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Enhancement of the Ion Neoclassical Heat Conduction Due to Eleectron and Superthermal Ion Energy-scattering Collisions

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Abstract

This paper considers the extra components of neoclassical ion heat conduction driven by energy scattering collisions with electrons and superthermal ions. These components are important in auxiliary heated discharges and can be *the* most important if the plasma ion distribution has an enhanced non-Maxwellian tail.

I. Introduction

In tokamak plasmas, due to pitch-angle scattering and ion orbits, passing ions at radius r with $v_{||} < 0$ are strongly coupled to passing ions of the same energy and $v_{||} > 0$ at $r+2\delta r$ with $\delta r \sim (r/R)^{1/2}\rho_{i\theta}$. The tendency for these two groups of ions to equalize their rates of heating leads to extra components of ion heat conduction (q_i) driven by $C_E(f_{i1})$ and $\partial C_E(f_{i0})/\partial r$, where C_E is the energy-scattering part of the collision operator.

Starting from the ion drift-kinetic equation, multiplying by $B/B_\theta v_{||}$ and taking the θ-average, one obtains the solubility condition. Integrating this equation with respect to μ

[a)]This paper is part of a joint paper (IAEA-CN-50/D-2-3-3) to be presented at the 12th IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research from 0 to the value μ_B at the boundary of the trapped region, the parts of the acceleration flow in pitch angle (J_α) second and third order in $\rho_{i\theta}/L$ are given by

$$\begin{aligned} F_{2p} &\equiv \overline{(J_\alpha)_2 \sin\alpha} \simeq \sigma v \left[(C_{iiE})_2 + C_{ieE}(f_0, f_{e0}) + C_{ibE}(f_0, f_{b0}) - \frac{\partial f_0}{\partial t} \right] \\ F_{3p} &\equiv \overline{(J_\alpha)_3 \sin\alpha} \simeq \sigma v \left[(C_{iiE})_3 + C_{ieE}(f_1, f_{e0}) + C_{ibE}(f_1, f_{b0}) - \frac{\partial f_1}{\partial t} \right] \end{aligned} \quad (1)$$

$Z_{\rm eff}$ has been assumed large ($\gtrsim 3$) so that f_1 is approximately constant in the passing ion region. $\alpha = \cos^{-1}(v_\|/v)$, $\sigma = v_\|/|v_\||$, the subscript p refers to the passing particle side of the boundary and b refers to the superthermal ions such as slowing down beam ions or α-particles. Neglecting the weak Pfirsh-Schluter component, there is no radial diffusion term in Eq. (1), since only passing particles are involved. A contribution from $C_{ie}(f_0, f_{e1})$ has been assumed small [1] and f_b is assumed to be isotropic in pitch angle so that f_{b1} is zero.

Just on the trapped particle side of the boundary the average of F_2 for the whole trapped ion orbit passing through this point, to third order accuracy, is

$$(F_2 + F_3)_{\rm Tr} = F_2 - \frac{v_\| h}{\Omega_\theta} F_2', \quad (2)$$

where $h \equiv 1 + (r/R)\cos\theta$ and the prime denotes the radial gradient. The orbit average of the F_3 terms of Eq. (1) is zero.

The fourth order part of the ion heat conduction is given by

$$q_{ir} = -\int \frac{d\theta}{2\pi} \int \frac{m_i v^2}{2} \frac{v_\| h}{\Omega_\theta} C_3 d^3 v, \quad (3)$$

and the contribution from the discontinuity in F_3 at the trapped ion boundary is given by the pitch angle scattering collision operator, $-v_\| \partial F_3 / B\partial\mu$. Using Eqs. (1) and (2), the μ-integral in Eq. (3) is given by

$$\sum_\sigma \int \frac{v_\|^2}{|v_\||} \frac{\partial F_3}{B\partial\mu} B d\mu \simeq -2 v_{\|B} \left(F_{3p} + \frac{v_{\|B}}{\Omega_\theta} F_2' \right),$$

with $v_{\|B} = v(2r/R)^{1/2}\cos(\theta/2)$ and

$$(q_{ir})_{\rm boundary} = \int d\theta \int m_i v^2 v_{\|B} \left(F_{3p} + \frac{v_{\|B}}{\Omega_\theta} F_2' \right) v\, dv. \quad (4)$$

II. The Contribution from Electron Collisions

Inserting the energy-scattering part of the C_{ei} operator into Eq. (1), Eqs. (1), (2) and (4) yield

$$(q_i)_{ie} = -26.7 \left(\frac{r}{R}\right) n_i \nu_e \frac{m_e}{m_i} \rho_{i\theta}^2 T_i' \left(1 - 0.48\frac{T_e}{T_i}\right) - \frac{28.6r}{R\Omega_\theta^2} \frac{\partial}{\partial r} \left[\frac{T_i}{m_i} n_i \nu_e \frac{m_e}{m_i} (T_e - T_i)\right], \qquad (5)$$

where ν_e is the standard electron collision frequency $16\pi^{1/2} n_e e^4 \ln \Lambda / 3m_e^2 v_{Te}^3$. The first term, coming from $C_{ie}(f_{i1})$, has already been published in Ref. [1], except that the $0.48T_e/T_i$ part was neglected there. The second term coming from $C_{ie}'(f_{i0})$ is a new result. The numerical coefficients are large in both terms because the ie-energy-scattering collision operator is strongly weighted towards high ion-energies. The two collision terms will also drive ion diffusion, and the increment in the radial electric field for ambipolarity will cause a small increase in $(q_i)_{ie}$ [1], which has been omitted here for simplicity. If T_i/T_e is sufficiently large compared with unity, $(q_i)_{ie}$ can be the dominant component in the ion heat conduction. Like the contribution to q_i due to ii-energy-scattering collisions [2], both terms in Eq. (1) are proportional to (r/R) and not $(r/R)^{1/2}$.

The heat conduction involved in Eq. (5) has the unusual characteristic that energy is taken from, and given back to, the electrons at each ion diffusion step length. Part of the energy involved is contained in the standard formula for the energy transfer from electrons to ions—the energy moment of $C_{ie}(f_{i0}, f_{e0})$. But for the electrons within approximately an ion poloidal Larmor radius of the magnetic axis, there is an extra electron energy loss coming from $C_{ie}(f_{i1}, f_{e0})$. The energy moment of this term is not zero since there are no trapped ions with $v_\parallel > 0$ and $f_{i1}(v_\parallel > 0)$ is zero. This energy is given back to the electrons for the last time in the ion loss cone region near the wall (or separatrix) where there are no trapped ions with $v_\parallel < 0$.

III. The Contribution From Superthermal Ion Collisions

Taking as an example slowing down beam ions, assuming that f_b is isotropic in pitch angle with all beam ion velocities satisfying $v_b \gg v_{Ti}$, the C_{ib} collision operator is

$$C_{ib}(f_{i0}) = \frac{4\pi n_b e^4 \ln\Lambda}{3m_i^2} \left\langle \frac{1}{v_b} \right\rangle \frac{1}{v^2} \frac{\partial}{\partial v} \left(v^2 \frac{\partial f_{i0}}{\partial v} \right), \tag{6}$$

where n_b is the beam ion density and $\langle 1/v_b \rangle$ is the average value of v_b^{-1} for these ions, and the rate at which the plasma ions are being heated per unit volume is

$$p_{ib} = \frac{4\pi n_i n_b e^4 \ln\Lambda \left\langle v_b^{-1} \right\rangle}{m_i}. \tag{7}$$

Inserting Eq. (6) into Eqs. (1), from Eqs. (1), (2), (4) and (7), the ion heat conduction caused by C_{ib} is

$$(q_i)_{ib} = 4.3 \left(\frac{r}{R} \right) p_{ib} \rho_{i\theta}^2 \frac{T_i'}{T_i} - \frac{9.5r}{R\Omega_\theta^2} \frac{\partial}{\partial r} \left(\frac{T_i}{m_i} p_{ib} \right). \tag{8}$$

Since there is an approximate cancellation of the T_i' terms in Eq. (8), the dominant term is proportional to P_{ib}'. This term will help explain the deterioration in tokamak energy containment with centralized auxiliary heating.

IV. The Effect of Non-Maxwellian f_0

If the ion distribution function has an enhanced non-Maxwellian tail, all components of the ion heat conduction are increased, but the increase is particularly large for the C_{ie} and C_{ib} components because of their strong weighting to high ion energies. Such non-Maxwellian distributions have been observed with both ohmically heated tokamak discharges and with neutral beam heated discharges [3]. In the latter case an enhanced tail is ensured, not only because of the presence of the slowing down beam ions, but also because of the effect on the background ions due to C_{ie} and C_{ib}. These two operators contain an extra v^3 compared with the C_{ii} term and hence must dominate at sufficiently large ion energy. Equating $(C_{ie}+C_{ib})(f_0)$ to zero for the distribution tail gives $\partial f_0/v\partial v = -m_i/T_H$ or $f_0 \sim \exp(-m_i v^2/2T_H)$ where

$$T_H = T_e + m_i p_{ib}/3n_i\nu_e m_e. \tag{9}$$

If the radial diffusion of the tail is dominated by ii and iz pitch angle scattering collisions, the exponential character of the tail is preserved as these ions diffuse outwards and T_H remains constant [3]. This causes the non-Maxwellian tail to become increasingly more important with radius since the temperature of the low energy part of f_0 (which is the part diffusing inwards) is decreasing with radius.

In the two component ion distributions observed on PDX with active charge exchange [4], the high energy part satisfies Eq. (9) at $r = 12$cm. It is not satisfied at $r = 26$cm; the values of the parameters indicate that the tail ions are diffusing too rapidly outwards in radius to reach equilibrium with the electron and beam-ion collisions, let alone thermalize with the inward diffusing low energy ions. In the TFTR supershots [5] the value quoted for the central ion temperatures satisfies Eq. (9). The ion heat conduction at $r = 26$cm in the PDX measurements due to ii and iz pitch angle scattering collision is four times the neoclassical heat conduction predicted using $\partial T_i/\partial r$ from the cold part of f_0. In addition, an approximately equal contribution is predicted due to C_{ie} and C_{ib}.

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