DOE/ET-53088-319 IFSR **#319**

Initial Evolution of Nonlinear Magnetic Islands in High Temperature Plasmas

M. Kotschenreuther
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

June 1988

Initial Evolution of Nonlinear Magnetic Islands in High Temperature Plasmas

M. Kotschenreuther
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

Abstract

The evolution of nonlinear magnetic islands is computed in the kinetic collisionality regime called the semicollisional regime, which is appropriate to present fusion confinement devices. Realistic effects are included, such as the presence of small external field errors, radial electric fields, and ω_*. When present simultaneously, these effects can greatly change the stability of small amplitude nonlinear islands. Islands with $\Delta' > 0$ can sometimes be prevented from growing to macroscopic size; it is also possible to produce moderate mode-number nonlinear instabilities in the plasma edge. Furthermore, island growth can be prevented by application of external fields with suitably chosen amplitude and frequency.

I. Introduction

The dynamics of magnetic islands in the nonlinear stage were first calculated in the classic paper by Rutherford.[1] Resonant magnetic perturbations become strongly nonlinear when their associated island width significantly exceeds the very thin linear tearing layer width.[1] Other investigators showed that kinetic effects[2] and diamagnetic drifts[3,4,5] do not modify the dynamics in the strongly nonlinear phase (even though they greatly change the linear phase).[6,7] Typical resonant magnetic perturbations from external sources (e.g., field errors) are large enough to produce nonlinear islands. Here, we examine islands with realistic levels of external perturbations in a realistic kinetic collisionality regime, the semicollisional regime,[6] and find that an external resonant perturbation in a kinetic regime can radically change nonlinear stability.

New driving mechanisms arise which can dominate the previously known nonlinear island driving forces from Δ', MHD curvature[8] and bootstrap currents[9,10] for small islands. The new effects can lead to either growth or damping depending on collisionality regime, the radial electric field, diamagnetic velocity, size of external resonant perturbation, magnitude of particle diffusion coefficient and initial conditions for the magnetic island. The nonlinear analytic calculation here is valid for island widths up to a poloidal gyroradius (depending on collisionality regime). The effects uncovered can determine whether islands grow from small amplitude to macroscopic size when $\Delta' > 0$ and in the presence of realistic field errors. It may be possible to prevent large island growth by using external coils to modify the external resonant fields. Also, enhanced transport near the plasma edge may arise from moderate poloidal mode-number (m) islands driven unstable by pressure and electric potential gradients together with moderate m field errors.

It is appropriate to use periodic slab geometry to describe the narrow island region around the rational surface at minor radius r_s, with x and y playing the role of minor radius and poloidal angle. The magnetic field is $B = B_0\hat{z} - \hat{z} \times \nabla A_z$, with A_z the vector potential.

We use fluid equations to describe the electrons, specifically the continuity equation, $\partial n/\partial t + \nabla \cdot (\mathrm{v}_E n) = \nabla_{||} j_{||}/e + D'\nabla_\perp^2 n$, and Ohm's law $\eta J = E_{||} + \nabla_{||} p/ne$, where n, j, p, D' and e are v_E are the electron density, current, pressure, diffusion coefficient and unsigned charge, $E_{||}$ and η are the parallel electric field and resistivity, and $\mathbf{v}_E$ is the $\mathbf{E} \times \mathbf{B}$ velocity. The electron temperature T_e is taken as constant in space for simplicity. As in the work of Hassam and Scott,[5] these are simplified and normalized as is appropriate for semicollisional drift-tearing modes localized near a rational surface with radius r_s. Define n_0 and dn_0/dx as the density and gradient of the original equilibrium at the rational surface. Define t as time normalized to $\omega_*^{-1} = eBn_0/cTk_y(dn_0/dr)$, where $k_y = m/r_s$, define $x = (r - r_s)/\Delta_D$, where $\Delta_D = (\omega_* n_0 e^2 \eta L_s^2/k_y^2 T_e)^{1/2}$ is the linear semicollisional tearing layer width, ξ is poloidal angle times m, $N = (n - n_0)/(\Delta_D dn_0/dx)$, $\phi/[T_e\Delta_D(dn_0/dr)/en_0]$, and $J = 4\pi L_s/cB\hat{\beta}$, where $\hat{\beta} = (4\pi n_0 T_e/B_0^2)[L_s(dn_0/dx)/n_0]^2$. We now consider the ion response to obtain ϕ.

The electrostatic potential can be split as $\phi = \phi_e + \delta\phi$, where ϕ_e is the global equilibrium potential which is present even without the island, and $\delta\phi$ is the perturbation caused by the island. Similarly, for the ion density, $n_i = n_{i0} + \delta n_i$. The ion density response is adiabatic, $\delta n/n_{i0} = e\delta\phi/T_i$, when the radial perturbation scale length of the tearing region is smaller than an ion gyroradius.[6] When toroidal neoclassical effects are included[10,11] for low frequency perturbations, the ion response is adiabatic for perturbations wider than this; depending on collisionality and other parameters, it is adiabatic up to a width of numerous gyroradii. Because of the neoclassical effects, the ion response is linear, and also the equilibrium electric field is unaffected, for islands widths up to several gyroradii. We limit our calculation to islands in this range, and use linear adiabatic ion response.

For the thin region near the rational surface, we may approximate the global equilibrium quantities by $\phi_0(x) = \phi_0(0) + x(d\phi_0(0)/dr)$ and $n_0(x) = n_0(0) + x(dn_0(0)/dr)$. Thus we have

$\phi = \phi_0(0) + T_i\delta n/N_q + x(d\phi_0/dr) = (\phi_0(0) - T_i n_0(0)/q_i) + T_i\delta n/n_0 q_i + xd(\phi_0 - T_i N'/q_i)/dr$.

Inserting this into Ohm's law and the continuity equation, we find

$$J = -\frac{\partial\psi}{\partial t} + (1+k)\nabla_{\|}N - (f+k)\frac{d\psi}{d\xi} \tag{1}$$

$$\frac{\partial N}{\partial t} + f\frac{dN}{d\xi} - \nabla_{\|}J = D\frac{d^2N}{dx^2} \tag{2}$$

$$\frac{\partial^2\psi}{\partial x^2} = -\hat{\beta}J\ , \tag{3}$$

where

$$\nabla_{\|} \equiv [(\partial\psi/\partial\xi)(\partial/\partial x) - (\partial\psi/\partial x)(\partial/\partial\xi)]\ ,$$

$DdD'/\omega_*\Delta_D^2$, $k = T_i/T_e$ and $f = en_0(d\phi_0/dr)/T_e(dn_0/dr)$. With no island, $\psi = -x^2/2$, $dN/dx = 1$, and $\omega_* = 1$. The $\mathbf{E}\times\mathbf{B}$ nonlinearity in the continuity equation vanishes for adiabatic response, leaving only the part from the equilibrium field. The terms proportional to f tend to induce rotation of ψ and N in ξ.

We solve these equations analytically with four approximations: 1) As in Rutherford's analysis, we use the constant ψ approximation with a single dominant harmonic; 2) We take the island width to be much larger than the linear layer width, so it is strongly nonlinear; 3) We take the growth rate to be much less than ω_*, (valid for low collisionality regimes); and 4) In order to analytically solve for the density evolution equation, we will require that there is strong density diffusion, $D/\Delta x_I^2 > (d\omega/dt)/\omega_*$. For island widths within the domain of this calculation, this is well-satisfied for empirically estimated values of D, and is true in cases of interest for much smaller neoclassical values of D.[13]

The first approximation above implies

$$\psi = -\frac{x^2}{x} + A(t)\cos[\xi + \theta(t)]\ . \tag{4}$$

Fourier decomposing Ampere's law and integrating in space, and defining $\psi_1 = \oint d\xi e^{i\xi}\psi$, we have the following

$$\frac{\partial\psi_1}{\partial x_+} - \frac{\partial\psi_1}{\partial x_-} = -\hat{\beta}\int_{-\infty}^{+\infty} dx \oint d\xi e^{i\xi} J \; . \tag{5}$$

An expression for $d\psi_1/dx_+ - \partial\psi_1/\partial x_-$ is found by matching to the MHD kink equation[12] in the region exterior to the island. The homogeneous boundary condition which is usually chosen for the kink equation at the plasma edge must be replaced by inhomogeneous ones to include the effect of fixed current sources outside the plasma, such as coil errors causing resonant fields. It is easily shown that in the "constant ψ" approximation

$$\frac{\partial\psi_1}{\partial x_+} - \frac{\partial\psi_1}{\partial x_-} = \Delta'\psi(r_s) + \psi_e'(r_s) \tag{6}$$

where $\psi_e' = \partial\psi_e/\partial x$, and ψ_e is the particular solution of the kink equation with appropriate inhomogeneous boundary conditions which has $\psi = 0$ at the rational surface. Note that $\Delta' = \mathbf{\Delta}'\Delta_D \sim 10^{-1} - 10^{-2}$, where $\mathbf{\Delta}'$ is the unnormalized slope change with units of inverse length. It is beyond the scope of this paper to solve the kink equation for ψ_e'; we merely note that for weak current gradients, in these normalized units $\psi_e' \sim g\psi_{e0}$, $g \equiv k_y\Delta_D$, where ψ_{e0} is the normalized ψ vacuum field error.

We define $\psi_{\text{ext}}' = gA_{\text{ext}}\cos(\xi)$. Using Eq. (4)-(6),

$$\Delta' A + gA_{\text{ext}}\cos\theta = -\hat{\beta}\int dx \oint \frac{d\xi}{\pi}\cos(\theta+\xi)J \tag{7}$$

$$gA_{\text{ext}}\sin\theta = -\hat{\beta}\int dx \oint \frac{d\xi}{\pi}\sin(\theta+\xi)J \; . \tag{8}$$

We will obtain J in terms of ψ and $\partial\psi/\partial t$ (i.e., $\partial A/\partial t$ and $\partial\theta/\partial t$) by solving Eq. (1)-(2), and insert the results into Eq. (7)-(8) to obtain evolution equations for A and θ.

Following Rutherford,[1] we change coordinates from x, ξ to ψ, ξ. Then

$$\nabla_{||}J = -\left(\frac{\partial J}{\partial\xi}\right)_\psi \left(\frac{\partial\psi}{\partial x}\right)_\xi \tag{9}$$

and $(\partial\psi/\partial x)_\xi = -x(\psi,\xi)$. Define a flux surface average

$$\langle Q(\psi,\xi)\rangle = \oint d\xi[Q/(\partial\psi/\partial x)_\xi] / \oint d\xi[Q/(\partial\psi/\partial x)_\xi] \; .$$

We now derive some exact consequences of Eq. (1)-(2). Using Eq. (9), and integrating Eq. (2) in ξ,

$$J = J_T + J_D + K(\psi) \tag{10}$$

where $J_T = \int d\xi(\partial N/\partial t - f\partial N/\partial\xi)/(\partial\psi/\partial x)_\xi$, $J_D = -\int d\xi\ D'(d^2N/dx^2)/(\partial\psi/\partial x)_\xi$, and $K(\psi)$ is the constant of integration. Other useful exact results follow from flux surface averaging Eq. (1)-(2),

$$\langle J \rangle = -\langle \partial\psi/\partial t \rangle \tag{11}$$

$$\langle \partial N/\partial t \rangle = D\left\langle d^2N/dx^2 \right\rangle . \tag{12}$$

With negligible diffusion, and negligible J_T and A_{ext}, Eq. (11) would exactly give Rutherford's result[1] for island evolution in the nonlinear regime. This was obtained previously by Drake and Lee:[2] semicollisional kinetic effects alone (which greatly modify the linear theory) do not modify nonlinear behavior of islands much larger than the linear layer width. Modified behavior results from J_T and J_D; to obtain these we now compute N.

Eliminating J from Eq. (1)-(2),

$$\nabla_\| \left[-\frac{\partial\psi}{\partial t} + (1+k)\nabla_\| N - (f+k)\frac{\partial\psi}{\partial\xi}\right] = \frac{\partial N}{\partial t} - f\frac{\partial N}{\partial\xi} - D'\frac{d^2N}{dx^2} . \tag{13}$$

We take time derivatives of order ω_* and $\mathbf{E}\times\mathbf{B}$ flows of order the diamagnetic velocity (i.e., $\partial/\partial t \sim 1$ and $f \sim 1$). Recall from Eq. (9) that $\nabla_\| \sim x$. For $D < 1$, the linear tearing layer width $\Delta x_L \sim 1$ in these units. It is simple to show that for $D > 1$, $\Delta x_L \sim D^{1/4}$. Consider a magnetic island with width Δx_I. In the island region, $\nabla_\| \sim \Delta x_I$, $A \sim \Delta x_I^2, \partial\psi/\partial t \sim \Delta x_I^2$ and $\partial\psi/\partial\xi \sim \Delta x_I^2$.

Now consider the nonlinear limit where $\Delta x_I \gg \Delta x_L$; the right-hand side of Eq. (13) is much smaller than the left, so to lowest order in Δx_I,

$$\nabla_\| \left[-\frac{\partial\psi}{\partial t} - (f+k)\frac{\partial\psi}{\partial\xi} + (1+k)\nabla_\| N\right] = 0 . \tag{14}$$

Note, however, that $\nabla_{\|}$ vanishes as the island x-point is approached. The left-hand side of Eq. (13) does not dominate there, and Eq. (14) is not valid in a layer around the separatrix. For $\Delta x_I \gg \Delta x_L$ this layer is thin, and we find it does not effect our results below.

Using Eq. (4), we have $\partial\psi/\partial t = (\partial A/\partial t)\cos(\xi+\theta) - (\partial\theta/\partial t)A\sin(\xi+\theta)$. For $\gamma \ll \omega_*$, we may neglect $\partial A/\partial t$ here. Then Eq. (14) implies

$$N = \frac{(\partial\theta/\partial t + f + k)}{1+k}x + G(\psi) \tag{15}$$

for an arbitrary function $G(\psi)$. Inserting Eq. (15) into Eq. (12), and taking $\gamma \ll \omega_*$ and $\dot{\omega}/\omega_* \ll D/\Delta x_I^2$, we find $\langle d^2G(\psi)/dx^2\rangle = 0$. Far from the island the density gradient dN/dx is unaffected by the island, so it must approach the equilibrium value. Together with $\langle d^2G/dx^2\rangle = 0$, this implies[8] that in the region outside the separatrix, $\psi < |A|$,

$$\frac{dG}{d\psi} = -\frac{1-(\partial\theta/\partial t) - f}{(1+k)\oint\frac{d\xi}{2\pi}x(\psi,\xi)} \,. \tag{16}$$

Because of the topology change inside the separatrix, and the physically necessary requirement that N be non-singular there, $\langle d^2G/dx^2\rangle = 0$ inside the island implies

$$\frac{dG}{d\psi} = 0 \,. \tag{17}$$

Recall Eq. (14) is not valid in a thin layer around the separatrix; the discontinuity between Eq. (16) and Eq. (17) is resolved in this layer. Note that $dN/d\psi$ is finite there; thus we find that a thin layer region will only make a small contribution to Eq. (7)-(8), and we may neglect it for $\Delta x_I \gg \Delta x_L$. Also note that Eq. (17) implies that the density is flattened inside a large stationary island with $f = 0$, as is physically necessary.

We now consider Eq. (8). It can be shown that J_T and $k(\psi)$ do not contribute to Eq. (8) by symmetry in ξ. Change coordinates $\int dx d\xi \to -\int d\psi d\xi/x$, use $-\sin(\xi-\theta)/x = (dx/d\xi)_\psi/A$ integrate by parts in ψ, use $\partial/\partial x = -x(\partial/\partial\psi)$, integrate by parts in ξ, and change variables from ξ to $w = -\psi/A$ to obtain

$$\int dx \oint \frac{d\xi}{\pi}\sin(\xi+\theta)J_D = -\frac{DI_0}{(1+k)\sqrt{A}}\left(1-\frac{\partial\theta}{\partial t} - f\right) \tag{18}$$

where $I_0 \simeq .12$ is the limit of $2\sqrt{2}\left[-2\sqrt{w} + \int_1^w dw' \left(2\pi / \oint d\xi \sqrt{w' + \cos\xi}\right)\right]$ as w approaches infinity.

We now compute the integral in Eq. (7). The current J_D does not contribute by its symmetry in ξ. We proceed to compute J_T. Using $\partial N/\partial t = (\partial N/\partial\psi)(\partial\psi/\partial t)$ and $\gamma \ll \omega_*$, the equation for J_T becomes $x(\partial J_T/\partial\xi)_\psi = (\partial\theta/\partial t + f)(\partial/\partial\xi)_x G(\psi)$. This has the same mathematical form as the expression for the current due to MHD interchange effects found in previous calculations,[8] and in previous investigations of semicollisional islands.[5] It has solution $J_T = (\partial\theta/\partial t + f)[dG(\psi)/d\psi](x - \langle x\rangle)$, so that

$$\int dx \oint \frac{d\xi}{\pi} \cos(\xi + \theta) J_T = -I_1 \frac{(\partial\theta/\partial t +)\,(1 - \partial\theta/\partial t - f)}{1+k} A^{1/2} \tag{19}$$

where

$$I_1 = 8\sqrt{2}\pi \int_1^\infty dW \left(\oint d\xi \cos\xi / \sqrt{w - \cos\xi}\right) \left(\oint d\xi \sqrt{w - \cos\xi}\right)^{-1} \left(\oint d\xi / \sqrt{w - \cos(\xi)}\right) \simeq 1.5$$

To compute the contribution from $K(\psi)$, we note that $\langle J_D\rangle = \langle J_T\rangle = 0$, so that Eq. (11) can be used as in Rutherford's calculation, giving $\int dx \oint d\xi K(\psi)\cos(\xi + \theta)/\pi = -I_2 A^{1/2}(\partial A/\partial t)$.

Combining these results we arrive at the evolution equation. Changing variables to the island half width $\Delta x_I = 2\sqrt{A}$, we have

$$I_2 \frac{\partial \Delta x_I}{\partial t} = \frac{\Delta'}{\hat\beta} + \frac{g\Delta x_e^2}{\hat\beta \Delta x_I^2}\cos\theta + \frac{|1-f|\sin\theta}{(1+k)\Delta x_s}\left(1 + \frac{|1-f|\Delta x_I \sin\theta}{\Delta x_s}\right) \tag{20}$$

$$\frac{\partial\theta}{\partial t} = 1 - f - \frac{|1-f|\Delta x_I \sin\theta}{\Delta x_s} \tag{21}$$

where $\Delta x_e^2 \equiv 4A_{\text{ext}}$ and $\Delta x_s \equiv 8I_0\hat\beta D|1-f|/(1+k)g\Delta x_e^2$; the quantity Δx_e is of order the vacuum island which would be caused by ψ_e'. The terms on the right in Eq. (20) arise respectively from the homogeneous exterior MHD kink effects; error fields; and J_T. As $\Delta x_e \to 0$ (no field errors), $\Delta x_s \to \infty$ and we recover exactly Rutherford's result for island width evolution driven by Δ'. For $\Delta x_e \to 0$, Eq. (21) gives island rotation at the Doppler-shifted ω_* frequency.

Now consider the limit where Δx_e is finite but $D \to 0$ (so $\Delta x_s \to 0$) and $f \to 0$. Then Eq. (21) implies that the island phase θ rapidly decays to the phase induced by ψ_e', $\theta \to 0$. The third term in Eq. (20) also approaches zero. Then for $\Delta' < 0$, Δx_I evolves to $\Delta x_e\sqrt{-g/\Delta'}$ on a resistive time scale. This is expected for resistive reconnection driven by A_{ext}; it has also been found previously that the externally driven island width increases as $\Delta' \to 0$. For $\Delta' > 0$, Eq. (20) shows that the island grows as long as it is thin enough to be described by Rutherford's analysis; the A_{ext} term provides an additional growth drive.

Novel behavior arises from finite ψ_e' and finite f and/or D. We analyze this by taking $\Delta\dot{x}_I/\Delta x_I \sim \gamma \ll \omega_*$. First, note that Eq. (21) implies that θ increases monotonically with time for $\Delta x_I < \Delta x_s$. Thus we can time average Eq. (20) over one periodic cycle in θ; $\int dt = \oint d\theta/(d\theta/dt)$, taking Δx_I and $\Delta\dot{x}_I$ to be constant over one cycle. For $\Delta x_I > \Delta x_s$, θ decays to a fixed, stable value on a time scale $\sim \omega_*$. Thus for $\Delta x_I > \Delta x_s$, we insert this value into Eq. (20). We obtain

$$I_2\frac{\Delta x_I}{dt} = \frac{\Delta'}{\hat{\beta}} + G_E(\Delta x_I) + G_I(\Delta x_I) \tag{22}$$

where

$$G_E(\Delta x_I) = \begin{cases} 0 & \Delta x_I < \Delta x_s \\ \left(\Delta x_e^2/\hat{\beta}\Delta x_I^2\right)\sqrt{1-\Delta x_s^2/\Delta x_I^2} & \Delta x_I > \Delta x_s \end{cases} \tag{23}$$

$$G_I(\Delta x_I) = \begin{cases} -2I_1 f(1-f)\Delta x_I/\left(\sqrt{\Delta x_s^2-\Delta x_I^2}+\Delta x_s\right)(1+k) & \Delta x_I < \Delta x_s \\ -2I_1 f(1-f)/\Delta x_I(1+k) & \Delta x_I > \Delta x_s \end{cases} \tag{24}$$

The G_E and G_I terms give the average driving effect of the external perturbation and inertial effects, respectively. They are continuous at $\Delta x_I = \Delta x_s$, but their derivatives are not.

The G_E term is positive for $\Delta x_I > \Delta x_s$, and induces growth there. It has the extraordinary property that it vanishes for $\Delta x_I < \Delta x_s$. Thus there is no tendency for external resonant perturbations to produce islands for $\Delta x_I < \Delta x_s$; this phenomenon has been termed "healing" by the present author. If $f = 0$ (so $G_I = 0$) and $\Delta' < 0$, an island will decay to very small values (of order the linear layer width where Eq. (14) breaks down), even though one expects ψ'_{ext} to produce a substantial island. This requires finite D. Direct numerical solution of Eq. (20)-(21) also shows this. The phenomenon is not limited to semicollisional tearing modes with small island widths; it has also been found for large external resonant perturbations in simulations of neoclassical MHD equations similar to Eq. (1)-(3), which will be reported elsewhere. Furthermore, note that if $-\Delta'$ exceeds the maximum value of $G_E(\Delta x_I)$ for $\Delta x_I > \Delta x_s$,

$$-\Delta' > 2\Delta x_e^2/3\sqrt{3}\Delta x_s^2 \; , \tag{25}$$

then all solutions decay to close to zero. If the reverse inequality holds, a stable steady-state solution exists with $\Delta x_I \sim \Delta x_e\sqrt{-g/\Delta'}$. If Eq. (25) does not hold, islands with initial widths close to $\Delta x_I\sqrt{-g/\Delta'}$ will evolve to this stable root; islands with initial widths $\lesssim \Delta x_s$ will decay to nearly zero.

Islands are healed in this way because a part of the current J_D is causing a magnetic perturbation to almost cancel the one from external sources. The phenomenon is akin to dielectric shielding. However, note that if $\Delta' > 0$, the islands do not decay if $f = 0$. We now consider the G_I term in Eq. (22).

The G_I term monotonically increases for $\Delta x_I \leq \Delta x_s$, and monotonically decreases toward zero for $\Delta x_I > \Delta x_s$. Consider Eq. (2) for $\Delta x_I < \Delta x_s$. (Even though $G_E(\Delta x_2) = 0$, the external resonant fields are still affecting island by the terms with Δx_s in Eq. (20)-(21), without these, G_I would vanish.) If $2I_1|f(1-f)/\Delta'| < 1$, (which is typically true), there is a critical island width where the Δ' and G_I terms are of equal magnitude

$$\Delta x_c/\Delta x_s = \frac{2|\Delta'|}{\sqrt{\Delta'^2 + 4I_1^2 f^2(1-f)^2}} \,. \tag{26}$$

For $\Delta x_I < \Delta x_c$, the Δ' term dominates, while for $\Delta x_c < \Delta x_I < \Delta x_s$ the G_E term dominates. It must be born in mind that Eq. (26) is only meaningful if $\Delta x_I > \Delta x_L$, the linear layer width.

For $\Delta x_I > \Delta x_s$, the G_E term may be of importance. If $\Delta x_e^2 > \Delta x_s \gg 2\hat{\beta} I_1 f(1-f)$, it dominates, and the behavior is as described in the paragraphs above on healing. For small semicollisional islands it is more typical for the G_E term to dominate. The Δ' term and G_I term are of the same magnitude when

$$\Delta x_2 = 2I_1 f(1-f)/\Delta' \,. \tag{27}$$

This expression is only valid if Δx_2 is not so large that a semicollisional treatment is no longer valid.

Thus, let us examine island dynamics from Eq. (22) in the case $G_E < G_I$. For $\Delta' > 0$ and $f(-1) < 0$, all driving terms are destabilizing, so the island grows. Conversely for $\Delta' < 0$ and $f(f-1) > 0$ the island always shrinks.

For $\Delta' > 0$ and $f(1-f) > 0$, the island with small amplitude $\Delta x_I < \Delta x_c$, grows from small amplitude to a steady state at Δx_c. For $\Delta x_c < \Delta x_I < \Delta x_2$, an island shrinks back to $\Delta x_I = \Delta x_c$. However, for $\Delta x_I > \Delta x_2$, an island grows (till its width exceeds the validity of our calculation). Conversely, if $\Delta' < 0$ and $f(1-f) < 0$, a nonlinear instability with finite threshold can occur. An island with $\Delta x_I < \Delta x_c$ shrinks, whereas one with $\Delta x_c < \Delta x_I < \Delta x_2$ grows to a steady state at $\Delta x_I = \Delta x_2$. A larger island decays back to Δx_2. Note that if $G_E \sim G_I$, additional steady state solutions are possible.

Now we examine the implications of these results to island formation in confinement devices. Consider what happens when a tokamak, which is in a macroscopic steady state after the current penetration and equilibration phase, undergoes further parameter changes, (e.g., heating, impurity influx) which change Δ' from negative to positive. Our analysis shows that this depends on both diffusion and radial electric field, and in addition, on the initial island width before the parameter change. The initial island width in the steady state can determine whether one expects to find a very small island as parameters are changed (e.g., $f(1-f) > 0$, initial $\Delta x_I = \Delta x_c$) or an evolution to a large macroscopic island ($f(1-f) > 0$, initial $\Delta x_I = \Delta x_2$). It is very difficult to assess a priori what the width is before the parameter change. Several steady state island widths are possible, depending on the present plasma parameers. It is incorrect to assume that the initial island width is even close to the naively predicted value from the external perturbation alone, $\Delta x_I = \Delta x_e/\sqrt{-\Delta'}$. The island width at any moment depends on the plasma time history starting from the initial current ramp up.

Because of this, it is quite possible for two similar plasma discharges with the same macroscopic current and pressure profiles to have very different MHD behavior. Sometimes the island may be "stuck" at very small amplitude, whereas for slightly different parameters it may grow beyond the semicollisional stage into the MHD stage, and evolve to saturation or disruption in the familiar way.[6,7,14]

We also note that it is possible for nonlinear moderate m instabilities to occur from field coil errors near the plasma edge. Without field coil errors, we note that an instability can arise from toroidal coupling of islands on different rational surfaces. The instability drive becvomes stronger as β is increased; this might lead to substantial magnetic stochasticity at the plasma edge.

Finally, this calculation implies that it may be possible to stabilize islands with $\Delta' > 0$ using externally produced resonant magnetic perturbations. If $f(1-f) > 0$, an external perturbation with an appropriate amplitude can create a strong barrier against growth for islands thin enough to be in the semicollisional regime. If the external perturbation is oscillated to a have finite rotation, one can always find a frequency so that in the rest frame of the magnetic perturbation (which is the relevant frame for this calculation) one has $f(1-f) > 0$. A similar stabilizing effect might be produced for moderate m islands arising in the plasma edge from any instability mechanism.

Acknowledgments

This work was supported by the U.S. Department of Energy contract #DE-FG05-80ET-53088.

References

1. P.H. Rutherford, Phys. Fluids **16**, 1903 (1973).

2. J.F. Drake and Y.C. Lee, Phys. Rev. Lett. **39**, (1977).

3. D.A. Monticello and R.B. White, Phys. Fluids **23**, 366 (1980).

4. D. Biskamp, Nucl. Fusion **19**, 777 (1979).

5. B. Scott and A.B. Hassam, Phys. Fluids **30**, 90 (1987).

6. J.F. Drake and Y.C. Lee, Phys. Fluids **20**, 1341 (1977).

7. J.F. Drake, T.M. Antonsen, A.B. Hassam, N.T. Gladd, Phys. Fluids **26**, 2509 (1983).

8. M. Kotschenreuther, R.D. Hazeltine, and P.J. Morrison, Phys. Fluids **28**, 294 (1985).

9. R. Carrera, R.D. Hazeltine, and M. Kotschenreuther, Phys. Fluids **29**, 899 (1986).

10. J.D. Callen, et al., in *Plasma Physics and Controlled Nuclear Fusion Research* 1986 (Proc. 11th Int. Conf., Kyoto, 1986) vol. 2, 157.

11. M. Kotschenrether, et al., in *Plasma Physics and Controlled Nuclear Fusion Research* 1986 (Proc. 11th Int. Conf., Kyoto 1986) vol. 2, 149.

12. R.D. Hazeltine and J.D. Meiss, Phys. Rep., **121**, vol. 2 (1985).

13. F.L. Hinton and R.D. Hazeltine, Rev. Mod. Phys. **48**, No. 2, Part ' (1976).

14. B.V. Waddell, B.A. Carreras, H.R. Hicks, and J.A. Holmes, Phys. Fluids **22**, 896 (1979).