MHD Alfvén Stability in Ignited Toroidal Plasmas

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ABSTRACT

The presence of fusion-product alpha particles in an ignition tokamak can significantly modify the stability behavior of the burning plasma. In this paper, the effects of toroidicity are retained in a theoretical description of two global-type shear Alfvén modes that can be destabilized by the alpha particles. Toroidicity can lead to stabilization of the global shear Alfvén eigenmode, but it induces a shear Alfvén gap eigenmode whose growth rate is even larger, although these modes may saturate nonlinearly. The stabilizing influence of highly energetic particles on low-frequency MHD ballooning and internal kink modes is also explored; in particular, ion diamagnetic frequency effects may stabilize fishbone oscillations, as has been observed with monster sawteeth in JET with ICRF heating.

1. GLOBAL ALFVÉN WAVE STABILITY WITH ALPHA PARTICLES

In an ignited tokamak plasma, high energy alpha particles are born with a radially peaked profile. The expansion free energy associated with the alpha particle density gradient can drive thermonuclear instabilities. In particular, global-type shear Alfvén waves can resonantly interact with the transiting alpha particles via inverse Landau damping to tap this free energy source and grow unstably.

There are two types of global Alfvén waves in a tokamak plasma. Both are characterized by having low to moderate toroidal and poloidal mode numbers (n and m, respectively) and by having no relevant mode rational surface, i.e., $k_\parallel \neq 0$. The first type, the so-called Global Alfvén Eigenmode (GAE), has its frequency just below the radial minimum of the shear Alfvén continuum. This mode has previously been examined only in cylindrical geometry, where it was found that the transit wave-particle resonant interaction with super-Alfvénic alpha particles could destabilize it, but with weak growth rates [1]. Here, we find that toroidicity induces coupling to electron

Landau-damped sideband modes, which causes the GAE modes with $n \neq 0$ to become stabilized at moderate values of the aspect ratio. Furthermore, the $n = 0$ GAE mode is found to be stable in toroidal geometry.

The second type of global Alfvén mode is termed the Toroidicity-Induced Shear Alfvén Eigenmode (TAE). This is a new global mode, induced purely by finite toroidicity. Its frequency lies within "gaps" in the shear Alfvén continuum, which are created by toroidal coupling. Previously, the existence of this mode was exhibited in the MHD fluid limit without alpha particles [2]. Here, we show that the TAE mode can be strongly destabilized by alpha particles in a burning plasma: it is much more unstable than the GAE mode, over a wider range of values for the alpha particle density scale length (which is a measure of the available free energy).

1.1. Global Alfvén Eigenmodes

In the present work, the effects of finite toroidicity were retained in the wave equation for the $n \neq 0$ GAE modes:

$$\nabla \times \nabla \times \vec{E} - \left(\frac{\omega}{c}\right)^2 \overleftrightarrow{\chi_f} \cdot \vec{E} = \left(\frac{\omega}{c}\right)^2 \left(\overleftrightarrow{\chi_k} + \overleftrightarrow{\chi_\alpha}\right) \cdot \vec{E}, \tag{1}$$

where the plasma response, described by the susceptibility tensor $\overleftrightarrow{\chi}$, has been separated into three parts. The fluid-like response of the background plasma is given by $\overleftrightarrow{\chi_f} = (c/v_A)^2 \left(\overleftrightarrow{I} - \vec{b}\vec{b}\right)$, where $\vec{b} = \vec{B}/B$ is the unit vector along the magnetic field. The kinetic response of the background plasma (in the low-frequency limit $\omega < \omega_{ci}$), given by $\overleftrightarrow{\chi_k}$, includes finite ion Larmor radius effects and Landau damping from parallel electron dynamics. The response of the low-density, highly energetic alpha particle component is given by $\overleftrightarrow{\chi_\alpha}$, which is derived from the drift kinetic equation (with trapped alpha particles neglected). Note that $\overleftrightarrow{\chi_\alpha}$ involves coupling to the $m \pm 1$ sidebands because the alpha particle magnetic drift velocity has poloidal variation; however, these sidebands contribute to a perturbed drift current that has the same poloidal mode number as the perturbed electric field $\vec{E}$. The dominant toroidal effects come from $\overleftrightarrow{\chi_f}$ and the $\nabla \times \nabla \times$ operator on the left-hand side of Eq. (1); these were derived, for concentric flux surfaces, in toroidal coordinates. The resulting coupled system of linear differential equations was truncated to a six-by-six system for three poloidal

modes of the two transverse $\vec{E}$-field components. These equations were solved numerically for a small, non-zero value of the density at the plasma edge $r = a$, in order to have finite Alfvén frequency $\omega_A = k_{\|m} v_A$, where $k_{\|m} = (n - m/q)/R$ and $v_A = B/\left(n_i m_i\right)^{1/2}$.

Figure 1 shows the growth rate, γ, as a function of the inverse aspect ratio, $\varepsilon = a/R$, for the $n = 1, m = -2$ GAE mode coupled to the sidebands $(1, -1)$ and $(1, 0)$. The real frequency of the mode is $\omega_r = 7.95 \times 10^6 \text{s}^{-1}$. The results in Fig. 1 were obtained for $L_\alpha/a = 0.25$, where L_α is the alpha particle density scale length, and for ignition parameters of $B = 10\,\text{T}$, $n_0 = 10^{21}\text{m}^{-3}$, and $T_{io} = T_{eo} = 30\,\text{keV}$. Because one of the sidebands has a shear Alfvén singularity, e.g., $\omega^2 = \omega_{A,m+1}^2(r)$, it experiences large electron Landau damping, which draws energy from the main mode through the toroidal coupling. The consequence is that the GAE, which is driven unstable in the presence of alpha particles at $\varepsilon = 0$ (i.e., in the cylindrical limit), can be stabilized by this sideband dissipation for $\varepsilon \gtrsim 0.1$.

In contrast to the previous cylindrical geometry calculations [1] which had found that the $n = 0$ GAE is more unstable than the GAE modes with $n \neq 0$, we find that the $n = 0$ GAE is stable in toroidal geometry. The reason is that the $\pm m$ poloidal harmonics for the $n = 0$ mode have very nearly the same amplitude, so that their contributions to the alpha particle-induced inverse Landau damping through the alpha particle diamagnetic frequency, $\omega_{*\alpha}$, which is proportional to m, tend to cancel. This result, which will be discussed also in the next section, was found from fully toroidal numerical calculations that used the NOVA-K nonvariational kinetic-MHD stability code [3].

1.2.   Toroidicity-Induced Shear Alfvén Eigenmode

The alpha particle-driven instability of the TAE mode was studied by considering an axisymmetric toroidal plasma consisting of core and alpha particle components, with $\beta_\alpha < \beta_c$ and $T_\alpha \gg T_c$. The ideal MHD description was adopted for the core plasma, and the drift kinetic description for the alpha particles. The alpha particles interact with the core plasma through a pressure gradient force on the core plasma fluid displacement $\vec{\xi}$, which is governed by the momentum balance equation:

$$\omega^2 \rho \vec{\xi} = \nabla \delta P_c + \nabla \cdot \delta \overleftrightarrow{P_\alpha} + \delta \vec{B} \times \vec{J} + \vec{B} \times \delta \vec{J}. \qquad (2)$$

Here, $\delta\vec{B}$, $\delta\vec{J}$, and δP_c are the perturbed magnetic field, current, and core plasma pressure, respectively. The perturbed alpha particle pressure tensor, $\delta\overleftrightarrow{P_\alpha}$, is obtained from the drift kinetic solution for the perturbed distribution function, δf_α, in which all bounce and transit resonances were included.

The resultant integro-differential eigenmode equations were solved in a general flux coordinate (ψ, θ, ζ) system for realistic numerical equilibria by means of a nonvariational kinetic-MHD stability code (NOVA-K) [3]. The alpha particle equilibrium distribution was taken to be uniform in pitch angle Λ and slowing down in energy ε: $F_{0\alpha} = C(\psi)\varepsilon^{-3/2}$ for $\varepsilon > \varepsilon_a$ and zero otherwise, with ε_a the alpha particle birth energy and $C(\psi)$ proportional to the alpha particle pressure $P_a(\psi)$, which is related as $P_a(\psi) \propto P^{7/2}(\psi)$ to the total plasma pressure $P(\psi)$.

The TAE modes exist with discrete frequencies inside the gap of the shear Alfvén toroidal continuum spectrum [2]. The gap is formed due to toroidal coupling of different poloidal harmonics. For example, the coupling of modes (n, m) and $(n, m+1)$ at $q(r) = (m+1/2)/n$ form a gap bounded by

$$\omega_\pm^2 = \omega_0^2 + \left\langle \frac{\gamma_s p B^2 \kappa_g^2}{|\nabla\psi|^2} \right\rangle \pm 2\omega_0^2 \left(\frac{r_0}{R} + \Delta'(r) \right). \tag{3}$$

Here, $\omega_0^2 = (v_A/2qR)^2$, $\kappa_s = 2\vec{\kappa}\cdot(\vec{B}\times\nabla\psi)/B^2$ is the geodesic curvature, and Δ measures the shift of the center of the flux surfaces from the magnetic axis, with $\Delta' = d\Delta/dr > 0$.

Figure 2 shows the growth rate, γ/ω_A, of the $n = 1$ fixed boundary TAE mode as a function of $\omega_{*\alpha}/\omega_A$ for a numerical equilibrium with circular plasma boundary. The pressure profile is $P(y) \propto (1 - y^2)^2$, with $y = \psi/\Delta\psi$ the normalized poloidal flux; the other parameters are $q(0) = 1.05$, $q(1) = 2.3$, $q'(0) = 36.12$, $q'(1) = 140$, and $R/a = 4$. The volume-averaged total beta is $\langle\beta\rangle = 1.89\%$ and the alpha particle beta is $\langle\beta_\alpha\rangle = 0.4\%$. Here, $\omega_{*\alpha}$ is the alpha particle diamagnetic drift frequency for $m = 1$ evaluated at $y^{1/2} = 0.5$, and $\omega_A = v_A(0)/q(1)R$. The real frequency for the mode, $\omega_r/\omega_A = -0.705$, lies within the continuum gap. The $m = 1$ and $m = 2$ harmonics have their maximum amplitude near $y^{1/2} = 0.5$; hence $\omega_{*\alpha}$ is a good measure of the available alpha particle free energy via inverse Landau damping. Figure 2 shows that $\omega_{*\alpha}$ is large enough to overcome the usual Landau damping when $\omega_{*\alpha}/\omega_A > 1.5$, and beyond this threshold the growth rate γ is almost linearly proportional to $\omega_{*\alpha}$. From these results, we can

conclude that for typical tokamak reactor parameters, the growth rate of the $n = 1$ TAE mode can easily be of the order of $10^{-2}\omega_r$, which is at least one order of magnitude larger than the GAE mode growth rates.

The stability properties of the global Alfvén waves can be interpreted with the use of the quadratic form derived from Eq. (2): $\omega^2 \delta K = \delta W_f + \delta W_{k,\alpha}$, where δK is the fluid inertial energy, δW_f is the total fluid potential energy, and $\delta W_{k,\alpha}$ is the alpha particle kinetic energy, which contains both bounce and transit resonances. For $\omega = \omega_r + i\gamma$, with $|\gamma| \ll |\omega_r|$, we obtain

$$\gamma \approx -\int d\psi \, d\Lambda \left(\frac{9\pi^2}{4}\frac{\hat{\tau}_t}{\delta K}\right) [\text{sign}(\omega_r) P_\alpha(\psi)]$$

$$\times \sum_{m,m',p=-\infty}^{\infty} \frac{\left[\omega_r - \omega_{*\alpha}^{(m)}\left(\frac{\omega_r/\omega_t}{p-nq}\right)^2\right]}{(p-nq)^2\omega_t^2} \text{Re}\left[\langle G_{m',p}\rangle^* \langle G_{m,p}\rangle\right]. \quad (4)$$

Here ω_t is the alpha particle transit frequency evaluated at $\varepsilon = \varepsilon_\alpha$, $\omega_{*\alpha}^{(m)} = m\omega_{*\alpha}$, $\hat{\tau}_t/v_\alpha$ is the alpha particle transit time, and $\langle G_{m,p}(\theta)\rangle$ is the transit average of $G_{m,p}$, which is proportional to the mode amplitude. Equation (4) shows that if $\omega_{*\alpha}^{(m)} < 0$, then γ is positive for $\omega_r < 0$, and vice versa. It is also clear that γ is linearly porportional to $\omega_{*\alpha}$, consistent with the numerical results from the NOVA-K code.

From Eq. (4), we can also see why the $n = 0$ GAE mode is stable with respect to the alpha particle free energy. Since the alpha particle effects are perturbative ($\beta_\alpha \ll 1$), the mode structure is well described by the ideal MHD toroidal wave functions, whose $(0,m)$ and $(0,-m)$ modes have equal amplitudes, $|\langle G_{m,p}(\theta)\rangle|^2 = |\langle G_{-m,p}(\theta)\rangle|^2$. Therefore, the $\omega_{*\alpha}^{(m)}$ term cancels with the $\omega_{*\alpha}^{(-m)}$ term, and the $n = 0$ GAE mode is thus stable. [More precisely, note that this symmetry among the mode amplitudes is broken by the $\omega_{*\alpha}$ terms themselves, so that instability for the $n = 0$ mode can be recovered for sufficiently large alpha particle drive. From a zero-dimensional mode coupling argument, we estimate that $n = 0$ instability can occur when $\gamma_{\alpha,m}/\omega_m \gtrsim O\left(\langle\varepsilon\rangle^{m+1}\right)$, where ω_m and $\gamma_{\alpha,m}$ are the real frequency and the alpha particle contribution to the cylindrical growth rate for mode $(0,m)$ and $\langle\varepsilon\rangle$ is the inverse aspect ratio averaged over the wave function. However, for the typical ignition parameters considered here, this condition is not satisfied (in particular, for $m = 1$ and 2, the most unstable

cylindrical modes), and hence the conclusion is valid that the $n = 0$ GAE is stable in toroidal plasmas.]

Finally, kinetic effects from the core plasma were not included in this analysis, because ion FLR effects are negligible and electron Landau damping on the TAE mode is about an order of magnitude smaller than the alpha particle-induced growth rate.

1.3. Nonlinear Saturation of the Toroidicity-Induced Shear Alfvén Mode

A general procedure was developed to describe the nonlinear saturation of modes like the TAE mode discussed earlier. It is assumed that the background plasma supports discrete non-overlapping modes with a positive wave energy $\omega \left[\partial G(\omega)/\partial \omega \right] |\xi|^2$, where ξ represents the wave field amplitude and $G(\omega) = 0$ is the dispersion relation. Without a source of instability, it is assumed that there is a power drain, $P_D(\omega)|\xi|^2$, due to background plasma dissipation mechanisms. Thus, the damping rate is given by $\gamma_d = P_D(\omega) \left[2\partial G(\omega)/\partial \omega \right]^{-1}$. Instability is assumed to arise from an injected source at high energy that slows down by a classical drag process. In the case of the Alfvén gap mode, the instability source arises from the spatial gradient in a group of resonant particles. The power introduced from the source is given by $P_S(\omega)|\xi|^2$ and instability arises if $P_S(\omega) > P_D(\omega)$, with a growth rate $\gamma = \left[P_S(\omega) - P_D(\omega) \right] \left[2\partial G(\omega)/\partial \omega \right]^{-1}$.

As the mode grows, the local spatial gradient is flattened, and if no source were present, the mode would saturate by the flattening of the spatial gradient in the vicinity of the resonant particles. However, with a source, new resonant particles come into resonance at the source's input rate. Hence, the flattening of the spatial gradient cannot saturate the instability by itself, but the flattening does reduce the growth rate. We have calculated the self-consistent slowing-down distribution in the presence of a finite amplitude wave and find that at large enough amplitude, the power, P_{NL}, transferred to the wave by the resonant particles is $P_{NL} \cong (\nu/\omega_b) P_S(\omega)|\xi|^2$, where ν is the input rate of the source and $\omega_b \propto m |\xi/L_s|^{1/2}$ is the trapped particle bounce frequency in the wave (as long as $\omega_b > \nu$), with L_s the shear length and m the poloidal mode number. This result indicates that saturation occurs when $\omega_b/\nu = P_S(\omega)/P_D(\omega)$. Since ν is small, this saturation result normally means that spatial diffusion

7

of resonant particles is localized, and they are not lost from the system. Detailed quantitative calculations using parameters of linear theory still need to be performed. Further study is also needed to assess large phase space gradients that arise in the resonant region, which may give rise to nonlinear instability.

2.   LOW-FREQUENCY MHD STABILITY WITH ENERGETIC PARTICLES

The stability of low-frequency MHD modes, such as ballooning and internal kink modes, can also be influenced by the presence of alpha particles (or a beam-injected or rf-heated energetic species). In contrast to the global Alfvén waves, these low-frequency modes tend to interact with the *trapped* component of the energetic particles, and the relevant wave-particle resonance occurs at their magnetic field curvature drift frequency (bounce-averaged).

2.1.   Stabilization of Ballooning Modes

When such resonant interaction is negligible—which requires a highly anisotropic distribution of energetic particles—the energetic trapped particles can actually stabilize ballooning modes by enhancing compressibility effects through their non-fluid behavior [4]. A detailed study of numerically generated equilibria for finite-aspect-ratio, anisotropic-pressure tokamak plasmas was able to demonstrate the global stabilization of high-mode-number ballooning modes that is due to energetic trapped particles. A procedure was developed to derive the optimal trapped particle pressure profile. Variations in aspect ratio and in plasma shape (circular, dee, and bean cross sections) were studied and found to have a strong impact on the conditions for stable access to the high-beta second stability regime. Studies with aspect ratio values of 5 and 10 suggest that the required hot particle beta value scales as $\beta_{\text{hot}} \propto a/R$. In particular, at $R/a = 5$, the minimum values for β_{hot} and $q(0)$ that are needed for simultaneous stability on all flux surfaces as the plasma beta is increased were found to depend on cross-sectional shape as $0.5\% = \beta_{\text{bean}} < \beta_{\text{dee}} < \beta_{\text{circle}} = 4.5\%$ and $1.25 = q(0)_{\text{bean}} < q(0)_{\text{dee}} < q(0)_{\text{circle}} = 1.5$.

8

The dependence of ballooning mode stability on finite aspect ratio in the presence of energetic trapped particles was also examined analytically by solving for the equilibrium in an expansion up to $O(a/R)^2$. Finite aspect ratio effects somewhat improve the stabilization. However, when the energetic particles become drift reversed as the beta increases, an additional region of ballooning instability is encountered. Also, the limits imposed by ripple loss are more stringent at high values of the aspect ratio than at low values [5].

2.2. Stabilization of the $m = n = 1$ Internal Kink

The $m = n = 1$ internal kink that can be excited by hot trapped particles through their curvature drift frequency resonance was found to possess a new stability regime when background plasma finite gyroradius effects are included. Experimentally, such destabilization by an energetic minority ion component has been observed in fishbone oscillations of tokamaks with neutral beam injection [6], whereas the stabilizing feature has recently been observed in "monster" sawteeth oscillations in JET with ICRF heating [7]. We have developed a stability theory that describes both types of behavior. The quantitative results thus far only accurately describe the case when the parallel electric field is zero. This leads to the dispersion relation

$$\hat{\omega}_A \left(\delta \hat{W}_f + \delta \hat{W}_k \right) - i\sqrt{\hat{\omega} \left(\hat{\omega} - \hat{\omega}_{*i} \right)} = 0, \tag{5}$$

where all frequencies with a caret are normalized to the deeply trapped hot particle drift frequency at its birth energy. (Note that here $\omega_A = v_A S/R$, at $q = 1$.) For δW_f we use the expression obtained by Bussac et al. [8]:

$$\delta \hat{W}_f = 3 \left[1 - q(0) \right] \left(\frac{13}{144} - \tilde{\beta}_p^2 \right) \quad \text{with} \quad \tilde{\beta}_p = - \left(\frac{8\pi}{B_p^2} \right) \int_0^{r_s} dr \left(\frac{r}{a} \right)^2 \left(\frac{dp}{dr} \right) \tag{6}$$

where B_p is the poloidal field at the $q = 1$ surface. For δW_k we used the expression obtained by Chen et al. [9] for deeply trapped particles:

$$\delta \hat{W}_k \cong \tilde{\beta}_h \hat{\omega} \ln \left(1 - \frac{1}{\hat{\omega}} \right) \quad \text{with} \quad \tilde{\beta}_h = - \left(\frac{R_0}{6} \right) \frac{\partial \langle \beta_{p,h} \rangle}{\partial r}, \tag{7}$$

where $\langle \beta_{p,h} \rangle$ is the poloidally averaged beta.

The marginal stability boundary for $\hat{\beta}_h = \tilde{\beta}_h \hat{\omega}_A$ as a function of $\hat{\omega}_{*i}$ is obtained from Eq. (5) and plotted in Fig. 3 for $\delta \hat{W}_f \hat{\omega}_A = 0, -0.01, -0.03,$

9

and -0.05. For $\delta\hat{W}_f\hat{\omega}_A < -0.087$, no stable region exists. Even though the ideal MHD internal kink would be unstable for these parameters, a substantial stability domain may exist, as seen in Fig. 3. As $\hat{\omega}_{*i}$ is increased, we find that the negative energy precessional mode [9] and the ω_{*i} mode [10] merge. As a result, the stable region exists only for small $\hat{\omega}_{*i}$ (< 0.5) in the ideal MHD limit.

Considering typical operating parameters of various tokamaks in which highly energetic particles are generated, our theory suggests that TFTR ($\hat{\omega}_{*i} \sim 0.8$) and PDX ($\hat{\omega}_{*i} \sim 0.4$) operate in a fishbone unstable region due to large ω_{*i} effects, whereas JET ($\hat{\omega}_{*i} \sim 0.2$) and DIII-D ($\hat{\omega}_{*i} \sim 0.1$) may lie in either the stable or unstable region, depending on the values of $\hat{\beta}_h$ and $\delta\hat{W}$.

3. SUMMARY AND CONCLUSIONS

The stability of the two types of global Alfvén waves in an ignited tokamak plasma has been studied analytically and numerically. Toroidicity tends to stabilize the $n \neq 0$ global Alfvén eigenmodes (GAE); and the $n = 0$ GAE is stable, to lowest order, when toroidal coupling between the $\pm m$ modes is taken into account. For the proposed CIT parameters, the GAE modes are therefore not problematic, and attention should be focused on the toroidicity-induced shear Alfvén eigenmodes (TAE), which are strongly destabilized via transit resonance with alpha particles. A theoretical framework for studying the nonlinear saturation of these toroidicity-induced modes was also described.

With respect to lower frequency MHD modes, highly energetic trapped particles are found to be able to stabilize ballooning modes and internal kink modes in the ideally unstable regime. The former result suggests the possibility of access to high-beta second stability; the latter result may be relevant to JET observations of monster sawteeth.

ACKNOWLEDGMENTS

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Figure Captions

1. Growth rate γ of the $n = 1$, $m = -2$ global Alfvén eigenmode coupled to the sidebands $(1, -1)$ and $(1, 0)$, as a function of the inverse aspect ratio a/R.

2. Growth rate γ for the $n = 1$ toroidicity-induced shear Alfvén eigenmode as a function of the alpha particle diamagnetic drift frequency $\omega_{*\alpha}$ (normalized to the shear Alfvén frequency ω_A).

3. Stability boundaries for the $n = 1$, $m = 1$ internal kink mode stabilized by ion diamagnetic frequency (ω_{*i}) effects in a tokamak plasma containing an energetic minority ion species (with beta value $\hat{\beta}_h$).
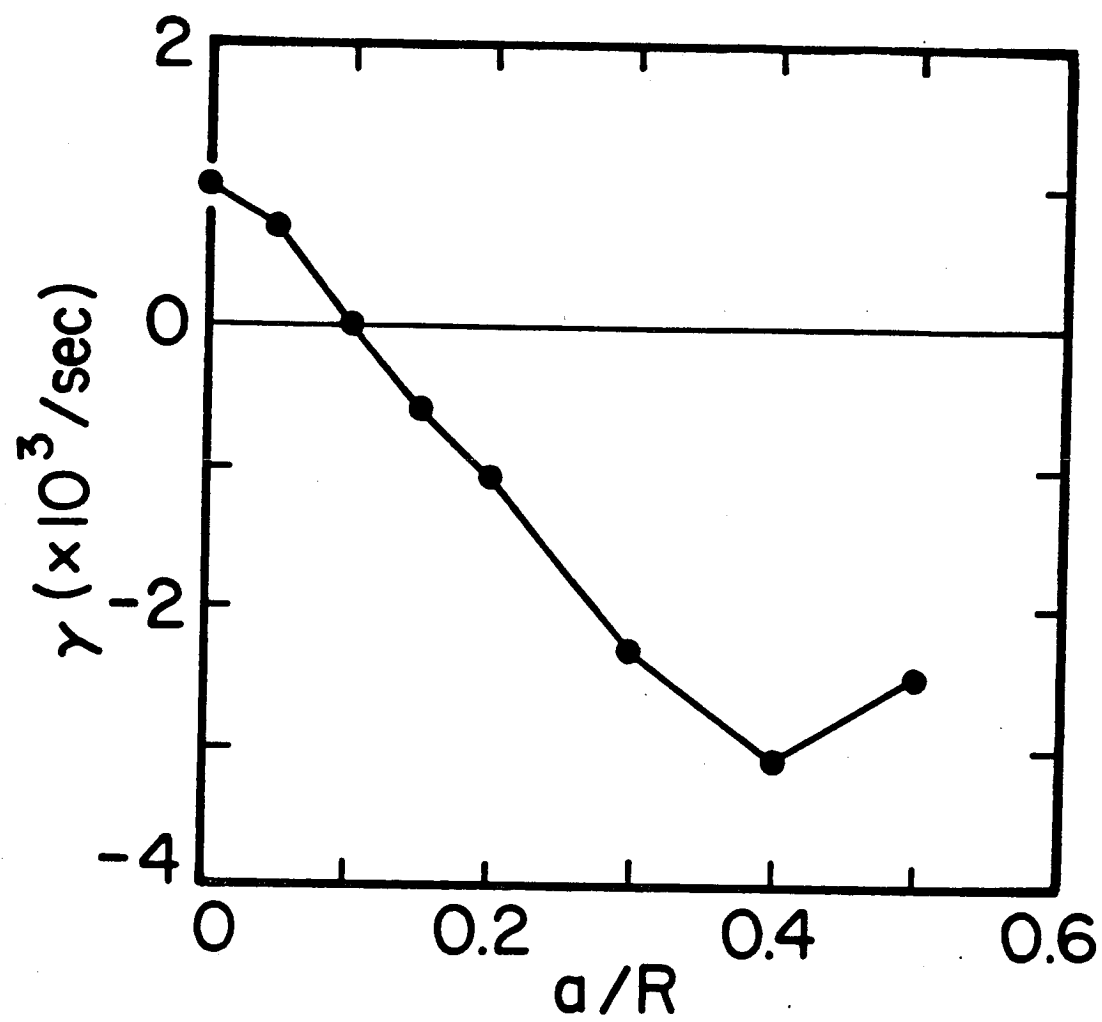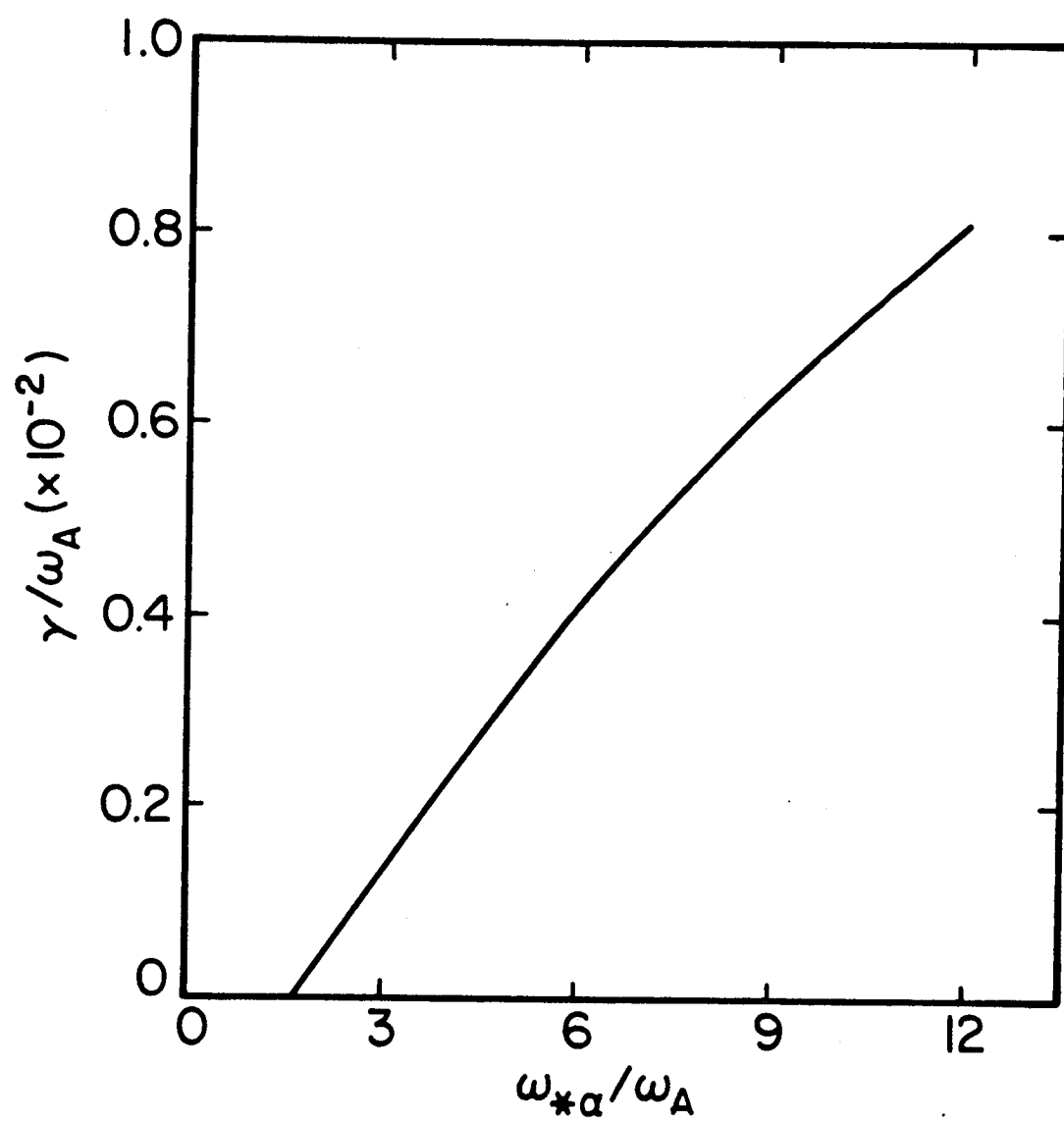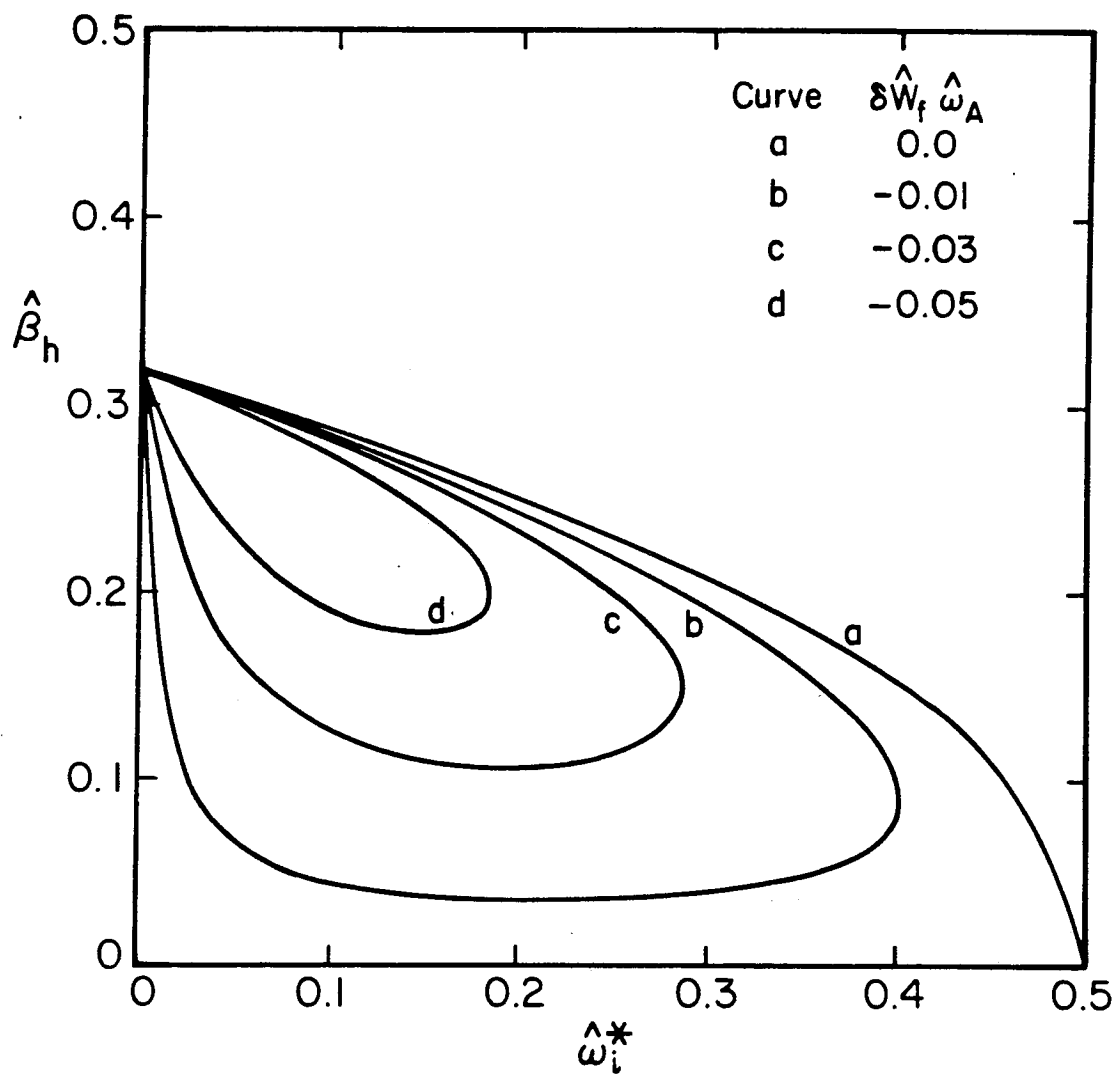
Fig. 1

Fig. 2

Fig. 3