DOE/ET-53088-318 IFSR **#318**

# U.S.–Japan Workshop on Plasma and Fluid Turbulence

Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

April 1988

## Table of Contents

## Opening Remarks

Welcome to the US-Japan Workshop on *Plasma and Fluid Turbulence*. Before I turn to introducing and welcoming the Japanese delegation, let me say a few words about this workshop and its organization.

This workshop is the third on the U.S. side and the sixth in the series of US-Japan workshops on fundamental physics problems in the nonlinear and statistical behavior of plasmas. This year, we chose the theme of fluid turbulence as the area of concentration. It is our pleasure, and to our benefit, to have Professor Harry Swinney and his group, that forms the Center for Nonlinear Dynamics at The University of Texas at Austin, join us as co-organizers of this year's US-Japan statistical physics workshop. Dr. Harry Swinney has played a key role in organizing the workshop and in bringing a number of leading fluid dynamicists to the meeting. Both the U.S. and Japanese participants wish to thank Dr. Swinney for his contribution to the meeting.

In putting together the program for the meeting, we have tried to bring a balance between theory, simulation and fluid-plasma experiments.

Today, we have three overview talks by leading contributors to the field. In the following days, we will hear more state-of-the-art talks in the problems of fluid and plasma turbulence and vortex structures. It is the hope of the organizers that by the close of the workshop Thursday evening, each participant will leave the workshop with not only a clearer perspective of the interrelations between the complex facets of the key issues in plasma and fluid turbulence, but with a few valuable ideas on how to better attack the difficult problems that remain in the way of understanding and predicting plasma and fluid motions.

Now, for Wednesday evening we have a planned special banquet with the distinguished Texas Historian, Professor Joseph Frantz, flying in from Corpus Christi State University to speak on *Reflections on Texas*. Dr. Frantz has been President of both the Western Historical Association and the Southern Historical Association. He was Director of the Oral Historical project under President Lyndon B. Johnson.

Those who have not yet registered for the meeting should do so at the break. Nonregistered guests may attend the banquet by paying $25.00. Saralyn Stewart will turn in the final count for attendance to the banquet about 5:00 p.m.

Now, let me introduce Professor Tadatsugu Hatori, of Nagoya University, as leader of the Japanese delegation which includes Professor Hazime Mori (Kyushu), Professor Mitushiro Nambu (Kyushu), Professor Masahiro Wakatani (Kyoto), and Dr. Michio Yamada (Kyoto).

Wendell Horton
Austin, Texas
December 1987

## US-JAPAN WORKSHOP PROGRAM

### *PLASMA AND FLUID TURBULENCE*

December 7-10, 1987
The University of Texas at Austin
Austin, Texas 78712
*(Workshop location: Hyatt Regency, Big Bend Foyer)*

**MONDAY, DECEMBER 7, 1987**

2:00 PM Opening Remarks: W. Horton and T. Hatori

**PLASMA AND ROTATING FLUID DYNAMICS**
Chairman: W. Horton

| | | |
|---|---|---|
| 2:10 PM | Hasegawa | *Relationships Between the Dynamics of Magnetized Plasmas and Rotating Neutral Fluids* |
| 3:10 PM | Leith | *Models of Turbulent Mixing* |
| 4:10 PM | BREAK | |
| 4:30 PM | Waltz | *Review of Numerical Simulation of Ion Temperature Gradient Mode Turbulence* |

**TUESDAY, DECEMBER 8, 1987**

Concurrent All Day Poster Session

**TURBULENCE THEORY**
Chairman: B. Taylor

| | | |
|---|---|---|
| 8:30 AM | Williams | *Application of Kraichnan's Decimated-Amplitude Scheme to the Betchov Model of Turbulence* |
| 9:00 AM | Yamada | *A Chaotic Model of Fully Developed Turbulence* |
| 10:00 AM | BREAK | |
| 10:20 AM | Kraichnan | *Recent Developments in Statistical Turbulence Theory* |
| 11:20 AM | Tam | *Turbulent Transport in Flow Between Concentric Cylinders* |
| 11:40 AM | LUNCH | |

**INSTABILITIES, CHAOS AND VORTEX DYNAMICS**
Chairman: C. Leith

| | | |
|---|---|---|
| 2:00 PM | Mori | *Anomalous Dynamic Scaling Properties of Chaos* |
| 3:00 PM | Patera | *Complex-Geometry Stability Theory: Numerics and Physics* |
| 4:00 PM | BREAK | |
| 4:20 PM | Nycander | *New Stationary Vortex Solutions of the Hasegawa-Mima Equation* |
| 4:50 PM | Horton | *Quasi-Coherent Transport by Vortices and Vortex-Wave Interactions* |
| 5:20 PM | Swinney | *Coherent Vortices in a Turbulent Quasi-Geostrophic Flow* |

## WEDNESDAY, DECEMBER 9, 1987

### TURBULENCE MEASUREMENTS AND THEORY

Chairman: H. Swinney

| | | |
|---|---|---|
| 8:30 AM | Bengtson | *Characteristic Fluctuations in the TEXT Tokamak* |
| 9:30 AM | Lumley | *Low Dimensional Model of the Wall Region of a Turbulent Boundary Layer* |
| 10:30 AM | BREAK | |
| 10:50 AM | Hatori | *MHD Cellular Automaton Model and Chaotic Magnetic Field* |
| 11:50 AM | Wootton | *Turbulence and Electron Heat Transport in TEXT* |
| 12:20 PM | LUNCH | |

### ANOMALOUS TRANSPORT

Chairman: H. Mori

| | | |
|---|---|---|
| 2:00 PM | Krommes | *Rigorous Upper Bounds on Turbulent Transport* |
| 3:00 PM | Nambu | *A New Mode Coupling Process in Plasma Turbulence* |
| 4:00 PM | BREAK | |
| 4:20 PM | W. Lee | *Temperature Gradient Driven Drift Wave Particle Simulations* |
| 4:50 PM | Mahajan/Zhang | *Anomalous Electron Transport* |

6:00 PM HYATT REGENCY - COCKTAILS (cash bar)
7:00 PM BANQUET - TEXAS BALLROOM I
8:00 PM Speaker: Professor Joseph Frantz
Former Director, Texas State Historical Association
Talk Title: *REFLECTIONS ON TEXAS*

## THURSDAY, DECEMBER 10, 1987

### RESISTIVE MHD TURBULENCE

Chairman: J. Meiss

| | | |
|---|---|---|
| 8:30 AM | Similon | *Intermittency Structures and Their Effect on the Transport of Magnetic Field in Turbulent Fluid Flows* |
| 9:30 AM | Strauss | *Turbulent Relaxation and Reconnection* |
| 10:30 AM | BREAK | |
| 10:50 AM | Montgomery | *Turbulent Relaxation of Driven Reversed Field Pinch* |
| 11:50 AM | Wakatani | *Relaxation and Transport Due to Resistive G Modes* |
| 12:50 PM | LUNCH | |

### INTERMITTENCY AND TURBULENCE

Chairman: T. Hatori

| | | |
|---|---|---|
| 2:30 PM | Mori | *Global Spectral Structure of Intermittent Chaos* |
| 3:30 PM | Drake | *Electron Temperature Gradient Driven Turbulence* |
| 4:30 PM | BREAK | |
| 4:50 PM | Terry | *Interaction of a Strong Vortex with Decaying Turbulence* |
| 5:50 PM | Bekki | *Electron Temperature Gradient Driven Convection* |
| 6:10 PM | Adjourn | |

*ELECTRON TEMPERATURE GRADIENT DRIVEN CONVECTION*

N. Bekki
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JAPAN

## ELECTRON TEMPERATURE GRADIENT DRIVEN CONVECTION

N. Bekki

In order to explain the anomalous transport of electrons in tokamaks, we propose a set of reduced nonlinear equations that describe the coupling of linearly unstable short wavelength $\rho_e$ electrostatic fluctuations driven by the electron temperature gradient $\eta_e$ to the longer wavelength $c/\omega_{p_e}$ electromagnetic fluctuations. The instability is the toroidal version of the $\eta_e$ driven lower hybrid drift mode in the presence of the electron temperature gradient emphasized by Guzdar[1] and Lee[2] for a slab. The growth rate $\gamma \sim v_e/(r_{T_e} R)^{1/2}$ is driven by the local bad curvature and the electron temperature gradient. Since the system of the reduced equations is weakly 3D, with $\nabla_\perp \gg \nabla_\parallel = \partial_z - \frac{1}{B}[\tilde{A}_\parallel, \;]$, we may consider the possibility of an inverse cascade $\rho_e \rightarrow c/\omega_{p_e}$ in the energy spectrum, as in the 2D Navier-Stokes turbulence. Such an inverse cascade may explain the free-energy source of the magnetic fluctuations $\tilde{A}_\parallel$. Previously, such magnetic fluctuations have been assumed, and then used to explain the neo-Alcator and Merezhkin-Mukhovatov type of scaling for $\chi_e$.

### References

1. P.N. Guzdar, C.S. Lin, J.W. Dong, and Y.C. Lee, Phys. Rev. Lett **57**, 2818 (1986).

2. Y.C. Lee, J.Q. Dong, P.N. Guzdar, and C.S. Liu, Phys. Fluids **30**, 1331 (1987).

# *CHARACTERISTIC FLUCTUATIONS IN THE TEXT TOKAMAK*

R.D. Bengtson

**Characteristic Fluctuations in the TEXT Tokamak.** Roger D. Bengtson, Physics Department and Fusion Research Center, University of Texas at Austin, USA. We present measurements of the turbulence in the TEXT tokamak using data from three techniques-Langmuir probes in the edge, heavy ion beam probe and FIR scattering. Measurements of the turbulent propagation in three dimensions are discussed and characteristic lengths and frequencies are given. Particle and energy transport due to fluctuations are compared with global measures of confinement. A set of observations of the turbulence characteristics are presented for comparison with theory.

## *ELECTRON TEMPERATURE GRADIENT DRIVEN TURBULENCE*

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*(Please write to the address above if interested in this work.)*

# *MUTUAL INFORMATION FOR NOISY MEASUREMENTS OF SHIFT MAPS**

A. Fraser

Mutual Information for Noisy Measurements of Shift Maps

Andrew M. Fraser

Physics Deptartment and Center for Nonlinear Dynamics the University of Texas, Austin, Tx 78712

We present an illustration of the superiority of mutual information (or marginal redundancy R') over conditional entropy H' for detecting deterministic aspects of noisy scalar time series. We use the shift map, $s_{n+1} = \phi(s_n) = 2s_n \bmod 1$, and compositions of $\phi$ with itself as examples. The metric entropy[1] $h_\mu(\phi)$ is 1 bit per iteration. Noisy measurements are simulated by adding a uniformly distributed noise term of width of $\eta$ to sequences generated by the maps.

REFERENCES

[1] A.N. Kolmogorov, "A new metric invariant of transient dynamical systems and automorphisms in Lebesgue spaces," Dolk. Akad. Nauk. SSSR, vol. 119 pp. 861-864, 1958; English summary in MR, vol. 21, pp. 386, 1960.

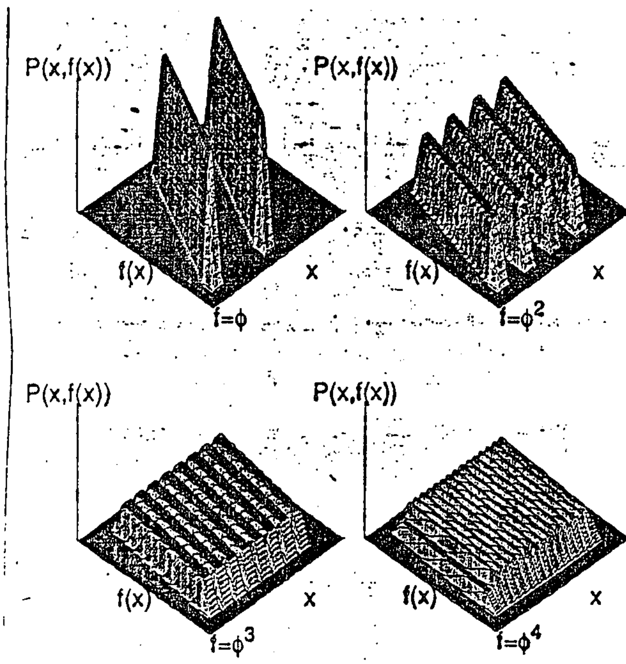


Fig. 1 Joint probability distributions of noisy measurements of 4 shift maps. In each case the partition grid $\varepsilon$=0.01 and the noise width $\eta$=0.1

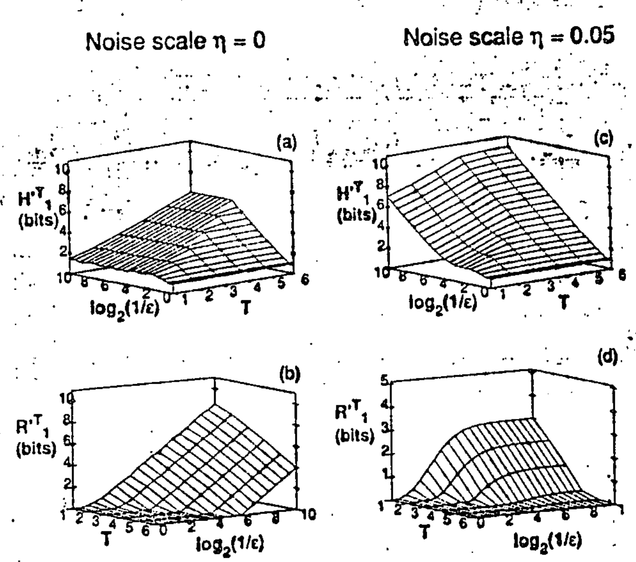


Fig. 2 The dependence of $R'^T_1$ and $H'^T_1$ on partition resolution $\varepsilon$, delay time T, and noise is shown. (a) and (b) are results for noise free measurements, while (c) and (d) show the effect of 5% measurement noise. Without noise $R'^T_1$ increases linearly with $\log(1/\varepsilon)$ and $H'^T_1$ is well behaved. When noise is added $R'^T_1$ behaves well and $H'^T_1$ diverges in the limit of fine patritions.

# SELF-ORGANIZATION OF ELECTROSTATIC TURBULENCE IN A CYLINDRICAL PLASMA

A. Hasegawa

**Self-Organization of Electrostatic Turbulence in a Cylindrical Plasma***

A. Hasegawa and M. Wakatani

On the basis of theory and computer simulations we show that electrostatic turbulence in a cylindrical plasma with magnetic shear and curvature self-organizes to form a macroscopic potential $\phi$ which depends only on the radial coordinate $r$ and is given by $\phi(r) \cong J_0(pr) + C_1r^2 + C_2$, where $C_1$ and $C_2$ are functions of constant $p$. A unique feature of the potential is the existence of a coaxial $\phi(r_0) = 0$ surface at $r_0 \cong 0.7a$, where $a$ is the radius of the cylinder. This surface is found to be fairly rigid and is considered to inhibit radial particle transport.

*Phys. Rev. Lett. **59**, 1581-1584 (1987).

*HEAT TRANSPORT DUE TO RESISTIVE G-MODES**

S. Hamaguchi

HEAT TRANSPORT DUE TO RESISTIVE g-MODES

S. Hamaguchi, E. Hameiri and H.R. Strauss

Courant Institute of Mathematical Sciences

New York University

We consider the anomalous heat transport caused by resistive g-mode fluctuations in a plasma. If we define the entropy s by $s = \log (p/\rho^{\gamma})$, where p is the pressure, $\rho$ is the mass density and $\gamma$ is the ratio of the specific heats, then the total heat flux is given by $\kappa\nabla T_0 - \frac{p_0}{\gamma-1} \langle s_1 \underset{\sim}{v}_1 \rangle$, the second term of which represents the anomalous heat transport. Here subscripts 0 and 1 denote the mean and fluctuating quantities respectively and $\langle\ \rangle$ denotes the average over the fluctuations. We have found that this anomalous heat transport is related to the so-called dynamo term, i.e., the electric field $\underset{\sim}{\varepsilon} = \langle \underset{\sim}{v}_1 \times \underset{\sim}{b}_1 \rangle$ caused by fluctuations, through the equation $\underset{\sim}{\varepsilon} \cdot \underset{\sim}{J}_0 + \frac{1}{\gamma-1} \rho_0 \langle \underset{\sim}{s}_1 \underset{\sim}{v}_1 \rangle \cdot \nabla T_0$ = negative quantity related to the energy dissipation; in particular, if the mean pressure gradient $\nabla p_0$ is much smaller than the mean temperature gradient $\nabla T_0$, then the anomalous heat transport can be written as $-K^2 \nabla T_0$ by using a non-negative function $K^2$ which depends on mean quantities, including $\nabla T_0$. Dependence of $K^2$ on $\nabla T_0$ has been shown by numerical simulation.

# *MHD CELLULAR AUTOMATON MODEL*

T. Hatori

# MHD CELLULAR AUTOMATON MODEL

Tadatsuga Hatori

Development of a special purpose computer to study a certain phenomena has been the new trend in the scientific research. It has been in fact realized in specific fields of physics such as simulations based upon the lattice gauge and Ising models. In the field of continuum dynamics, the lattice gas cellular automaton has become a candidate of the special purpose computer. The basic theoretical framework for the Navier-Stokes equation has been developed recently,[1] and the two-dimensional simulations have already been performed. More recently, MHD cellular automata models have also been developed. There are two MHD CA models in the two-dimensional configuration, Montgomery-Doolen model (MD model)[2] and Chen-Matthaeus model.[3] In this workshop, we proposed a generalized MD model which has an advantage that the processes are deterministic, while both MD and CM models are nondeterministic.

In MD model, the MHD particles are distributed in the hexagonal lattice systems. The particle has integer variables, $\alpha(1, 2, \ldots, 6)$ and $\sigma(1, 0, -1)$, where $\alpha$ denotes 6 velocities $\hat{e}_\alpha$ with $|\hat{e}_\alpha| = 1$ directed to nearest neighbor cites, and $\sigma$ stands for porality, up, neutral, and down. Physical meaning of $r$ is such that $nA_z = \sum_{\text{s.c.}} \sigma^{(i)}$, where $n$ is a particle number density, $A_z$ the $z$-component of vector potential and the sum is over all cites in the super-cell. Time evolution of the many particle system is governed by the same collision rule for velocities as in the FHP model[4] for Navier-Stokes equation, and by the collision rule for $\sigma$ index that all possible transitions which conserve total $\sigma$, $\sum_i \sigma^{(i)}$, $i$ denoting different colliding particles. Reduction to the macroscopic dynamics, in this case 2D MHD, from the microscopic many body system composed of artificial MHD particles, has been given[2] applying the Chapman-Enskog expansion. The transport coefficients, magnetic diffusivity and kinematic viscosity, are given analytically.[5]

The MHD particle in the present model has more freedom characterized by four parameters, $r_1$, $r_2$, $\bar{B}$, and $\ell$, than in MD model. The shape of the particle is an annulus with inner and outer radius $r_1$ and $r_2$, and with length $\ell$, where there exists an azimuthal magnetic field $\bar{B}/r$ ($r$ is radial coordinate) and the associated surface current. Especially when $\pi\bar{B}(r_2^2 - r_1^2)/2 = 1$ and $\ell = \infty$, the present model is identical to the MD model. The equipartition spectrum in the high wavenumber region, the Alfvén-mode dominant region, and the quantization of Lorentz force, determine parameters.

It is possible to apply the present model to the Strauss equation by taking MHD particle with finite length $\ell$ and by properly incorporating the field line stretching term.

## References


1. U. Frisch et al., Complex Systems **1**, 649 (1987).
2. D. Montgomery and G.D. Doolen, Complex Systems **1**, 831 (1987).
3. H. Chen and W. Matthaeus, Phys. Rev. Lett. **58**, 1865 (1987).
4. U. Frisch, B. Hasslacher and Y. Pomeau, Phys. Rev. Lett. **56**, 1505 (1986).
5. T. Hatori and D. Montgomery, Complex Systems **1**, 735 (1987).

# DRIFT WAVE VORTICES AND ANOMALOUS TRANSPORT

W. Horton

# Drift Wave Vortices and Anomalous Transport

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**Abstract**

Theory and computer simulations are used to describe the inelastic vortex-vortex and vortex-wave interactions that lead to the quasi-coherent transport of plasma across the magnetic field. Monopole and dipole drift wave vortices with radii $r_0$ large compared with the ion inertial scale length $\rho_s$ are shown to produce transport at the rate $un_v \int d\sigma(b) \leq n_v v_{de} r_0$ where $n_v$ is the vortex line density and $d\sigma(b)$ is the inelastic collision cross-section for impact parameter $b$. The transport during collisions and mergings is evaluated from the evolution of a passively convected scalar concentration of test particles.

In contrast to small amplitude drift waves, which spread out and lose strength as they travel, the solitary drift wave vortices are coherent, self-sustaining packets that retain their strength over long distances. These coherent vortices trap particles that are consequently also transported over long distances. The anomalous transport of plasma is determined from the net flux $\langle v_x f \rangle$ of a test particle field $f(x, y, t)$ being passively $\mathbf{E} \times \mathbf{B}$ convected by the nonlinear drift wave flows. Depending on the particular application, the passive field $f$ may represent the temperature, the poloidal flux function, or the density of charged particle species.

The vortices studied are of two types: the monopole vortex representing a net excess in the local charge density and the dipole vortex representing a local polarization in the local charge density. A vortex of scale $k^{-1}$ and amplitude $\Phi_k$ is characterized by the internal circulation rate $\Omega_E(k) = ck_x k_y \Phi_k / B$. The vortex turnover time is $2\pi/\Omega_E(k)$.

The vortex behavior of self-binding and the trapping of the drift wave wake sets in when $\Omega_E(k) > \omega_k$ the linear drift wave frequency. In the case of monopoles the transition from wave packet behavior for $\Omega_E \ll \omega_k$ to a wakeless vortex behavior $\Omega_E \gg \omega_k$ is not sharp but, nevertheless, is clearly shown by the simulations when $R_E = \Omega_E/\omega_k \simeq 1 - 2$.

# *NOISE REDUCTION IN DYNAMICAL SYSTEMS**

E. Kostelich

— from Turbulence Meeting Dec 7-11.
471-3105

# Noise Reduction in Dynamical Systems

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and
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Institute for Physical Science and Technology
and Department of Mathematics
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The analysis of chaotic experimental data typically involves the analysis of attractors reconstructed from scalar time series. Noise in the time series causes errors in the location of points on the reconstructed attractor. This work describes how the dynamics on the attractor can be used to identify and correct errors in trajectories.

Our approach deals with a spatial problem (i.e., the phase space attractor). The dynamical information of interest is not localized in a time or frequency domain; hence, traditional methods of time series analysis are not applicable.

The procedure has two steps. First, a linear approximation $L_k(x_k)$ is computed of the unknown map $f$ which takes th $k$th to the $(k+1)$st attractor point. This is done by examining the images of a collection $\{y_i\}$ of points in a small ball about $x_k$ and finding a least-squares estimate of a matrix $A_k$ and vector $b_k$ such that $f(y_i) \approx A_k(y_i) + b_k$. (Here $A_k$ is an approximation to the Jacobian matrix of $f$ at $x_k$ and $b_k$ is a translation term. The subscript merely emphasizes the dependence of $A$ and $b$ on $x_k$.)

The second step consists of examining "windows" of points on trajectories. The idea is to replace the observed window $W = \{x_k, \ldots, x_{k+w}\}$ with a new trajectory $\hat{W} = \{\hat{x}_k, \ldots, \hat{x}_{k+w}\}$ such that each point $x_j$ in $\hat{W}$ is close to the corresponding point in $W$ and best satisfies the relation $x_j = L_{j-1}(x_{j-1})$ in a least-squares sense.

Numerical experiments on the Henon attractor suggest that the procedure can reduce noise levels by a factor of ten or more.

## *RECENT DEVELOPMENTS IN STATISTICAL TURBULENCE THEORY*

R. Kraichnan

# RECENT DEVELOPMENTS IN STATISTICAL TURBULENCE THEORY

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## ABSTRACT

Fourth-order moments play an important role in recent turbulence studies. These moments are needed to describe some physically interesting phenomena shown by computer studies of homogeneous and shear Navier-Stokes turbulence: for example, enhancement of helicity fluctuations and suppression of the mean-square value of the total nonlinear term. Moreover, the Howard-Busse bounding theory leans crucially on fourth-order moments.

If reliable and computationally feasible predictions are to made about fourth-order moments, it is clear that traditional turbulence closures, like those of the direct-interaction family, must be re-thought. Already at the level of second-order moments such closures have unknown error levels and are very hard to compute except in highly symmetric geometries.

In this talk some relations will be brought out among the Howard-Busse theory, decimation under symmetry constraints, and the realizability constraints on moments of the many-time distribution. There is reason to hope that these tools in combination can lead to practicable turbulence approximations which yield both upper and lower bounds on turbulence statistics and thereby offer internal error estimates.

87 Dec 06

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## *RIGOROUS UPPER BOUNDS ON TURBULENT TRANSPORT*

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# *GYROKINETIC PARTICLE SIMULATION OF ION TEMPERATURE GRADIENT DRIFT INSTABILITIES*

W. Lee

US/Japan Workshop on Plasma and Fluid Turbulence
December 7-11, 1987, Austin, Texas

# Gyrokinetic Particle Simulation of Ion Temperature Gradient Drift Instabilities

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Ion temperature gradient drift instabilities ($\eta_i$-modes) in slab geometry have been studied utilizing the newly-developed gyrokinetic particle simulation techniques.[1] The purpose of the investigation is to gain better understanding of the saturation and transport of these modes in a system where the saturation spectrum is dominated by only a few semi-coherent modes. To this end, a two-dimensional simulation with and without shear has been carried out, in which the poloidal wavenumber is restricted to $k_\theta \rho_s \lessapprox 0.4$. The results indicate that, in the case of shear, the modes with the largest saturation amplitude are those with $k_\theta \rho_s = \pm 0.4$ and $n = 0$, where n is the radial eigenmode number. A perturbative analysis, based on a three-wave mode coupling model, gives the saturation amplitude at the rational surface ($k_\parallel = 0$) as

$$|e\Phi/T_e| \approx (|\omega_\ell + i\gamma_\ell|/\Omega_i) \,/\, (2k_r\rho_s k_\theta \rho_s),$$

where $\omega_\ell$ and $\gamma_\ell$ are the real frequency and the growth rate,

$$k_r\rho_s \approx [\omega_* \gamma_\ell L_n/(2|\omega_\ell + i\gamma_\ell|^2 L_s)]^{1/2}$$

is the radial wavenumber (normalized to $\rho_s \equiv c_s/\Omega_i$), $\omega_*$ is the diamagnetic drift frequency, and $L_n$ and $L_s$ are the density and shear scale lengths, respectively. For the shear-free cases, the saturation amplitude has the same parameter dependence and $k_r$ is determined by the simulation box size. In addition, the nonlinear E×B convection is found to be the mechanism responsible for the saturation through the generation of the zero-frequency ion current and pressure responses. The corresponding quasilinear ion thermal diffusivity can then be expressed as

$$\chi_i \approx \gamma_\ell / k_r^2.$$

However, the steady state $\chi_i$ depends critically on the dissipation (collisionless or collisional) of the system. Its exact scaling is still under investigation.

[1]W. W. Lee, J. Comput. Phys. 72, 243 (1987).

## *MODELS OF TURBULENT MIXING*

C. Leith

# Models of Turbulent Mixing

C. E. Leith
Lawrence Livermore National Laboratory
December 1987

Interest in computing the mixing that occurs at unstable interfaces has led to the use of inhomogeneous turbulence models. A simple example is the Rayleigh-Taylor unstable interface whose behavior was examined in rocket rig experiments by Read and analyzed by Youngs. After an initial period of classical exponential growth, disturbances soon reach a nonlinearly saturated state of rising bubbles and falling spikes. At a late stage mixing is dominated by a self-similar process involving competition between rising bubbles. It is this final stage that can be modeled as turbulence.

The popular and notorious K-epsilon model has been generalized to treat compressible turbulence through use of closed mass-averaged equations. A Rayleigh-Taylor source term arises that effectively converts internal energy into turbulent kinetic energy to drive the mixing process. By suitable choice of empirical model constants, consistent with many aerodynamic applications, it is possible to simulate well the observed buoyant mixing process described by Youngs.

The simulation of shock induced turbulence and mixing at an interface is more difficult. Both Soviet and French experiments show that significant turbulent mixing occurs only after the passage of a second shock. A simpler question has been asked first. How does the K-epsilon model treat the interaction of a shock running through turbulence in a single gas? It is found that the eddy viscosity in the model spreads out the shock front, the gas is therefore only adiabatically compressed, and the extra energy required by Hugoniot jump conditions appears as a considerable enhancement of the turbulent kinetic energy level. An open question is whether this is a model artifact. Some experimental evidence supports qualitatively the model behavior.

Mixing induced by the Kelvin-Helmholtz shear instability is rather easily modeled by a K-epsilon model and has been for many years. What the model does not capture are the coherent eddy structures observed in shear layers. Instead it treats all eddies as averaged turbulence no matter how big they may be.

Preliminary experiments have been carried out with a new subgrid turbulence model in which there is included the random forcing of resolved scales of motion by subgrid turbulence along with the usual diffusive effects. In such a model a particular simulation is treated as one possible realization of the flow field. When such stochastic backscatter is applied to the shear problem, coherent eddies appear as soon as the mixing layer becomes wide enough that they can be resolved. These are now Monte Carlo flow calculations. If one starts with a different seed in the random number generator, eddies will form again but in a different place.

# *LOW-DIMENSIONAL MODEL OF THE WALL REGION OF A TURBULENT BOUNDARY LAYER*

J. Lumley

# Low-dimensional model of the wall region of a turbulent boundary layer

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Abstract.

This work was done in collaboration with Nadine Aubry, of the same department, and with Phillip Holmes and Emily Stone of the Department of Theoretical and Applied Mechanics; Holmes is also affiliated with the Department of Mathematics and the Center for Applied Mathematics. We have modeled the wall region of a turbulent boundary layer by expanding the instantaneous field in so-called empirical eigenfunctions, as permitted by the proper orthogonal decomposition theorem ( Lumley [1967,1980]). We truncate the representation to obtain low dimensional sets of ordinary differential equations, from the Navier Stokes equations, via Galerkin projection. The experimentally determined eigenfunctions of Herzog [1986] are used; these are in the form of streamwise rolls. Our model equations represent the dynamical behavior of these rolls. We show that these equations exhibit intermittency, which we analyze using the methods of dynamical systems theory, as well as a chaotic regime. We argue that this behavior captures major aspects of the ejection and bursting events associated with streamwise vortex pairs which have been observed in experimental work (Kline et al [1967]). We show that although this bursting behavior is produced autonomously in the wall region, and the structure and duration of the bursts is determined there, the pressure signal from the outer part of the boundary layer triggers the bursts, and determines their average frequency. The analysis and conclusions drawn appear to be among the first to provide a reasonably coherent link between low dimensional chaotic dynamics and a realistic turbulent open flow system.

Acknowledgements:

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# ANAMOLOUS ELECTRON ENERGY TRANSPORT IN TOKAMAKS

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**Anomalous Electron Energy Transport in Tokamaks**

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A local turbulent electron energy transport coefficient

$$D^{(T)} \sim \frac{\omega_*}{k_\perp^2}$$

with $\omega_* \sim (1+\alpha\eta_e)(T_e c k_\perp / eBL_n), (\alpha \sim O(1))$, and $k_\perp \sim \omega_{pe}/c$ is obtained by analyzing the general turbulent transport formula. Following arguments have been essential for the choice:

(1) Since the dominant energy loss in tokamak is mainly through the electron channel, the electromagnetic fluctuations are expected to be a better candidate than the pure electrostatic ones for the explanation of this phenomenon.

(2) The magnetic fluctuations can not rise to superthermal levels unless the strong shear damping effect is somehow supressed. Thus the inverse correlation time of the turbulence is dominated by $\omega \sim \omega_* \sim k_\perp^2 D_\perp^{(T)}$, rather than by $\langle k_\parallel \rangle v_e$.

(3) The magnetic fluctuation implied by the estimate of the transport is amenable to the simple physical interpretation that the saturation is obtained when the perpendicular phase velocity equals the electron drift velocity. At the saturation, the linear term in the drift kinetic equation is balanced by the nonlinear term.

Comparison of this formula with tokamak experiments yields fairly good agreement.

## *LABORATORY MODEL OF A PLANETARY EASTWARD JET**

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# *TURBULENT RELAXATION OF DRIVEN REVERSED FIELD PINCH*

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TURBULENT RELAXATION OF DRIVEN REVERSED FIELD PINCH

D. Montgomery

Analytical analyses of linear plasma instabilities and their nonlinear consequences are always highly approximate, may assume symmetries which are absent in real life, and usually ignore the effects of other instabilities. For these reasons, numerical methods which intend to predict the dynamics of high-temperature magnetofluids must be flexible and robust enough to follow large qualitative changes in the plasma configuration over time, and to respond to the appearance of irregularly-shaped features having initially unexcited length and time scales. These requirements are illustrated by some recent computational studies motivated by the reversed-field pinch.[1]

A previously-described, pseudo-spectral, three-dimensional MHD code is used to compute the dynamical behavior of a channel of magnetofluid carrying an axial current and magnetic flux. This situation contains the essential MHD behavior of the reversed-field-pinch ("RFP"). An externally-imposed electric field is applied to an initially current-free magnetofluid, and drives currents which rise and eventually fluctuate about values corresponding to pinch ratios $\theta \simeq 1.3$, 2.2, and 4.5. A period of violent turbulence leads to an approximately force-free core, surrounded by an active MHD boundary layer which is not force-free. A steady state is reached which can apparently be sustained indefinitely ($\geq$ several hundred Alfven transit times). The turbulence level and time variability in the steady state increases with increasing $\theta$. The average toroidal magnetic field at the wall reverses for the $\theta \simeq 2.2$ and 4.5 runs, but not for the $\theta \simeq 1.3$ one. Negative toroidal current filaments are observed. The Lundquist numbers are of the order of a few hundred.

[1] J.P. Dahlburg, D. Montgomery, G.D. Doolen, and L. Turner, Phys. Rev. Lett. 57, 428 (1986); J. Plasma Phys. 37, 299 (1987); and Los Alamos Report LA-UR 87-3320 (submitted to J. Plasma Phys., 1987).

## *ANOMALOUS DYNAMIC SCALING PROPERTIES OF CHAOS*

**AND**

## *GLOBAL SPECTRAL STRUCTURE OF INTERMITTENT CHAOS*

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# *A NEW MODE COUPLING PROCESS IN PLASMA TURBULENCE*

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# A New Mode Coupling Process in Plasma Turbulence

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## ABSTRACT

The role of a new mode coupling effect (plasma-maser) in plasma turbulence is studied. The new maser effect, the idea that the resonant electrons with the low-frequency mode can amplify the high-frequency mode, does not require population inversion of electrons. The generation mechanism of the new mode coupling process is best understood from the point of view of the high-frequency dissipative forces[1]. The new maser effect always coexists with Landau resonance. The forced plasma-maser interaction model reduces to a conservative Lotka-Volterra system. A chaotic behavior of the forced Lotka-Volterra system is obtained[2]. The laboratory experiment[3] is consistent with the theoretical prediction. The new mode coupling process has potential importance in attempting to interpret numerous radio phenomena in space[4] and laboratory.

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# *NEW STATIONARY VORTEX SOLUTIONS OF THE HASEGAWA-MIMA EQUATION*

J. Nycander

NEW STATIONARY VORTEX SOLUTIONS OF THE HASEGAWA-MIMA EQUATION

J. Nycander

The Hasegawa-Mima equation is an important model equation in nonlinear two-dimensional fluid dynamics, and describes both driftwaves in plasmas and Rossby waves in the atmosphere. An interesting exact solution having the shape of a propagating dipole vortex was found in 1976 by Larichev and Rezuik. It has since then been widely studied, and it has also been shown that a great number of similar equations have the same kind of solution.

In this paper two families of explicit solutions of the Hasegawa-Mima equation are obtained by perturbation analysis. In the first case the usual dipole vortex is used as the zero order solution, and new solutions which are close to but distinctly different from it are found. In the new solutions the separatrix is noncircular and the vorticity in the inner region is a nonlinear functional of the stramfunction. In the second case the dispersive term in the equation is treated as a small parameter, and a radially symmetric solution (a monopole vortex) is used as the zero-order approximation.

Both families of solutions are found to be infinite and contain an arbitrary function. A recent general proof of the existence of infinitely many stationary solutions containing an arbitrary function is also examined and found to be invalid.

# *COMPLEX-GEOMETRY STABILITY THEORY: NUMERICS AND PHYSICS*

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*NONLINEAR ASPECTS OF TURBULENCE IN PLASMA AND NEUTRAL FLUID**

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# NONLINEAR ASPECTS OF TURBULENCE IN PLASMA AND NEUTRAL FLUID

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Fluctuations measured in a transitioning wake and in the turbulent edge of a tokamak plasma have been analyzed by means of a simple model consisting of a quadratically nonlinear system of the form

$$Y_f = L_f X_f + \frac{1}{2} \sum_{\substack{f_1, f_2 \\ f=f_1+f_2}} Q_f^{f_1,f_2} X_{f_1} X_{f_2} . \tag{1}$$

The spectral components of the input $X_f$ and output $Y_f$ are the Fourier transforms of the raw fluctuation data. The linear and quadratic transfer functions $L_f$ and $Q_f^{f_1,f_2}$ are estimated by higher order spectral analysis [1].

We have analyzed the temporal fluctuation of a **transitioning flow in the wake** of a thin flat plate, which is collected at two adjacent downstream positions with hot wire probes. The amplitude of the quadratic transfer function is shown in Figure 1a for the downstream location of $15cm$.

The quadratic transfer function of the experimental data shows strongest interaction efficiency for coupling between high frequency components (several times the fundamental frequency) which alter the energy at the low frequency components below the fundamental frequency. The interaction process shows only gradual change with downstream position.

The quadratic coherency, which is related to the quadratic transfer function,

$$\gamma_Q^2(f_1, f_2) = \sum_{\substack{f_1, f_2 \\ f=f_1+f_2}} \frac{1}{4} |Q_f^{f_1,f_2}|^2 \frac{\langle |X_{f_1} X_{f_2}|^2 \rangle}{\langle Y_f Y_f^* \rangle} , \tag{2}$$

measures the relative amount of quadratic interaction. Figure 1b shows that the dominant change of output power through quadratic interaction results from the interaction of the fundamental frequency with the first harmonic $2f_0$ generating the second harmonic frequency $3f_0$. This process is local. Only a few centimeters further upstream the $f_0, f_0, 2f_0$ interaction triplet dominates the coherency spectrum.

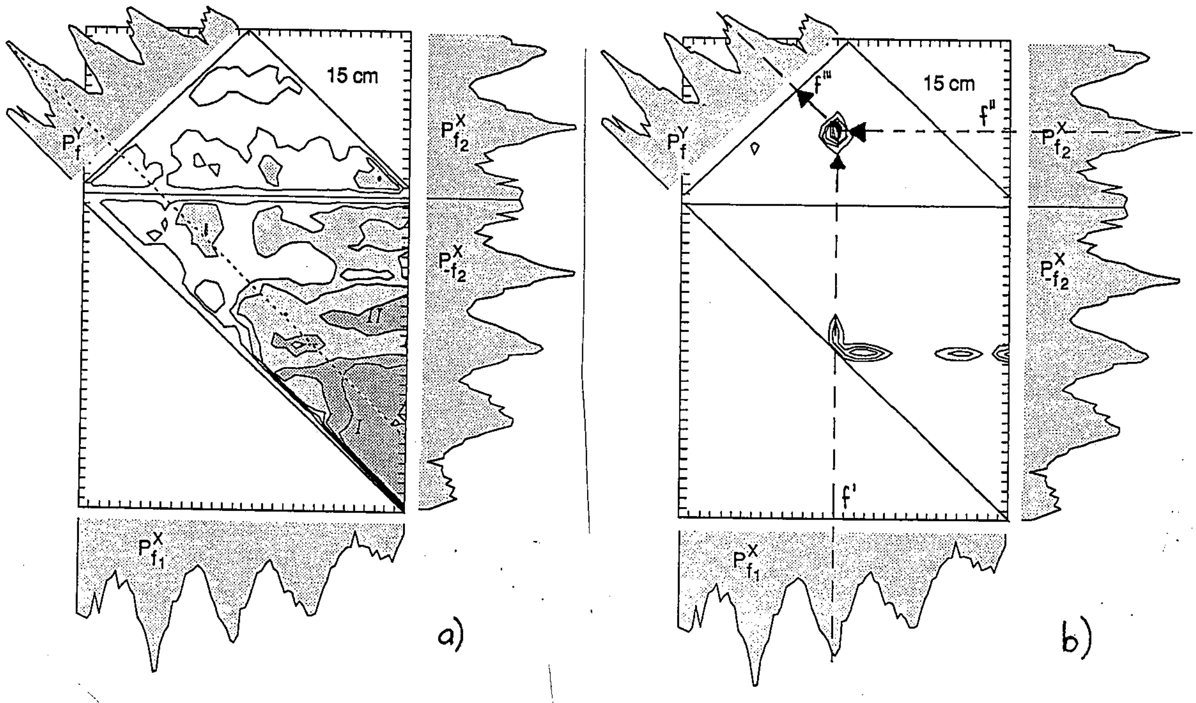


Figure 1: a) Quadratic transfer function $Q_f^{f_1,f_2}$ in the transitioning flow at distance $x = 15cm$ from the plate. The corresponding axes show the power spectra of the input signal $P_f^X$ and output signal $P_f^Y$. b) Local cross-bicoherency $\gamma_Q^2(f_1, f_2)$ for $x = 15cm$. The maximal value at this downstream location occurres through interaction of the fundamental frequency $f''$ and the first harmonic frequency $f'$ changing the output signal at the second harmonic frequency $f'''$.

The results extracted from the fluctuations in a wake can be compared with the spectral evolution with downstream position. For all position measured in the wake we find a good qualitative agreement between the change of the power spectrum with downstream position and the local change in energy (i.e. the local coherency $\gamma_Q^2(f_1, f_2)$) through three wave interactions. Because the coupling properties do not change abruptly in the region where progressively higher harmonics of the fundamental mode and the filling in of the valleys takes place, this process seems mainly to be a result of spectral redistribution of energy which is available in the interacting modes.

In a similar way the turbulence in the **edge plasma of the TEXT tokamak** is analysed. The result is however brought one step further in that the energy transfer between the spectral component is computed. For the dominant components of the spectrum we find that the energy exchange between the modes of frequency $f_1, f_2, f_1 + f_2$ is approximately conserved. The results indicate an energy cascading through three wave interaction away from the dominant peaks mainly toward lower frequency components and in a smaller amount toward higher frequency components. The qualitative agreement with a conservation law merely has to be interpreted as a test of the method and not as a test of the conservation law. This because the gain or loss of energy of each frequency component through interaction with the other partners is measured individually and thus depends on the quality of the data and the model.

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# *FLUX CONVERSION AND THE TAYLOR STATE**

N. Salingaros

Flux Conversion and the Taylor State. N. A. Salingaros, The University of Texas at San Antonio

The popular Bessel-function model succeeds in describing the magnetic profiles of the reversed-field pinch, and in predicting the value 1.20 for the critical pinch parameter $\theta \approx 1.40$. This success, coupled with the variational arguments of Chandrasekhar and Woltjer on minimum-energy static magnetic configurations, has guided the development of the spheromak. The original variational arguments are, however, in error (Int. J. Mod. Phys. B 1 (1987), 1329-1349), so that the success of the Taylor model has to be explained on other terms. It was proposed (Hadronic J. 10 (1987), 109-116) that energy equipartition between orthogonal magnetic modes, which the Bessel-function model satisfies, is a basic requirement for stability. Further, it is experimentally observed that any departure from equipartition induces spontaneus flux conversion in the plasma. This flux conversion mechanism can be postulated as one of the significant components of the plasma relaxation process. From this assumption, one can derive a better theoretical value of 1.41 for the critical pinch parameter of the reversed-field pinch (loc. cit.). Work in progress examines spheromak plasmas from this alternative conceptual viewpoint.

# *INTERMITTENCY STRUCTURES AND THEIR EFFECT ON THE TRANSPORT OF MAGNETIC FIELD IN TURBULENT FLUID FLOW*

P. Similon

Intermittency Structures and their Effect on the Transport of Magnetic Field in Turbulent Fluid Flow.

P. L. Similon, Cornell University.

The problem of magnetic field behavior in a turbulent flow arises in particular in the context of stellar dynamo. Some of the issues are which are the mechanisms generating magnetic fields, and what is the structure of the magnetic field, in space and in time.

Passive turbulent convection of scalar or magnetic fields has been the subject of extensive work, but still poses a challenge to theories. For instance, standard closure theories, such as Kraichnan's Direct-Interaction Approximation (DIA), do not adequately take into account helicity correlations (related to fourth-order velocity moments), and predict erroneously the identity of magnetic field diffusion to scalar diffusion, in mirror-symmetric isotropic turbulence. Robert Kraichnan[1,2] pointed out these deficiencies, and analysed the so-called $\alpha^2$-effect ($\alpha$ is a second order moment proportional to the mean helicity), using a second averaging procedure on the DIA, as well as by direct numerical simulation. It was then shown that magnetic diffusivity could even be negative, in mirror-symmetric isotropic turbulence, provided that the correlation length and time of helicity fluctuations exceeded the eddy size and turnover time.

I investigate here the effect of the correlations between eddies of neighboring size on the passive field evolution. Such correlations occur because of the strong memory which exists during the decay of an eddy of a given size into smaller eddies, as part of the turbulent cascade process. Because of the localization of the interaction, simultaneously in position space and in Fourier space, this effect is very difficult to describe with low order moment closure theories. As an alternative, I use a model of turbulence somewhat reminiscent of the $\beta$-model of Frisch, Sulem and Nelkin.[3] The turbulent flow is assumed to be composed of a hierarchy of vortices of all scales, characterized by their size, orientation, location, intensity, and helicity. The turbulent cascade is modelled as a Markov process, where transition probabilities regulate the decay of large vortices into smaller vortices. Thus, one maintains a strong correlation

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between vortices of immediately neighboring scales, although memory gets lost after further cascading steps.

The simplicity of this turbulence model allows the analysis, without the use of approximate closures, of the convection of a tensor field (such as a scalar field or magnetic field), Lie-dragged in the flow[4]. In particular, I derive the renormalization group equation which is satisfied by the average response function of the tensor field. The fixed point of the transformation can be numerically determined, given the vortices which compose the turbulence. It is finally shown that the memory of the transition from larger to smaller eddies enhances the dynamo action and reduces the magnetic field diffusivity.

[4]Differential Geometry concepts are particularly convenient for this problem, and can be found, for instance, in B. Schutz, "Geometrical Methods oh Mathematical Physics", Cambridge University Press, New York (1980).

# *TURBULENT RELAXATION AND RECONNECTION*

H. Strauss

TURBULENT RELAXATION AND RECONNECTION

H. Strauss

Current carrying plasmas can be unstable to tearing modes. The modes cause a flattening or relaxation of the current gradient. This process can be described as an anomalous electron viscosity or hyper-resistivity.

To calculate the hyper-resistivity, reduced MHD equations are used (Strauss, Phys. Fluids 1984, 1985). Numerical solutions of the equations agree with full MHD computations and show sustained magnetic field reversal for RFP parameters. The hyper-resistivity which affects the mean field evolution can be calculated quasilinearly. The hyper-resistive term conserves magnetic helicity while dissipating energy. Taylor's relaxed force free state is obtained in the limit if infinite hyper-resistivity. To get agreement with the experimentally observed relation between field reversal and current, one finds that the hyper-resistivity is comparable to the resistivity. The hyper-resistivity vanishes at a conducting wall, allowing the current to have a large gradient there. On the basis of this theory, RFP experiments with a resistive wall would be expected to behave badly.

Using a form of direct interaction approximation, it can be shown that the effect of tearing modes on each other can also be described by hyper-resistivity. This gives a larger tearing mode growth rate. With this turbulent growth rate, the hyper-resistivity scales as the cube of the r.m.s. magnetic fluctuation amplitude (Strauss, Phys. Fluids 1986).

These results can be applied to reconnection. In Sweet-Parker reconnection, a current sheet is formed by the merging of magnetic fields of opposite signs. The merging rate scales as the square root of the resistivity, which is quite slow. Many attempts have been made to find a mechanism which gives a faster rate. It seems likely that the current sheet could be tearing mode unstable. If the magnetic field has a component parallel to the current, the sheet could disrupt. Calculating the merging rate using the turbulent hyper-resistivity, gives a rate proportional to the r.m.s. magnetic fluctuation level and a macroscopic Alfvén frequency. This merging rate is consistent with the time scale of solar flares (Strauss, Ap. J. 1988) and might also explain fast sawtooth crashes of tokamak internal kink modes.

# *THE GREAT RED SPOT OF JUPITER*

H. Swinney

U.S.-Japan Workshop Plasma and Fluid Turbulence
Austin, Texas, 7-10 December 1987

# The Great Red Spot of Jupiter

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Large isolated stable vortices have long been observed in the turbulent Jovian atmosphere. Such vortices have also been observed recently on Saturn, but the origin of such vortices has remained an unsolved problem in fluid dynamics. A laboratory experiment has been designed to mimic the conditions of a planetary atmosphere—a turbulent two-dimensional flow is produced in a rapidly rotating annulus.[1] It is found that a single stable vortex forms in the annulus for a wide range of conditions. The experimental results support a model of the Great Red Spot recently proposed by Marcus.[2]

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1. J. Sommeria, S.D. Meyers, and H.L. Swinney, submitted to Nature.
2. P. S. Marcus, submitted to Nature.

*BICRITICALITY AND MODE INTERACTION IN FLOWS BETWEEN COUNTERROTATING CYLINDERS**

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# *TURBULENCE TRANSPORT IN FLOW BETWEEN CONCENTRIC CYLINDERS*

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# Turbulence Transport in Flow Between Concentric Cylinders

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**Abstract**

We have studied mass transport in a turbulent flow with large coherent structures — turbulent Taylor vortices.[1] A pulse of dye is injected in fluid contained between concentric cylinders (with the inner one rotating and the outer one fixed), and the time dependence of the dye concentration at two axial positions is then determined from optical absorption measurements. The measurements have been made for radius ratios $\eta$ ranging from 0.494 to 0.875, and at Reynolds numbers R ranging from 50 to 1000 times that corresponding to the onset of Taylor vortex flow. Transport in the axial direction is found to be modeled very well by a one-dimensional diffusion process with the reflections at the ends of the annulus and the finite injection time of the dye are taken into account. (The time scale for the transport in the radial and azimuthal directions is short compared to that in the axial direction.) The effective axial diffusion coefficient D in the parameter range studied is of the order of 1 $cm^2/s$ or greater, orders of magnitude larger than molecular diffusion coefficients. For a fixed R and $\eta$, D increases linearly with the axial wavelength $\lambda$ of the Taylor vortices. The Reynolds number dependence of the wavelength-independent scaled diffusion coefficient, $D^* = (2d/\lambda)D$ [where d is the gap between the cylinders], is described by a power law, $D^* \propto R^{\beta}$. Measurements for different parameter regions yield $0.69 < \beta < 0.86$, while theory suggests a larger value, $\beta = 1$.

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*INTERACTION OF A STRONG VORTEX WITH DECAYING TURBULENCE*

P. Terry

# Interaction of a Strong Vortex with Decaying Turbulence

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The evolution of a localized, axially symmetric vortex under the action of shear stresses assosciated with decaying two-dimensional turbulent vorticity which is inhomogeneous in the presence of the vortex is studied analytically. For a vortex which is sufficiently strong relative to the coefficient of turbulent eddy viscosity, it is shown that turbulent fluctuations in the vortex interior localize to the vortex periphery. Similarly, the coefficient of diffusion differs from zero only in a narrow layer at the vortex periphery and is small compared to the coefficient of eddy viscosity. Precise conditions for the vortex strength in terms of its turnover rate relative to the eddy damping rate for turbulent fluctuations of scale both larger and smaller than the vortex are given.

Presented at the US-Japan workshop on Plasma and Fluid Turbulence, December 1987

## *A DIAGNOSTIC TOOL FOR SPATIOTEMPORAL CHAOS**

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*RELAXATION AND TRANSPORT DUE TO RESISTIVE G-MODES*

M. Wakatani

# RELAXATION AND TRANSPORT DUE TO RESISTIVE $g$-MODES

M. WAKATANI, H. SUGAMA and M. YAGI
A. HASEGAWA

In the Heliotron $E$ high beta experiment pressure-driven internal disruption was observed.[1] The main characteristics are explained in the context of the nonlinear evolution of the resistive interchange modes[2] based on the reduced MHD equations for stellarator/heliotron.[3,4]

Transport analysis of Heliotron $E$ plasmas showed that anomalous transport exists in the edge region.[5] The characteristics of the edge turbulence in Heliotron $E$ observed by using Langmuir probes[6] are similar to those in tokamaks.[7] This result suggests that physical mechanism to explain the edge turbulence may be universal in toroidal plasmas confined by sheared magnetic field and equilibrium plasma current does not play an important role, since current free plasmas can be confined in Heliotron $E$. In the Heliotron $E$ plasma, a probable candidate to explain the edge turbulence is the resistive interchange modes coupled to the resistive driftwaves. Three-dimensional numerical calculations show a kind of self-organized structure due to energy condensation at $m = 0/n = 0$ mode.[8] The model equations are extended to include parallel ion motion which does not change characteristics of the self-organized structure significantly. Poloidal shear flow can be implemented in the same model equation which makes it possible to study a Kelvin-Helmholtz instability.

Finally turbulent transport due to $g$-modes is discussed by using the scale invariance theory.[9] It was shown that the scale invariance, the mixing length theory and the renormalized theory give the same transport scaling.[10,11] It is pointed out that the scaling for the poloidal mode number is crucial in the comparison between the different approaches. An application of the scale invariance approach to the Navier-Stokes equation is given.

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# REVIEW OF ION TEMPERATURE GRADIENT MODE TURBULENCE

R. Waltz

# Review of Ion Temperature Gradient Mode Turbulence*

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It is believed that there are two mechanisms for the turbulent transport in the core of tokamaks below the beta critical for ideal MHD ballooning instability: trapped electron modes and ion temperature gradient (ITG) modes. This talk reviews the second for an audience accustomed to dealing with classical hydrodynamic turbulence. It covers the long history of the ITG-mode as a linear instability and the recent progress [1,2] in the numerical simulation of the turbulence among such modes and its resulting transport. It emphasizes the fluid nature of the ITG-modes and, in the case of toroidal geometry, the similarity and differences with the Rayleigh-Taylor overturning modes in a gravitationally stratified fluid. The problem of dealing with dissipation at short wave lengths (the problem of high Reynolds number) is addressed in terms of its minimal role in determining the transport provided by the long wave lengths. The general importance of scaling laws, mixing length roles, and quasilinear theory to understanding turbulent transport in magnetically confined plasmas is stressed.

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# *APPLICATION OF KRAICHNAN'S DECIMATED-AMPLITUDE SCHEME TO THE BETCHOV MODEL OF TURBULENCE*

T. Williams and E. Tracy

Abstract of talk given Tuesday December 8, 1987. Talk given by E. Tracy.

# Application of Kraichnan's Decimated-Amplitude Scheme to the Betchov Model of Turbulence

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A recent test of the Decimated Amplitude Scheme (DAS) of Kraichnan[1] was reported[2]. The DAS was applied to a random coupling model of turbulence originally introduced by Betchov[3] as a test of the Direct Interaction Approximation (DIA). A brief review of the DAS and the Betchov model was presented. In the DAS the large system of equations describing a turbulent system is replaced by a much smaller 'sample set' of equations. The couplings to degrees of freedom which have been removed in the decimation are modeled by stochastic forces. In order to capture the correct statistical behavior of the original large system these stochastic forces must satisfy constraints. Through the use of a small Betchov system, forced stochastically under appropriate statistical constraints, it was shown that the DAS can accurately compute the autocorrelation function of a much larger Betchov system. In ref.(2) the larger Betchov system contained three times as many degrees of freedom as the smaller system(3:1 decimation). In this talk, further results (based on a new constraint scheme, different from that used in ref.(2)) were reported where the decimation strength has been pushed much higher. Accurate results have been obtained for decimation strengths of up to $10^6$:1.

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## *TURBULENCE AND ELECTRON HEAT TRANSPORT IN TEXT*

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*A CHAOTIC MODEL OF FULLY DEVELOPED TURBULENCE*

M. Yamada

# A Chaotic Model of Fully Developed Turbulence

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Properties of fully developed turbulence still lack an appropriate interpretation in the strange attractor theory in spite of recent advances in the theory of chaotic dynamical systems. In particular the scaling property in the inertial range has not yet been understood as any characteristic of strange attractor, mostly because numerical techniques so far developed are not powerful for high-dimensional attractors of partial differential equations which require much larger computer capacity than present ones. At the present stage, one of the possible strategies is to investigate in detail a chaotic model system which is tractable in size and has the scaling property similar to real fluid turbulence.

We study a model equation of two-dimensional turbulence which was first proposed by Gledzer. The model is originally devised to have both the $k^{-3}$ and $k^{-5/3}$ spectra in the enstrophy cascade and the energy inverse cascade ranges respectively as a steady solution. The model is constructed in wavenumber space descretized by octaves. One real scalar variable which stands for Fourier components in the shell is associated with each wavenumber. The evolution equation of each scalar variable is quadratically nonlinear and connected with two preceding and two succeeding ones. The nonlinear interaction conserves both the energy and the enstrophy. We introduce constant forcing terms and viscous dissipation terms to keep the system stationary (ref.1).

We investigate unsteady solutions of the model equation numerically. With an arbitrary chosen initial condition and after an initial transient period, unsteady but apparently stationary state is realized. In this stationary state we obtain the time-averaged energy spectrum, which is normalized following the enstrophy cascade theory by Batchelor, Kraichnan and Leith (ref.2). The normalized spectra for several values of viscosity agree well to each other, and at the same time the enstrophy flux is faily constant in the inertial range where the energy spectrum taken $k^{-3}$-form.

We calculated all the Lyapounov exponents and the Lyapounov vectors in the stationary state. In each case we calculated, some of the Lyapounov exponents are positive, indicaing that the system is in a chaotic state. In the inertial range the spectrum of the first Lyapounov vector takes a power-form like $k^{-1.7}$. The dependence of of the Kaplan-Yorke dimension D on the viscosity is consistent with the enstrophy cascade theory in the sense that $2^D$ is proportional to $k_d$ where $k_d$ is the dissipation wavenumber.

The Lyapounov exponents, which are ordered as $\lambda_1>\lambda_2>\cdots>\lambda_N$, themselves have a scaling property. The curve made of the points,

$$( j/D, \Sigma_{i=1}^{j}\lambda_i/H) \text{ for } j\leq D \text{ (interior of the attractor (ref.3))},$$

for each value of viscosity agrees well to each other, where H is the sum of the positive Lyapounov exponents. On the other hand, the Lyapounov exponents for j>>D (exterior of the attractor)coincide well to corresponding viscous dissipation rates. It is interesting that the distribution function of the Lyapounov exponent obtained numerically appears to have a singularity at null Lyapounov exponent (ref.4).

A relation is observed between the inertial range and the Lyapounov vectors: The Lyapounov vectors which are associated with positive Lyapounov exponents have their support in the enstrophy containing range, while Lyapounov vectors with null and negative exponents in the interior of the attractor ($j\leq D$) do in the inertial range. Therefore, in this model equation, the inertial range is characterized as the support of Lyapounov vectors which correspond to non-positive Lyapounov exponents in the interior of the attractor.

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