \left(\frac{cE_\parallel C_n L_s}{BL_n\chi_n}\right)^{4/3} \left(\frac{L_s}{k_\theta}\right)^{2/3} \frac{R^2 q^2}{r^2} S_M \tau_A^{-1}, \tag{22}$$

$$W_n \sim \left(\frac{cE_\parallel C_n}{BL_n\chi_n k_\theta^2}\right)^{1/3} L_s, \tag{23}$$

$$D_n^{\mathrm{ML}} \sim \left(\frac{cE_\parallel C_n L_s Rq}{BL_n\chi_n k_\theta r}\right)^2 \frac{L_s^2}{\tau_A}. \tag{24}$$

Since $b_n/b_t \sim O(m_e/M)$, these modes cannot be neglected ($\gamma_n \gg \gamma_t$), even in a relatively high temperature region.

In general, tokamaks operate in the regime where L_t and L_n are comparable and same in sign. Therefore, both driving terms contribute to the instability. In the limit of $b_t = b_n = 0$,

$$\gamma^{5/4} = \gamma_t^{5/4} + \gamma_n^{5/4},$$

i.e., the density gradient acts to enhance the growth rate. In a more relevant, large b_t and b_n limit, γ is expected to be slightly bigger than γ_n (Eq. (22)), since $b_t \gg b_n$. When two gradients have different sign, these modes will grow more slowly.

IV. Nonlinear Theory

Having discussed the linear properties of the basic NRGDT model, we investigate the nonlinear evolution and saturation of NRGDT evolving from linearly unstable neoclassical rippling modes. Energy-like quantities (E_K, E_t, and E_n) are defined. These are required to be stationary at saturation. The condition $\partial E_K/\partial t = 0$ leads to the current decoupling, an important feature common to turbulence[1,4,5] evolving from the linear rippling instabilties.[6–9] Two remaining conditions determine the level of the turbulence. We use the standard one-point DIA theory[20] to iteratively renormalize the dominant $E \times B$ convective nonlinearities. This procedure shows that amplitude-dependent radial diffusions are induced by convective nonlinearities. The spectrum-dependent radial diffusion broadens radial scales of modes. This, in turn, facilitates coupling to $\chi_\parallel$ and χ_n dissipation as a means for saturation. This saturation mechanism persists without, and dominates over, the quasilinear flattening of the background gradients.

To study the nonlinear dynamics of fluctuations, it is customary to define energy-like integrals which are quadratic in fluctuation quantities as follows:

$$E_K = \frac{1}{2}\int d\tau\, |\nabla_\perp \tilde{\phi}|^2,$$
$$E_t = \frac{1}{2}\int d\tau\, |\tilde{T}_e|^2,$$
$$E_n = \frac{1}{2}\int d\tau\, |\tilde{n}|^2.$$

These will evolve as:

$$\frac{\partial E_K}{\partial t} = -\int d\tau\, \frac{B^2}{\rho c^2}\tilde{\phi}^* \nabla_\parallel \tilde{J}_\parallel, \tag{25}$$
$$\frac{\partial E_t}{\partial t} = -\int d\tau\, \left[\frac{c}{B}\tilde{T}_e^* \mathbf{b}\times\nabla\tilde{\phi}\cdot\nabla T_e + \chi_\parallel |\nabla_\parallel \tilde{T}_e|^2\right], \tag{26}$$
$$\frac{\partial E_n}{\partial t} = -\int d\tau\, \left[\frac{c}{B}\tilde{n}^* \mathbf{b}\times\nabla\tilde{\phi}\cdot\nabla n_0 + \chi_n |\nabla_\parallel \tilde{n}|^2 - \frac{1}{e}\tilde{n}^*\nabla_\parallel \tilde{J}_\parallel - \frac{n_0}{\mu_i M}\tilde{n}^*\nabla_\parallel^2 \tilde{T}_e\right]. \tag{27}$$

Nonlinear mode coupling terms do not appear explicitly the spectrum-summed energy evolution since they only represent energy transfer between fluctuations at different scales. Destabilizing sources are the resistivity perturbations produced by the relaxation of the background temperature and density gradients. The magnetic field line bending, thermal conduction, and viscous drags are stabilizing. We note that the current decoupling approximation ($\tilde{J}_\parallel$ is negligible where $\tilde{\phi}$ is significant) is a sufficient condition for $\partial E_t/\partial t = 0$. Also, the simulation results[1,4] support this condition showing that $\tilde{\phi}$, $\tilde{T}_e$, and $\tilde{n}$ (equivalent to $\tilde{Z}_{\text{eff}}$ in Ref. 4) are sharply skewed off the rational surface where $\tilde{J}_\parallel$ is peaked. Therefore, we can neglect $\tilde{J}_\parallel$ in the region of interest. This condition (current decoupling) greatly simplifies the governing equations because it renders the vorticity evolution equation irrelevant. This decoupled system is also referred to as the *current convective* instability.[8] Indeed, in the nonlinear regime, RGDT reverts from its manifestation as a rippling mode to that of a current convective instability. This is accomplished by the fact that nonlinear effects resolve singularities at the mode rational surface. We study the nonlinear evoluton of NRGDT for this case.

We use a standard one-point DIA theory[1,4,20] to describe the nonlinear evolution of NRGDT. The driven fields, ($\tilde{n}^{(2)}_{\mathbf{k}''}$ and $\tilde{T}^{(2)}_{e,\mathbf{k}''}$), which are 'second order' in fluctuations, are calculated from Eqs. (12) and (13). These are produced by the direct beating between the test ($\mathbf{k}$) and the background ($\mathbf{k}'$) modes where $\mathbf{k}'' = \mathbf{k} + \mathbf{k}'$. Subdominant contributions from other driven fields are neglected. The driven fields are then substituted into the $E \times B$ convective nonlinearity to yield amplitude-dependent diffusivities. The coupled, renormalized equatons are

$$\frac{\partial}{\partial t}\tilde{T}_{e,\mathbf{k}} - \chi_\parallel \nabla_\parallel^2 \tilde{T}_{e,\mathbf{k}} - \frac{\partial}{\partial x} D^t_{\mathbf{k}} \frac{\partial}{\partial x}\tilde{T}_{e,\mathbf{k}} = \frac{cE_\parallel}{B}\frac{k_\theta T_e}{k_\parallel L_t}\left(C_t \frac{\tilde{T}_{e,\mathbf{k}}}{T_e} + C_n \frac{\tilde{n}_{\mathbf{k}}}{n_0}\right) - i\omega^t_{*e} T_e \left(\frac{\tilde{T}_{e,\mathbf{k}}}{T_e} + \frac{\tilde{n}_{\mathbf{k}}}{n_0}\right), \tag{28}$$

$$\frac{\partial}{\partial t}\tilde{n}_{\mathbf{k}} - \chi_n \nabla_\parallel^2 \tilde{n}_{\mathbf{k}} - \frac{\partial}{\partial x} D^n_{\mathbf{k}} \frac{\partial}{\partial x}\tilde{n}_{\mathbf{k}} - \frac{n_0}{\mu_i M}\nabla_\parallel^2 \tilde{T}_{e,\mathbf{k}} = \frac{cE_\parallel}{B}\frac{k_\theta n_0}{k_\parallel L_n}\left(C_t \frac{\tilde{T}_{e,bfk}}{T_e} + C_n \frac{\tilde{n}_{\mathbf{k}}}{n_0}\right) - i\omega^n_{*e} n_0 \left(\frac{\tilde{T}_{e,\mathbf{k}}}{T_e} + \frac{\tilde{n}_{\mathbf{k}}}{n_0}\right). \tag{29}$$

Here,

$$\omega^t_{*e} = k_\theta c T_e / eBL_t,$$

$$\omega_{*e}^{n} = k_\theta c T_e / eBL_n,$$
$$D_{\mathbf{k}}^{t} = \sum_{\mathbf{k}'} \left(\Gamma_{\mathbf{k}''}^{t}\right)^{-1} \left| \frac{c}{B} k'_\theta \tilde{\phi}_{\mathbf{k}'} \right|^2, \tag{30}$$
$$D_{\mathbf{k}}^{n} = \sum_{\mathbf{k}'} \left(\Gamma_{\mathbf{k}''}^{n}\right)^{-1} \left| \frac{c}{B} k'_\theta \tilde{\phi}_{\mathbf{k}'} \right|^2. \tag{31}$$

The propagators are

$$(\Gamma_{\mathbf{k}''}^{t})^{-1} = \mathrm{Re}\left[\gamma_{\mathbf{k}''} + \Delta\omega_{\mathbf{k}''}^{t} - \chi_\parallel \nabla_\parallel^2 + i\omega_{*e}^{t}(k'')\right]^{-1}, \tag{32}$$
$$(\Gamma_{\mathbf{k}''}^{n})^{-1} = \mathrm{Re}\left[\gamma_{\mathbf{k}''} + \Delta\omega_{\mathbf{k}''}^{n} - \chi_n \nabla_\parallel^2 + i\omega_{*e}^{n}(k'')\right]^{-1}. \tag{33}$$

Here, $\mathrm{Re}[(\cdots)]$ denotes the real part of $(\cdots)$, and $\Delta\omega_{\mathbf{k}''}$ is the nonlinear decorrelation rate, which effectively limits the coherence time of nonliner interaction.

It is interesting to note that one-point DIA theory generically does not conserve the total energy. However, in energy spectrum evolution,[21] incoherent emission as well as coherent damping act to conserve energy. Indeed, the local (in $\mathbf{k}$) competition of these two effects yields a spectrum *flow rate* comparable to the damping rate predicted by one-point theory. This feature underlies the validity of one-point theory here. It is worthwhile to draw analogy between the propagtor in energy-conserving two-point theory and the propagator in one-point theory. By neglecting $\tilde{n}$ for simplicity, one can write for $\tilde{T}_e$:

$$\frac{\partial \tilde{T}_{e,\mathbf{k}}}{\partial t} + i\Omega_{\mathbf{k}} \tilde{T}_{e,\mathbf{k}} = \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \Lambda_{\mathbf{k}',\mathbf{k}''} \tilde{\phi}_{\mathbf{k}'} \tilde{T}_{e,\mathbf{k}''}, \tag{34}$$

where,

$$\Omega_{\mathbf{k}} = \omega_{*e}^{t} - i\left(\chi_\parallel k_\parallel^2 - \frac{C_t k_\theta \eta J_\parallel}{k_\parallel B L_t}\right),$$
$$\Lambda_{\mathbf{k}',\mathbf{k}''} = \frac{c}{B}\mathbf{k}' \times \mathbf{k}''.$$

Nonlinear terms conserve energy since $\int d\tau\, \tilde{T}_e \mathbf{b} \times \nabla\tilde{\phi} \cdot \nabla\tilde{T}_e = 0$. By multiplying $\tilde{T}_{e,\mathbf{k}}^{*}$ to Eq. (34), one finds the evolution of the energy spectrum:

$$\begin{aligned} &\frac{\partial}{\partial t}\left\langle \tilde{T}_e^2 \right\rangle_{\mathbf{k}} - 2\,\mathrm{Im}(\Omega_{\mathbf{k}})\left\langle \tilde{T}_e^2 \right\rangle_{\mathbf{k}} \\ &\quad = \mathrm{Re} \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \Lambda_{\mathbf{k}',\mathbf{k}''} \left\langle \tilde{T}_{e,\mathbf{k}}^{*} \tilde{\phi}_{\mathbf{k}'} \tilde{T}_{e,\mathbf{k}''} \right\rangle \\ &\quad \simeq \mathrm{Re} \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \Lambda_{\mathbf{k}',\mathbf{k}''} \left(\left\langle \tilde{T}_{e,\mathbf{k}}^{*(2)} \tilde{\phi}_{\mathbf{k}'} \tilde{T}_{e,\mathbf{k}''} \right\rangle + \left\langle \tilde{T}_{e,\mathbf{k}}^{*} \tilde{\phi}_{\mathbf{k}'} \tilde{T}_{e,\mathbf{k}''}^{(2)} \right\rangle \right). \end{aligned} \tag{35}$$

Here, $\langle(\cdots)\rangle$ denotes ensemble-average. The first term in the right-hand-side of Eq. (35) denotes the incoherent emission and the second term denotes the coherent damping. The contributions from $\langle \tilde{T}^*_{e,\mathbf{k}}\tilde{\phi}^{(2)}_{\mathbf{k}'}\tilde{T}_{e,\mathbf{k}''}\rangle$ are similar to the second term, and are neglected for simplicity. From Eq. (34), $\tilde{T}^{*(2)}_{e,\mathbf{k}}$ satisfies:

$$\tilde{T}^{*(2)}_{e,\mathbf{k}} = \int^t dt'\, e^{-(i\Omega^*_{\mathbf{k}}+\Delta\omega_{\mathbf{k}})(t-t')}\Lambda^*_{\mathbf{k}',\mathbf{k}''}\tilde{\phi}^*_{\mathbf{k}'}(t')\tilde{T}^*_{e,\mathbf{k}''}(t').$$

Here, $\Delta\omega_{\mathbf{k}}$ reflects nonlinear scrambling due to interaction with modes other than $\mathbf{k}'$ and $\mathbf{k}''$. We further assume that the spectrum evolution time-scale is much larger than the nonlinear scrambling time-scale to approximate:

$$\langle\tilde{\phi}^*_{\mathbf{k}'}(t')\tilde{\phi}_{\mathbf{k}'}(t)\rangle = \left\langle\tilde{\phi}^2(t)\right\rangle_{\mathbf{k}'} e^{-(i\Omega_{\mathbf{k}'}+\Delta\omega_{\mathbf{k}'})(t-t')}.$$

Therefore, Eq. (35) can be written as

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\langle\tilde{T}^2_e\right\rangle_{\mathbf{k}} - 2\,\mathrm{Im}(\Omega_{\mathbf{k}})\left\langle\tilde{T}^2_e\right\rangle_{\mathbf{k}} \\
&\quad\simeq \mathrm{Re}\sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''}\int^t dt'\,\Lambda_{\mathbf{k}',\mathbf{k}''}\left[\Lambda^*_{\mathbf{k}',\mathbf{k}''}\left\langle\tilde{\phi}^2(t)\right\rangle_{\mathbf{k}'}\left\langle\tilde{T}^2_e(t)\right\rangle_{\mathbf{k}''} + \Lambda_{-\mathbf{k}',\mathbf{k}}\left\langle\tilde{\phi}^2(t)\right\rangle_{\mathbf{k}'}\left\langle\tilde{T}^2_e(t)\right\rangle_{\mathbf{k}}\right] \\
&\qquad\times e^{-\{i(\Omega^*_{\mathbf{k}}+\Omega_{\mathbf{k}'}+\Omega_{\mathbf{k}''})+\Delta\omega_{\mathbf{k}}+\Delta\omega_{\mathbf{k}'}+\Delta\omega_{\mathbf{k}''}\}(t-t')} \\
&\quad\simeq \mathrm{Re}\sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''}\Lambda_{\mathbf{k}',\mathbf{k}''}\frac{\Lambda^*_{\mathbf{k}',\mathbf{k}''}\left\langle\tilde{\phi}^2(t)\right\rangle_{\mathbf{k}'}\left\langle\tilde{T}^2_e(t)\right\rangle_{\mathbf{k}''} + \Lambda_{-\mathbf{k}',\mathbf{k}}\left\langle\tilde{\phi}^2(t)\right\rangle_{\mathbf{k}'}\left\langle\tilde{T}^2_e(t)\right\rangle_{\mathbf{k}}}{i(\Omega^*_{\mathbf{k}}+\Omega_{\mathbf{k}'}+\Omega_{\mathbf{k}''})+\Delta\omega_{\mathbf{k}}+\Delta\omega_{\mathbf{k}'}+\Delta\omega_{\mathbf{k}''}}. \quad (36)
\end{aligned}$$

Note that the terms on the right hand side of Eq. (36) are comparable as noted above. Also, the nonlinear decorrelation rate can be recursively defined as

$$\Delta\omega_{\mathbf{k}} \simeq \mathrm{Re}\sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''}\frac{\Lambda_{\mathbf{k}',\mathbf{k}''}\Lambda_{-\mathbf{k}',\mathbf{k}}\left\langle\tilde{\phi}^2(t)\right\rangle_{\mathbf{k}'}}{i(\Omega^*_{\mathbf{k}}+\Omega_{\mathbf{k}'}+\Omega_{\mathbf{k}''})+\Delta\omega_{\mathbf{k}}+\Delta\omega_{\mathbf{k}'}+\Delta\omega_{\mathbf{k}''}}.$$

Therefore, the propagator can be written as:

$$(\Gamma^t_{\mathbf{k}''})^{-1} = \mathrm{Re}\left[i(\Omega^*_{\mathbf{k}}+\Omega_{\mathbf{k}'}+\Omega_{\mathbf{k}''})+\Delta\omega_{\mathbf{k}}+\Delta\omega_{\mathbf{k}'}+\Delta\omega_{\mathbf{k}''}\right]^{-1}. \quad (37)$$

When nonlinear interactions are negligible, the interaction time becomes infinite, and we recover the *weak turbulence* limit. However, since rippling modes are nondispersive

($\omega_{\mathbf{k}''} \simeq \omega_{\mathbf{k}}+\omega_{\mathbf{k}'}$), NRGDT is always in the *strong turbulence* regime, and the real frequency and ω_{*e} effects play no role. Also, the radial diffusivities can be written as:

$$D_{\mathbf{k}}^t = \frac{L_s^2 \langle \tilde{V}_r^2 \rangle}{\chi_\| \overline{k}_\theta^2 (\Delta^t)^2}, \tag{38}$$

$$D_{\mathbf{k}}^n = \frac{L_s^2 \langle \tilde{V}_r^2 \rangle}{\chi_n \overline{k}_\theta^2 (\Delta^n)^2}. \tag{39}$$

Here,

$$\tilde{V}_r^2 = \sum_{\mathbf{k}} \left| \frac{c}{B} k_\theta \tilde{\phi}_{\mathbf{k}} \right|^2, \tag{40}$$

and $\overline{k}_\theta$ is the spectrum-averaged mean poloidal wavenumber. Also, Δ^t and Δ^n are the nonlinear radial scale lengths for the temperature and the density fluctuations, respectively, evaluated at $\overline{k}_\theta$.

We now determine the level of the turbulence which mediates the energy cascade from the dominant large scale modes to small scale modes, thus leading to the saturation of NRGDT. To find the dependences of Δ^t and Δ^n on the level of turbulence, we asymptotically balance (at large x) the radial turbulent diffusions with parallel conduction terms, yielding:

$$\left(\Delta_{\mathbf{k}}^t\right)^4 = \frac{L_s^2 D_{\mathbf{k}}^t}{\chi_\| k_\theta^2}, \tag{41}$$

$$\left(\Delta_{\mathbf{k}}^n\right)^4 = \frac{L_s^2 D_{\mathbf{k}}^n}{\chi_n k_\theta^2}. \tag{42}$$

We note that Δ^t and Δ^n increase as the level of the turbulence increases. When the turbulence level reaches the point where the dissipation sink ($\chi_\| \nabla_\|^2$ and $\chi_n \nabla_\|^2$) balances the linear instability source, the saturated state can be attained.

Following Ref. 8, we multiply $\int d\tau\, \tilde{n}^*$ to Eqs. (28) and (29), yielding:

$$\langle \tilde{n} | L_1 | \tilde{T}_e \rangle \langle \tilde{n} | \tilde{T}_e \rangle + \langle \tilde{n} | L_2 | \tilde{n} \rangle \langle \tilde{n} | \tilde{n} \rangle = 0, \tag{43}$$

$$\langle \tilde{n} | L_3 | \tilde{T}_e \rangle \langle \tilde{n} | \tilde{T}_e \rangle + \langle \tilde{n} | L_4 | \tilde{n} \rangle \langle \tilde{n} | \tilde{n} \rangle = 0. \tag{44}$$

Here,

$$\langle\tilde{n}|\, L\, |\tilde{T}_e\rangle = \frac{\int d\tau\, \tilde{n}^* L \tilde{T}_e}{\langle\tilde{n}|\tilde{T}_e\rangle},$$

$$\langle\tilde{n}|\tilde{T}_e\rangle = \int d\tau\, \tilde{n}^* \tilde{T}_e,$$

$$L_1 = \chi_\parallel \frac{k_\theta^2}{L_s^2} x^2 - D_{\mathrm{k}}^t \frac{\partial^2}{\partial x^2} - C_t \frac{cE_\parallel L_s}{BL_t} \frac{1}{x},$$

$$L_2 = -\frac{T_e}{n_0} C_n \frac{cE_\parallel L_s}{BL_n} \frac{1}{x},$$

$$L_3 = \frac{n_0 k_\theta^2}{\mu_i M L_s^2} x^2 - \frac{n_0}{T_e} C_t \frac{cE_\parallel L_s}{BL_t} \frac{1}{x},$$

$$L_4 = \chi_n \frac{k_\theta^2}{L_s^2} x^2 - D_{\mathrm{k}}^n \frac{\partial^2}{\partial x^2} - C_n \frac{cE_\parallel L_s}{BL_n} \frac{1}{x},$$

where we have required $\partial/\partial t = 0$ for saturation. We note that $\tilde{T}_e$ and $\tilde{n}$ are skewed about the rational surface, and that they are negligible at $x = 0$. Therefore, integrations involving $1/x$ remains finite. Also, when cross products are involved in integration, Δ^t should be taken as the characteristic width, since $\Delta^t < \Delta^n$. For Eqs. (43) and (44) to have a nontrivial solution, the determinant should vanish, i.e.,

$$\begin{vmatrix} \tilde{V}_r - C_t cE_\parallel L_s/BL_t & -C_n cE_\parallel L_s/BL_n \\ T_e \tilde{V}_r/\chi_\parallel \mu_i M - C_t cE_\parallel L_s/BL_t & \tilde{V}_r - C_n cE_\parallel L_s/BL_n \end{vmatrix} = 0. \tag{45}$$

The solution is

$$\tilde{V}_r = \frac{cE_\parallel L_s}{BL_n} \left[C_n \left(1 + \frac{T_e}{\chi_\parallel \mu_i M} \right) + C_t \eta_e \right]. \tag{46}$$

As expected, the density gradient acts to enhance the turbulence level, in comparision to the case with only the temperature gradient.[1] However, the way the gradient drives combine $[(C_n/L_n) + (C_t/L_t)]$, is interesting because one may naively expect the temperature gradient drive contribution to be suppressed, due to the large parallel thermal conduction.

From Eqs. (38)-(42), and (46), we obtain

$$\Delta^t = \left[\frac{cE_\parallel L_s}{BL_n} (C_n + C_t \eta_e) \right]^{\frac{1}{3}} \left(\chi_\parallel \frac{\overline{k}_\theta^2}{L_s^2} \right)^{-\frac{1}{3}}, \tag{47}$$

$$\Delta^n = \left[\frac{cE_\parallel L_s}{BL_n} (C_n + C_t \eta_e) \right]^{\frac{1}{3}} \left(\chi_n \frac{\overline{k}_\theta^2}{L_s^2} \right)^{-\frac{1}{3}}. \tag{48}$$

We have used $\chi_\parallel \gg T_e/\mu_i M$. Turbulent radial diffusivities are then given by

$$D^t = \left[\frac{cE_\parallel L_s}{BL_n}(C_n + C_t\eta_e)\right]^{\frac{4}{3}} \left(\chi_\parallel \frac{\overline{k}_\theta^2}{L_s^2}\right)^{-\frac{1}{3}}, \tag{49}$$

$$D^n = \left[\frac{cE_\parallel L_s}{BL_n}(C_n + C_t\eta_e)\right]^{\frac{4}{3}} \left(\chi_n \frac{\overline{k}_\theta^2}{L_s^2}\right)^{-\frac{1}{3}}. \tag{50}$$

Since $\Delta^n \gg \Delta^t$, and Δ^n controls the particle flux, the thermal flux due to NRGDT is dominated by convection ($D^n \gg D^t$). By comparing Eq. (49) for $L_n \to \infty$ (or $C_n = 0$) with Eq. (20), we find that the prediction of $D_t^{\rm ML}$ according to the mixing length principle differs substantially from the more consistent nonlinear result. The same can be said when Eq. (50) for $L_t \to \infty$ is compared with Eq. (24). However, Δ^t and Δ^n compare well with W^t and W^n, respectively, in limiting cases. Therefore, we conclude that the nonlinear decorrelation time-scale decouples from the linear growth time-scale, in contradiction to the mixing length prediction. Also, since L_n has the same sign as L_t in tokamak discharges, the density gradient acts to enhance the turbulence level. The density gradient drive is most relevant near $\nu_{*e} \sim 1$, where C_n has a maximum, and C_t has a minimum. The density gradient drive becomes also important for a steep density profile ($\eta_e \ll 1$). Thus, the region where resistivity-gradient-driven turbulence is significantly excited broadens inward from the edge of the plasma. The particle diffusivity in Eq. (50) has an interesting scaling which may offer further insights into understanding of confinement properties in H-mode[22] tokamak edge, in particular, ELMs in DIII-D tokamak.[17] The confinement improvement in the H-mode after L→H transition is due to the formation of a transport barrier just inside the separatrix flux surface.[23] It has been also suggested[24] that the strong global shear near separatrix region is responsible for the reduction of turbulent transport due to resistive MHD type of modes and the creation of a transport barrier. This favorable role of global shear is also supported by the limiter H-mode results in JFT-2M,[25] where the quality of H-mode is better for higher elongation. After L→H transition, the plasma density at the edge rises continuously. In DIII-D, the electron density profile changes dramatically, and becomes very flat (sometimes even hollow) with a sharp gradient at transport barrier region. It is interesting to note that T_e at the edge remaions relatively low, implying a higher value of ν_{*e} for H-mode compared to L-mode. This unique feature of DIII-D H-mode profiles at the edge (sharp L_n, high n_0, $\nu_{*e} > 1$, modest T_e) is usually destroyed by ELMs, resulting in a large outward particle flux. Interestingly, from Eq. (50), we have

$$D^n \propto \frac{n_0^{1/3}}{T_e^{5/6}} \left[\frac{C_n(\nu_{*e})}{L_n} + \frac{C_t(\nu_{*e})}{L_t} \right]^{4/3},$$

which indicates a rapid increase in turbulent particle flux as the density and density gradient are increased as ν_{*e} passes through unity from below. To this end, it should be noted that recent experiments have established that the ideal ballooning stability limits edge pressure gradients but have not identified the physical mechanism of the giant ELMs. Indeed, it should be mentioned that resistive ballooning modes[26] are also a candidate for explaining ELM activity.

Now, we derive the saturation amplitude of various fluctuations. Balancing the destabilizing gradient with the stabilizing parallel conduction (this is *effective* mixing length theory in that the nonlinear mode width is used) yields:

$$\frac{\tilde{T}_e}{T_e} = \frac{L_s^2 \tilde{V}_r}{\chi_\parallel \overline{k}_\theta^2 (\Delta^t)^2 L_t}, \tag{51}$$

$$\frac{\tilde{n}}{n_0} = \frac{L_s^2 \tilde{V}_r}{\chi_n \overline{k}_\theta^2 (\Delta^n)^2 L_n}. \tag{52}$$

By combining Eqs. (51) and (52) with Eqs. (47) and (48), we obtain the temperature and density fluctuation levels:

$$\frac{\tilde{T}_e}{T_e} = \frac{\Delta^t}{L_t} = \frac{1}{L_t} \left[\frac{cE_\parallel L_s}{BL_n} (C_n + C_t \eta_e) \right]^{\frac{1}{3}} \left(\chi_\parallel \frac{\overline{k}_\theta^2}{L_s^2} \right)^{-\frac{1}{3}}, \tag{53}$$

$$\frac{\tilde{n}}{n_0} = \frac{\Delta^n}{L_n} = \frac{1}{L_n} \left[\frac{cE_\parallel L_s}{BL_n} (C_n + C_t \eta_e) \right]^{\frac{1}{3}} \left(\chi_n \frac{\overline{k}_\theta^2}{L_s^2} \right)^{-\frac{1}{3}}. \tag{54}$$

Usually, since $\Delta^n \gg \Delta^t$,

$$\left(\frac{\tilde{T}_e}{T_e} \right) \Big/ \left(\frac{\tilde{n}}{n_0} \right) = \eta_e \left(\frac{\chi_n}{\chi_\parallel} \right)^{\frac{1}{3}} < 1,$$

as experimentally observed in tokamaks.[2] The electrostatic potential fluctuation level is given by

$$\frac{e\tilde{\phi}}{T_e} = \frac{eE_\parallel L_s}{T_e \overline{k}_\theta L_n} (C_n + C_t \eta_e). \tag{55}$$

We note that unlike drift wave turbulence,[16] the Boltzmann relation, $e\tilde{\phi}/T_e = \tilde{n}/n_0$, does not hold in NRGDT, which is also consistent with experimental results.[2]

Unlike $\tilde{T}_e$ and $\tilde{n}$, $\tilde{J}_\parallel$ has nonvanishing components near the rational surface:

$$\frac{\tilde{J}_\parallel}{J_\parallel} \simeq C_t \frac{\tilde{T}_e}{T_e} + C_n \frac{\tilde{n}}{n_0}. \tag{56}$$

Therefore, magnetic fluctuations can be induced in NRGDT. Integrating Eq. (56) using the constant-ψ approximation,[7] we obtain

$$\Delta' \tilde{\psi} = \frac{4\pi}{c} J_\parallel \left(C_t \Delta^t \frac{\tilde{T}_e}{T_e} + C_n \Delta^n \frac{\tilde{n}}{n_0} \right),$$

where ψ is the parallel component of the vector potential. For large poloidal mode numbers, $\Delta' \simeq -2k_\theta$, and therefore,

$$\frac{\tilde{B}_r}{B} = \frac{2\pi J_\parallel}{cBL_n} \left[\frac{cE_\parallel L_s}{BL_n} (C_n + C_t \eta_e) \right]^{\frac{2}{3}} \left(\chi_n \frac{\bar{k}_\theta^{-2}}{L_s^2} \right)^{-\frac{2}{3}} \left[C_n + C_t \eta_e \left(\frac{\chi_n}{\chi_\parallel} \right)^{\frac{2}{3}} \right]. \tag{57}$$

This magnetic fluctuation level is too feeble to induce any significant electron heat transport due to stochastic magnetic fields.[27] The electron heat transport according to NRGDT is mainly due to thermal convection.

V. Summary and Conclusions

We have incorporated the effects of the neoclassical correction to the resistivity[12,13] into the evolution of rippling modes.[6–9] Through the density dependence of the neoclassical resistivity, rippling modes can tap the density gradient free energy source as well as the usual temperature gradient free energy source. The density coupling (C_n) to the neoclassical resistivity is strong when $\nu_{*,e} \sim 1$. Although C_n is smaller than C_t, the density gradient is much more effective free energy source, since conduction of density fluctuations along flux surfaces provided by neoclassical viscous force is less efficient than the parallel thermal conduction. Thus, the region where the neoclassical rippling modes are significantly unstable (linearly) extends further inside from the edge. The growth rate and the linear mode width are calculated in limiting cases.

The nonlinearly saturated state is characterized by current decoupling: asymmetry further develops to make other fluctuations skewed around the singular surface where $\tilde{J}_{\|}$ is localized. Mathematically, this is a sufficient condition for a stationary E_K. Two remaining stationary conditions determine the level of turbulence. The saturation mechanism is nonlinear mode coupling between long wavelength modes and stable short wavelength modes. It is shown that NRGDT is always in the *strong turbulence* regime, and that ω and ω_{*e} do not play important roles in nonlinear evolution. In one-point DIA theory,[20] the nonlinear damping is expressed in terms of fluctuation-spectrum-dependent radial diffusion, acting to broaden the radial scale of modes, thus leading to more effective parallel dissipation of fluctuations. The renormalized coupled equations are solved by treating the turbulent diffusivities as an eigenvalues. The principal results are as follows:

(i) The density gradient enhances the turbulence level in comparision to the case with only temperature gradient,[1] as expected from the linear theory.

(ii) The density fluctuation becomes much broader than the temperature fluctuation.

(iii) Turbulent radial diffusivities (D^t and D^n) are different from mixing length estimates ($\gamma^t(W^t)^2$ and $\gamma^n(W^n)^2$) both qualitatively and quantitatively, i.e.,:

$$D^t = \left[\frac{cE_{\|}L_s}{BL_n}(C_n + C_t\eta_e)\right]^{\frac{4}{3}} \left(\chi_{\|}\frac{\overline{k}_\theta^2}{L_s^2}\right)^{-\frac{1}{3}},$$

$$D^n = \left[\frac{cE_{\|}L_s}{BL_n}(C_n + C_t\eta_e)\right]^{\frac{4}{3}} \left(\chi_n\frac{\overline{k}_\theta^2}{L_s^2}\right)^{-\frac{1}{3}}.$$

We note that the recent nonlinear study of resistive interchange modes[28] and neoclassical pressure-gradient-driven turbulence[29] revealed only quantitative diffference between nonlinear results and mixing length estimates. Here, current decoupling, a novel feature which modifies the mode structure drastically in NRGDT, is responsible for this more significant discrepancy.

(iv) The density fluctuation level is higher than the temperature fluctuation level and the Boltzmann relation does not hold:

$$\frac{\tilde{n}}{n_0} = \frac{1}{L_n}\left[\frac{cE_{\|}L_s}{BL_n}(C_n + C_t\eta_e)\right]^{\frac{1}{3}} \left(\chi_n\frac{\overline{k}_\theta^2}{L_s^2}\right)^{-\frac{1}{3}},$$

$$\frac{\tilde{T}_e}{T_e} = \frac{1}{L_t}\left[\frac{cE_{\|}L_s}{BL_n}(C_n + C_t\eta_e)\right]^{\frac{1}{3}} \left(\chi_{\|}\frac{\overline{k}_\theta^2}{L_s^2}\right)^{-\frac{1}{3}},$$

$$\frac{e\tilde{\phi}}{T_e} = \frac{eE_{\|}L_s}{T_e\overline{k}_\theta L_n}(C_n + C_t\eta_e).$$

(v) The magnetic component of NRGDT is calculated to be too feeble to be significant.

(vi) The region where RGDT applies is extended into $\nu_{*,e} \leq 1$ regimes. Real frequency and ω_* effects do not play a significant role in the nonlinear evolution.

(vii) Some speculations into ELMs in DIII-D H-mode plasmas[17] are developed in the context of this model.

Other potentially important neoclassical corrections which we neglected in this study due to the omission of the poloidal component of the first order perpendicular flows are the bootstrap current[12,13] in Ohm's law and neoclassical viscous damping in the vorticity equation. These effects combine to excite neoclassical pressure-gradient-driven instabilities[18,19]. These instabilities are significant when β (β is the ratio of the kinetic pressure to the magnetic pressure) is large, while the present study is limited to a low-β case. Study of turbulence evolving from neoclassical pressure-gradient-driven instabilities is reported in another publication.[29] Modification of the current decoupling in presence of these neoclassical corrections will be studied later.

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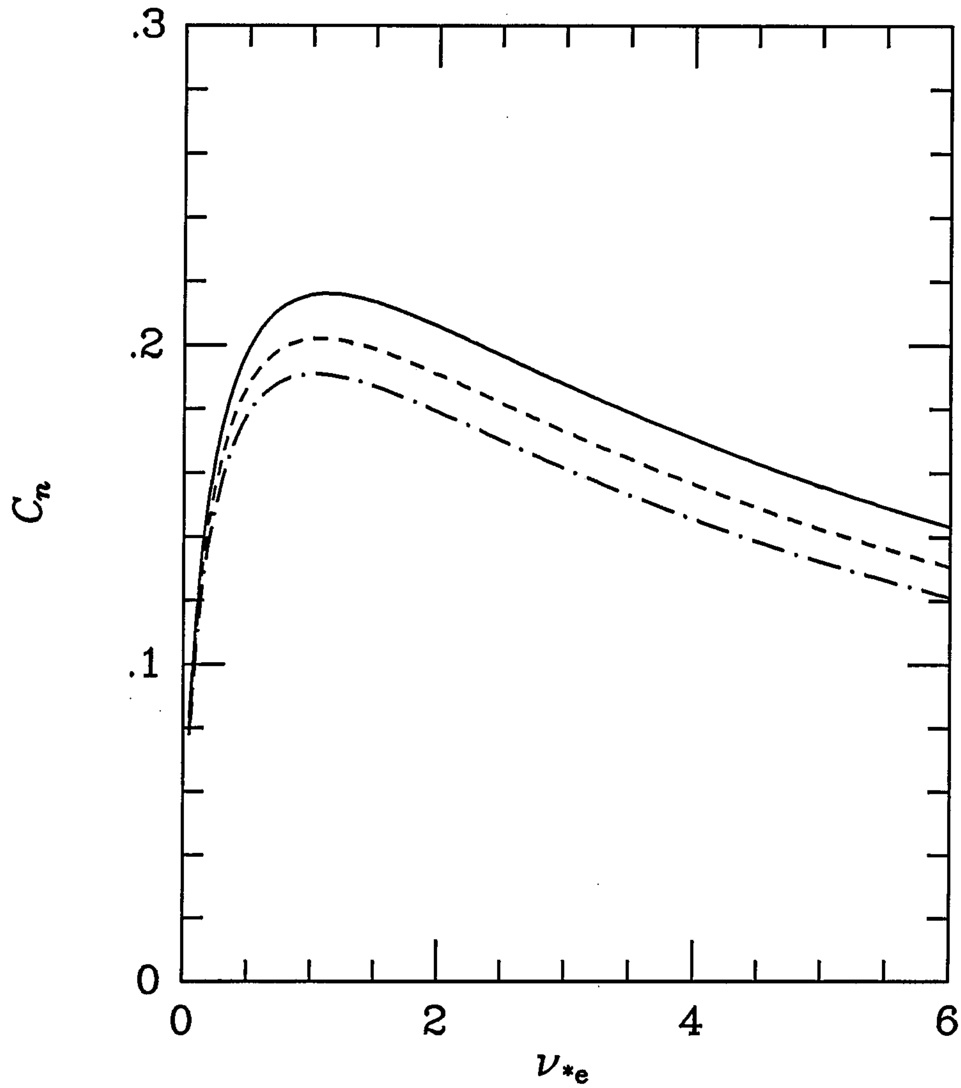

FIG. 1. Plot of the density coupling coefficient to the neoclassical resistivity ($C_n = -\partial \ln \eta_{\rm nc} / \partial \ln n_e$), in terms of ν_{*e} for (i) ϵ=1/4 (solid line), (ii) ϵ=1/5 (dashed line), and (iii) ϵ=1/6 (dash-dotted line).