DOE/ET-53088-343 IFSR#343

Diffusive Processes in the Cross-Field Flow of Intense Plasma Beams

B. Newberger and *N. Rostoker**
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

September 1988

* University of California, Irvine

Diffusive Processes in the Cross-Field Flow of Intense Plasma Beams

B. Newberger and N. Rostoker[a)]
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

Abstract

We consider magnetic field diffusion in the presence of strongly magnetized electrons ($\omega_{ce}\tau_{co} > 1$) as a mechanism for the rapid field penetration observed in cross-field flows of high-β plasma beams. The diffusion has been investigated in several cases which are amenable to analytic solution. The flux penetration times are found to be insensitive to the particular configuration. Comparison with two experiments is made. Agreement within the limits of the experiments is found. Both require an anomalous collision rate which is consistent with observed fluctuations in one case but apparently not the other.

[a)] University of California, Irvine

I. Introduction

Active injections of intense beams of neutral plasmas (sometimes alternatively called plasmoids or plasma jets) into the near earth space environment are of interest in the simulation of phenomena associated with naturally occurring events. These events include aurorae, magnetospheric substorms and comets, and the phenomena include wave generation and emission, particle precipitations associated with these[1] and the interaction of the solar wind with cometary bodies.[2] In order to interpret these active injection experiments, the dynamics of the flow of the jet across the geomagnetic field must be understood. To this end, laboratory experiments investigating the cross-field flow of plasma beams are also being conducted.[3,4]

Of particular interest, both in space and the laboratory, are jets of sufficient intensity that the ram kinetic energy, ρv^2 exceeds the magnetic pressure, $B^2/4\pi$ of the ambient field; this is called the high-β regime. In this case, the conventional picture holds that the external field will be excluded from the interior of the jet and the jet propagates by plowing the ambient field aside. A significant finding in both the space based and laboratory experiments is an anomalously rapid penetration of the ambient field into the plasma beam. The penetration rates considerably exceed those based on a classical collision frequency. Some evidence exists for enhanced levels of turbulence in some of the active injection experiments,[5] and mechanisms for an anomalous collision frequency have been suggested.[6] Recent laboratory experiments[4] also have observed field fluctuations although their interpretations are not yet complete. An anomalous resistivity could give an enhanced field diffusion. However, it has been noted by Rostoker and co-workers that even with an anomalous collision frequency ν_{ei}^* which could be expected based on observed fluctuations, the electrons are magnetized, $\Omega_e/\nu_{ei}^* > 1$.

In this paper, we obtain estimates of the diffusion times to be expected, based on the anomalous resistivity in the presence of magnetized electrons. These estimates are obtained from analytic solutions of the magnetic diffusion equation in several cases. In general, the electron magnetization makes the diffusion equation nonlinear. We have obtained an approximate solution in the slab limit. By comparing the diffusion times from this solution with those obtained by imposing linearity on the diffusion equation in both the slab and cylindrical limits, we can independently get some measure of influence of geometry and nonlinearity. It is found that both increase the diffusion time over the linear theory in the slab. The differences are measurable in principle but probably not within experimental uncertainties in practice. In considering some particular experimental situation, it is likely to be reasonably safe to use the simple linear slab estimates. The differences are expected to be comparable to the factors of 1.5-2.5 found here. We have also evaluated the absolute diffusion times for several cases of interest to space and laboratory experiments.The diffusion times are consistent with observations. We will now describe the model and present our solutions.

II. Diffusion Model

The transport in the beam is taken to be standard Braginskii, with collision times reduced by turbulence from the Coulomb value. The transport model neglects the effect of inertia and this imposes a constraint on the solution which, because we are not considering time-harmonic phenomena, takes the form $\Omega_e \tau_D > 1$, where Ω_e is the electron gyrofrequency and τ_D is the diffusion time. Diffusion times based on Coulomb collisions alone are too fast to satisfy this condition and the model breaks down. With an anomalous collision rate, the inequality can be satisfied. The jet is taken to be homogeneous so the ∇P term does not appear and thermoelectric terms are neglected. In this case, the currents in the beam are related to the electric fields by

$$J = \sigma_{\perp} \overline{E}'_{\perp} + \sigma_H \overline{E}' \times \hat{n} \qquad \text{where}$$

$$\overline{E}' = \overline{E} + \overline{\beta} \times \overline{B} \ ,$$

$$\overline{\beta} = v/c\,\hat{z} \tag{1}$$

is the jet velocity and taken to be along the z-axis,

$$\sigma_\perp = \frac{\sigma_0}{1 + (\Omega_e \tau_{ei}^*)^2} \ , \tag{2}$$

the Pedersen conductivity,[8]

$$\sigma_H = \Omega_e \tau_{ei}^* \sigma_\perp \ , \tag{3}$$

the Hall conductivity, where

$$\sigma_0 = \frac{\omega_{pe}^2 \tau_{ei}^*}{4\pi} \ . \tag{4}$$

We further define $\overline{E}_\perp = \overline{E}_\wedge + E_z \hat{z}$ and we see $J_{||} = 0$.

It is observed in the experiments that the motion of the beam particles are essentially force free in the beam frame. This implies $\overline{E}_\wedge = -\hat{\beta} \times \overline{B}$ and therefore $\overline{E}_\perp$ is parallel to the z-axis. Ampere's law in cylindrical geometry give the three equations:

$$4\pi J_r \ + \ \frac{\partial E_r}{\partial t} = 0 \tag{5}$$

$$4\pi J_\theta \ + \ \frac{\partial E_\theta}{\partial t} = 0 \tag{6}$$

$$\frac{4\pi J_z}{c} \ = \ \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \frac{1}{r}\frac{\partial B_r}{\partial \theta} \ . \tag{7}$$

The z-component of displacement current has been neglected relative to the conduction current. This constrains the solution and must be checked *a posteriori*. In the r and θ component equations, the terms in the spatial derivatives of B vanish identically by virtue of the symmetry. The current responsible for the diamagnetism is a Pedersen current[8] driven by the electric field induced by the penetrating flux. The self-consistency is obtained from

Faraday's law which closes the system. After some straightforward algebra a pair of coupled nonlinear diffusion equations for the r, θ components of B are obtained:

$$-\frac{\partial b_r}{\partial \tau} = \frac{1}{\rho}\frac{\partial}{\partial \theta}\left[(1+\kappa b^2)\left(\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho b_\theta) - 1/\rho\,\frac{\partial b_r}{\partial \theta}\right)\right] \tag{8}$$

$$\frac{\partial b_\theta}{\partial \tau} = \frac{\partial}{\partial \rho}\left[(1+\kappa b^2)\left(\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho b_\theta) - 1/\rho\,\frac{\partial b_r}{\partial \theta}\right)\right] \tag{9}$$

where dimensionless variables have been introduced by the definitions

$$\rho \equiv r/a, \quad \tau \equiv t/\tau_D, \quad \mathrm{b} \equiv \mathbf{B}/B_0, \quad \tau_D \equiv 4\pi a^2 \sigma_0/c^2 \ ,$$

$$\kappa = (\Omega_{e0}\tau_{ei}^{*})^2 \quad \text{and} \quad \Omega_{e0} = eB_0/mc \ .$$

Here B_0 is the magnitude of the applied magnetic field far from the beam where $B = B_x$; initial condition is then

$$\left.\begin{aligned} b_r &= (1-1/\rho^2)\cos\theta \qquad (10)\\ b_\theta &= -(1-1/\rho^2)\sin\theta \qquad (11)\end{aligned}\right\}\rho > 1 \ .$$

The boundary condition at $\rho = 1$ is found by matching to a vacuum solution for $\rho > 1$ which satisfies for all time

$$\left.\begin{aligned} b_r &\to \cos\theta \qquad (12)\\ b_\theta &\to -\sin\theta \qquad (13)\end{aligned}\right\}\rho \to \infty \ .$$

In the slab limit, the pair of equations, Eq. (8) and (9), reduce to a single equation for b_x

$$\partial/\partial x\left[\left(1+\kappa b^2\right)\frac{\partial b_x}{\partial x}\right] = \frac{\partial b_x}{\partial \tau} \tag{14}$$

subject to the boundary condition $b_x = 1$ at $x = \pm 1/2$ and the initial condition $b_x = 0$, $-1/2 < x < 1/2$.

We will now consider the solution of the cylindrical system in the linear approximation and the solution of the nonlinear slab model.

III. Linear Solution in Cylindrical Geometry

The solution of the problem in cylindrical geometry can be more readily obtained by introducing a scalar function, χ through the definition

$$\mathbf{b} = \hat{z} \times \nabla \chi . \tag{15}$$

(χ is essentially the z-component of the vector potential.) In terms of χ, Eq. (8)-(9) become

$$\partial/\partial\rho \left\{ (1 + \kappa \nabla\chi \cdot \nabla\chi) \left[\frac{1}{\rho} \frac{\partial}{\partial\rho} (\rho \, \partial\chi/\partial\rho) + 1/\rho \, \frac{\partial^2\chi}{\partial\theta^2} \right] \right\} = -\frac{\partial}{\partial\rho} \frac{\partial\chi}{\partial\tau} \tag{16}$$

$$1/\rho \, \partial/\partial\theta \left\{ (1 + \kappa \nabla\chi \cdot \nabla\chi) \left[1/\rho \, \frac{\partial}{\partial\rho} (\partial\chi/\partial\rho) + 1/\rho \, \partial^2\chi/p\theta^2 \right] \right\} = -\frac{\partial}{\rho\partial\theta} \frac{\partial\chi}{\partial\tau} . \tag{17}$$

These equations can each be integrated once. Because the integrated equations resulting from Eq. (16)-(17) are the same and χ must satisfy given boundary conditions the arbitrary functions, otherwise resulting from the integration, vanish. Thus χ satisfies the nonlinear diffusion equation

$$\partial\chi/\partial\tau = (1 + \kappa \nabla\chi \cdot \nabla\chi) \nabla^2 \chi . \tag{18}$$

The boundary condition at $\rho = 1$ is obtained from a solution in the vacuum region of Laplace's equation

$$\nabla^2 \chi = 0 \tag{19}$$

which satisfies

$$\chi \to -\rho \sin\theta \quad \text{as} \quad \rho \to \infty . \tag{20}$$

That is our boundary condition is imposed by the essentially cartesian nature of our magnetic field system. Hence, this problem is different from the standard mixed problem solved in the textbooks. Thus the solution is only obtained by solving simultaneously Eq. (18) for $\rho < 1$ and Eq. (19) for $\rho > 1$ and matching the solutions at $\rho = 1$. We do not know how to do this in general but will obtain the solution in the linear approximation where Eq. (18) is replaced by

$$\partial \chi / \partial \tau \;=\; \hat{\kappa} \nabla^2 \chi \; , \tag{21}$$

$$\hat{\kappa} \;\equiv\; 1 + \kappa \approx \kappa \quad \text{since typically} \quad \kappa \gg 1 \; .$$

In the outer region, $\rho > 1$ a solution of Laplace's equation satisfying the condition, Eq. (20) is

$$\chi = -\rho \sin\theta + \sum_\ell a_\ell(\tau')\rho^{-\ell} \sin \ell\theta, \quad \tau' = \kappa\tau \; . \tag{22}$$

(The $\cos \ell\theta$ terms turn out to be unnecessary.)

In the interior $\rho < 1$, the solution is found by taking the Laplace transform of Eq. (21). The solution is

$$\overline{\chi}(s, \rho, \theta) = \sum_\ell c_\ell(s) \sin \ell\theta \, I_\ell(\sqrt{s}\,\rho) \; , \tag{23}$$

where I_ℓ is the modified Bessel function of the first kind, and the transform variable is s. Taking the Laplace transform of Eq. (22) gives for $\rho > 1$

$$\overline{\chi}(s, \rho, \theta) = \frac{\rho \sin\theta}{s} + \sum_\ell \overline{a}_\ell(s)\rho^{-\ell} \sin \ell\theta \; . \tag{24}$$

Now matching $\partial\overline{\chi}/\partial\rho$, $\partial\overline{\chi}/\partial\theta$ from Eq. (23) and (24) at $\rho = 1$ gives ℓ pairs of equations for the $\overline{a}_\ell, c_\ell$ which are satisfied by

$$\overline{a}_\ell \;=\; c_\ell = 0 \quad \text{for} \quad \ell > 1 \quad \text{and}$$

$$c_1(s) \;=\; \frac{-2/s}{\sqrt{s}\, I_0(\sqrt{s})} \tag{25}$$

$$\overline{a}_1(s) \;=\; -\frac{(I_1(\sqrt{s}) - \sqrt{s}\, I_1'(\sqrt{s}))}{s^{3/2} I_0(\sqrt{s})} \; , \tag{26}$$

where in $\overline{a}_1$ the prime denotes differentiation with respect to the argument of the Bessel function.

These can be substituted back into their respective equations for $\overline{X}$ and the standard Bromwich inversions of the Laplace transforms done. The solutions are then $\rho < 1$,

$$\chi = \left(-\rho + 4\sum_n \frac{e^{-\alpha_{on}^2\tau'} J_1(\alpha_{on}\rho)}{\alpha_{on}^2 J_1(\alpha_{on})}\right) \sin\theta \ , \tag{27}$$

$$b_r = \left\{-1 + \frac{4}{\rho}\sum_n \frac{e^{-\alpha_{on}^2\tau'} J_1(\alpha_{on}\rho)}{\alpha_{on}^2 J_1(\alpha_{on})}\right\} \cos\theta \ , \tag{28}$$

$$b_\theta = \left\{-1 + 4\sum_n \frac{e^{-\alpha_{on}^2\tau'} \left[J_0(\alpha_{on}\rho) - 1/\alpha_{on}\rho\, J_1(\alpha_{on}\rho)\right]}{\alpha_{on} J_1(\alpha_{on})}\right\} \sin\theta \tag{29}$$

where α_{oj} is the j^{th} zero of J_0 and the Bessel function notation is standard.

For $\rho > 1$

$$\chi = -\rho\sin\theta - 4\sum_n \frac{e^{-\alpha_{on}^2\tau'}}{\alpha_{on}^2} \frac{\sin\theta}{\rho} \tag{30}$$

$$b_r = \cos\theta + 4\sum_n \frac{e^{-\alpha_{on}^2\tau'}}{\alpha_{on}^2} \frac{\cos\theta}{\rho^2} \tag{31}$$

$$b_\theta = -\sin\theta + 4\sum_n \frac{e^{-\alpha_{on}^2\tau'}}{\alpha_{on}^2} \frac{\sin\theta}{\rho^2} \ . \tag{32}$$

We will use these to obtain numerical estimates of diffusion times following our discussion of the nonlinear diffusion in the slab.

IV. Nonlinear Diffusion in a Slab

In this case, the evolution of the field inside the beam is given by Eq. (14):

$$\frac{\partial}{\partial x}\left[\left(1 + \kappa b^2\right)\frac{\partial b}{\partial x}\right] = \frac{\partial b}{\partial \tau} \ . \tag{14}$$

Nonlinear diffusion equations of this kind are known to produce solutions with front-like behavior. Because typical cases of interest have $\kappa \gg 1$, we neglect the constant term relative to κb^2 in the diffusion coefficient. This is not quite right just at the front but the effect is the elimination of a small foot ($\sim 1/\sqrt{\kappa}$) right at its leading edge. The finite thickness of the jet means self-similar solutions do not exist for our problem. However, until the fronts from the two edges meet, neither edge can know about the other and the problem is the same as the semi-infinite one. Thus we use the self-similar form of the solution $b = (1 - x/\delta(\tau'))^{1/2}$ where $\delta(\tau)$ will be determined by imposing an integral constraint on the solution; the method of moments.[9] The resulting constraint equation is an ordinary differential equation in time for $\delta(\tau)$. It will be found to be proportional to $\tau^{1/2}$ which is consistent with self-similarity. Putting $\tau' = \kappa\tau$ and integrating Eq. (14) in x over $(0, \delta)$ we have

$$\frac{\partial \Phi}{\partial \tau'} = b^2 \, \partial b/\partial x \Big|_0^{\delta}, \quad \text{where} \quad \Phi \equiv \int_0^{\delta} b \, dx \; . \tag{33}$$

With $b(x, \tau') = b_0(1 - x/\delta(\tau'))^{1/2}$, this becomes

$$\frac{d\delta^2(\tau')}{d\tau'} = \frac{3b_0^2}{2} \; . \tag{34}$$

The boundary condition at $x = 0 \Rightarrow b_0 = 1$, at the solution is

$$\delta(\tau') \;=\; \sqrt{\frac{3}{2}}\,\tau^{1/2} \qquad \text{and} \tag{35}$$

$$b(x, \tau') \;=\; \left(1 - \sqrt{\frac{2}{3\tau'}}\,x\right)^{1/2} \qquad 0 < x < 1/2 \; . \tag{36}$$

This solution breaks down when the fronts meet in the center of the slab at $x = 1/2$. This occurs at a time $\tau_p' = 1/6$. After this time, the solution is no longer self-similar. We will again use the moment method to obtain an approximate solution in the case $\tau' > \tau_p$.

It is convenient in this case to shift the coordinate axis by half a unit and place the origin in the center of the slab. The solution is symmetric about the origin. It is also convenient to rescale so that the boundary conditions are again at $x = \pm 1$. To return to the original system, $x \to 2x - 1$ in the solution. We also shift the time origin by τ_p', and scale by a factor of four, to have, for $\overline{\tau} = 4\tau > 0$:

$$\frac{\partial b}{\partial \overline{\tau}} = \frac{\partial}{\partial x}\left(b^2 \frac{\partial b}{\partial x}\right) \; . \tag{37}$$

We look for a solution of the form

$$b = \left(\beta_0(\overline{\tau}) + \beta_1(\overline{\tau})x^2\right)^\alpha \ . \tag{38}$$

This is the simplest solution with the appropriate symmetry about $x = 0$, the required smoothness at $x = 0$ and which will let us impose the necessary physical constraints. One constraint is the moment integral. This will give a differential equation in time for either $\beta_0(\overline{\tau})$ or $\beta_1(\overline{\tau})$. The other is determined from the boundary condition:

$$b(x = \pm 1) = 1 \quad \text{which implies}$$

$$\beta_0(\overline{\tau}) + \beta_1(\overline{\tau}) = 1 \quad \text{for all } \overline{\tau} \geq 0 \ . \tag{39}$$

Furthermore, at $\overline{\tau} = 0$, $b(x = 0) = 0$. That is, just as the diffusion fronts meet, the field at the slab center is zero. This implies $\beta_1(0) = 1$. The second constraint is a condition on $\frac{\partial b}{\partial x}$ at the slab edges at $\overline{\tau} = 0$. This is essentially a constraint on the "flux" (actually edge current density) which does not instantly change when the diffusion fronts meet. This will fix the value of α as we now show. From Eq. (38)

$$\begin{aligned} \frac{\partial b}{\partial x} &= 2\alpha\beta_1 x\left(\beta_0(\overline{\tau}) + \beta_1(\overline{\tau})x^2\right)^{\alpha-1} \\ &= -2\alpha\beta_1(0) \quad \text{at} \quad \overline{\tau} = 0, \ x = -1 \ . \end{aligned} \tag{40}$$

From the similarity solution, written in the present coordinate system, for $-1 \leq x \leq 0$

$$b(x, \overline{\tau} = 0) = (-x)^{1/2} \tag{41}$$

$$\Rightarrow$$

$$\left.\frac{\partial b}{\partial x}\right|_{x=-1} = -1/2(-x)^{-1/2}\Big|_{x=-1} = -1/2 \ . \tag{42}$$

Therefore

$$-2\alpha\beta_1(0) = -1/2 \ ; \ \Rightarrow \alpha = 1/4 \ . \tag{43}$$

In fact, with $\alpha = 1/4$, the second derivatives at the edges also match. Thus we have

$$b(x,\overline{\tau}) = \left(1 - \beta_1(\overline{\tau})(1-x^2)\right)^{1/4} . \tag{44}$$

The moment integral gives

$$\frac{\partial}{\partial \overline{\tau}} \int_0^1 b\,dz = b^2 \frac{\partial b}{\partial z}\bigg|_0^1 = \frac{\beta_1(\overline{\tau})}{2} . \tag{45}$$

Now

$$\int_0^1 b\,dz = \int_0^1 \left(1 + \beta_1(\overline{\tau})(x^2-1)\right)^{1/4} dx . \tag{46}$$

This does not have a closed form expression. However, we know that for $\overline{\tau} > 0$, $\beta_1 < 1$ and also clearly for $x \in (0,1)$, $1 - x^2 < 1$ so we will approximate the integrand by the first two significant terms in its Taylor expansion. The differential equation which results is

$$-1/6\,\frac{d\beta_1}{d\overline{\tau}} - \frac{\beta_1}{10}\frac{d\beta_1}{d\overline{\tau}} - \frac{\beta_1}{2} = 0 . \tag{47}$$

This can be solved by quadratures and a transcendental equation for $\beta_1(\overline{\tau})$ results:

$$\beta_1 \exp\left(3/5(\beta_1 - 1)\right) = \exp(-3\overline{\tau}) . \tag{48}$$

To solve this, we again expand to $O(\beta_1)^2$ and find, on solving the resulting quadratic

$$\beta_1(\overline{\tau}) = 1/3\left\{-1 + [1 + 15\exp(-3\overline{\tau})]^{1/2}\right\} . \tag{49}$$

This solution has the appropriate limiting values:

$$\text{For} \qquad \overline{\tau} = 0\ , \quad \beta_1(0) = 1/3\left\{-1 + [16]^{1/2}\right\} = 1$$

and as $\overline{\tau} \to \infty$ $\beta_1 \to 0$. The full solution is then, in the original system of units with $0 < x < 1$:

For $0 < \tau < 1/6\kappa$

$$b(x,\tau) \quad = \quad \left(1 - \left(\frac{2}{3\kappa\tau}\right)^{1/2} x\right)^{1/2} \qquad 0 < x < 1/2 \tag{50a}$$

$$= \left(1 - \left(\frac{2}{3\kappa\tau}\right)^{1/2}(1-x)\right)^{1/2} \quad 1/2 \leq x \leq 1 \tag{50b}$$

$$1/6\kappa < \tau$$

$$b(x,\tau) = \left\{1 - \frac{4}{3}\left[-1 + \left(1 + 15e^{-12\kappa(\tau - 1/6\kappa)}\right)^{1/2}\right]x(1-x)\right\}^{1/4} . \tag{50c}$$

These solutions are sketched in Fig. 1 as a function of x for several values of τ'. In the next section, we discuss numerical estimates of diffusion times from the models considered. We will also compare these with a solution of the slab model in the linear limit.[10]

V. Numerical Results

Here we will use our results of the previous sections to obtain some estimates of the penetration times of the field. First we will make some relative estimates simply to have some measure of the effect of the different physics we have been considering. We will then use parameter values appropriate for the laboratory experiments of Ref. 4 and space experiment of Ref. 5 to obtain some absolute diffusion times.

In the following experiments, the penetration of the field is determined by the decay of the diamagnetic signal and a concurrent rise of the polarization field as measured on floating Langmuir probes.[4] The experiments in space generally observe the field within the jet by means of satellite borne instruments.[1,5] As a basis of comparison, both with experiment and the different theoretical solutions, we adopt the central field 90% return time (the time at which $b(x = 1/2) = 0.9$) as a measure of the diffusion time. There is no particular justification for this. Given the experimental uncertainties (finite beam risetimes, approximate geometries, etc.) and the relative insensitivity of the diffusion times to the central field fractional value if it is sufficiently near to one, this seems to be as reasonable measure as any. From Eq. (2) of Ref. 10, for example, for the slab in the linear approximation and Eq. (49)

in the nonlinear case, we find

$$\tau'_{.90} = 0.26 \qquad \text{(linear)}$$

$$\tau'_{.90} = 0.30 \qquad \text{(nonlinear)} .$$

The closeness of these values is striking considering the difference in the diffusion profiles at early times. (Compare Fig. 1 with Fig. 8 of Ref. 10, for example. This also shows that the evolution of the linear problem at late times and the nonlinear solution, Eq. (50), at late times have very similar profiles). In the cylindrical case, Eqs. (28) and (29) give

$$\tau'_{.90} = 0.48 .$$

The diffusion in this case is significantly slower, by about a factor of two, than that in the linear case. (This is a consequence of the planar geometry in the large ρ limit. If a dipolar field were composed at the edge of a cylindrical plasma jet, the diffusion times would be much closer to the slab values.) In view of the slab results, it seems reasonable to expect that, even in the cylindrical case, the nonlinear diffusion times will not differ substantially from the linear case. Numerical solution of the nonlinear diffusion have proved to be frustrating and difficult in that parabolic solvers typically are based on an iterative application of elliptic integrators. The front-like solutions of the nonlinear diffusion are a problem for these methods.

We now return to the dimensional form of the diffusion times by reintroducing the scaling $\tau' \rightarrow \kappa t/\tau_D$ and evaluating these for the UC Irvine experiments,[3,4] and the AMPTE artificial comet experiment. The diffusion model is not applicable to the Porcupine experiments in that the Pedersen current does not dominate the displacement current in that case, even with an anomalous collision frequency.

In the experiments at the University of California-Irvine, a neutralized ion beam of several hundred keV energy and density $n \simeq 3 \times 10^{11}\text{cm}^{-3}$ was injected across a magnetic field whose value was varied from several tens to several hundreds of Gauss. The details of these experiments have been reported elsewhere (Refs. 4 and 11-13). For the plasma parameters there, the classical collision frequency $\nu_{ei} \approx 10^4/\,\mathrm{sec}$ and the displacement currents are not negligible compared to the conduction currents in the model. If diffusive processes are to be responsible, an anomalous transport must be taking place and evidence of electrostatic turbulent fluctuations has been observed in the experiments.[11,13] The fluctuations are higher frequency than would be expected of the lower hybrid; they are more in the ion acoustic range. The role of these turbulent fluctuations has not yet been investigated in detail experimentally. However, turbulence of the ion acoustic type could be expected to produce an anomalous collision frequency $\nu_{ei}^{*} \sim \omega_{pi}$ which is, for the parameters of the UC-I experiments, somewhat larger than the anomalous collision frequency of Ref. 6. If we use this value of ν_{ei}^{*}, with $B_0 = 200\,\mathrm{G}$, $a = 10\,\mathrm{cm}$, we find

$$
\begin{aligned}
\tau_D &\simeq 1.5 \times 10^{-7}\,\mathrm{sec} \\
\kappa &= 24 \\
t_{0.90} &\simeq 3\mathrm{ns}\ .
\end{aligned}
$$

This diffusion is very fast but is within the condition imposed by the neglect of the z-component of the displacement current. This result is qualitatively consistent with the experimental observation in which fast diffusion was seen relative to the $0.5\mu\,\mathrm{sec}$ duration of the beam.

We now consider the AMPTE artificial comet experiment.[2] Here a rapid penetration of the interplanetary magnetic field was observed as well. If a diffusive process is to be responsible, some turbulent enhancement of electron-ion collisions is also required here too. If we again look to an ion-acoustic enhanced resistivity reasonable agreement with the experimental return time is obtained. The parameters[15] are $n = 1.2 \times 10^4\,\mathrm{cm}^{-3}$, $B_0 = 1.3 \times 10^{-3}\,\mathrm{G}$,

$a = 80\,\mathrm{km}$ and a plasma composed of $B_a^+(A = 137)$. If we take $\nu_{ei}^* \sim \omega_{pi}$, then

$$\kappa \simeq 3.5$$

$$t_{.90} \simeq 30 \text{ sec },$$

compared to an experimental field return time of 17 sec. The value of κ only marginally satisfies our requirement $\kappa > 1$. But, as the anomalous collision frequency is only an order of magnitude estimate in any case, the values derived there must be considered in the same way. Nevertheless, it would appear on the face of it that diffusive transport of the field would be a plausible mechanism for the observed rapid field penetration. Unfortunately the real puzzle (and problem) for the model lies in the observation of fluctuations in the ion-acoustic range with amplitudes too small to be expected to lead to anomalous resistivity. This has led to the suggestion that hydromagnetic mechanisms may be responsible.[15] However, it seems that these should be subject to interchange instabilities which have been found to be able to grow even in the case of unmagnetized ions. Indeed, an interchange instability of this general kind is now believed to be responsible for the fast field penetration in numerical simulations[16] done by our colleagues of high-β plasma beams crossing a magnetic field in vacuum. In these simulations, the plasma temperature was sufficiently high (ion masses were artificially low, a standard technique in simulations) that ion acoustic instability was not expected and no evidence of it was found.

VI. Summary and Conclusions

In this work, we have considered the diffusive transport of magnetic fields in the limit in which the electrons are strongly magnetized and applied the results to the problem of field penetration into high-β plasma beams. Because the magnetization of electrons makes the diffusion coefficient nonlinear, the solution of the problem is non-trivial and numerical methods can have difficulties even in simple geometry.[17] We have constructed analytic solutions under different approximations and have shown that while there are differences in the details of the solutions, the gross measures obtained are quite similar. In particular, measures of the diffusion times are the same to within any reasonably expected experimental determination and are about 50% of the values simple scaling arguments would give. We have applied these to two experimental cases of plasma beams in a transverse field. The diffusion times in the laboratory experiment qualitatively agree with the observations which essentially establish an upper bound to the field penetration time. There is reasonably good quantitative agreement with the space experiment (AMPTE) penetration time. Both require the existence of an anomalous collision frequency. Given the strong diamagnetic currents which must flow to shield the field, it is not unreasonable to expect such. This is consistent with the laboratory observations but not the experiment in space. The fluctuations in the laboratory experiments need to be investigated in more detail in order to come to quantitative conclusions. These could be supported with some numerical simulations as well. The space experiment remains a puzzle. We anticipate that some simulations extending the work of Ref. 16 will provide some insight into this problem. In general, the microscopic dynamics in the boundary layer between the plasma beam and field is likely to be where the action is. The physics of this region is complex and needs considerably more work. Interestingly, this was an area of interest early in the magnetic confinement fusion program and is again becoming the focus of increased attention.

Acknowledgment

We wish to thank Dr. Frank Wessel for many discussions of his experimental results. One of us (B.N.) would like to thank Dr. W.R. Shanahan for several helpful discussions and the results of his unpublished research.

This work was supported by the U.S. Department of Energy contract #DE-FG05-80ET-53088, and the Air Force of Scientific Research under grant 88-0300.

References

1. G. Haerendel and R.Z. Sagdeev, Adv. Space Res. **1**, 29 (1981).

2. A. Valenzuela *et al.*, Nature **320**, 700 (1986).

3. F.J. Wessel, *et al.*, Proc. SPIE-Int. Soc. Opt. Eng. V828, Paper 38 (1988).

4. R. Hong, F.J. Wessel, J. Song, A. Fisher and N. Rostoker, J. Appl. Phys. **64**, 73 (1988).

5. B. Hausler, *et al.*, J. Geophys. Res. **91**, 287 (1986).

6. E.V. Mishin, R.A. Treumann and V.Ya. Kapitanov, J. Geophys. Res. **91**, 10183 (1986).

7. S.I. Braginskii, in *Reviews of Plasma Physics*, edited by M.A. Leontovich (Consultants Bureau, NY, 1985) Vol. 1, Chap. 3, pp. 205-311.

8. Bruno Rossi and Stanislaw Olbert, "Introduction to the Physics of Space" (McGraw-Hill, NY, 1970), Chap. 13, p. 394.

9. W.F. Ames, "Nonlinear Partial Differential Equations in Engineering" (Academic Press, NY, 1965) pp. 249-256.

10. H.S. Carslaw and J.C. Jaeger, *Conduction of Heat in Solids* (Oxford, London, 1947) Chap. 3, p. 83.

11. J. Song, F.J. Wessel, A. Fisher and N. Rostoker, Conference Record, 1988 IEEE Int. Conf. on Plasma Science, Seattle Wash. 1988, p. 107.

12. F.J. Wessel, *et al.*, submitted to Phys. Fluids.

13. F.J. Wessel, A. Fisher, N. Rostoker, and J. Song, Proc. 7th Int. Conf. on High-Power Particle Beams, Karlsruhe FRG, 1988, to appear.

14. A.A. Galeev and R.Z. Sagdeev, in Handbook of Plasma Physics – Basic Plasma Physics edited by A.A. Galeev and R.N. Sudan (North Holland, Amsterdam, 1984) Vol. 2, Chap. 6.1, pp. 271-303.

15. G. Haerendel, G. Paschmann, W. Baumjohann, and C.W. Carlson, Nature **320**, 720 (1986).

16. T. Tajima, J. Koga, and T. Fujinami, EOS **68**, 1400 (1987).

17. Robert D. Richtmyer and K.W. Morton, *Difference Methods for Initial-Value Problems* 2nd ed. (Interscience, NY, 1967) pp. 201-206.

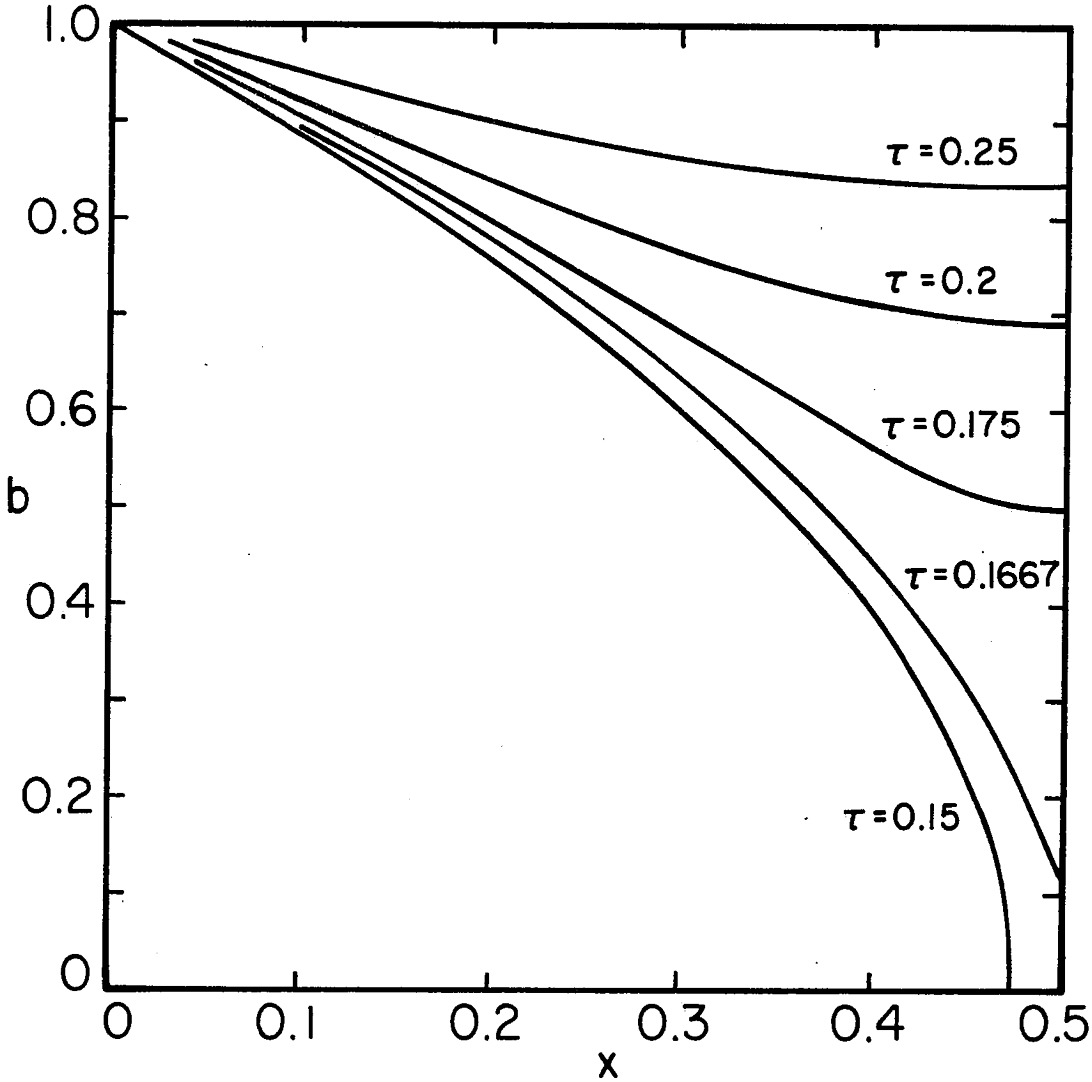

Fig. 1