DOE/ET-53088-312 IFSR **#312**

Drift Wave Vortices and Anomalous Transport

W. Horton

Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

April 1988

Drift Wave Vortices and Anomalous Transport

W. Horton
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

Abstract

Theory and computer simulations are used to describe the inelastic vortex-vortex and vortex-wave interactions that lead to the quasi-coherent transport of plasma across a constant magnetic field. Monopole and dipole drift wave vortices with radii r_0 large compared with the ion inertial scale length ρ_s are shown to produce transport at the rate $un_v \int d\sigma(b) \leq n_v v_{de} r_0$ where n_v is the vortex line density and $d\sigma(b)$ is the inelastic collision cross-section for impact parameter b. The transport during collisions and mergings is evaluated from the evolution of a passively convected scalar concentration of test particles.

I. Introduction

There has been numerous studies[1–6] of the anomalous plasma transport produced by drift waves from the point of view of turbulence represented by a spectrum $\mathbf{k}\omega$ of wave fluctuations. Typically, weak turbulence theory[1–4] or renormalized turbulence theory[4–6] based on a short correlation time and weak field correlations is used to calculate the anomalous transport due to the fluctuation spectrum. In the present work we consider transport arising in the opposite limit where coherent or quasi-coherent vortices[7] interact through vortex collisions and mergers.[8,9] During these interactions $\mathbf{E} \times \mathbf{B}$ streamlines reconnect allowing the rapid transport of plasma from one vortex center to another as well as the loss of some plasma to the wake of emitted drift waves.

In contrast to small amplitude drift waves, which spread out and lose strength as they travel, the solitary drift wave vortices are coherent, self-sustaining packets that retain their strength over long distances. These coherent vortices trap particles[10] that are consequently also transported over long distances.

The anomalous transport of plasma is determined from the net flux $\langle v_x f \rangle$ of a test particle field $f(x, y, t)$ being passively $\mathbf{E} \times \mathbf{B}$ convected by the nonlinear drift wave flows. Depending on the particular application, the passive field f may represent the temperature, the poloidal flux function, or the density of charged particle species. Using a typical coupling of the test field into the drift wave equation, an estimate is made for the maximum value of the coupling below which the the passive approximation remains valid during the time of a typical vortex collision.

The vortices studied are of two types: the monopole vortex representing a net excess in the local charge density and the dipole vortex representing a local polarization in the local charge density. A vortex of scale k^{-1} and amplitude Φ_k is characterized by the internal circulation rate $\Omega_E(k) = ck_x k_y \Phi_k / B$. The vortex turnover time is $2\pi/\Omega_E(k)$.

The vortex behavior of self-binding and the trapping of the drift wave wake sets in when $\Omega_E(k) > \omega_k$ the linear drift wave frequency. In the case of monopoles the transition from wave packet behavior for $\Omega_E \ll \omega_k$ to a wakeless vortex behavior $\Omega_E \gg \omega_k$ is not sharp but, nevertheless, is clearly shown by the simulations when $R_E = \Omega_E/\omega_k \simeq 1-2$. For dipoles the transition to the wakeless solitary drift wave is sharp and given by the theoretical formula for the two parameter (r_0, u) family of Bessel function dipole vortices.[7] Here r_0 is the radius of the inner negative curvature part of the vortex field, and u is the speed of propagation. The properties of the dipole vortex solutions are extensively developed in Ref. 7 and will not be repeated here. We note that the minimum amplitude $\Phi_m(r_0, u)$ for the dipole vortex solution is just above the corresponding mixing length level amplitude with r_0 taking the role of $1/k_x$ in the mixing length formula. For example, the dipole vortex amplitude given in Eq. (26) of Ref. 7 is $c\Phi_m/B = 1.28 v_{de} r_0$ where v_{de} is the electron diamagnetic drift speed.

The dipole vortex structure is the natural, long-time, finite amplitude solution for the problem in the absence of a sheared $\mathbf{E} \times \mathbf{B}$ flow and the monopole is the natural, finite amplitude solution in the presence of a sheared flow. The relevant sheared $\mathbf{E} \times \mathbf{B}$ flow can arise from neighboring vortices or long wavelength waves $(\lambda_\perp \gg r_0)$ in which the vortices are formed. Dipole vortices are produced in the turbulent wake of a 2D fluid flowing past an obstacle. The double sheared flow in the wake of the obstacle produces alternating signed vortices which are observed to merge into dipole vortices.[11]

Dipole vortex collisions have been studied before in both numerical simulations[8,9] and experiments[11–13] with neutral fluids. The numerical studies have considered the zero impact parameter $b = 0$ collisions which are elastic in nature and show no net transport of test particles across the magnetic surface. Here we show that the maximum inelasticity and net transport occurs for the impact parameter $b \simeq r_0$ where r_0 is the radius is the larger of the two colliding dipole vortices. In an example shown here, the final state after the collision is a newly formed single dipole vortex along with a monopole vortex and a drift wave wake.

During the course of the vortex collisions the passively convected field develops very steep gradients within the vortex due to the merging and mixing of the combined vortical flows. The steep gradients are then acted on by the background classical diffusivity to produce a rapid transport across the magnetic surfaces within the vortex. In the limit of vanishing background diffusivity the collisions produce very fine scale granulations and associated fluctuations due to the area preserving mixing within the vortices. Without a background diffusivity the energy or enstrophy moment (mean square gradient) associated with the passive field diverges in time due to the continual mixing into finer scales. In the simulations we limit the increase of the gradients with a background diffusivity D such that the Peclet number $P = \tilde{v}\ell/D = c\Phi/BD \leq 10^4$ which is typical of a tokamak plasma. This limit on P prevents the number of Fourier modes required for resolving the field from exceeding $(256)^2$.

The vortex dynamics itself can be computed in the Hamiltonian or inviscid limit (Reynolds number $R_{en} = \tilde{v}\ell/\nu \to \infty$) without an appreciable change in the enstrophy ($\Delta U/U \lesssim 10^{-4}$) or energy ($\Delta E/E \lesssim 10^{-6}$) during the course of a typical collision $t_{\rm col} \sim 4r_0/\Delta u < 100\rho_s/v_{de}$. Thus, the self-consistent velocity field maintains its smoothness during the interactions and mergings.

The study of large scale vortex dynamics can be further motivated by the observation that the drift mode fluctuation measurements indicate that the maximum fluctuation amplitudes occur at the longest length scales where $\pi/k_\perp \sim r_0 \gg \rho_s$. Furthermore, the measured amplitudes may be sufficiently high[3] so as to put the fluctuations in the regime $R_E = \Omega_E(k)/\omega_k > 1$ where theory and simulations predict that the vortex features of the dynamics dominates over dispersive wave features. The neglect of magnetic shear and profile variation limit the maximum vortex size to $r_0 \leq \min(r_n, L_s\rho_s/r_n)$ in the present work.

The dominance of the large scale modes in the measured fluctuations may be understood from the quasi-2D nature of the magnetohydrodynamic system. Simulations for 2D hydrodynamics and magnetohydrodynamics show that fluctuation energy is transformed to the largest scale of the system. In quasi-2D systems where $\nabla_\perp \gg \nabla_\parallel$ there remains a strong tendency to build up large scale fluctuations. In tokamak experiments the largest scales that are not strongly damped are determined by the magnetic shear which is not included in the present study.

In Sec. II the dynamical equations, conservation laws, and dimensionless parameters of the study are given. In Sec. III the properties and dynamics of the drift wave vortices are given. In Sec. IV the dynamics and anomalous transport during inelastic collisions with merging and scattering processes are studied. In Sec. V the anomalous transport model for a vortex gas is developed and compared with the transport model from weak turbulence theory. In Sec. VI the summary and conclusions are given.

II. Drift Wave and Transport Equations

In the dissipationless limit the basic model equation for drift waves is the Hasegawa-Mima equation.[1] The same basic nonlinear wave equation describes the slow, nearly incompressible motions of shallow rotating fluids. The rapidly rotating neutral fluid experiments serve as analog simulations of drift waves described by the Hasegawa-Mima equation. The analog of the $\mathbf{E} \times \mathbf{B}$ plasma drift velocity in the rotating fluid is the geostrophic flow $\mathbf{v}_\perp = (g/f)\hat{\mathbf{z}} \times \nabla h(x,y,t)$ arising from the balance of the Coriolis force $f\mathbf{v} \times \hat{\mathbf{z}}$ with $f \equiv 2\Omega_v = 2\times$ (vertical component of the angular rotation frequency) with the pressure gradient $\nabla p = \rho g \nabla h$. Here $h(x,y,t)$ is the variable part of the fluid depth $H = H_0(x) + h(x,y,t)$. The analog of the plasma quasineutrality condition $\nabla \cdot \mathbf{j} = 0$ determining the dynamics of the electrostatic plasma potential $\Phi(x,y,t)$ is the height integrated continuity equation for the neutral fluid which gives $dH/dt + H\nabla \cdot \mathbf{v}_\perp = 0$ where the compressible part of the flow velocity $\mathbf{v}^{(2)}$ is given by the balance of the Coriolis force with the inertial acceleration giving $\mathbf{v}_\perp^{(2)} = f^{-1}\hat{\mathbf{z}} \times d\mathbf{v}/dt = -(g/f^2)d\nabla h/dt$. The non-geostrophic flow $\mathbf{v}^{(2)}$ is the analog of the plasma polarization current. The scale length for wave dispersion follows from these compressible inertial drift velocities and is $\rho_s = c(m_i T_e)^{1/2}/eB = c_s/\omega_{ci}$ for the plasma drift wave and $\rho_g = (gH)^{1/2}/f$ for the Rossby wave. Here the ion acoustic speed $c_s = (T_e/m_i)^{1/2}$ is analogous to the gravity wave speed $c_g = (gH)^{1/2}$.

The derivation and procedure for solving the dissipative drift wave equation used here is given in Terry and Horton.[5] The dissipative drift wave equation differs from the Hasegawa-Mima equation in the presence of electron dissipation. The plasma density fluctuations given by

$$\delta n_e(\mathbf{k}, t) = n_{e0}\left[1 + i\delta_0 k_y(c_1 - k_\perp^2)\right]\left[e\phi(\mathbf{k}, t)/T_e\right]$$

effects the nonlinear mode coupling in the Hasegawa-Mima, equation due to the presence of the $\mathbf{v}_E \cdot \nabla n_e$ nonlinearity in the electron continuity equation. It is the $\mathbf{v}_E \cdot \nabla n_e$ nonlinearity that causes saturation of the drift wave turbulence at the mixing length level. Recently, Kono and Miyashita[14] report simulations of dissipative drift wave turbulence that evolves through an inverse cascade to a final state containing a large amplitude dipolar vortex structure.

Statistical turbulence theory was compared with strong ($\delta_0 \simeq 0.1$) driven turbulence in Ref. 6. With strong driving the vortices appear unimportant compared with short lifetime fluctuations: in the long-time limit with reduced values of δ_0 long-lived vortex structures emerge from the turbulence.

The plasma equations used in the present study are conveniently written in the usual dimensionless variables as

$$(1+\mathcal{L})\frac{\partial\varphi(x,y,t)}{\partial t} + v_d\frac{\partial\varphi}{\partial y} + \left[\frac{\partial\varphi}{\partial x}\frac{\partial}{\partial y}(\mathcal{L}\varphi) - \frac{\partial\varphi}{\partial y}\frac{\partial}{\partial x}(\mathcal{L}\varphi)\right] + \nu\nabla^4\varphi = 0 \tag{1}$$

with

$$\mathcal{L} = -\nabla^2 + \delta_0(c_1 + \nabla^2)\partial_y$$

and the convection of the passive scalar distribution f

$$\frac{\partial f(x,y,t)}{\partial t} + \frac{\partial\varphi}{\partial x}\frac{\partial f}{\partial y} - \frac{\partial\varphi}{\partial y}\frac{\partial f}{\partial x} - D\nabla^2 f = 0\ . \tag{2}$$

The equations are solved using fifth and sixth order Runge-Kutta variable time stepping and a truncated Fourier series transformation $(x, y) \leftrightarrow (k_y, k_y)$ at each time step. The principal computational improvements since the earlier turbulence work[6] are the use of the CRAY II and the vectorized (VCFT) fast Fourier transform. The runs presented are from either a $(85)^2$ k space with $N = 3612$ $\varphi_{\mathbf{k}}(t)$ modes or $(171)^2$ k space with $N = 14620$ $\varphi_{\mathbf{k}}(t)$ modes using the $(128)^2$ and $(256)^2$ FFT respectively. Typical CPU times for a vortex collision experiment are 50 min CPU/100 r_n/c_s or 200 min CPU/100 r_n/c_s, respectively.

The symmetry property of Eq. (1) is that for every solution $\varphi(x, y, t)$ there is a reflected solution $\overline{\varphi}$ with $\overline{\varphi} = -\varphi(-x, y, t)$. The nonlinear equation does not have the symmetric solutions $\varphi(-x, y, t) = \varphi(x, y, t)$ of the linear wave equation.

Equations (1) and (2) are written in the usual dimensionless drift wave units of ρ_s for x, y and $r_n/c_s = \rho_s/v_d$ for t and with a constant $k_{\|}r_n$ assumption. The electrostatic potential Φ is given in terms of $\varphi(x, y, t)$ by $e\Phi/T_e = (\rho_s/r_n)\varphi(x, y, t)$. For quasi-geostrophic dynamics the equation (1) follows for x, y in units of the Rossby radius ρ_g, time in units of ρ_g/v_R with $v_R = (H_0 g/f)\partial_x \ell n(f/H_0) = c_g(\rho_g/L)$ where $c_g = (gH_0)^{1/2}$ and the amplitude $h/H_0 = (\rho_g/H)\varphi$. The dissipation coefficient ν for the plasma arises from the ion-ion collisional viscosity $\nu = 0.3(r_n\nu_{ii}/c_s)(T_i/T_e)$ and for the neutral fluid $\nu = \nu_{\text{vis}}/\rho_g v_R$ from viscosity. The dimensionless diffusion coefficient for test electrons is $D = (m_e/m_i)(r_n\nu_{ei}/c_s) \sim (m_e/m_i)^{1/2}\nu$ and for test ion is $D \sim \nu$. For typical tokamak plasmas $\nu \sim 10^{-3}$ and $D \simeq 10^{-4}$.

The transport equation (2) is integrated simultaneously with the vorticity equation (1) to determine the convection of plasma test particle field. Depending on the application, the passive plasma field $f(x, y, t)$ may be the distribution of test particles, the temperature or the poloidal flux function. Other studies with f actively coupled back into the vorticity equation will be reported later. Here we note that a typical form of the coupling for the back reaction of f onto φ is the addition to the right-hand side of Eq. (1) of the term $-g(\partial f/\partial y)$ or $B_p\partial_y(\nabla^2 f)$. When the back reaction coefficient exceeds a certain critical value the vorticity

$B_p\partial_y(\nabla^2 f)$. When the back reaction coefficient exceeds a certain critical value the vorticity equation is destabilized. Below this value the effect of the back reaction is small provided $(u - v_d)\|\varphi\| \sim k_\perp^2 v_d \|\varphi\| \gg g\|f\|$.

Here we define the characteristic dimensionless numbers describing the state of the system governed by Eqs. (1) and (2). A fluctuation of amplitude $\Phi_{\mathbf{k}}$ and scale $\mathbf{k} = (k_x, k_y)$ has a linear frequency of oscillation $\omega_{\mathbf{k}}$ and a nonlinear frequency of circulation $\Omega_E(\mathbf{k})$ given by

$$\begin{aligned} \omega_{\mathbf{k}} &= \frac{k_y v_d}{1 + k_\perp^2 \rho_s^2} \longrightarrow \frac{k_y}{1 + k_\perp^2}\left[\frac{c_s}{r_n}\right] \\ \Omega_E(\mathbf{k}) &= \frac{c k_x k_y}{B}\Phi_{\mathbf{k}} \longrightarrow k_x k_y \varphi_{\mathbf{k}}\left[\frac{c_s}{r_n}\right] . \end{aligned} \tag{3}$$

The ratio of the vortex rotation frequency $\Omega_E(\mathbf{k})$ to the linear frequency $\omega_{\mathbf{k}}$ is defined as the dissipationless $\mathbf{E} \times \mathbf{B}$ Reynolds number R_E and is given by

$$R_E = \frac{\Omega_E(\mathbf{k})}{\omega_{\mathbf{k}}} = k_x r_n \left(\frac{e\Phi_{\mathbf{k}}}{T_e}\right) = k_x \varphi_{\mathbf{k}} . \tag{4}$$

The magnitude of R_E measures both the balance of nonlinearity in Eq. (1) with wave dispersion and the balance of nonlinearity in Eq. (2) with the convection over the linear gradient $r_f^{-1} = -\partial_x \ell n f(x)$. The mixing length level is defined by $R_E \sim 1$ and the self-binding vortex dynamics occurs for $R_E \gtrsim 1$.

Conservation Laws

The drift wave equation (1) in the dissipationless limit poses three conservation laws: mass, energy, and enstrophy. The equation itself is a statement of local momentum conservation.

(1) Mass Conservation

Integrating (1) over all x, y gives

$$\frac{d}{dt}\varphi_{\mathbf{k}=0} = \frac{d}{dt}\int \varphi dx dy = 0 \tag{5}$$

which is equivalent to the conservation of mass since the plasma density is $N = N_0\Big[1+(\rho_s/r_n)$ $(\delta n/n - x)\Big]$ and $\delta n/n = \varphi + \delta_0(c_1 - \nabla^2)\partial_y\varphi$, and thus $\int dxdy\, N = \int dxdy\, \varphi + \text{const.}$

(2) Energy Conservation

The local energy density $w(x,y,t)$ is the sum of the electrostatic potential energy $\frac{1}{2}e\delta n_e\varphi \sim \frac{1}{2}\varphi^2$ and the ion kinetic energy $\frac{1}{2}m_i n_0 v_E^2 \sim \frac{1}{2}(\nabla\varphi)^2$. In the dimensionless variables

$$w(x,y,t) = \frac{1}{2}\left[\varphi^2 + (\nabla\varphi)^2\right] \tag{6}$$

in units of $w_f = n_e T_e(\rho_s/r_n)^2$. The local conservation of energy is derived using the energy flux

$$\mathbf{F}_w = \frac{1}{2}\mathrm{v}_d\varphi^2 - \varphi\nabla\frac{\partial\varphi}{\partial t} - \nabla^2\varphi\hat{\mathbf{z}}\times\nabla(\varphi^2/2) \tag{7}$$

in units of $v_{de}T_e(\rho_s/r_n)^2$, and the conservation law is

$$\frac{\partial w}{\partial t} + \nabla\cdot\mathbf{F}_w = -\nu\varphi\nabla^4\varphi \tag{8}$$

derived using Eq. (1). Thus, $E = \int dxdy\, w = \text{const.}$ when $\oint_c \mathbf{F}_w \cdot d\boldsymbol{\ell}\times\hat{\mathbf{z}} = 0$ and $\nu = 0$.

(3) Potential Enstrophy Conservation

The potential vorticity for Eq. (1) is

$$\Pi = \nabla^2\varphi - \varphi + v_d x \tag{9}$$

and in the dissipationless (Hasegawa-Mima) limit the dynamical equation[1] becomes

$$d\Pi/dt = 0\ . \tag{10}$$

Equation (10) is Ertel's theorem for the conservation of potential vorticity as given in the Appendix of Ref. 7. From the convection of Π, the conservation law

$$\int \Pi^2 dxdy = \text{ const.}$$

and the energy conservation law $E = \int w dx dy = \text{const.}$ suggests the definition of the local enstrophy density

$$u(x,y,t) = \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}(\nabla^2\varphi)^2 \tag{11}$$

and the enstrophy flux

$$\mathbf{F}_u = \mathbf{v}_d(\nabla\varphi)^2 - \nabla\varphi\left(\frac{\partial\varphi}{\partial t} + v_d\frac{\partial\varphi}{\partial y}\right) + \frac{1}{2}(\nabla^2\varphi)^2\hat{\mathbf{z}}\times\nabla\varphi \; . \tag{12}$$

The dynamical equation (1) leads to the conservation law

$$\frac{\partial u}{\partial t} + \nabla\cdot\mathbf{F}_u = -\nu\nabla^2\varphi\nabla^4\varphi \tag{13}$$

with $U = \int dx dy\, u = \text{const.}$ when $\oint_c \mathbf{F}_u \cdot d\boldsymbol{\ell}\times\hat{\mathbf{z}} = 0$ and $\nu = 0$.

The dynamical equation (1) is a constraint (the vorticity equation) derived from the local momentum balance equation. The integration of (1) over y leads to the Reynolds stress

$$\pi(x) = \overline{v_x v_y} \equiv -\int \frac{dy}{L_y}\frac{\partial\varphi}{\partial x}\frac{\partial\varphi}{\partial y} \tag{14}$$

governing the flux of y-momentum from x to $x + dx$. In the regime $\nabla^2\varphi \gg \varphi$ the mean flow

$$\overline{v}_y(x,t) \equiv \int \frac{dy}{L_y}\frac{\partial\varphi}{\partial x}(x,y,t) \tag{15}$$

satisfies the simple transport equation

$$\frac{\partial\overline{v}_y}{\partial t} + \frac{\partial\pi}{\partial x} = \nu\frac{\partial^2\overline{v}_y}{\partial x^2} \tag{16}$$

with the kinetic energy transferred from the mean flow $\frac{1}{2}\int \overline{v}_y^2 dx$ to the wave and vortex flow $\varphi(x,y,t)$ given by

$$T = \int \overline{v}_y\frac{\partial\pi}{\partial x}dx \; . \tag{17}$$

The transfer of momenta in the drift regime ($\nabla^2\varphi < \varphi$) is complicated by the existence of a mean $\overline{j}_x B_z\hat{\mathbf{y}}$ force due to closure of the current loops $\nabla\cdot\mathbf{j} = \nabla_\perp\cdot\mathbf{j}_\perp + \nabla_{||}\mathrm{j}_{||} = 0$. The mean current $\overline{j}_x = -\int^x dx'\overline{\nabla_{||}j_{||}}$ with $\nabla_{||}j_{||} = -(ne^2/T_e)\left(\partial_t\Phi + v_{de}\partial_y\Phi\right)$ levels to

$\overline{j}_x = -\partial_t \int_0^x dx' \overline{\varphi}(x', t)$. With the dimensionless $\overline{j}_x B_z$ acceleration, the momentum balance Eq. (16) is modified to

$$\frac{\partial \overline{v}_y}{\partial t} + \frac{\partial \pi}{\partial x} = \partial_t \int_0^x dx' \overline{\varphi}(x', t) + \nu \frac{\partial^2 \overline{v}_y}{\partial x^2}.$$

Here we note that the effect of a large scale sheared flow $\overline{v}_y(x)$ with $r_0 \partial \overline{v}_y / \partial x \gtrsim v_{de}$ is the rapid transformation of the dipole vortex into a monopole vortex. The transformation occurs due to the stagnation point introduced on the counter streaming side of the dipole. The sensitivity of the dipole vortices to a large scale ($\gtrsim r_0$) sheared flow may explain the pre-dominance of monopoles in many laboratory experiments.

The conservation of potential vorticity $d\Pi/dt = 0$ in Eq. (10) implies the existence of an infinite number of constants of the motion of the form $\int G(\Pi) dx\, dy =$ constant which might suggest that the system is integrable. In contrast, the numerical simulation clearly suggest that the presence of the drift wave component makes the vortex collisions inelastic. Mathematical results consistent with the simulations are reported by Zakharov and Schulman[15] showing that the existence of infinitely many invariants does not imply the integrability of such Hamiltonian field equations. They suggest that additional constraints on the wave spectrum are required for integrability.

III. Drift Wave Vortex Dynamics

A. Monopoles and Wake Fields

In the 2D Euler equation the axisymmetric monopole vortex is an exact motionless solution, and the dipole vortex is an exact solution that translates with a constant speed in an arbitrary direction. The Euler equation has no preferred direction and no linear waves. An experimental example of a gas of Euler equation-like dipole vortices is given in the soap film experiments and computer simulations of Couder and Basdevant.[11]

In contrast to the Euler equation, the 2D drift wave equation has a continuous spectrum of linear wave modes

$$\omega_{\mathbf{k}} = \frac{\mathbf{k}\cdot\mathbf{v}_d}{1+k_\perp^2\rho_s^2} \tag{18}$$

with $\mathbf{v}_d = -(cT_e/eB)\hat{\mathbf{b}}_0 \times \nabla \ell n\, n_e$ giving the preferred direction and velocity of wave propagation. Arbitrary small amplitude $R_E \ll 1$ structures propagate along $x = x_0$, $y \cong y_0 + (\partial\omega/\partial k_y)t$ with the wave dispersion given by

$$\varphi_w(x,y,t) = \sum_{\mathbf{k}} \varphi_{\mathbf{k}} \exp(ik_x x + ik_y y - i\omega_{\mathbf{k}} t) \tag{19}$$

where $\varphi_{-\mathbf{k}} = \varphi_{\mathbf{k}}^*$. The central peak of φ_w given by Eq. 19 loses strength due to the spread of wave energy given by $\Delta x^2 \approx (\partial^2\omega/\partial \mathrm{k}^2)t \simeq \rho_s^2\omega_{k_0}t$ which is a strong effect except for very large-scale structures.

The characteristic propagation of a general initial field $\varphi(x,y,0)$, however, depends sensitively on its amplitude and scale size $1/k_\perp$. The ratio of the nonlinear binding compared with the wave dispersion is given by R_E in Eq. (4). For $R_E \lesssim 1$ we observe the dispersion of the wave packet as given by (19). For $R_E \gtrsim 1$ the nonlinear self-binding dominates producing a long-lived localized packet. The typical self-binding behavior for a monopolar structure is shown in Fig. 1 for $R_E = 10$. This self-binding structure occurs above the transition value $R_E \sim 1$.

For structures with $R_E \gtrsim 1$, the binding nonlinearity captures much of the wave energy due to the nonlinear feedback that occurs in the structure when the circulation time $2\pi/\Omega_E$ is less than the oscillation time $2\pi/\omega_k$. The typical $R_E \gtrsim 1$ structures are described as long lived monopole and dipole vortices with localized fields of the form $\varphi_v(x, y-ut) = \varphi_v(r,\theta)$ where $r = [x^2+(y-ut)^2]^{1/2}$ and $r\cos\theta = x$. For $R_E \gtrsim 1$ the computed fields are qualitatively better described by the combined field $\varphi = \varphi_v + \varphi_w$ of vortices and waves than by either individual component.

The steady-state symmetric monopole $\varphi(r)$, while long-lived, is not an exact solution of Eq. (1) due to the linear terms which drive up an asymmetric x dependence to φ. The dipole solution has the correct x asymmetry to balance the linear wave terms and is an exact, long-time solution to the Hamiltonian wave equation. The positive potential monopolar-like solution that develops from $\varphi(r)$ has a shelf on the positive x (low density) side with a smaller flow velocity and a steeper gradient, higher flow velocity on the high density side. The negative monopolar-like solution has the shelf and steeper gradient sides reversed. The reversal follows from the symmetry property of Eq. (1) with the reflected solution $\overline{\varphi} = -\varphi(-x, y, t)$.

The nonlinear localization of the wave field into a vortex field is a very important mechanism for minimizing the effects of wave diffraction and wave scattering by inhomogeneities in the system. For drift waves in a sheared magnetic field the local radial dependence of the parallel gradient described by $k_{\|} = k_y(x - x_0)/L_s$ is typically the most important inhomogeneity for the wave. Wave packets with $R_E \ll 1$ spread from $x = x_0$ to $x = x_i$ where $k_{\|} = k_y(x_i - x_0)/L_s \equiv \omega_k/v_i$ at which point the wave energy is rapidly absorbed by resonant thermal velocity v_i ions. The distance Δx to the absorption region is $\Delta x_i = L_s(\omega/k_y v_i) \cong \rho_s(L_s/r_n)$. The numerical experiment in Fig. 1 contrasts the spreading of the linear and nonlinear disturbance. While shear is not included in Fig. 1 the result tends to support the calculation in Meiss and Horton[7] with the asymptotic matching method find that the nonlinear localization of the vortex energy E_v to $r_0 \lesssim \Delta x_i = \rho_s(L_s/r_n)$ strongly reduces the ion Landau resonant absorption of the structure. The approximate calculation in Ref. 7 gives the decay rate $dE_v/dt \simeq -E_v \exp\left(-\frac{\pi L_s}{r_n}\left|1 - \frac{v_d}{u}\right|\right)$. The nonlinear suppression of shear damping is also investigated in the simulation of Biskamp and Walter[16] by solving the drift wave-ion acoustic wave equations in the sheared slab model.

In Fig. 1 is an example comparing the linear $R_E \ll 1$ behavior with the strongly nonlinear $R_E \sim 10 \gg 1$ propagation of the same size initial disturbance which clearly illustrates these binding or self-focusing effects. We choose the initial disturbance as a gaussian monopole

$$\varphi(x,y,0) = A\exp\left(-\frac{x^2+y^2}{r_0^2}\right) \tag{20}$$

with $r_0 = 6.0[\rho_s]$. Down the left side of Fig. 1 is shown the evolution in the linear regime $R_E \ll 1$ and down the right side of Fig. 1 is shown the evolution in the nonlinear regime with $\Omega_E(k) = 5$ and $\omega_k = 0.5$ for $R_E = 10$.

From the left column in Fig. 1 we see that the long wavelength components of Eq. (1) propagate with a speed near v_d but that the small scale components have nearly zero speed. The waves propagate symmetrically in x giving an approximately parabolic envelope to the expanding wake $\varphi(x^2, y - v_d t, t)$. In contrast, the nonlinear evolution given in the right-hand column of Fig. 1 develops an asymmetrical x dependence. The growth of the asymmetry and the weak wave emission are limited after a few rotation periods with the reformed structure propagating as a vortex $\varphi_v(x, y - ut)$ for as long a time as computed in the dissipationless system.

Considering the effects of inhomogeneity and wave energy dispersion in Fig. 1 we see that by $tc_s/r_n = 24$ only three contours of φ remain in the central wave front and considerable wave energy has reached to $\Delta x \cong 20\rho_s$ where magnetic shear of the strength $L_s/r_n = 10$ to 20 will cause strong wave absorption. In contrast, the nonlinear vortex at the same time has only the first contour level expanded slightly outside the initial gaussian perturbation. At the time $tc_s/r_n = 60$ (not shown) most of the wave energy has been dispersed out of the initial structure in the linear system. We note that the long wavelength head of the linear wave packet keeps up with the vortex, but the rest of the wave energy propagates at much slower speed. Since the passively convected field $f(x,y,t)$, or distribution of test particles, is trapped in the nonlinear vortex, the transport along $\mathbf{v}_d$ of the trapped particles is nearly complete in the nonlinear vortex. For the linear wave packet the net transport is essentially zero.

Repeating this experiment for various values of A and r_0, we find that the transition to the monopole vortex behavior is continuous with R_E, but that pronounced self-trapping has set in at $R_E \simeq 1.5$ to 2.0.

B. Coalescence of Drift Wave Vortices

One of the most important processes in the nonlinear dynamics of vortices is the merging or coalescence of like-signed regions of vorticity. In the merging process streamlines reconnect producing a mixing of the transported fields previously trapped in the two separate vortices. Since the merging process itself happens in about one vortex circulation period $2\pi/\Omega_E(k)$ and the reconnection quickly mixes the fields f_1 and f_2 over the new larger vortex, the coalescence of vortices produces a rapid, local transport mechanism. Nezlin[17] gives a review of the types of vortices, turbulence, and coalescence phenomena found in the rotating water tank experiments.

The coalescence of two-dimensional vortices with cores of constant vorticity for the Euler equation is shown by Overman and Zabusky[18] to occur for equal strength vortices when the separation of the vortices is less than a critical value of $d_* = 3.2R$ where R is the core radius of the constant vorticity vortex. Asymmetric merging of Euler vortices proceeds as shown in Melander et al.[19] The merging of vortices is observed in the nonlinear stage of the Kelvin-Helmholtz instability where the local growth rate for merging can be predicted by the linear stability analysis of the period doubling perturbation of the periodic vortex chain as given by Pierrehumbert and Widnall[20] and Horton et al.[21]

In the case of the drift wave-Rossby wave equation, the coalescence process is observed in the rotating neutral fluid experiments. Recent rotating water tank experiments by Swinney et al.[22] and Griffiths and Hopfinger[12,13] show the importance of vortex merging in $\mathbf{E} \times \mathbf{B}$ or geostrophic vortices which are governed by equations close to the drift wave Eqs. (1) and (2). Swinney et al.[22] report that merging of up to five vortices into one vortex takes place when the pumping rate driving the fluctuating flow is sufficiently strong. Griffiths reports that two anti-cyclonic (positive h and negative φ) vortices again only merge for separations $d \lesssim 3.3R$ where as cyclonic (negative h and positive φ) vortices merge for larger separations-distances $d \lesssim 5R$ where the upper limit was given by the size of vessel and wall effects.

For the drift wave equation (1) the positive φ perturbation corresponds to negative h which is a low pressure cyclonic disturbance. The reversal of the signs from cyclonic (negative h) to positive φ follows from the transformation required to take the β-plane equation [Eqs. (1) and (2) of Ref. 9 or Eq. (6.3.21) of Ref. 23]

$$\frac{\partial \zeta}{\partial t} - \gamma^2 \frac{\partial \psi}{\partial t} + b\frac{\partial \psi}{\partial x} + [\psi, \zeta] = -k\nabla^4 \zeta \tag{21}$$

with

$$\zeta = \nabla^2 \psi \tag{22}$$

into the drift wave equation. The transformation is $x \to -y$, $y \to x$ (rotation by $\pi/2$) and $\psi \to -\varphi$ for $b = v_d = 1$. Thus, the positive potential vortices correspond to the cyclonic vortices on the β-plane.

For a simple, generic drift wave coalescence experiment we take two positive, gaussian vortices displaced in radius by $d = 2x_0$ and $\mathbf{E} \times \mathbf{B}$ rotation frequencies somewhat greater than unity. In Fig. 2 we take

$$\varphi(x, y, t=0) = A_1 \exp\left[-\frac{(x+x_0)^2 + y^2}{r_0^2}\right] + A_2 \exp\left[-\frac{(x-x_0)^2 + y^2}{r_0^2}\right] \tag{23}$$

with $A_1 = A_2 \equiv 50$, $r_0 = 2[\rho_s]$ so that the maximum (isolated) vorticity is $\zeta_0 = -4A/r_0^2 = 50$ and the rotation frequency at the $1/e$ radius of $\varphi_\pm(r_\pm = r_0)$ is $\Omega_E = -2A/r_0^2 e = 9.2$. In the experiment shown in Fig. 2 we choose $d = 2x_0 = 8[\rho_s]$ so that the overlapping of the potentials is

$$\varphi(0,0,0)/A_1 = 2\exp(-x_0^2/r_0^2) = 0.037\ . \tag{24}$$

Figure 2 shows that immediately after releasing the initial condition in frame (a), the mutual convection forms the substantial $m = 2$ distortion of the structure shown in frame

(b) at $tc_s/r_n = 4$ ($\Omega_E t_b = 37$) and the reconnection of the streamlines in the coalescence is essentially complete in frame (c) at $tc_s/r_n = 6$ ($\Omega_E t_c = 55$). During the coalescence the circularization of the resulting vortex and the separation from the drift wave wake takes about 5 to 10 rotation periods in this experiment. The resulting monopole vortex has $\Omega_E \lesssim 9.0$ and is shown at $c_s t/r_n = 60$ in frame (d) along with its wake field which has max $|\varphi_{\rm wave}| \lesssim 0.1$.

The positive $A_{1,2}$ merger shown in Fig. 2 corresponds to the cyclonic merger reported experimentally to occur for large relative separations. When the direction of the vortex circulation is reversed with $A_1 = A_2 = -50$, Eq. (1) predicts the merger occurs for exactly the same d over r_0 rotation. The pictures obtained with negative $A_{1,2}$ is the x-reflection of the positive $A_{1,2}$ mergers. Thus, for both positive and negative gaussian drift wave vortices the critical separation distance d^* is about $d^*/r_0 \simeq 5.5/2 \simeq 2.8$, similar to or somewhat smaller than that reported by Griffiths and Hopfinger for the anticyclonic merger. The difference is discussed further in the conclusions.

It is interesting to note that the $m = 2$ coalescence instability appears to be the time reversed evolution of the well-known $m = 2$ instability of a rotating plasma as can be seen qualitatively by comparing frame (c) in Fig. 2 with the flow lines for the $m = 2$ instability of the drift modes in the rotating (tandem mirror) plasma in Fig. 14 of Liu et al.[24]

C. Dipole Vortex Formation From Pairing

Two oppositely charged drift wave vortices can also merge to form a dipole vortex through a process we may call pairing or coupling.[11]. Since drift waves are emitted during the pairing, the inelastic merging process is analogous to the atomic process of radiative recombination. The presence of the drift wave component of the excitation spectrum is particularly important in the pairing process since it is through the shedding of energy and enstrophy in the radiated wake that the vortices can be modified to produce a bound state characteristic of the dipole vortex state. In the absence of drift wave radiation the invariants of the equation of motion would exclude the formation of the dipole vortex. Another way of expressing the difference is to note that when the shelves on the monopolar constituents extend inward toward each other the coupling process is weaker than when the steep gradient sides come together.

For the drift wave system the invariants are still sufficiently restrictive to prevent the complete overlapping and thus neutralization of the charge separation or vorticity ($\nabla^2\varphi$) contained in the neighboring lobes of the dipole vortex. Since the dipole vortex state is known to be an exceptionally stable state, the overall process is well thought of as a radiative recombination with the cross-section σ for pairing comparable to the vortex diameter.

For a generic drift wave pairing or recombination experiment we use Eq. (23) with $A_2 = -A_1 = 50$ and $r_0 = 2.0$ giving $\zeta_0 = .50$ and $\Omega_E(r_\pm = r_0) = 9.2$. In the experiment shown we choose $d = 2x_0 = 14[\rho_s]$ so that the initial overlap at $x = 0, y = 0$ is $\varphi_1(0,0,0) = A\exp\left(-(x_0/2)^2\right) = 4.8 \times 10^{-4}$. The experiment is shown in Fig. 3 with a fixed contour interval of $\Delta\varphi = 4.0$.

After launching the initial condition shown in Fig. 3 the mutual convection advances the vortex cores with a speed $u \gtrsim v_d$ while leaving a wake of oscillations of amplitude of order unity as shown in (3b) at $tc_s/r_n = 20$ ($\Omega_E t = 184$). The pulling together of the vortex cores is a rather slow process compared with that in the corresponding like-signed merging. At time $tc_s/r_n = 20$ the core separation is $d_{20} \simeq 8$ and at $tc_s/r_n = 60$, shown in (3c), the pulling together is nearly complete with $d_{60} = 6.0$. The final separation of the dipole centers is $d \simeq 6[\rho_s]$ — one half the initial separation giving an average attracting velocity of $\dot{x}_{12} \sim -6/60 = -0.1[v_d]$ during the drift along the magnetic surface with $\dot{y} \simeq 60/60 \sim 1.0[v_d]$ during the pairing process.

Reversing the signs of the initial monopoles ($A_1 = -A_2 = 50$) produces a different result. With negative potential on the left and the positive potential on the right the monopoles do not pull together to form a dipole pair. Repeating the experiment in Fig. 3 with this sign reversal we find that at $t = 60r_n/c_s$ the monopoles have slightly separated to $d \cong 15\rho_s$ from the initial $d_0 = 12\rho_s$. The physical difference between the two configurations is given by the magnitude of the potential vorticity Π. With the polarity leading to coupling the $\varphi - \nabla^2\varphi$ contribution partially cancels (small Π) the ambient density gradient $v_d x$ whereas, with the repelling polarity the solitary wave adds to the density gradient (large Π). Another way of stating the difference is that when the shelves of the monopoles extend toward each other they inhibit the coupling process.

D. The Dipole Vortex

The dipole vortex is an exact solitary wave solution to the Hamiltonian or dissipationless drift wave equation in the limit of constant equilibrium gradients. Although the conditions required for natural formation of the dipoles are not well known, the solitary wave properties of the dipole solutions have been the subject of numerous theoretical, numerical, and experimental studies. The dipole vortices have been created and studied in several neutral fluid experiments. In the present work the dipolar vortices are used extensively as initial conditions.

The dipole vortex solitary wave is a linear relation between excess charge density or vorticity $\nabla^2\varphi$ and potential φ propagating with speed u parallel to $\mathbf{v}_d$. Outside the circular boundary $r > a$ the vortex solution decays exponentially $\exp(-kr)$ as given by

$$\nabla^2\varphi = k^2\varphi \qquad \text{for} \qquad r \geq a \tag{25}$$

with

$$k^2 = 1 - \frac{v_d}{u} > 0 \ . \tag{26}$$

The condition for a localized solution given in Eq. (26) defines the vortex velocity regions $u > v_d$ and $uv_d < 0$ that are the complements of the wave phase velocity region given by Eq. (18) with $v_{ph} = \omega/k_y = v_d(1 + k_\perp^2)^{-1}$ and $k_\perp^2 > 0$. Inside the circle $r = a$ the solution has negative curvature for regularity at the origin with

$$\nabla^2\varphi + p^2\varphi = \Big[(1+p^2)u - v_d\Big]\, x = qx \ . \tag{27}$$

The solutions of Eqs. (25) and (27) with continuous φ and $\nabla^2\varphi$ is

$$\begin{aligned} \varphi_{dp} &= -\frac{qa}{k^2+p^2}\frac{K_1(kr)}{K_1(ka)}\cos\theta \quad r \geq a \\ &= qa\left\{\frac{k^2}{k^2+p^2}\frac{J_1(pr)}{J_1(ka)} - \frac{r}{a}\right\}\cos\theta \quad r \geq a \ . \end{aligned} \tag{28}$$

The condition that $v_\theta = \partial\varphi/\partial r$ be continuous at $r = a$ determines $p = p(ka)$ through

$$\frac{1}{ka}\frac{K_2(ka)}{K_1(ka)} + \frac{1}{pa}\frac{J_2(pa)}{J_1(pa)} = 0 \ . \tag{29}$$

In this analysis $x = r\cos\theta$ and $y - ut = r\sin\theta$. The solution (28) is a two parameter (a, u) family of dipole vortices for $R_E > 1$ just as the waves (18)-(19) are a two parameter (k_x, k_y) family for $R_E \ll 1$. The properties of the dipole vortices such as the energy $E(a, u)$ and enstrophy $U(a, u)$, and the dynamical form factor $S(\mathrm{k}, \omega) = \delta(\omega - k_y u)S(k_x, k_y)$ are given in Ref. 7.

The dipole vortex requires a critical amplitude $\varphi_m(a, u)$ for its formation. The critical amplitude is determined from Eq. (28) and occurs near $r \cong a/2$. For $ka \ll 1$ the value is

$$\varphi_m = 1.28 v_d a \tag{30}$$

which, with the identification $k_x \cong 1/a$, yields the critical $\mathbf{E} \times \mathbf{B}$ Reynolds number

$$R_E = k_x k_y \varphi_m / k_y v_d = 1.28 \ . \tag{31}$$

We often use somewhat large amplitude dipoles in the numerical experiments to reduce their exterior scale size and increase their robustness.

E. Stability of the Dipole Vortex

The stability of the dipole vortex structure has been investigated previously both numerically and analytically. Probably the most convincing demonstration of the stability and robustness of the dipoles is given by their ability to reconstruct their initial form after the strong distortions that occur during the zero impact parameter $b = 0$ head-on and overtaking collisions first shown by Makino et al.[8] and investigated in more detail by McWilliams and Zabusky.[9]

McWilliams and Zabusky state that the zero impact parameter head-on collision "is remarkably non-destructive" as is clearly shown in their Fig. 9. They report that the longer the overlap or collision time the stronger the merging tendency with the extreme example of a smaller diameter dipole vortex being absorbed by the larger diameter dipole during an overtaking collision (their case 6, Fig. 8). In the work reported here we emphasize the effect of impact parameters b of order the vortex radius, and the fact that the Bessel function solutions are not unique to the solitary vortex properties of the nonlinear system.

Here we examine the stability and the attraction of the flow to a dipole vortex state by its ability to adjust to distortions in the initial flow distribution. Since the dipole vortex formulas (26)-(27) are complicated, it is useful to consider the replacement of the $J_1(pr)$-Bessel function by the truncated polynomial expansion $J_1(pr) \rightarrow (pr/2)(1 - p^2r^2/j_1^2)$ and to match this approximate solution to the asymptotic form $K_1(kr)$ of the exterior solution. The resulting eigenvalue equation for continuous φ and $\nabla^2\varphi$ has the solution

$$pr_0 = \left(\frac{1+4a_1}{1/j_2^2 + 4a_1/j_1^2}\right)^{1/2} \quad \text{where} \quad a_1 = \frac{K_2}{kr_0K_1} \simeq \begin{cases} \frac{2+kr_0}{(kr_0)^2} & \text{for} \quad kr_0 \ll 1 \\ \frac{1}{kr_0} & \text{for} \quad kr_0 \gg 1. \end{cases}$$

We find a rapid readjustment in several rotation periods with the initial perturbed dipole vortex adjust to the usual state. The adjustment can require the ejection of small amplitude waves in some cases.

The $J_1(pr) - K_1(kr)$ solutions (27)-(28) of the Rossby-Drift Wave equation are not unique since they assume linearity in the of the $\Pi = \nabla^2\varphi - \varphi + x$ versus $\varphi - ux$ functional and a circular boundary $r = a$. We find that good dipole solitary waves are created from the nearby initial condition

$$\varphi(x, y, 0) = Ar\cos\theta\, e^{-kr} \tag{32}$$

with $k = (1 - v_d/u)^{1/2}$ and $A \gtrsim 3v_d/k$. The curvature of this function is continuous and changes sign at $r_1 = 3/k$ which defines the effective radius a of the corresponding Bessel function dipole vortex. The amplitude dependence of the speed of the dipole vortex generated from Eq. (32) is slightly weaker than that of the Bessel function vortex. For example, with $u = 2v_d$ we observe that $u \cong v_d(1 + .012A)$ for the vortex evolving from Eq. (32), whereas the corresponding Bessel function vortex with $a = 6\rho_s$ has $u \cong v_d(1 + .017A)$.

Our investigations of the stability of the dipole vortices indicate that the most dangerous perturbation to the dipole is an ambient shear flow with $\delta\varphi = v_E' x^2/2$. The sheared flow places a stagnation point ($\times$ point) in the flow at a distance $\Delta x = u/v_E'$ on the side of the dipole with opposing flows. When this $\times$ point is within the distance $u/v_E' < \max(1/k, 2r_0)$ of the vortex center, the lobe of the vortex with the stagnation point opens up dumping its vorticity into the counterstreaming plasma. The remaining lobe forms a shear flow driven monopole vortex as given, for example, in Horton et al.[20]

To consider whether the dipole vortices prefer the circular shape of the Bessel function solutions, we have to replace $r(x, y)$ with $r = (x^2 + e^2y^2)^{1/2}$ in Eq. (32) and run with $e = 0.5$ and $e = 2.0$ and $A = 50$. In both cases an approximately circular dipole propagates out of the initial data and leaves a wake behind. The elliptical perturbation with the long axis parallel to $\hat{y}(e = 0.5)$ produces less distortion than the $e = 2.0$ distortion with the long axis parallel to x. Both disturbances form a Bessel function-like dipole vortex and a secondary wave field.

IV. Anomalous Transport During Inelastic Vortex Collisions

In the first studies of the dipole vortex collisions, the elastic nature of the collisions was emphasized. Makino et al.[8] investigate head-on and overtaking collisions with zero impact parameter and conclude that the collisions are nearly elastic. McWilliams and Zabusky[9] extend the study of dipole vortex interactions with a set of eight collision experiments still using zero impact parameter and show that while some (head-on) collisions are essentially elastic, other collisions are strongly inelastic. They report observing a fusion or capture event during an overtaking collision where like regions of vorticity permanently merge during the collision, and a fission event splitting apart one dipole during a head-on collision where opposite signs of vorticity come together. We also find this event for $u_1 = 2v_d, u_2 = -v_d$ and $b = 0$.

In our studies we are particularly interested in collisions that give rise to anomalous transport across the magnetic field. This transport appears to be strongest for inelastic collisions due to the permanent reconnection of the streamlines that takes place during the inelastic collisions.

Furthermore, the symmetry present in the zero impact parameter collisions reduces the net anomalous transport. We find that the largest net radial transport due to collisions occurs when the impact parameter b is comparable to the vortex radius r_0. In collisions with $b \sim r_0$ the symmetry of the vortex field is broken and like regions of vorticity merge permanently into a new final state. During the merging, the plasma, initially trapped in separate vortices, mix over the core of the new vortex. We observe large net values of radial flux $\langle \delta f v_x \rangle \lesssim -r_0 u \, (d\bar{f}/dx)$ during this type of collision.

As an example we consider the collision of the two vortices shown in Fig. 4. Both initial dipoles, with opposite polarity, have the radius $a = 6\rho_s$ for their circular boundaries. The impact parameter defined by the relative displacement of the dipole centers in the x-direction is taken as $b = 5\rho_s$. The amplitude and structure of the dipoles follows from Eqs. (26)-(27) for the chosen speeds $u_1 = 2v_d$ and $u_2 = -v_d$. The negative φ with positive vorticity $\nabla^2\varphi$ regions meet in the collision and merge into a single strong vortex as shown in Figs. 4(a). The merging event is similar to that shown in Fig. 2 with a strong $m = 2$ distortion followed by re-circularization.

During the collision the left-hand region of positive φ with negative vorticity is rotated under the newly formed, larger positive vortex ($\nabla^2\varphi > 0$) and couples to it while growing in strength. This process forms a new dipole vortex with approximately the same circular boundary $a' \simeq 6\rho_s$ and strength as the original $u_2 = -v_d$ dipole during $t = 32 - 40$ and shown at $t = 80$.

The newly created dipole ($t > 30$) contains a mixture of plasma from the original dipoles and is created with its axis rotated by about 45° from the direction of the magnetic surface. Plasma trapped in the new vortex is transported across the magnetic surface by $\Delta x \simeq 2r_0 \simeq 10\rho_s$ in a time of order $\Delta t \simeq 20r_n/c_s$ as seen in Figs. 4(b). The newly formed dipole propagates with $\dot{y} \simeq 0.5v_d$ and its axis slowly rotates with $\dot{\theta} \simeq -.03\,\mathrm{rad/s}$ continuing to rotate past the usual aligned position ($t = 80$). The monopole propagates with speed $\dot{y} \cong 1.0v_d$ and contains plasma that moved from $x \cong -a$ to $x \cong 0$ during the course of the collision.

The perturbation δf to the distribution f of test particles in the vortex field is shown in Fig. 4(b). The plasma is assumed to have a uniform background gradient $d\overline{f}/dx \equiv -\overline{f}/r_f = \mathrm{const.}$ across the magnetic surface. The evolution of δf is computed from

$$\left(\partial_t + \mathbf{v}_E \cdot \nabla - D\nabla^2\right)\delta f = -\frac{\partial\varphi}{\partial y}\frac{\partial\overline{f}}{\partial x} \tag{33}$$

and shown in Figs. 4(b) for $D = 0.05$. Other experiments with $D\nabla^2 \to -D_2\nabla^4$ have been used to test the small D limit of the convective transport.

Integrating Eq. (2) over y gives the net convective flux $\Gamma(x)$ across the magnetic surface at x with

$$\frac{\partial \overline{f}(x)}{\partial t} + \frac{\partial}{\partial x}\left(\Gamma(x) - D\frac{\partial \overline{f}}{\partial x}\right) = 0 \tag{34}$$

where $\Gamma(x) = \overline{nv_x} = -L_y^{-1}\int dy \partial_y \varphi \delta f$. The total or global flux Γ_T is given by

$$\Gamma_T = \int \frac{dx}{L_x}\Gamma(x) = -\int \frac{dxdy}{L_x L_y}\frac{\partial \varphi}{\partial y}\delta f = \sum_{\mathbf{k}} ik_y \varphi_{\mathbf{k}}^{*} \delta f_{\mathbf{k}} \tag{35}$$

and is evaluated as a function of time during the collision. The approximate formula for Γ_T following from Eqs. (33) and (35) is

$$\Gamma_T \cong \sum_{\mathbf{k}\omega} \frac{k_y^2 |\phi_{\mathbf{k}\omega}|^2}{-i\left(\omega - \Omega_E(\mathbf{k})\right) + \mathbf{k}_\perp^2 D}\frac{\partial \overline{f}}{\partial x} \tag{36}$$

where $\Omega_E(\mathrm{k})$ is the vortex rotation frequency used as a local estimate of $\mathbf{v}_E \cdot \nabla \delta f_k \simeq i\Omega_E(\mathrm{k})\delta f_k$. The exact calculation of Γ requires the solution of the second order partial differential Eq. (33) for δf. The solution for Γ_T of a single sinusoidal wave is given in Rosenbluth et al.[24] For two drift waves the stochastic diffusion[10] at $D = 0$ occurs for $R_E \lesssim 1$.

For isolated vortices the convection of δf evolves to $\mathbf{v}_E \cdot \nabla \delta f = 0$ giving rise to the stationary trapped particle distribution

$$\delta f(x, y - ut) = \frac{\varphi}{u}\frac{\partial \overline{f}}{\partial x} \tag{37}$$

with $\varphi(x, y - ut)$ given by Eqs. (28)-(29) for isolated dipole vortices. This stationary distribution gives perfect transport of the trapped plasma along the magnetic surface with speed u and zero net transport across the magnetic surface since

$$\Gamma(x) = \langle nv_x \rangle = -\frac{1}{u}\frac{\partial \overline{f}}{\partial x}\int_1^2 \frac{dy}{L_y}\varphi\frac{\partial \varphi}{\partial y} = 0$$

between any points 1,2 where $\varphi_1^2 = \varphi_2^2$. In the presence of the small background diffusivity $D(P = \ell v_E/D \gg 1)$ there is a residual diffusion varying as $D_x = (Dav_d)^{1/2}$ due to the sharp boundary layer gradients occurring between the trapped and passing particles.

The distribution in Eq. (37) and shown in Fig. (4b) at $t = 0$ for $\partial \overline{f}/\partial x < 0$ describes an excess of density $\delta f > 0$ (solid lines) in the upper positive vortex ($\nabla^2 \varphi > 0$) coming down ($u_2 < 0$) and a density deficit in the lower positive vortex going up ($u_1 > 0$).

During the collision the regions of positive and negative δf are brought together creating sharp gradients ($\gg |\delta f|/r_0$) in the merged vortex as shown at $t = 8r_n/c_s$. After one vortex turnover time these regions are mixed with further turnovers producing a spiraled filamentary structure of positive and negative δf. With the estimate that the gradients of δf build up as $k_\perp^2 \simeq \Omega_E^2 t^2/r_0^2$ the time for the background diffusivity to stop the filamentation is $t_d = (r_0/D\Omega_E^2)^{1/3}$ whereupon the distribution again evolves to a state with $\mathbf{v}_E \cdot \nabla \delta f \simeq 0$ and with δf given again by Eq. (37) and shown in Fig. 4(b) at $t = 80r_n/c_s$.

Although this description of the effect of the collision on the test particle distribution is overly simplified, as seen from Fig. 4(b), where it appears that a substantial number of particles are lost during the collision, it describes the essential features of the collisional interaction. Based on this description that states there is a complete mixing of the trapped plasma over the vortex size r_0 each collision, assuming the time between collisions τ_v is long compared with interaction time $\Delta t \lesssim 10r_n/c_s$, we construct a model for the quasi-coherent transport of plasma induced by vortex collisions in Sec. V.

During the collision in Fig. 4 the net volume averaged flux $\Gamma_T = \langle v_x \delta f \rangle$ reaches a value of $\simeq -0.1 r_0 v_d (\partial \overline{f}/\partial x)$ which is about 10 times larger than the residual background flux between collisions.

The measure of the filamentation of δf occurring during the collision is $U_f(t)$

$$U_f \equiv \frac{1}{2} \int \frac{dxdy}{L_x L_y} (\nabla \delta f)^2 = \frac{1}{2} \left\langle (\nabla \delta f)^2 \right\rangle . \tag{38}$$

Before the collision U_f is constant at $U_f \cong 2.5$. During the collision U_f increases to a peak value of ~ 8.0 and then decays to about 3.0 after the collision.

Increasing the impact parameter b while keeping other parameters fixed in the experiment in Fig. 4 shows that the final state consisting of a new dipole and a monopole occurs until $b \gtrsim b_* \simeq 10\rho_s \lesssim 2r_0$. An experiment with $b = 2r_0 = 12\rho_s$ shows a kind of resonance in which the quadrupole interaction appears strong. In this case we find that Out to $t \simeq 50 r_n/c_s$ the four vortices retain their identity but remain grouped together with an inner vortex spacing of $r_{ij} \sim 2r_0$. The four vortices propagate together with a reduced speed $\dot{y} \simeq 0.6 v_d$. In this last experiment (not shown) the diffusion of f was taken with $-D_2 k_\perp^4 = -0.01 k_\perp^4$ replacing $-k_\perp^2 D$. The pulse of volume integrated $\langle \delta f v_x \rangle$ reached $-0.6(c_s \rho_s df/dx)$ with a width of $\Delta t = 3r_n/c_s$. The value of U_f increased from 2.5 to 10 and decays back to 3.0.

V. Quasi-Coherent Transport from Vortex Collisions

The vortex collision experiments in Sec. IV show a fast, transient plasma transport over the vortex radius $r_0 \gg \rho_s$ during a vortex collision. It remains, however, to determine the net plasma transport than can occur due to the vortex-vortex interactions.

Let us consider a gas of vortices with line density $n_v = N_v/L_x L_y$ where L_x and L_y are the dimensions of the periodic 2D box $V = L_x L_y$ and N_v is the number of vortices in the box. We define the average distance r_{ij} between the vortices by $\pi r_{ij}^2 N_v = L_x L_y$ or $r_{ij} = (\pi n_v)^{-1/2}$. For N_v vortices with average radius r_0 the packing fraction f_p is defined by

$$f_p = \frac{N_v \pi r_0^2}{L_x L_y} = \pi n_v r_0^2 = \frac{r_0^2}{r_{ij}^2} \tag{39}$$

with the restriction that $f_p < 1$. The fractional area perpendicular to $\mathbf{B}$ covered by vortices is f_p.

For an electrostatic potential field

$$\varphi = \sum_{i=1}^{N_v} \varphi_i + \sum_{\mathbf{k}} \varphi_{\mathbf{k}}^w \tag{40}$$

composed of vortices and waves (see the Appendix for details) the energy density w in the drift wave field φ compared with free energy density $w_f = nT_e(\rho_s/r_n)^2$ has a vortex and wave component given by

$$\frac{w}{w_f} = f_p\overline{A_v^2}\left(1+\frac{\rho_s^2}{r_0^2}\right) + (\tilde{\varphi}^w)^2\left(1+\overline{k}_\perp^2\rho_s^2\right) \tag{41}$$

where $\overline{A_v^2}$ is the average (dimensionless) vortex amplitude

$$\overline{A_v^2} = \frac{1}{N_v}\sum_{i=1}^{N_v} A_i^2 \tag{42}$$

and $\tilde{\varphi}^w$ is the root-mean-square wave amplitude

$$\tilde{\varphi}^w = \left(\sum_{\mathbf{k}} |\varphi_{\mathbf{k}}^w|^2\right)^{1/2} . \tag{43}$$

In obtaining Eq. (41) the waves and vortices are assumed uncorrelated.

The ratio of the energy in the vortices to the waves can be estimated for a spectrum of waves saturated at the mixing length level $\tilde{\varphi}^w = \alpha^{1/2}/\overline{k}_x$ (corresponding to $e\Phi/T_e \sim \alpha^{1/2}/\overline{k}_x r_n$) and noting that the dimensionless vortex amplitude satisfies $A_v \gtrsim ur_0$ for self-binding. We use the dimensionless constant[2] $\alpha < 1$ to describe the saturation level of those fluctuations not strong enough to form vortices. From these amplitudes the ratio of the energy densities in vortices to waves is given by

$$E^v/E^w \simeq f_p\overline{k}_x^2 r_0^2 . \tag{44}$$

Since both simulations and experiments show that large scale fluctuations condense into vortices, we may expect that $\overline{k}_x^2 r_0^2 \gg 1$. Thus, for a packing fraction $f_p \gtrsim 1/\overline{k}_x^2 r_0^2$ the energy density in the vortices is equal to or greater than the wave energy density. The appropriate division of the fluctuation spectrum into waves and vortices is discussed in Sec. IIIA and is based on the onset of self-binding that occurs for $R_E \gtrsim 1-2$.

From the vortex collision experiments in Sec. IV we find that the cross-section $\sigma(b)$ for strong inelastic collisions is peaked at $b \simeq r_0$ where $\sigma_{\max} \simeq 2r_0$ and is negligible for $b \gtrsim 3r_0$. This range is comparable with the coalescence range reported by Overman and Zabusky[18] and Griffiths and Hopfinger.[12]

The mean-free-path $\lambda_{\rm mfp}$ between vortex collisions is defined by

$$n_v \lambda_{\rm mfp} \sigma = 1 \tag{45}$$

and with $\sigma = 2r_0$, where r_0 is the average vortex radius, the mean-free-path is $\lambda_{\rm mfp} = 1/2r_0 n_v$. The time τ between collisions is

$$\tau u n_v \sigma = 1$$

and the vortex-vortex collision frequency is

$$\nu_v = \frac{1}{\tau} = n_v u \sigma \simeq 2 n_v r_0 v_d \tag{46}$$

taking the average vortex speed as $u \gtrsim v_d$. The numerical experiments indicate that the vortex-vortex collisions should lead to an effective diffusion D_v of the vortices given by

$$D_v = \nu_v r_0^2 \equiv n_v r_0^2 v_d \sigma \tag{47}$$

which increases as r_0^3 at fixed n_v and v_d. The plasma trapped in a vortex is $\pi r_0^2 n_e$ per unit length, and the total trapped fraction of the plasma density is $n_e \pi r_0^2 N_v / L_x L_y = f_p n_e$. Thus the net plasma diffusivity D_p from vortex trapped plasma is related to the vortex diffusion D_v by

$$D_p = f_p D_v = f_p^2 (u\sigma/\pi) \cong f_p^2 r_0 v_d \tag{48}$$

where $r_0 v_d = (r_0/r_n)(cT_e/eB)$.

Adding the transport from the uncorrelated wave field $\tilde{\varphi}^w$ we obtain that the total plasma diffusivity is given by

$$D = f_p^2 r_0 v_d + \alpha \overline{\lambda}_x \tau_c \tag{49}$$

where the vortex induced diffusion arises from structures with $R_E \gtrsim 1$ or $c\Phi_v/B > r_0 v_d$ and the wave diffusion from $c\varphi^w/B \simeq \lambda_x v_d \alpha^{1/2}$ with α the empirical mixing length saturation constant describing the broadband fluctuations. In dimensional units the diffusion is given by

$$D_p = f_p^2 \frac{r_0}{r_n} \frac{cT_e}{eB} + \alpha \frac{\overline{\lambda}_x}{r_n} \frac{cT_e}{eB} \,. \tag{50}$$

The strong packing fraction square f_p^2 dependence of the vortex diffusion component in Eq. (50) arises from the increases of the vortex-vortex interaction rate with f_p and the increasing fraction of plasma trapped in the vortices with f_p.

A theoretical determination of the packing fraction f_p and average vortex size r_0 appears to be an important, unsolved problem. Both f_p and r_0 are associated with rate of buildup of wave energy at large scales due to mode coupling or the inverse cascade of fluctuation energy from active regions of scale ρ_s or smaller to large scales of order r_0. The rate of damping of the large scale motion and the effect of magnetic shear may both influence the final value of r_0. For sufficiently energetic system the mean value of r_0 may be as large as r_n which could account for the large amplitude, perhaps coherent, structures observed in the edge regions of some plasmas. The experiments of Couder and Basdevant[11] indicate the possibility of high packing fractions under certain conditions.

VI. Conclusion

The nonlinear drift wave-Rossby wave studies show that for a potential fluctuation with an amplitude large enough that the $\mathbf{E} \times \mathbf{B}$ rotation rate $\Omega_E(\mathbf{k})$ around the fluctuation of scale $\mathbf{k}_\perp$ is greater than the linear wave oscillation frequency $\omega_*(\mathbf{k})$ the convective nonlinearity of the plasma vorticity produces a self-binding in the fluctuation. The self-binding is shown to allow the fluctuation to propagate over long distances without the loss of energy that occurs in the same space scale, but small amplitude disturbance. The amplitudes required for the onset of self-binding are shown to be just above the equivalent mixing length levels where the role of the mean radial wavelength $\overline{\lambda}_x$ in the mixing length formula is replaced with the vortex diameter $2r_0$.

We show that monopolar perturbations from a local charge excess in the plasma or a local high or low pressure in the Rossby wave equation trap any passively convected field and coherently transport the field for long distances. An initial monopole develops an antisymmetric dipolar component due to the v_d term and propagates over long distances without appreciable loss of energy.

With regard to the interactions we show that liked signed gaussian monopoles merge for separations similar to those reported by Overman and Zabusky[18] for piece-wise constant Euler vortices. Merger occurs when the separation d is less than three times the $1/e$ radius of the vortex potential $d^* \simeq 2.6r_0$. We find that there is no difference in the merger of two positive and two negative vortices within the context of the Hasegawa-Mima equation in plane-slab geometry with constant coefficients. The identity of the dynamics for positive and negative merges follows from the symmetry of the equation under the transformation $\varphi \to -\varphi$ and $x \to -x$. Since the two positive vortices have their shelf on the positive x side while the two negative vortices have their shelf on the negative x side, the merger may occur at different separations when the right and left sides have different properties as in cylindrical geometry due to $1/r$ or from nonuniform $v_d(x)$ or $\beta(x)$. These effects may account for the different critical distances observed by Griffiths and Hopfinger where $d_* \sim 3.0r_0$ for anticyclonic and $d_* \gtrsim 5r_0$ for cyclonic vortices.

The conditions for the coupling of opposite signed vortices is shown to depend on the polarity of the electric field or the internal flow velocity with respect to the drift term v_d or β-term. Since the monopolar structures develop an x-asymmetry in the form of a shelf with lower velocities on one side, the internal x-gradient of φ and the associated internal flow velocity is stronger for the case when the polarity of the couple is such that the steep gradients comes together. This is the polarity with the internal flow parallel to v_d. We observe that coupling to form a dipole occurs for considerably larger separations (for a fixed amplitude) for the polarity where the shelves are external and the steep gradients are internal to the couple.

We emphasize, and study by integrating a passively convected scalar field, that when like signed vortex regions merge there is a rapid reconnection of flow lines and the mixing of plasma properties previously localized to different spatial regions. For long-lived dipole vortices the inelastic vortex collisions resulting in the merging of vorticity produce strong mixing. The cross-section σ for the inelastic merging and mixing is a maximum for the impact parameter b comparable with the dipole vortex radius r_0. The zero impact parameter collisions studied earlier[8,9] are more elastic and produce much less radial or latitudinal transport than those with $b \sim r_0$.

Based on our simulations we develop a model for the rate of plasma transport produced by vortex interactions for a system with a given vortex line density n_v (number of vortices per unit area) and of mean radius r_0. The packing fraction $f_p = n_v \pi r_0^2$ is assumed sufficiently small that binary collisions dominate. We compare the vortex interaction induced transport with the wave induced transport. For comparable energy densities in vortices and wave fluctuations we estimate that for $r_0 \gg \overline{\lambda}_x$ the vortex induced transport dominates the wave induced transport for $f_p > 1/(\overline{k}_x r_0)^{1/2}$.

Much work remains to determine the formulas governing and the overall importance of the vortex-wave transport problem. These studies serve to point out the real possibility for the practical importance of long-lived, large scale $r_0 \gg \rho_s$ or ρ_g vortices in determining the transport and the large scale fluctuation spectrum in quasi 2D systems such as magnetized plasma or rapidly rotating neutral fluids.

The importance and prevalence of 2D vortical flows arises from the self-binding or self-focusing nature of these finite amplitude ($R_E \gtrsim 1$) structures. The self-focusing of the structure allows it to avoid the destructive effects of system inhomogeneities leading to diffraction and energy absorption such as by Landau damping induced by magnetic shear, or for Rossby vortices by bottom topography, which are so effective in limiting the lifetime of small amplitude fluctuations. Further research is required to determine the limits of the 2D approximation to the 3D systems and the limits on the critical magnetic shear and other inhomogeneity parameters for the validity of the 2D vortex description.

Acknowledgments

The author gratefully acknowledges the work of Lee Leonard and Xiang Su during the course of this investigation. He also acknowledges helpful discussions with J. D. Meiss and K. Nozaki. The work was supported by the U.S. Department of Energy contract #DE-FG05-80ET-53088.

Appendix A – Moments of a Distribution of Vortices and Waves

Here we give the three invariant moments of the Hamiltonian dynamics for a model distribution of vortices and incoherent waves. For simplicity we parameterize each vortex $i = 1$ to N_v as in Eq. (20) with amplitude A_i and radius $r_i \equiv 1/\gamma_i$. The waves are given by $\varphi^w_{\mathbf{k}}$ and are assumed to be the Fourier modes of the difference field $\varphi - \varphi^v = \varphi - \sum_{i=1}^{N_v} \varphi^v_i$. At a given time the total field is represented by

$$\varphi = \sum_{i=1}^{N_v} \varphi^v_i(x - x_i, y - y_i) + \sum_{\mathbf{k}} \varphi^w_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \tag{A.1}$$

where the waves $\varphi^w_{-\mathbf{k}} = (\varphi^w_{\mathbf{k}})^*$ are uncorrelated with φ^v_i.

We calculate the three moments defined in Eqs. (5)-(13) as follows:

Mass Moment

$$M = \int dxdy\varphi = \pi \sum_{i=1}^{N_v} A_i r_i^2 + L_x L_y \varphi^w_{\mathbf{k}=0} \tag{A.2}$$

Energy Moment.

$$E = \sum_{i=1}^{N_v} E_i + \sum_{i<j}^{N_v} E_{ij} + L_x L_y \sum_{\mathbf{k}} (1 + k^2)|\varphi^w_{\mathbf{k}}|^2 \tag{A.3}$$

where the vortex energy is

$$E_i = \frac{\pi A_i^2}{4\gamma_i^2}(1 + 2\gamma_i^2) = \frac{\pi}{2} A_i^2 \left(1 + \frac{1}{2} r_i^2\right) \tag{A.4}$$

and E_{ij} is the vortex interaction energy which is only simple in the case $\gamma_i = \gamma_j = \gamma$. For this case the interaction energy E_{ij} is the function of the separation

$$r_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2$$

given by

$$E_{ij} = \frac{\pi}{2} \frac{A_i A_j}{\gamma^2} \left(1 + 2\gamma^2 - \gamma^4 r_{ij}^2\right) \exp\left(-\frac{\gamma^2 r_{ij}^2}{2}\right) . \tag{A.5}$$

The interaction energy has local extrema at

$$r_{ij} = 0 \qquad \text{and} \qquad r_{ij} = (1+4\gamma^2)^{1/2}/\gamma^2 \tag{A.6}$$

and vanishes ($E_{ij} = 0$) at

$$r_{ij} = \infty \qquad \text{and} \qquad r_{ij} = \frac{(1+2\gamma^2)^{1/2}}{\gamma^2}\,. \tag{A.7}$$

Enstrophy Moment

$$U = \sum_{i=1}^{N_v} U_i + \sum_{i<j}^{N_v} U_{ij} + L_x L_y \sum_{\mathbf{k}} k^2(1+k^2)|\varphi_{\mathbf{k}}^w|^2 \tag{A.8}$$

where

$$U_i = \frac{\pi A_i^2}{2}(1+4\gamma_i^2) \tag{A.9}$$

and the interaction enstrophy is only simple when $\gamma = \gamma_i = \gamma_j$ where

$$U_{ij} = \pi A_i A_j \left[1 + 4\gamma^2 - \left(\frac{\gamma^2}{2} + 4\gamma^4\right) r_{ij}^2 + \frac{1}{2}\gamma^6 r_{ij}^4\right] \exp\left(-\frac{\gamma^2 r_{ij}^2}{2}\right)\,. \tag{A.10}$$

The interaction enstrophy has extrema at

$$\begin{aligned} r_{ij} &= 0 \\ r_{ij} &= \left[\frac{1+12\gamma^2}{2\gamma^4} - \frac{[2+(1+12\gamma^2)^2]^{1/2}}{2\sqrt{3}\gamma^4}\right]^{1/2} \end{aligned} \tag{A.11}$$

and

$$r_{ij} = \left[\frac{1+12\gamma^2}{2\gamma^4} + \frac{[2+(1+12\gamma^2)^2]^{1/2}}{2\sqrt{3}\gamma^4}\right]^{1/2}\,. \tag{A.12}$$

The interaction enstrophy vanishes at

$$\begin{aligned} r_{ij} &= \infty \\ r_{ij} &= \left[\frac{1+8\gamma^2}{2\gamma^4} + \frac{[1+(1+(8\gamma^2+1)^2]^{1/2}}{2\sqrt{2}\gamma^4}\right]^{1/2} \end{aligned} \tag{A.13}$$

and

$$r_{ij} = \left[\frac{1+8\gamma^2}{2\gamma^4} - \frac{[1+(1+8\gamma^2)^2]^{1/2}}{2\sqrt{2}\gamma^4}\right]^{1/2} .$$

The packing fraction f_p for this model field is defined by

$$f_p = \sum_{i=1}^{N_v} \frac{\pi r_i^2}{L_x L_y} = \frac{N_v \pi r_0^2}{L_x L_y} = \pi r_0^2 n_v$$

where we define the average vortex radius r_0 by $r_0 = \left(\frac{1}{N_v}\sum_{i=1}^{N_v} r_i^2\right)^{1/2}$ and the line density of vortices by $n_v = N_v/L_x L_y$.

The line mass density is then

$$m = M/L_x L_y = f_p \overline{A} + \varphi^w_{\mathbf{k}=0}$$

where $\overline{A} = \sum A_i r_i^2 / \sum r_i^2$ and $\varphi^w_{\mathbf{k}=0} = 0$ by definition.

The line energy and enstrophy density are complicated functions that depend on the pair correlation $P_v(r_{ij})$ function of the vortices. For a dilute, uniform distribution where $\left\langle r_{ij}^2 \right\rangle = L_x L_y / \pi N_v = (\pi n_v)^{-1}$ the interactions E_{ij}, U_{ij} are weak. In this limit we have

$$E/L_x L_y \cong f_p A^2 \left(1 + \frac{2}{r_0^2}\right) + \sum_{\mathbf{k}} (1+k^2)|\varphi^w_{\mathbf{k}}|^2$$

and

$$U/L_x L_y \cong \frac{f_p A^2}{r_0^2}\left(1 + \frac{4}{r_0^2}\right) + \sum_{\mathbf{k}} k^2(1+k^2)|\varphi^w_k|^2$$

where A^2 is an average of A_i^2 over the distribution of vortices.

The presence of the vortices can strongly change the skewness and kurtosis of the fluctuation field. For small f_p the third and fourth order momentums are

$$\begin{aligned}
\left\langle \varphi^3 \right\rangle &\cong \sum_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}} \varphi^w_{-\mathbf{k}} \cdot \varphi^w_{\mathbf{k}_1} \varphi^w_{\mathbf{k}_2} + \sum_{i=1}^{N_v} A_i \sum_{\mathbf{k}} |\varphi^w_{\mathbf{k}}|^2 \\
\left\langle \varphi^4 \right\rangle &\cong 3\left(\sum_{\mathbf{k}} |\varphi^w_{\mathbf{k}_1}|^2\right)^2 + f_p \overline{A}^2 \sum_{\mathbf{k}} |\varphi_k|^2 + f_p A^4 \\
\left\langle \varphi^2 \right\rangle &= f_p A^2 + \sum_{\mathbf{k}} |\varphi^w_{\mathbf{k}}|^2 .
\end{aligned}$$

Thus, the Kurtosis is

$$K = \frac{f_p A^4 + f A^2 \langle \varphi^2 \rangle + 3 \langle \varphi^2 \rangle^2}{(f_p A^2 + \langle \varphi^2 \rangle)^2}.$$

For $f_p A^2 > \langle \varphi^2 \rangle$

$$K \simeq \frac{1}{f_p} = \frac{\langle r_{ij}^2 \rangle}{r_0^2}$$

and $K \cong 3$ for $f_p A^2 < \langle \varphi^2 \rangle$.

Assuming that the decomposition in Eq. (A.1) with a quasi-normal distribution for φ^w is valid and that suitable ordinary differential equations could be derived, the original problem of $2N$ **k** space equations would be reduced to $5N_v$ equations for A_i, r_i, x_i, y_i and $2N_w$ equations for $\varphi^w_{\mathbf{k}}$. The reduction would be significant when N_w, $N_v \ll N$. Since we know that even in the limit of $N_w = 0$ that point vortices and modulated point vortices fail, except when used in large numbers, to describe the collisions between distributed vortices and, furthermore, in the drift wave collision the numbers N_v and N_w change during a collision, the prospect for a quantitative reduced description based on small N_w and N_v seems rather dim.

References

1. A. Hasegawa and K. Mima, Phys. Rev. Lett. **39**, 205 (1977) and Phys. Fluids **21**, 87 (1978).

2. W. Horton and R.D. Estes, Nucl. Fusion **19**, 203 (1979).

3. P.C. Liewer, Nucl. Fusion **25**, 543 (1985).

4. W. Horton, in *Handbook of Plasma Physics II,* edited by M.N. Rosenbluth and R.Z. Sagdeev, p. 383 (1984).

5. P.W. Terry and W. Horton, Phys. Fluids **25**, 491 (1982).

6. W. Horton, Phys. Fluids **29**, 1491 (1986).

7. J.D. Meiss and W. Horton, Phys. Fluids **26**, 990 (1983).

8. M. Makino, T. Kamimura, and T. Tanuiti, J. Phys. Soc. Japan, Vol. , 980 (1981).

9. J.C. McWilliams and N.J. Zabusky, Geophys. Astrophys. Dynamics **19**, 207 (1982).

10. W. Horton, Plasma Physics and Controlled Fusion **27**, 937 (1985).

11. Y. Couder and C. Basdevant, J. Fluid Mech. **173**, 225 (1986).

12. R.W. Griffiths and E.J. Hopfinger, J. Fluid Mech. **178**, 73 (1987).

13. R.W. Griffiths and E.J. Hopfinger, J. Fluid Mech. **173**, 501 (1986).

14. M. Kono and E. Miyashita, Phys. Fluids **31**, 326 (1988).

15. V. E. Zakharov and E. I. Schulman, Physica D **29**, 283 (1988).]

16. D. Biskamp and M. Walter, Physics Letts. **109A**, 34 (1985).

17. M.V. Nezlin, Sov. Phys. Usp. **29**, 807 (1986). [Usp. Fiz. Nauk **150**, 3 (1986)].

18. E.A. Overman and N.J. Zabusky, Phys. Fluids 25, 1297 (1982).

19. M.V. Melander, J.C. McWilliams, and N.J. Zabusky, J. Fluid Mech. **178**, 137 (1987); Phys. Fluids **30**, 2610 (1987).

20. R.T. Pierrehumbert and S.E. Widnall, J. Fluid Mech. **114**, 59 (1982).

21. W. Horton, T. Tajima, and T. Kamimura, Phys. Fluids **30**, 3485 (1987).

22. H.L Swinney, J. Sommerica and S.D. Meyers, *Nature* 331, 216 (1988).

23. J. Pedlosky, *Geophysical Fluid Dynamics* (Springer-Verlag, New York 1979), p. 328.

24. J. Liu, W. Horton, and J.E. Sedlak, Phys. Fluids **30**, 467 (1987).

25. M.N. Rosenbluth, H.L. Berk, I. Doxas and W. Horton, Phys. Fluids **30**, 2636 (1987).

Figure Captions

1. Dispersion and self-binding of a gaussian-φ drift wave monopole. (a) dispersive propagation in the linear regime (b) self-binding in the nonlinear regime.

2. Coalescence of like-signed vorticity regions produced by two neighboring positive (charge filaments) gaussian monopoles.

3. Pairing of opposite signed vorticity regions produced by neighboring positive and negative (charge filaments) gaussian monopoles.

4. Collision of opposite polarity drift wave vortices with impact parameter $b = 5\rho_s$ and dipole radius $a = 6\rho_s$. (a) contours of potential with the contour interval $\Delta\varphi = 2.0$ (b) contours for perturbed distribution δf with contour interval $\Delta\delta f = 2.0$. Positive values of the functions are solid lines and negative values are dashed.