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Theory of Neoclassical Pressure-Gradient-Driven Turbulence

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Abstract

The nonlinear evolution and saturation of neoclassical pressure-gradient-driven turbulence (NPGDT), evolving from linearly unstable bootstrap current modes, are investigated. The theoretical model is based on "neoclassical MHD equations" which are valid in the banana-plateau regimes of collisionality. Modes with poloidal wavelengths shorter than radial wavelengths are shown to be suppressed. From nonlinear saturation conditions, the turbulent pressure diffusivity is determined as an eigenvalue of the renormalized equations. Levels and radial scales of turbulence are determined from the pressure diffusivity and are shown to exceed mixing length estimates by powers of a nonlinear enhancement factor. The problem of the electron heat transport due to stochastic magnetic fields driven by NPGDT is revisited. The reconsideration of the radial structure of magnetic flutter leads to estimates of the electron heat transport and magnetic fluctuation levels which differ qualitatively and quantitatively from previous calculations.

I. Introduction

Resistive interchange-ballooning instabilities[1] have been successful in explaining experimentally observed anomalous thermal transport in stellarators,[2] reversed-field pinches,[3] and the auxiliary heated high-β ISX-B tokamak.[4] However, these theories are based on a short mean free path description of the plasma, valid only in a Pfirsch-Schlüter regime where the collisional mean free path is much shorter than the parallel scale length (i.e., connection length). This makes the application of these models to present-day tokamak operational regimes rather dubious since the temperatures are so high that plasmas are in low-collisionality, banana-plateau regimes.

Recently, so-called "neoclassical MHD equations" have been developed[5] to treat the low collisionality regime. It has been shown that these equations support interchange-like instabilities when $(dP/dr)(dq/dr) < 0$, where P and q are the pressure and the safety factor, respectively. In a related study,[6] a kinetic treatment of the dynamics of trapped particles reached similar conclusions. Moreover, these neoclassical MHD equations have well-known neoclassical properties such as: a bootstrap current[7] in Ohm's law as the consequence of the interaction of the pressure gradient with the radial excursion of trapped particles, an enhancement of the inertia[8] (by a factor of B^2/B_θ^2) because of an additional polarization drift, neoclassical diffusion fluxes,[9] the Ware pinch,[10] and the neoclassical correction to the resistivity.[9] Experimental evidence for the bootstrap current has been observed in TFTR[11] as well as in a toroidal multipole.[12] The neoclassical correction to the resistivity has important implications for the dynamics of resistivity-gradient-driven instabilities[13] (rippling modes) since the neoclassical resistivity is a function of the density as well as the temperature. This will be discussed in another publication.

In this paper, we study the nonlinear saturation of neoclassical pressure-gradient-driven turbulence (NPGDT), evolving from linear interchange-like instabilities described by the neoclassical MHD equations. The structure of the neoclassical pressure-gradient-driven instabilities is similar to that of resistive interchange modes.[2] However, it should be noted that neoclassical modes are somewhat more virulent, in that they are unstable for arbitrary β, S_M, etc. (β is the ratio of the kinetic pressure to the magnetic pressure and S_M is magnetic Reynolds number). Renormalized equations are derived. They are solved, yielding the amplitude-dependent nonlinear viscosity (or diffusivity) as an eigenvalue. A similar treatment for resistive interchange modes[2] led to the conclusion that although mixing-length estimates correctly predict the parameter scalings in leading order, they significantly underestimate the magnitude. Our study shows that the same result holds for NPGDT.

The remainder of the paper is organized as follows. In Sec. II, neoclassical MHD equations are summarized and corrections to the ususal resistive MHD equations due to the inclusion of the parallel viscous stress tensor effects will be discussed. The parallel viscosity due to trapped particles leads to a bootstrap current and a neoclassical correction to the resistivity. The perpendicular component of the parallel viscous force (the cross viscosity) leads to additional viscous damping and a neoclassical enhancement of the polarization drift. Energetic trapped particle effects,[14] which are significant in experiments[15] with perpendicular neutral beam injection or resonant radio frequency heating, are also incorporated.

In Sec. III, linear analysis reveals various branches of the neoclassical resistive interchange instability. Without neoclassical or hot trapped particle effects, resistive interchange modes[2] become unstable due to the interaction of the pressure gradient with the average unfavorable curvature. These modes are stable in tokamaks because of the minimum-B configuration. However, when the hot particle population trapped in the unfavorable curvature region becomes significant, energetic trapped-particle-driven resistive interchange modes[16] are unstable. When plasmas are in the banana-plateau regime of the collisionality, neoclassical effects are dominant and neoclassical pressure-gradient-driven modes[5,6] (bootstrap current modes) become unstable. NPGDT evolves from these modes. In all cases, when the radial correlation length is much smaller than the poloidal correlation length, the growth rate scales like $\gamma \sim S_M^{-1/3}\tau_A^{-1}$ where τ_A is the poloidal Alfvén transit time. In the case of resistive interchange modes,[2] it has been shown that when the poloidal correlation length becomes shorter than the radial mode width, one can recover "fast-interchange" modes where the growth rate is independent of the resistivity. However, because of their rather unusual structure, bootstrap current modes are stabilized in the short poloidal wavelength limit. The neoclassical correction to the resistivity is responsible for suppressing small scale bootstrap current modes.

In Sec. IV, nonlinear study of NPGDT is presented. Nonlinear saturation occurrs when the nonlinear energy cascade from linearly unstable, long wavelength modes to the short wavelength dissipation range balances the linear source. This cascade is mediated by $E \times B$ convective shearing. As to dominant long wavelength modes, the saturation mechanism can be viewed as a radial diffusion, which balances the driving forces, naturally leading to the broadening of these modes. The amplitude-dependent turbulent diffusivity is obtained from the renormalized equations (i.e., as an eigenvalue of the renormalized eigenvalue problem at saturation) and is significantly larger than the mixing-length estimate. Other fluctuation quantities are also evaluated, showing significant enhancement over mixing-length estimates.

In Sec. V, the problem of the electron heat transport due to stochastic magnetic fields driven by NPGDT is revisited. The radial structure of magnetic flutter[17] associated with NPGDT is reconsidered. The bootstrap current contribution to anomalous transport, which is unique to NPGDT, is also included in the determination of magnetic fluctation levels. Thus, a cross correlation between the pressure fluctuation and the electrostatic potential fluctuation appears. Results show both qualitative and quantitative differences from previous calculations[4] of χ_e (χ_e is the electron heat diffusivity). A recent study [18] of χ_e due to reistive interchange modes reached the same conclusion, as expected because of the similarity in the structure of the resistive interchange modes and NPGDT.

Finally, Sec. VI contains a summary and conclusions.

II. Basic Equations

The neoclassical MHD equations[5] are employed in order to study the structure of neoclassical pressure-gradient-driven turbulence (NPGDT). They are valid in the long mean-free path (banana-plateau) regime. A comprehensive theory of the extension of resistive MHD equations, which are valid only in the short mean-free path (Pfirsch-Schlüter) regime, into a low collisionality regime can be found in Ref. 5. In this section, we summarize the results necessary for the nonlinear study of NPGDT, and discuss the differences between the two models.

The major correction to the momentum balance equation is the addition of a large parallel stress tensor due to the friction between trapped and circulating particles. The flux-surface averaged neoclassical parallel Ohm's law is then

$$E_{\parallel} = \frac{1}{c}\frac{\partial \psi}{\partial t} - \mathbf{b}\cdot\nabla\phi = \eta_{\rm sp}\left(1 + \frac{\mu_e}{\alpha_e \nu_e}\right) J_{\parallel} + \eta_{\rm sp}\frac{\mu_e}{\alpha_e \nu_e}\frac{c}{B_\theta}\frac{dP}{dr}. \tag{1}$$

Here, $E_{\parallel}$ is the parallel electric field, $\mathbf{b} = \mathbf{B}/|\mathbf{B}_0|$, ψ is the parallel component of the magnetic potential, ϕ is the electrostatic potential, $J_{\parallel}$ is the parallel current, and $\eta_{\rm sp}$ is the classical Spitzer resistivity (which is a function of the electron temperature only). For $Z_{\rm eff} \simeq 1$, $\alpha_e \simeq 0.51$ and

$$\mu_e = \frac{2.3\sqrt{\epsilon}\,\nu_e}{1 + 1.02\nu_{*e}^{1/2} + 1.07\nu_{*e}}. \tag{2}$$

Here ϵ is the inverse aspect ratio, ν_e is the electron collision rate and $\nu_{*e} = \nu_e/\epsilon^{3/2}\omega_{\rm be}$, $\omega_{\rm be}$ being an electron bounce frequency. Recalling that the classical Ohm's law in resistive MHD is

$$E_{\parallel} = \eta_{\rm sp} J_{\parallel},$$

it is easy to see that the Spitzer resistivity is replaced by the effective neoclassical resistivity,

$$\eta_{\rm neo} = \eta_{\rm sp}\left(1 + \frac{\mu_e}{\alpha_e \nu_e}\right). \tag{3}$$

It is interesting to note that since $\eta_{\rm neo}$ is a function of the density as well as the temperature, rippling modes[13], which tap the free energy in the resistivity gradient, can now be driven by a density gradient. A nonlinear study, incorporating density dynamics in the evolution of resistivity-gradient-driven turbulence, indicates that the density gradient acts to enhance the turbulence level for normal tokamaks profiles (i.e., $n_e' T_e' > 0$) where n_e and T_e are the electron density and the temperature, respectively, and $n_e' = dn_e/dr$, etc.. A detailed study of the neoclassical resistivity-gradient-driven turbulence will be presented in another publication. The bootstrap current term in Eq. (1), $J_{\rm bt} = (\mu_e/\alpha_e \nu_e)(c/B_\theta) dP/dr$, which does not have an analog in the classical Ohm's law should also be noted. The bootstrap current arises due to the fact that ion poloidal current vanishes because of large ion viscous drag and electron viscous drag reduces electron poloidal current.

The vorticity evolution equation, derived from the charge neutrality condition, is

$$\frac{d}{dt}\left(1+\frac{B^2}{B_\theta^2}\right)\nabla_\perp^2 \tilde{\phi} = \frac{B^2}{c^2 \rho}\nabla_\| \tilde{J}_\| + \frac{B}{c\rho}\frac{B}{B_\theta}\frac{d}{dr}\nabla_\| \tilde{P} + \frac{B}{c\rho}\mathbf{b}\times\mathbf{K}\cdot\nabla(2\tilde{P}+\tilde{P}_{\|,h}+\tilde{P}_{\perp,h}), \quad (4)$$

where $\mathbf{K} = \mathbf{b}\cdot\nabla\mathbf{b}$ is the magnetic curvature, the subscript 'h' denotes the hot particle component, and $d/dt = \partial/\partial t + (c/B)\mathbf{b}\times\nabla\phi\cdot\nabla$ denotes the total convective derivative. The left-hand-side of Eq. (4) denotes the parallel component of the vorticity, which arises from the polarization drift. Note that the inertia is enhanced by a factor of B^2/B_θ^2. The first term in the right-hand-side of Eq. (4) represents field line diffusion (field line bending) and the second term represents viscous damping due to anisotropies in the perturbed pressure. The latter is an analog of the bootstrap current and should be kept pairwise with it. The interaction of these two terms is crucial in driving neoclassical pressure-gradient-driven instabilities.[5,6] Finally, the third term denotes the interaction of the pressure with the magnetic curvature. Note that hot particles, treated separately from the core or warm particles, do not contribute to viscous damping because the collision rate for hot components is very small. Their contribution to the bootstrap current is also negligible since $J_{{\rm bt},h} \sim -(c/B_\theta)dP_{\|,h}/dr$, so that in the present ordering scheme, $\epsilon^2 P \sim \epsilon P_{\perp,h} \sim P_{\|,h}$.

For the core plasma,

$$\frac{d\tilde{P}}{dt} \simeq \frac{\partial \tilde{P}}{\partial t} + \frac{c}{B}\mathbf{b}\times\nabla\tilde{\phi}\cdot\nabla P = 0, \quad (5)$$

i.e., the convective response to $E\times B$ flow is used. When hot particle effects and the curvature interaction are ignored, Eqs. (1), (4), and (5) constitute the basic equations for theoretical model of NPGDT. When the hot trapped particle population becomes significant, as in the perpendicular neutral beam injection experiment,[15] the perturbed distribution function is obtained from a drift kinetic description of hot particle dynamics. A complete description can be found elsewhere.[14] Taking a second moment of the nonadiabatic part of the perturbed distribution function of trapped particles yields the hot particle pressures:

$$\begin{pmatrix} \tilde{P}_{\perp,h} \\ \tilde{P}_{\parallel,h} \end{pmatrix} = -2^{\frac{5}{2}}\pi e_h B \int_{B_{\max}^{-1}}^{B_{\min}^{-1}} d\alpha (1-\alpha B)^{1/2} \int dE \begin{Bmatrix} \alpha B/2(1-\alpha B) \\ 1 \end{Bmatrix} \frac{E^{\frac{3}{2}} Q_h}{\omega - \overline{\omega}_{dh}} \overline{\left(\phi - \frac{v_\parallel}{c}\psi\right)}. \tag{6}$$

Here,

$$\begin{aligned} Q_h &= \left(\omega \frac{\partial}{\partial E} + \hat{\omega}_{*h}\right) F_{0h}, \\ \hat{\omega}_{*h} &= -\frac{i}{\Omega_h} \mathbf{b} \cdot \times \nabla \ln F_{0h} \cdot \nabla, \\ \omega_{dh} &= -i \frac{\mathbf{b} \times (v_\perp^2 \nabla \ln B + 2 v_\parallel^2 \mathbf{K})}{2\Omega_h} \cdot \nabla, \end{aligned}$$

$\Omega_h = e_h B / m_h c$ is the cyclotron frequency, $\alpha = \mu/E$ is the pitch angle, $\mu = v_\perp^2/2B$ is the magnetic moment, $E = v^2/2$ is the energy per unit mass, F_{0h} is the unperturbed hot particle distribution function, and $\overline{(\cdots)}$ denotes bounce averaging between turning points, i.e.,

$$\overline{(\cdots)} = \frac{\int (dl/v_\parallel)(\cdots)}{\int dl/v_\parallel}.$$

Also in a low-β approximation, $\nabla \ln B \simeq \mathbf{K}$. Now, Eqs. (1), (4), (5), and (6) constitute the basic equations for NPGDT with hot particle and curvature effects. The linear theory of this system will be studied in the next section.

III. Linear Theory

Having introduced the basic theoretical model, we study the linear stability of this model in this section. To render the theory analytically tractable, two important approximations are made. First, as a consequence of the electrostatic approximation, the neoclassical Ohm's law is simplified and **b** reduces to the unit vector of the equilibrium magnetic field. Second, by neglecting the rippling effect ($\tilde{\eta}$ evolution), the density and the temperature evolutions need not be treated separately, and the model consists of evolution equations for $\tilde{\phi}$, $\tilde{j}_{\|}$, and $\tilde{P}$.

With these approximations, one finds two coupled equations for Φ and $J_{\|}$ by solving for $\tilde{P}$, i.e.,

$$k_{\|}\Phi = i\eta J_{\|} + \delta_e \frac{c}{B_\theta}\frac{c\eta}{B\gamma} k_\theta \frac{dP}{dr}\frac{d\Phi}{dx}, \tag{7}$$

$$\begin{aligned}\rho\gamma^2\left(1+\frac{B^2}{B_\theta^2}\right)\nabla_\perp^2\Phi =& i\frac{B^2\gamma}{c^2}k_{\|}J_{\|} + \frac{B}{B_\theta}k_\theta\frac{dP}{dr}\frac{d}{dx}k_{\|}\Phi + 2k_\theta^2\langle K_\omega\rangle\Phi \\ & - 2^{\frac{5}{2}}\pi e_h k_\theta \frac{B^2}{c}\int d\alpha \frac{K_\omega \alpha B}{2(1-\alpha B)^{\frac{1}{2}}}\int dE \frac{\omega E^{\frac{3}{2}} Q_h}{\omega - \overline{\omega}_{dh}}\Phi.\end{aligned} \tag{8}$$

Here,

$$\delta_e = \frac{\mu_e}{\alpha_e \nu_e}\Big/\left(1+\frac{\mu_e}{\alpha_e\nu_e}\right),$$

$\eta = \eta_{\rm neo}$,K_ω is the total curvature, $\Phi = \langle\tilde{\phi}\rangle$, and $J_{\|} = \langle\tilde{j}_{\|}\rangle$ where $\langle(\cdots)\rangle$ denotes flux surface averaging,

$$\langle(\cdots)\rangle = \frac{\int (dl/B)(\cdots)}{\int dl/B}.$$

Therefore, coefficients are also flux surface averaged. Also, perturbed quantities are Fourier analyzed according to:

$$\tilde{\phi} = \sum_{m,n}\tilde{\phi}(x)e^{i\omega t}\exp\left[i\left(n\frac{z}{R_0} - m\theta\right)\right],$$

where m and n are poloidal and toroidal mode numbers, respectively, R_0 is the major radius of tokamaks, z is the coordinate along the major axis, and $x = r - r_s$ is the displacement from the mode rational surface [where $q(r_s) = m/n$]. In Eqs. (7) and (8), the analysis assumes two different spatial scales: the longer parallel length scale, related to the connection length, and the shorter perpendicular length scale, related to the resistive layer width. Perturbed quantities are averaged over along the slowly varying parallel direction, such that the equations are in the fast variable x. This analysis is valid since these modes are extended along the magnetic field lines. Also, one can write $k_{\parallel} = k_\theta x/L_s$ where $L_s = Rq/\hat{s}$ is the shear length, and $\hat{s} = rq'/q$ is the shear parameter. If one further assumes that the radial variation is much faster than the poloidal variation, the approximation $\nabla_\perp^2 \simeq \partial^2/\partial x^2$ is possible. Then, solving Eqs. (7) and (8) for Φ yields

$$I\gamma^2 \frac{d^2}{dx^2}\Phi - F_B \gamma x^2 \Phi + (N_V - F_{BB}) x \frac{d}{dx}\Phi + (N_V - K + K_H)\Phi = 0. \tag{9}$$

Here,

$$\begin{aligned}
I &= 1 + \frac{B^2}{B_\theta^2}, \\
F_B &= \frac{B^2 k_\theta^2}{c^2 \rho \eta L_s^2}, \\
N_V &= \frac{B k_\theta^2}{B_\theta \rho L_s}\left(-\frac{dP}{dr}\right), \\
F_{BB} &= \delta_e N_V, \\
K &= 2\langle K_\omega \rangle \frac{k_\theta}{\rho}\left(-\frac{dP}{dr}\right).
\end{aligned}$$

Physical interpretation for these terms is as follows: I represents the inertia of the fluid, F_B is the field-line bending, F_{BB} is an additional field-line bending due to the presence of the bootstrap current, and N_V is the neoclassical viscous damping which represents anisotropy in the perturbed pressure, which, in turn, drives NPGDT when combined with F_{BB}. Also, K denotes the interaction of the core plasma pressure gradient with the curvature, which drives the resistive interchange mode,[2] important in stellarators. However, this has a stabilizing effect in tokamaks because of their average favorable curvature (minimum-B configuration). Finally, K_H represents the interaction of trapped hot particle pressure with the curvature, driving energetic trapped-particle-induced resistive interchange modes.[16] The calculation of K_H is simplified when F_{0h} is approximated as slowing-down distribution, corresponding to a neutral beam injected at energy E_m,

$$F_{0h} = \frac{C_0}{E_m} E^{-3/2} \delta(\alpha - \alpha_0),$$

for $E \leq E_m$, where α_0 is related to the injection angle. This choice of F_{0h} yields:

$$K_H = \frac{\hat{\omega}_{*h}}{\hat{\omega}_{dh}} \beta_{ph} \hat{I}_0(\omega). \tag{10}$$

Here, $\hat{\omega}_{dh} = \overline{\omega}_{dh}/E$, β_{ph} is the poloidal β for hot particles, and I_0 is a function of complete elliptic integrals. For modes growing faster than hot particle precessional time, K_H is independent of the growth rate.[16] Then, the general solution for Eq. (9) can be written as

$$\Phi(x) = \Phi_{0\mathbf{k}} e^{-x^2/2W_\mathbf{k}^2} H_l(ax).$$

Here, H_l is a Hermite polynomial, l is an integer,

$$a = \left(\frac{(N_V - F_{BB})^2}{4\gamma^4 I^2} + \frac{F_B}{\gamma I} \right)^{\frac{1}{4}},$$

$$W_\mathbf{k}^{-2} = \left(\frac{(N_V - F_{BB})^2}{4\gamma^4 I^2} + \frac{F_B}{\gamma I} \right)^{\frac{1}{2}} + \frac{N_V - F_{BB}}{2\gamma^2 I}.$$

The eigenvalue condition is:

$$\frac{N_V + F_{BB} - K + K_H}{2\gamma^2} = (2l+1) I^{1/2} \left(\frac{N_V - F_{BB}}{4\gamma^2} + \frac{F_B}{\gamma} \right)^{\frac{1}{2}}.$$

The $l = 0$ modes are centrally localized and

$$\Phi(x) = \Phi_{0\mathbf{k}} e^{-x^2/2W_\mathbf{k}^2}. \tag{11}$$

Now, $\Phi_{0\mathbf{k}}$ is the magnitude of the electrostatic potential of the mode $\mathbf{k}$, which cannot be determined from the linear theory, and $W_\mathbf{k}$ is the radial width of the mode. By substituting Eq. (11) into Eq. (9), one finds two coupled equation for γ and $W_\mathbf{k}$, i.e.,

$$I \frac{\gamma^2}{W_\mathbf{k}^4} = F_B \gamma + \frac{N_V - F_{BB}}{W_\mathbf{k}^2}, \tag{12}$$

$$I \frac{\gamma^2}{W_\mathbf{k}^2} = N_V + K_H - K. \tag{13}$$

These centrally localized modes are dominant, in that their growth rate exceeds that of higher modes due to field line-bending stabilization (of higher, broader modes). Various instability branches emerge immediately from these relations.

A. Resistive Interchange Modes

In the absence of neoclassical effects ($N_V = F_{BB} = 0,\ I = 1$) and trapped energetic particles ($K_H = 0$), resistive interchange modes, which might play an important role in anomalous heat transport in stellerators[2] and reversed-field pinches,[3] are excited by the interaction of the pressure gradient with the average unfavorable curvature, where,

$$\gamma = (-K)^{2/3}/F_B^{1/3} \sim k_\theta^{2/3} S_M^{-1/3} \tau_A^{-1}. \tag{14}$$

Here, $S_M = \tau_R/\tau_A$ is magnetic Reynolds number, $\tau_R = r^2/\eta c^2$ is the resistive time scale, and $\tau_A = Rq/v_A$ is the poloidal Alfvén transit time, v_A being the Alfvén speed. Instability of these modes requires $K \sim \langle K_w \rangle < 0$ which is not satisfied in tokamaks because of their minimum-B configuration. As a result, these modes are supressed in tokamaks.

B. Energetic Trapped-Particle-Driven Resistive Interchange Modes

When neoclassical effects are ignored ($N_V = F_{BB} = 0,\ \ I = 1$), energetic trapped-particle-driven interchange modes[16] (due to hot particles trapped in the outside region for which $\omega_{*h}/\omega_{dh} > 0$) are excited even though tokamaks have average favorable curvature. The growth rate for these modes is

$$\gamma = \left(\frac{\hat{\omega}_{*h}}{\hat{\omega}_{dh}}\beta_{ph}\hat{I}_0 - K\right)^{2/3} \Big/ F_B^{1/3} \sim S_M^{-1/3}\tau_A^{-1}. \tag{15}$$

There is a threshold for these instabilities given by

$$\beta_{ph} > \frac{\hat{\omega}_{dh}K}{\hat{\omega}_{*h}\hat{I}_0} \qquad (16).$$

We have considered only the fluid limit here. These modes are purely growing in this limit and are not resonant with the precessional drift frequency of hot particles. Detailed results when these modes are resonant with the hot particle precession are described in Ref. 16.

C. Neoclassical Pressure-Gradien- Driven Modes

In the absence of curvature effects ($K = K_H = 0$), effects from bootstrap currents and neoclassical viscous damping, which are absent in a short mean free path description of the plasma, combine to destabilize the pressure-gradient-driven modes in the banana-plateau collisionality regime[5,6] (bootstrap current instabilities) yielding:

$$\gamma = \left(\frac{N_V F_{BB}}{I F_B}\right)^{1/3} \sim k_\theta^{2/3} S_M^{-1/3} \tau_A^{-1}. \tag{17}$$

Interestingly, both the neoclassical MHD description[5] and the kinetic treatment[6] lead to the same result. Note that $I = B^2/B_\theta^2$ in this case. To be unstable, both N_V and F_{BB} are required to be positive. This is equivalent to $q'P' < 0$, as is satisfied in tokamaks. It is also interesting to note that γ is independent of $\hat{s}$, in contrast to previous cases where the shear has stabilizing effects. However, the effect of the shear here is to limit the radial width of the modes. These modes are of particular interest because they have fast growth rates, and they may play an important role in anomalous electron heat transport in tokamaks. NPGDT evolving from these modes and electron heat transport associated with magnetic fluctuations in the nonlinearly saturated state are the main topics of this paper, and will be be discussed later in Sec. IV and in Sec. V, respectively.

D. Compelete Description

When we retain all of effects discussed earlier, we obtain

$$\gamma = (F_{BB} + K_H - K)^{1/3} (N_V + K_H - K)^{1/3} / (I F_B)^{1/3} \sim S_M^{-1/3} \tau_A^{-1}. \tag{18}$$

The instability condition is $F_{BB} + K_H > K$ and $N_V + K_H > K$. Bootstrap current interaction with viscous damping, as well as hot particles trapped in the unfavorable curvature region favors instability, while the core pressure gradient interaction with average curvature favors stability. Since the inequality, N_V, $F_{BB} > K$, K_H is usually satisfied for the hot, core region of tokamaks, previously mentioned case **C** (bootstrap current modes) is of the greatest interest. It is worthwhile to note that all the modes treated here are of resistive interchange-ballooning family, in the sense that all growth rates scale as $\gamma \sim S_M^{-1/3} \tau_A^{-1}$.

Now, concentrating on neoclassical pressure gradient driven modes, one can rewrite Eq. (17) as

$$\gamma = \left[\frac{c^2\eta}{B^2\rho}k_\theta^2\delta_e\left|\frac{dP}{dr}\right|^2\right]^{1/3}$$
$$= \left(\frac{\delta_e}{16\pi}\right)^{1/3}\left(\frac{\epsilon}{q}\beta_p\right)^{2/3}\left(\frac{r^2k_\theta}{L_P}\right)^{2/3}S_M^{-1/3}\tau_A^{-1}, \tag{19}$$

where $L_P = -(d\ln P/dr)^{-1}$ is the pressure scale length. Then, from Eqs. (12) and (13), the radial mode extent is

$$W_{\mathbf{k}} = \left(\frac{B}{B_\theta}\right)^{1/2}\delta_e^{1/3}\left[\frac{c^4\rho\eta^2L_s^3}{B^4k_\theta^2}\left|\frac{dP}{dr}\right|\right]^{1/6}$$
$$= \sqrt{2}\left(\frac{\epsilon}{q}\beta_p\right)^{1/6}\left(\frac{\delta_e}{16\pi}\frac{r}{k_\theta L_P^2}\right)^{1/3}S_M^{-1/3}(L_sL_P)^{1/2}. \tag{20}$$

It is important to note that we recover the "slow interchange" regime, where γ is proportional to $\eta^{1/3}$, in the limit where $\nabla_\perp^2 \simeq \partial^2/\partial x^2$. This is satisfied only for low-m modes, i.e., $W_{\mathbf{k}}^2 \ll k_\theta^{-2}$. In Ref. 2, it was found that there are also "fast interchange" regime where γ is independent of η, for very large m. However, since the structure of neoclassical pressure gradient modes is somewhat different from resistive interchange modes (i.e., both bootstrap current and viscous damping contribute to the former, while the pressure gradient in bad curvature is the only source for the latter), one finds that there are no instabilities for large m, where $k_\theta \ll \partial^2/\partial x^2$. This is due to the fact that $\delta_e > 1$, which leads to an unphysical situation of radially increasing eigenfunction for growing modes.

IV. Nonlinear Theory

In this section, we investigate nonlinear evolution and saturation of NPGDT evolving from linear neoclassical pressure gradient driven instabilities. Dominant $E \times B$ drift convective nonlinearities are replaced by spectrum-dependent turbulent diffusion coefficients using standard one-point renormalization theory.[19] The renormalized equations are solved as an eigenvalue problem to determine transport coefficients at saturation. The pressure diffusivity and fluctuation levels are found to exceed mixing-length estimates by powers of an enhancement factor obtained from the eigenvalue analysis.

Before describing the nonlinear closure scheme, we define two energy-like quantities. By eliminating $\tilde{J}_{\|}$, we obtain two coupled equations for Φ and $\Pi = \langle \tilde{P} \rangle$:

$$\rho \frac{B^2}{B_\theta^2} \frac{d}{dt} \nabla_\perp^2 \Phi = -\frac{B^2}{c^2 \eta} \nabla_\|^2 \Phi - \delta_e \frac{B^2}{c B_\theta} \nabla_\| \frac{d}{dr} \Pi + \frac{B^2}{c \rho B_\theta} \frac{d}{dr} \nabla_\| \Pi + \rho \mu \nabla_\perp^2 \nabla_\perp^2 \Phi, \qquad (21)$$

$$\frac{d}{dt} \Pi = \frac{c}{B} \frac{dP}{dr} \mathbf{b} \times \hat{\mathbf{r}} \cdot \nabla \Phi + \chi_\perp \nabla_\perp^2 \Pi. \qquad (22)$$

Here, μ and $\chi_\perp$ are added to provide energy sinks at small scales for $\nabla_\perp^2 \Phi$ and Π, respectively. They serve as short-wavelength disssipation to allow a nonlinearly saturated, turbulent state. However, they do not affect the growth of large scale modes. As usual, we define E_K and E_P as

$$E_K = \frac{1}{2} \int d^3x\, \rho |\nabla_\perp \Phi|^2,$$

$$E_P = \frac{1}{2} \int d^3x\, \Pi^2.$$

The evolution of these quantities is determined by:

$$\begin{aligned} \frac{d}{dt} E_K = & - \int d^3x\, \frac{B_\theta^2}{c^2 \eta} \left| \nabla_\| \Phi \right|^2 + \int d^3x\, \frac{B_\theta}{c} \delta_e \Phi \nabla_\| \frac{d}{dr} \Phi \\ & - \int d^3x\, \frac{B_\theta}{c} \Phi \frac{d}{dr} \nabla_\| \Phi - \int d^3x\, \rho \mu \frac{B_\theta^2}{B^2} \left| \nabla_\perp^2 \Phi \right|^2, \end{aligned} \qquad (23)$$

$$\frac{d}{dt} E_P = - \int d^3x\, \frac{c}{B} \frac{dP}{dr} \Pi\, \mathbf{b} \times \hat{\mathbf{r}} \cdot \nabla \Phi - \int d^3x\, \chi_\perp \left| \nabla_\perp \Pi \right|^2. \qquad (24)$$

The interaction of bootstrap currents with neoclassical viscous damping drives the kinetic energy (E_K), part of which is dissipated by viscosity (μ) and part of which is transformed into magnetic energy which, in turn, is dissipated by magnetic field diffusion (i.e., resistivity). The averaged pressure fluctuation (E_P) is driven by relaxation of equilibrium pressure gradients and damped by small scale dissipation due to $\chi_\perp$. A saturated state is attained when there is energy transfer, mediated by nonlinear three wave couplings, from the low-m energy source to the high-m dissipation region. To build a complete picture of the saturated state, two point theory, which allows the calculation of the fluctuation spectrum, must be utilized. However, in this paper, we study the saturation mechanism of low-m modes using the standard one-point theory where nonlinear interactions are renormalized to yield spectrum-dependent turbulent diffusivities. The saturation condition ($\gamma = 0$) then determines these diffusivities.

Detailed treatment of the standard one-point theory is described elsewhere[2] and we only summarize basic steps here. Noting that the dominant nonlinearity comes from $E \times B$ convection, one can write the nonlinearity as: for the field $A_{\mathbf{k}}$ (in our case, $A_{\mathbf{k}}$ denotes $\nabla_\perp^2 \Phi_{\mathbf{k}}$ or $\Pi_{\mathbf{k}}$),

$$N(A_{\mathbf{k}}) = i\frac{c}{B}\frac{\partial}{\partial r}\left[\sum_{\mathbf{k}'} k'_\theta(\Phi_{-\mathbf{k}'}A_{\mathbf{k}''} - \Phi_{\mathbf{k}''}A_{-\mathbf{k}'})\right] + i\frac{c}{B}k_\theta \sum_{\mathbf{k}'}\left(A_{\mathbf{k}''}\frac{\partial \Phi_{-\mathbf{k}'}}{\partial r} - \Phi_{\mathbf{k}''}\frac{\partial A_{-\mathbf{k}'}}{\partial r}\right), \tag{25}$$

where $\mathbf{k}'' = \mathbf{k} + \mathbf{k}'$. Now, the total convective derivative can be written as

$$\frac{d}{dt}A_{\mathbf{k}} = \frac{\partial}{\partial t}A_{\mathbf{k}} + N(A_{\mathbf{k}}). \tag{26}$$

We renormalize $N(A_{\mathbf{k}})$ by iteratively substituting the nonlinearly driven fluctuations ($A^{(2)}_{\mathbf{k}''}$ and $\Phi^{(2)}_{\mathbf{k}''}$) resulting from the direct beat interaction of test modes ($\Phi^{(1)}_{\mathbf{k}}$ and $A^{(1)}_{\mathbf{k}}$) with background modes ($\Phi^{(1)}_{\mathbf{k}'}$ and $A^{(1)}_{\mathbf{k}'}$) to extract the piece which is phase coherent with test modes. Here, the superscripts "(1)" and "(2)" denote first and second order in perturbed quantities, respectively. The renormalization can be simplified when one notes that $\Phi^{(2)}_{\mathbf{k}''}$ is spatially smoother than $\nabla_\perp^2\Phi^{(2)}_{\mathbf{k}''}$ or $\Pi^{(2)}_{\mathbf{k}''}$. The smoothness is because $\Phi^{(2)}_{\mathbf{k}''}$ is obtained from the inversion of the eigenmode operator, involving complicated spatial convolutions. By neglecting $\Phi^{(2)}_{\mathbf{k}''}$, we can write equations for the driven fields as:

$$A^{(2)}_{\mathbf{k}''} = \Gamma^{-1}(A_{\mathbf{k}''})i\frac{c}{B}\left(k_\theta\Phi^{(1)}_{\mathbf{k}}\frac{\partial A^{(1)}_{\mathbf{k}'}}{\partial r} + k'_\theta\Phi^{(1)}_{\mathbf{k}'}\frac{\partial A^{(1)}_{\mathbf{k}}}{\partial r} - k'_\theta A^{(1)}_{\mathbf{k}'}\frac{\partial \Phi^{(1)}_{\mathbf{k}}}{\partial r} - k_\theta A^{(1)}_{\mathbf{k}}\frac{\partial \Phi^{(1)}_{\mathbf{k}'}}{\partial r}\right), \tag{27}$$

where $\Gamma^{-1}(A_{\mathbf{k}''})$ is the propagator for $A_{\mathbf{k}''}$,

$$\Gamma(\nabla_\perp^2\Phi_{\mathbf{k}''}) = \gamma_{\mathbf{k}''} + \Delta\omega(\nabla_\perp^2\Phi_{\mathbf{k}''}) - \mu\nabla_\perp^2, \tag{28}$$

$$\Gamma(\Pi_{\mathbf{k}''}) = \gamma_{\mathbf{k}''} + \Delta\omega(\Pi_{\mathbf{k}''}) - \chi_\perp\nabla_\perp^2. \tag{29}$$

Here, $\gamma_{\mathbf{k}''}$ is the growth rate, which vanishes at saturation. Also, $\Delta\omega(A_{\mathbf{k}''})$ is the nonlinear decorrrelation rate which effectivelely limits the coherence time of nonlinear interactions. Since Π is not straightforwardly related to Φ, Π is convected by Φ and it is easy to calculate $N(\Pi_{\mathbf{k}})$ as:

$$N(\Pi_{\mathbf{k}}) = -\frac{\partial}{\partial x}D_{\mathbf{k}}^{xx}\frac{\partial}{\partial x}\Pi_{\mathbf{k}} + k_\theta^2 D_{\mathbf{k}}^{yy}\Pi_{\mathbf{k}}, \tag{30}$$

where

$$\begin{aligned} D_{\mathbf{k}}^{xx} &= \frac{c^2}{B^2}\sum_{\mathbf{k}'}|k'_\theta\Phi_{\mathbf{k}'}|^2\Big/\Gamma(\Pi_{\mathbf{k}''}), \\ D_{\mathbf{k}}^{yy} &= \frac{c^2}{B^2}\sum_{\mathbf{k}'}\left|\frac{\partial\Phi_{\mathbf{k}'}}{\partial r}\right|^2\Big/\Gamma(\Pi_{\mathbf{k}''}). \end{aligned} \tag{31}$$

We have used symmetry arguments to eliminate terms such as $\sum_{\mathbf{k}'}\Phi_{-\mathbf{k}'}(\partial\Phi_{\mathbf{k}'}/\partial r)$, etc.. Physically, $D_{\mathbf{k}}^{xx}$ and $D_{\mathbf{k}}^{yy}$ are interpreted as turbulent radial and poloidal pressure diffusivities, respectively.

To calculate $N(\nabla_\perp^2\Phi)$, one cannot simply neglect $\Phi_{\mathbf{k}''}^{(2)}$ in Eq. (25) because the vorticity is directly related to Φ. Equivalently, the vorticity is not simply convected by Φ, so that its back-reaction on Φ must be considered, as well. To simplify this consideration, we note that for a continuum of localized modes, one can replace the sum in $\mathbf{k}$-space by $\int dm'\,|m'q'/q^2|\int dx'$, where $q' = dq/dr$. Then, $N(\nabla_\perp^2\Phi_{\mathbf{k}})$ can be written as

$$N(\nabla_\perp^2\Phi_{\mathbf{k}}) \simeq i\frac{c}{B}\frac{\partial}{\partial x}\sum_{\mathbf{k}'}k'_\theta\frac{m^2+2mm'}{m''^2}\Phi_{-\mathbf{k}'}^{(1)}\nabla_\perp^2\Phi_{\mathbf{k}''}^{(2)} + i\frac{c}{B}k_\theta\sum_{\mathbf{k}'}\frac{m^2+2mm'}{m''^2}\frac{\partial\Phi_{-\mathbf{k}'}^{(1)}}{\partial r}\nabla_\perp^2\Phi_{\mathbf{k}''}^{(2)}. \tag{32}$$

We have used $dx''/dx' = m''/m'$, near the mode rational surface of $\mathbf{k}$. By substituting Eq. (27) into Eq. (32), we get

$$N(\nabla_\perp^2\Phi_{\mathbf{k}}) = -\frac{\partial}{\partial x}\mu_{\mathbf{k}}^{xx}\frac{\partial}{\partial x}\nabla_\perp^2\Phi_{\mathbf{k}} + k_\theta^2\mu_{\mathbf{k}}^{yy}\nabla_\perp^2\Phi_{\mathbf{k}} - \frac{\partial}{\partial x}C_{\mathbf{k}}^{xx}\frac{\partial}{\partial x}\Phi_{\mathbf{k}} + k_\theta^2 C_{\mathbf{k}}^{yy}\Phi_{\mathbf{k}}, \tag{33}$$

where

$$\mu_{\mathbf{k}}^{xx} = \frac{c^2}{B^2}\sum_{\mathbf{k}'}\frac{m^2}{m''^2}|\Phi_{\mathbf{k}'}|^2\Big/\Gamma(\nabla_\perp^2\Phi_{\mathbf{k}''}),$$
$$\mu_{\mathbf{k}}^{yy} = \frac{c^2}{B^2}\sum_{\mathbf{k}'}\frac{m^2}{m''^2}\left|\frac{\partial\Phi_{\mathbf{k}'}}{\partial r}\right|^2\Big/\Gamma(\nabla_\perp^2\Phi_{\mathbf{k}''}),$$
$$C_{\mathbf{k}}^{xx} = \frac{c^2}{B^2}\sum_{\mathbf{k}'}\frac{m^2}{m''^2}k'_\theta|\nabla_\perp\Phi_{\mathbf{k}'}|^2\Big/\Gamma(\nabla_\perp^2\Phi_{\mathbf{k}''}), \tag{34}$$
$$C_{\mathbf{k}}^{yy} = \frac{c^2}{B^2}\sum_{\mathbf{k}'}\frac{m^2}{m''^2}\left|\nabla_\perp\frac{\partial\Phi_{\mathbf{k}'}}{\partial r}\right|\Big/\Gamma(\nabla_\perp^2\Phi_{\mathbf{k}''}).$$

Here, odd moments in x' as well as in k_θ-space vanish because of symmetry considerations. The factor of m^2/m''^2 (in comparision to Eq. (31)) represents the effect of the back-reaction of the vorticity on the convecting fluid. Physical intrepretations of these coefficients are as follows. $\mu_{\mathbf{k}}^{xx}$ and $\mu_{\mathbf{k}}^{yy}$ are nonlinear radial and poloidal diffusivities of the vorticity, respectively, and act as effective sinks for E_K. $C_{\mathbf{k}}^{xx}$ and $C_{\mathbf{k}}^{yy}$ are their kinetic energy-conserving counterpart, and act as destabilizing energy sources.

Now, we determine how much energy outflow from long wavelength modes due to nonlinear multiple-helicity interactions is needed to balance the growth of these modes. When low-m modes dominate in the energy spectrum, turbulent radial diffusions are most effective stabilizing effects. Thus at saturation, the low-m modes satisfy:

$$\frac{\partial}{\partial x}\mu_{\mathbf{k}}^{xx}\frac{\partial}{\partial x}\nabla_\perp^2\Phi_{\mathbf{k}} - \frac{B_\theta^2}{c^2\rho\eta}\nabla_\parallel^2\Phi_{\mathbf{k}} - \frac{B_\theta}{c\rho}\delta_e\nabla_\parallel\frac{d}{dr}\Pi_{\mathbf{k}} + \frac{B_\theta}{c\rho}\frac{d}{dr}\nabla_\parallel\Pi_{\mathbf{k}} = 0, \tag{35}$$

$$\frac{\partial}{\partial x}D_{\mathbf{k}}^{xx}\frac{\partial}{\partial x}\Pi_{\mathbf{k}} + ik_\theta\frac{c}{B}\frac{dP}{dr}\Phi_{\mathbf{k}} = 0. \tag{36}$$

By solving these eigenmode equations, we obtain $D_{\mathbf{k}}^{xx}$ and $\mu_{\mathbf{k}}^{xx}$ as x-independent eigenvalues, required for the saturation of low-m modes. The level of turbulence is then determined from $D_{\mathbf{k}}^{xx}$ and $\mu_{\mathbf{k}}^{xx}$. In this treatment, nonlinear multiple-helicity interactions and significant overlap between different helicity modes are crucial. We solve this eigenvalue problem by Fourier transforming, i.e.,

$$\Phi(u) = (2\pi)^{-1/2}\int dx\,\Phi_{\mathbf{k}}(x)e^{iux}, \quad \text{etc..}$$

Then, from Eqs. (35) and (36),

$$F_B u^2 \frac{d^2}{du^2}\Phi(u) + \frac{F_{BB} - N_V}{D_{\mathbf{k}}^{xx}} u \frac{d}{du}\Phi(u) + \left(\frac{2N_V - F_{BB}}{D_{\mathbf{k}}^{xx}} - \mu_{\mathbf{k}}^{xx} I u^6 \right) \Phi(u) = 0. \quad (37)$$

Here, $\nabla_\perp^2 \simeq \partial/\partial x^2$ is used again. The solutions are Bessel functions of integer order,[20]

$$\Phi(u) = u^b Z_\nu(\lambda u^3),$$

where

$$1 - 2b = \frac{F_{BB} - N_V}{F_B D_{\mathbf{k}}^{xx}}.$$

For localized modes $\nu = 0$, and an eigenvalue condition for this case is

$$b^2 = (2N_V - F_{BB})/F_B D_{\mathbf{k}}^{xx},$$

or

$$1 + \left(\frac{F_{BB} - N_V}{F_B D_{\mathbf{k}}^{xx}} \right)^2 + 2\frac{F_{BB} - 3N_V}{F_B D_{\mathbf{k}}^{xx}} = 0 \quad (38)$$

Note that $\gamma W_{\mathbf{k}}^2 = F_{BB}/F_B$ is the mixing length estimate of $D_{\mathbf{k}}^{xx}$ obtained by balancing the linear growth rate and the turbulent diffision ($D_{\mathbf{k}}/W_{\mathbf{k}}^2$). Thus, if we put

$$D_{\mathbf{k}}^{xx} = \Lambda^2 \frac{F_{BB}}{F_B}, \quad (39)$$

we can interprete Λ as a nonlinear diffusion enhancement factor. Equation (37) is thus an eigenvalue equation for Λ:

$$\Lambda^2 = 3\delta_e^{-1} - 1 + 2\left[\delta_e^{-1}\left(2\delta_e^{-1} - 1\right)\right]^{1/2}. \quad (40)$$

We have chosen the + sign to eliminate the unphysical situation of zero diffusion as $\mu_e/\alpha_e\nu_e \to \infty$. Thus, $\Lambda^2 \geq 4$ is always satisfied. It is interesting to note that non-linear interactions always act to enhance the pressure diffusivity over the mixing length estimate ($\Lambda > 1$). This is consistent with the notion that the stabilizing mechanism for low-m modes is turbulent radial diffusion, thus leading to broadening of the modes. Also, although $\Lambda^2 \to \infty$ at very high temperature ($\mu_e/\alpha_e\nu_e \to 0$), the pressure diffusivity remains finite.

From Eq. (39), we can evaluate saturation levels of perturbed quantities. We follow the standard procedure[2] by approximating $\Gamma(\nabla_\perp^2\Phi_{\mathbf{k}}) \simeq \mu_{\mathbf{k}}^{xx}/\Delta_\Phi^2$ and $\Gamma(\Pi_{\mathbf{k}}) \simeq D_{\mathbf{k}}^{xx}/\Delta_\Pi^2$, where Δ_Φ and Δ_Π are nonlinear mode widths of Φ and Π, respectively. We thus find

$$\mu_{\mathbf{k}}^{xx} \simeq \frac{m^2}{\overline{m^2}}\frac{\overline{\Delta_\Phi}}{\overline{\Delta_\Pi}} D_{\mathbf{k}}^{xx}, \tag{41}$$

$$\overline{V}_r^2 \simeq \frac{(D_{\mathbf{k}}^{xx})^2}{\langle\Delta_\Pi^2\rangle_{\mathrm{rms}}}. \tag{42}$$

Here, $\overline{(\cdots)}$ denotes spectrum-averaged value of $(\cdots)$, i.e.,

$$\overline{\Delta}_\Phi^2 = \sum_{\mathbf{k}} \left|x\Phi_{\mathbf{k}}\right|^2 \Big/ \sum_{\mathbf{k}} \left|\Phi_{\mathbf{k}}\right|^2, \quad \text{etc..}$$

Also, $V_r = ck_\theta\Phi/B$ is the radial $E\times B$ velocity. Then, asymtotically balancing nonlinear viscosity with field-line bending yields:

$$\Delta_\Phi^6 = \mu_{\mathbf{k}}^{xx}\frac{c^2\rho\eta L_s^2}{B_\theta k_\theta^2}, \tag{43}$$

while balancing nonlinear pressure diffusion with the source (the pressure gradient) gives:

$$D_{\mathbf{k}}^{xx}\frac{\overline{\Pi}}{\overline{\Delta}_\Pi^2} = \overline{V}_r\left|\frac{dP}{dr}\right|. \tag{44}$$

It is interesting to note that by combining Eqs. (42) and (44), one finds

$$\frac{\overline{\Pi}}{P} = \frac{\overline{\Delta}_\Pi}{L_P},$$

which is just the mixing-length estimate of the pressure fluctuation. However, since Δ_Π is different from its linear value, $W_{\mathbf{k}}$, due to mode broadening, this relation should be understood in its nonlinear sense. Finally, balancing nonlinear viscosity with destabilizing neoclassical viscosity near the rational surface gives:

$$\mu_{\mathbf{k}}^{xx}\frac{\langle V_r^2\rangle_{\mathrm{rms}}^{1/2}}{\langle\Delta_\Phi^4\rangle_{\mathrm{rms}}} = \frac{B_\theta k_\theta^2}{B\rho L_s}\langle\Pi^2\rangle_{\mathrm{rms}}^{1/2}. \tag{45}$$

Equations (39) - (45) completely determine levels and radial scales of turbulence, i.e.,:

$$
\begin{aligned}
\overline{\Delta}_\Phi &= \overline{W}_{\mathbf{k}}\Lambda^{1/6},\\
\Delta_\Pi &= W_{\mathbf{k}}\Lambda^{7/6},\\
\overline{V}_r &= \overline{\gamma}\overline{W}_{\mathbf{k}}\Lambda^{5/3},\\
\overline{\Pi} &= \frac{P}{L_P}\overline{W}_{\mathbf{k}}\Lambda^{7/6},\\
D_{\mathbf{k}}^{xx} &= \overline{\gamma}\overline{W}_{\mathbf{k}}^2\Lambda^2\\
&= \Lambda^2\frac{1}{8\pi}\frac{\epsilon}{q}\beta_p c^2\eta\delta_e\frac{L_s}{L_P},\\
\mu_{\mathbf{k}}^{xx} &= \frac{m^2}{\overline{m}^2}\overline{\gamma}\overline{W}_{\mathbf{k}}^2\Lambda.
\end{aligned}
\tag{46}
$$

Our results are quite similar to those in Ref. 2. This is as expected, because of similarities in the structure of basic equations. It is shown there that analytic results agree well with nonlinear multiple-helicity simulations. It is interesting to note that in the nonlinearly saturated state, $\Delta_\Pi > \Delta_\Phi$, which is a consequence of the direct dependence of the vorticity on the convecting $E \times B$ velocity ($D^{xx} > \mu^{xx}$). Also, because Λ is only weakly dependent of plasma parameters, nonlinear results have the same parameter dependences as the mixing-length results, but show considerable enhancement in the magnitude.

V. Electron Heat Transport

In this section, we evaluate electron heat diffusivity due to stochastic magnetic fields[21] in satuated NPGDT. Magnetic fluctuations in NPGDT are coupled to electrostatic modes through the parallel Ohm's law.

From the electron drift kinetic equation[22] with magnetic flutter nonlinearity, one can calculate χ_e yielding:

$$\chi_e = \sum_{\mathbf{k}} v_\parallel^2 |\tilde{b}_{r,\mathbf{k}}|^2 \mathrm{Re}\left[i \left(\omega - \omega_D - k_\parallel v_\parallel + \frac{i}{\tau_{c\mathbf{k}}} \right)^{-1} \right]. \tag{47}$$

Here, χ_e is electron heat diffusivity, ω is the mode frequency, ω_D is the drift frequency, $\tilde{b}_{r,\mathbf{k}} = \tilde{B}_{r,\mathbf{k}}/B_0$, and $\tau_{c\mathbf{k}}$ is the nonlinear decorelation time, representing the nonlinear interaction term. Also, $\mathrm{Re}[(\cdots)]$ denotes the real part of $(\cdots)$. In the quasilinear approximation, where $1/\tau_{c\mathbf{k}}$ is neglected, Eq. (47) reduces to the χ_e of Ref. 22. In the more relevant, so-called strong turbulence regime, where nonlinear mode couplings become dominant, we can approximate Eq. (47) to yield

$$\chi_e = \sum_{\mathbf{k}} v_\parallel^2 |\tilde{b}_{r,\mathbf{k}}|^2 \tau_{c\mathbf{k}}. \tag{48}$$

To find $\tilde{b}_{r,\mathbf{k}}$, we use the neoclassical Ohm's law and Ampere's law;

$$\tilde{b}_{r,\mathbf{k}} = \frac{\mathbf{b} \times \hat{\mathbf{r}}}{B} \cdot \nabla \tilde{\psi}_{\mathbf{k}}, \tag{49}$$

where,

$$\tilde{\psi}_{\mathbf{k}} = \frac{4\pi}{c\eta} \nabla_\perp^{-2} \nabla_\parallel \Phi_{\mathbf{k}} + \frac{4\pi}{B_\theta} \delta_e \nabla_\perp^{-2} \frac{d}{dr} \Pi_{\mathbf{k}}. \tag{50}$$

By Fourier transforming, we can approximate $\tau_{c\mathbf{k}}$ by

$$\tau_{c\mathbf{k}}^{-1} = \chi_e \overline{u}^2, \tag{51}$$

where

$$\overline{u} \equiv \left(\frac{\sum_{\mathbf{k}} u^2 |\tilde{\psi}_{\mathbf{k}}|^2}{\sum_{\mathbf{k}} |\tilde{\psi}_{\mathbf{k}}|^2} \right)^{1/2}. \tag{52}$$

To evaluate $\overline{u}^2$, we note that

$$\sum_{\mathbf{k}} = \int dm' \left| \frac{m'q'}{q^2} \right| \int dx' = \frac{Rr}{L_s} \int dk_\theta \, |k_\theta| \int dx, \tag{53}$$

where R is the major radius of the tokamak. By assuming that nonlinear interactions change the radial correlation lengths without significant deformation of the linear eigenfunctions, we can write

$$\Phi(u) = \Delta_\Phi \left[C_\Phi \overline{\Phi}^2 S_\Phi(k_\theta)\right]^{1/2} \exp\left(-\frac{1}{2}\Delta_\Phi^2 u^2\right),$$
$$\Pi(u) = i\Delta_\Pi \left[C_\Pi \overline{\Pi}^2 S_\Pi(k_\theta)\right]^{1/2} \exp\left(-\frac{1}{2}\Delta_\Pi^2 u^2\right). \tag{54}$$

Here, S_Φ and S_Π represent k_θ-spectrum of Φ and Π, respectively. Also, C_Φ and C_Π satisfy spectrum-averaged fluctuation level normalization conditions,

$$\sum_{\mathbf{k}} |\Phi_{\mathbf{k}}|^2 = \overline{\Phi}^2,$$
$$\sum_{\mathbf{k}} |\Pi_{\mathbf{k}}|^2 = \overline{\Pi}^2,$$

to yield

$$C_\Phi = \left[\pi^{1/2} r R \int dk_\theta \, \frac{|k_\theta|}{L_s} S_\Phi(k_\theta)\Delta_\Phi\right]^{-1},$$
$$C_\Pi = \left[\pi^{1/2} r R \int dk_\theta \, \frac{|k_\theta|}{L_s} S_\Pi(k_\theta)\Delta_\Pi\right]^{-1}. \tag{55}$$

From Eqs. (50), (53)-(55), we obtain

$$\overline{u}^2 = \frac{I_1}{I_2}. \tag{56}$$

Here,

$$I_1 = \left(\frac{4\pi}{c\eta L_s}\right)^2 C_\Phi \overline{\Phi}^2 \frac{Rr}{L_s} \int dk_\theta \, |k_\theta|^3 S_\Phi(k_\theta)\Delta_\Phi^5 I_1^{\Phi\Phi}$$
$$+ \left(\frac{4\pi}{B_\theta}\delta_e\right)^2 C_\Pi \overline{\Pi}^2 \frac{Rr}{L_s} \int dk_\theta \, |k_\theta| S_\Pi(k_\theta)\Delta_\Pi I_1^{\Pi\Pi}$$
$$- 2(4\pi)^2 \delta_e \frac{(C_\Phi C_\Pi)^{1/2}\overline{\Phi\Pi}}{cB_\theta \eta L_s} \frac{Rr}{L_s} \int dk_\theta \, |k_\theta|^2 S_\Phi^{1/2} S_\Pi^{1/2} \Delta_\Phi^3 I_1^{\Phi\Pi}, \tag{57}$$

$$I_2 = \left(\frac{4\pi}{c\eta L_s}\right)^2 C_\Phi \overline{\Phi}^2 \frac{Rr}{L_s} \int dk_\theta \, |k_\theta|^3 S_\Phi(k_\theta)\Delta_\Phi^7 I_2^{\Phi\Phi}$$
$$+ \left(\frac{4\pi}{B_\theta}\delta_e\right)^2 C_\Pi \overline{\Pi}^2 \frac{Rr}{L_s} \int dk_\theta \, |k_\theta| S_\Pi(k_\theta)\Delta_\Pi^3 I_2^{\Pi\Pi}$$
$$- 2(4\pi)^2 \delta_e \frac{(C_\Phi C_\Pi)^{1/2}\overline{\Phi\Pi}}{cB_\theta \eta L_s} \frac{Rr}{L_s} \int dk_\theta \, |k_\theta|^2 S_\Phi^{1/2} S_\Pi^{1/2} \Delta_\Phi^3 \Delta_\Pi^2 I_2^{\Phi\Pi}, \tag{58}$$

where

$$
\begin{aligned}
I_1^{\Phi\Phi} &= \int_{-\infty}^{+\infty} d\alpha \, \frac{\alpha^4 e^{-\alpha^2}}{(\alpha^2 + k_\theta^2 \Delta_\Phi^2)^2}, \\
I_1^{\Pi\Pi} &= \int_{-\infty}^{+\infty} d\alpha \, \frac{\alpha^4 e^{-\alpha^2}}{(\alpha + k_\theta^2 \Delta_\Pi^2)^2}, \\
I_1^{\Phi\Pi} &= \int_{-\infty}^{+\infty} d\alpha \, \frac{\alpha^4 \exp\left[-\alpha^2 \left(1 + \Delta_\Phi^2/\Delta_\Pi^2\right)/2\right]}{(\alpha^2 + k_\theta^2 \Delta_\Pi^2)^2}, \\
I_2^{\Phi\Phi} &= \int_{-\infty}^{+\infty} d\alpha \, \frac{\alpha^2 e^{-\alpha^2}}{(\alpha^2 + k_\theta^2 \Delta_\Phi^2)^2}, \\
I_2^{\Pi\Pi} &= \int_{-\infty}^{+\infty} d\alpha \, \frac{\alpha^2 e^{-\alpha^2}}{(\alpha + k_\theta^2 \Delta_\Pi^2)^2}, \\
I_2^{\Phi\Pi} &= \int_{-\infty}^{+\infty} d\alpha \, \frac{\alpha^2 \exp\left[-\alpha^2 \left(1 + \Delta_\Phi^2/\Delta_\Pi^2\right)/2\right]}{(\alpha^2 + k_\theta^2 \Delta_\Pi^2)^2}.
\end{aligned} \tag{59}
$$

These integrals can be evaluated analytically, yielding:

$$
\begin{aligned}
\int_{-\infty}^{+\infty} d\alpha \, \frac{\alpha^4 e^{-\mu^2\alpha^2}}{(\alpha^2 + \nu^2)^2} &= \frac{\sqrt{\pi}}{\mu}(1 + \mu^2\nu^2) - \frac{\pi}{2}\nu(3 + 2\mu^2\nu^2)e^{\mu^2\nu^2}\left[1 - \mathrm{Erf}(\mu\nu)\right], \\
\int_{-\infty}^{+\infty} d\alpha \, \frac{\alpha^2 e^{-\mu^2\alpha^2}}{(\alpha^2 + \nu^2)^2} &= \frac{\pi}{2\nu}(1 + 2\mu^2\nu^2)e^{\mu^2\nu^2}\left[1 - \mathrm{Erf}(\mu\nu)\right] - \mu\sqrt{\pi},
\end{aligned}
$$

where $\mathrm{Erf}(\mu\nu)$ is an error function,

$$
\mathrm{Erf}(\mu\nu) = \frac{2\nu}{\sqrt{\pi}} \int_0^\mu d\alpha \, e^{-\nu^2\alpha^2}.
$$

In the limit where low-m modes are dominant, we obtain

$$
\begin{aligned}
I_1 &= \left(\frac{4\pi \overline{k}_\theta}{c\eta L_s}\right)^2 \overline{\Delta}_\Phi^4 \overline{\Phi}^2 + \left(\frac{4\pi}{B_\theta}\delta_e\right)^2 \overline{\Pi}^2 \\
&\quad - 2\frac{(4\pi)^2 \overline{k}_\theta}{cB_\theta \eta L_s}(\overline{\Delta}_\Phi^5 \overline{\Delta}_\Pi^{-1})^{1/2} \overline{\Phi\Pi}^2 \left[2\left(1 + \frac{\overline{\Delta}_\Phi^2}{\overline{\Delta}_\Pi^2}\right)^{-1}\right]^{1/2}, \\
I_2 &= \frac{8\pi^{5/2}}{c^2 \overline{k}_\theta}\left[\frac{\overline{k}_\theta}{\eta L_s}\overline{\Delta}_\Phi^{5/2}\overline{\Phi} - \frac{c}{B_\theta}\delta_e \overline{\Delta}_\Pi^{-1/2}\overline{\Pi}\right]^2.
\end{aligned} \tag{60}
$$

A simple mixing length would suggest[4] $\overline{u}^2 \sim \overline{W}_k^{-2}$. This approximation would be relevant if NPGDT has a tearing parity or if magnetic fluctuations have Gaussian structure. However, NPGDT has a twisting parity, or negligible magnetic fluctuations near $x = 0$. By using the turbulence level determined in Sec. IV, we find $\overline{u}^2 \sim \overline{k}_\theta/\overline{W}_{\mathbf{k}}$, when Λ is large. Therfore, the radial structure of magnetic fluctuations plays an important role when we apply the mixing length principle.

By substituting Eqs. (49)-(53), and (56) into Eq. (48), we obtain

$$\begin{aligned}\chi_e^2 &= v_\parallel^2 \overline{u}^2 \sum_{\mathbf{k}} \left| i \frac{4\pi}{Bc\eta L_s} \frac{k_\theta^2}{(u^2 + k_\theta^2)} \frac{d}{du} \Phi(u) + \frac{4\pi}{BB_\theta} \delta_e \frac{uk_\theta}{(u^2 + k_\theta^2)} \Pi(u) \right|^2 \\ &= \left(\frac{v_\parallel \overline{k}_\theta}{B} \right)^2 \frac{(I_2)^2}{I_1}. \end{aligned} \tag{61}$$

During the linearly growing stage, χ_e due to magnetic fluctuations is negligible because contributions from Φ and Π flutuations are almost equal in magnitude but opposite in sign (i.e., $I_2 \simeq 0$). However, in the nonlinearly saturated state, the contribution from the bootstap current becomes dominant over that from the inductive current because of different power dependences on Λ.. When Λ is large, χ_e can thus be written as

$$\chi_e = 4.6 \times 10^{-2} |v_\parallel| L_s \left(\frac{\epsilon}{q} \beta_p \right)^{4/3} \delta_e^{5/3} \left(\frac{r}{\overline{k}_\theta L_P^2} \right)^{2/3} S_M^{-2/3} \Lambda^{7/3}. \tag{62}$$

Parameter scalings of χ_e are

$$\chi_e \sim \hat{s}^{-1} S_M^{-2/3} \beta^{4/3} L_P^{-4/3} T_e^{1/2}. \tag{63}$$

It is interesting to compare these results with those of Ref. 4. There, the mixing length theory is used, thereby neglecting the contribution from integrals due to the radial structure of magnetic fluctuations. If the same approximation is applied to NPGDT, one finds that the scaling of χ_e would be

$$\chi_e \sim \hat{s}^{-3/2} S_M^{-1} \beta^{3/2} L_P^{-3/2} T_e^{1/2}.$$

Hence, both quantitative and qualitative differences are apparent. Note that because of the similarity in the structure of NPGDT and resistive-interchange modes,[2] the same procedure can be applied to resistive-interchange modes. Recent study[19] of the electron heat transport due to resistive-interchange modes reached conclusions similar to the present study.

Finally, the magnitudes of the radial and the poloidal magnetic fluctuation levels are given by

$$\frac{\overline{\tilde{B}_r}}{B} = 0.11 \left(\frac{\epsilon}{q}\beta_p\right)^{5/6} \delta_e^{3/2} \left(\frac{r^2 L_s^3}{L_P^5}\right)^{1/4} S_M^{-1/2} \Lambda^{7/4},$$

$$\frac{\overline{\tilde{B}_\theta}}{B} = 9.6 \times 10^{-2} \left(\frac{\epsilon}{q}\beta_p\right)^{7/6} \delta_e^{4/3} \left(\frac{r^2 L_s^3}{\overline{k}_\theta^2 L_P^7}\right)^{1/6} S_M^{-1/3} \Lambda^{7/6}.$$

Thus, magnetic fluctuation levels and associated electron thermal conduction are enhanced by increasing β_p and a sharper pressure gradient, but are suppressed by strong shear.

VI. Summary

From neoclassical resistive MHD equations[5] including hot, trapped particle effects, we have identified various branches of instability: stable resistive-interchange modes,[2] which are due to the average unfavorable curvature in the absence of neoclassical effects; energetic trapped-particle-driven resistive-interchange modes,[16] destabilized when there is a sufficient population of hot trapped particles; and neoclassical pressure-gradient-driven modes[5,6] (bootstrap current modes), which become important when neoclassical effects are dominant. The linear study of NPGDT shows that small scale bootstrap current modes are stabilized.

Nonlinear saturation of NPGDT is achieved via nonlinear coupling of unstable, long-wavelength modes to stable, short-wavelength modes, thus balancing the linear free energy source with its ultimate sink, the viscosity and $\chi_\perp$. Renormalized equations are obtained. The turbulence level is determined from nonlinear steady state condition. The principal results are as follows:

(i) The pressure diffusivity is obtained as an eigenvalue of renormalized equations at saturation, yielding:

$$D_p = \frac{1}{8\pi}\frac{\epsilon}{q}\beta_p c^2 \eta \delta_e \frac{L_s}{L_P}\Lambda^2.$$

This value is larger than the mixing length estimate of D_P by a factor of Λ^2 (Λ is always greater than 1; see Eq. (40)).

(ii) At saturation, the radial scale of pressure fluctuations differ from that of electrostatic potential fluctuations, i.e.,

$$\Delta_\Pi = \sqrt{2}\left(\frac{\epsilon}{q}\beta_p\right)^{1/6}\left(\frac{1}{16\pi}\frac{\delta_e r}{k_\theta L_P^2}\right)^{1/3} S_M^{-1/3}(L_s L_P)^{1/2}\Lambda^{7/6},$$

$$\Delta_\Phi = \sqrt{2}\left(\frac{\epsilon}{q}\beta_p\right)^{1/6}\left(\frac{1}{16\pi}\frac{\delta_e r}{k_\theta L_P^2}\right)^{1/3} S_M^{-1/3}(L_s L_P)^{1/2}\Lambda^{1/6}.$$

(iii) The levels of turbulence are determined from the diffusivities, i.e.:

$$\frac{e\tilde{\phi}}{T_e} = \sqrt{2}\left(\frac{\delta_e}{16\pi}\right)^{2/3}\left(\frac{\epsilon}{q}\beta_p\right)^{5/6}\left(\frac{r^5\overline{k}_\theta}{L_P}\right)^{1/3} S_M^{-2/3}\frac{(L_s L_P)^{1/2}}{\rho_s}\frac{\tau_A}{\overline{k}_\theta c_s}\Lambda^{5/6},$$

$$\frac{\tilde{P}}{P} = \sqrt{2}\left(\frac{\epsilon}{q}\beta_p\right)^{1/6}\left(\frac{\delta_e}{16\pi}\frac{r}{\overline{k}_\theta L_P^2}\right)^{1/3}\left(\frac{L_s}{L_P}\right)^{1/2} S_M^{-1/3}\Lambda^{7/6}.$$

(iv) Electron heat transport due to stochastic magnetic fields induced by NPGDT has been estimated. The radial structure of magnetic fluctuations has been taken into account, yielding:

$$\chi_e = 4.6 \times 10^{-2} |v_\parallel| L_s \left(\frac{r}{\overline{k}_\theta L_P} \right)^{2/3} S_M^{-2/3} \Lambda^{7/3}.$$

(v) Magnetic fluctuation levels are also determined:

$$\frac{\overline{\tilde{B}_r}}{B} = 0.11 \left(\frac{\epsilon}{q} \beta_p \right)^{5/6} \delta_e^{3/2} \left(\frac{r^2 L_s^3}{L_P^5} \right)^{1/4} S_M^{-1/2} \Lambda^{7/4},$$

$$\frac{\overline{\tilde{B}_\theta}}{B} = 9.6 \times 10^{-2} \left(\frac{\epsilon}{q} \beta_p \right)^{7/6} \delta_e^{4/3} \left(\frac{r^2 L_s^3}{\overline{k}_\theta^2 L_P^7} \right)^{1/6} S_M^{-1/3} \Lambda^{7/6}.$$

We note that while the resistive MHD turbulence[1] is relevant only in the cold edge of the plasma, the region where NPGDT can be applied is extended over wide zone between the center and the edge of the plasma. Also, as the pressure is increased with additional heating, NPGDT becomes aggravated. Therefore, NPGDT may have some bearings on confinement degradation during the L-mode.[23] Furthermore, since NPGDT shows similar behavior[24] to resistive turbulence under the influence of strong shear, it can be suppressed at the edge of an H-mode discharge,[25] thus providing improved confinement behavior.

So far, we have investigated the turbulence evolving from instabilities with the twisting parity. Neoclassical tearing instabilities and the transport associated with them will be discussed later.

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