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Abstract

The nature and role of nonwavelike incoherent fluctuations in Vlasov plasma turbulence and transport are considered. In particular, electrostatic drift holes, which are localized, self-binding incoherent fluctuations giving large negative skewness, are described in detail. Both the three dimensional structure and dynamics of drift holes are discussed. The important role of incoherent fluctuations in plasma transport is underscored with an example, that of transport in drift-Alfvén turbulence.

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Introduction

The transport of heat and particles by plasma turbulence is regarded as an important process in both laboratory and astrophysical plasmas. It is generally assumed that plasma turbulence results from instabilities wherein waves of initially infinitesimal amplitude grow up to fluctuation levels necessary for the balance of input energy with nonlinear transfer. Thus it has been natural to describe plasma turbulence as an ensemble of randomly phased, finite amplitude waves. Such descriptions, however, fail to account for incoherent mode coupling which manifests itself in the formation of localized fluctuations propagating ballistically in even weakly turbulent plasmas.[1-3] Such fluctuations have been shown to be nonlinearly unstable under rather general conditions[2-5], allowing relaxation and transport which is independent and quite distinct from that associated with linear stability. In this paper, a particular type of localized incoherent fluctuation relevant to inhomogeneous magnetized plasmas in 3-D geometries, the electrostatic drift hole[6], is described in detail. It is then argued that the self-consistent inclusion of incoherent fluctuations in transport calculations can seriously alter the predictions of such calculations[7].

Nonwavelike fluctuations in a plasma may be characterized in terms of two complimentary paradigms. The first paradigm is that of clumps[1-3]. Similar to the eddies of neutral fluid turbulence, clumps are localized phase space density fluctuations which decay due to the action of turbulent shear stresses. The lifetime of these fluctuations exceeds the correlation time of the turbulence (defined as the correlation time of the scattering potential) when the clump scale is less than the correlation length of the potential. Because of their localization and lifetime, clumps behave like a decaying macroparticle. The fact that they are directly and nonlinearly driven by gradients in the average distribution means that they affect both plasma stability and transport[2,3]. Moreover, clumps emit into collective modes precisely the same way in which a test charge emits into damped collective resonances in a discrete plasma description[8]. Such a process can account for the frequency linewidths of power spectra measured at fixed wavenumber in tokamak experiments[9-10,2-3].

Clumps are statistical in nature. They are described as a peak in the two-point phase space density correlation at small relative separation. The turbulent decay of clumps can be described within the framework of quasi-Guassian statistics. Thus, standard two-point closures model clump evolution. On the other hand, these statistical assumptions do not facilitate the incorporation of effects of the self-electric field associated with a localized clump of charged particles. Simple estimates show that such fields can be sufficient to trap the particles that produced them[11], leading to the possibility of very long fluctuation lifetimes.

The inclusion of self-electric field effects leads to the second paradigm, phase space density holes. Holes are a localized depression in phase space density in which the particles making up the fluctuation are self-trapped by the potential which they (and passing particles shielding the fluctuation) produce. The electrostatic self-trapping requires that the phase space density fluctuation be negative in the case of positive plasma shielding response. Hence, holes are strongly non-Guassian, having significant skewness[12].

A single isolated hole has effectively an infinite lifetime due to the self-binding produced by the potential. In the presence of other holes, shear stresses arising from the neighboring holes compete with self-binding by creating perturbations which permit detrapping of the trapped particles. If the density of holes in the available phase space volume is sufficient for the turbulent shear stresses to dominate electrostatic self-binding, the fluctuations can be described by the clump formalism[12]. In this case self-binding is negligible, turbulent stresses generate fluctuations of both sighs, skewness is thus small, and the statistics become quasi-Guassian. Consequently, clumps and holes may be viewed as complimentay paradigms for incoherent fluctuations which range from quasi-Guassian (clumps) to intermittent (holes). The control parameter spanning the spectrum of incoherent fluctuations is the packing fraction[12].

In this paper, phase space density holes in a three-dimensional magnetized plasma are described. Low frequency turbulence is considered. Hence, it is assumed that the drift kinetic equation (DKE) governs the electron dynamics and that a warm, low frequency response describes the ion dynamics. A single isolated hole representing a maximum entropy configuration of the system is investigated. Such a hole represents a 'most probable' fluctuation of the system. The physical scales characterizing the hole are determined by the maximum entropy condition. Thus, properties of the turbulence deriving from fluctuation scales, such as mixing lengths, are based on nonlinear statistical properties as opposed to linear eigenmode features.

Phase space density holes grow nonlinearly. Free energy is provided by gradients in the average distribution. This process is incorporated in the basic dynamical equation, the DKE, which through conservation of phase space density along particle trajectories relates the dynamics of phase space density fluctuations to the relaxation of the average distribution. Self-consistency constraints, in this case, quasineutrality, enter into the physics of the instability and effectively link growth of a hole in one species to dissipation in the opposite species. In this paper the nonlinear growth rate is derived, saturation mechanisms are identified, and saturated fluctuation levels are obtained.

In the last section of this paper, the relation described by the DKE between fluctuation dynamics, in particular the dynamics of generic incoherent fluctuations, and relaxation is used to study transport. It is shown that the self-consistent inclusion of incoherent fluctuations involving both the electrostatic potential and magnetic field in a model describing drift-Alfvén turbulence lead to the conclusion that electrostatic fluctuations *alone* regulate the transport[7]. This result is strikingly different from quasilinear theory, which does not account for incoherent fluctuations.

Electrostatic Drift Hole Structure

Consider a neutral plasma consisting of ions and electrons in a shearless slab with magnetic field in the $\hat{z}$ direction. Let the equilibrium density have a gradient in the $\hat{x}$ direction. We are interested in low frequency, long parallel wavelength fluctuations typical of drift waves, i.e., $\omega < \Omega_i$, $v_{ti} < \omega/k_\parallel < v_{te}$, $k_\perp \sim \rho_s^{-1}$, $k_\parallel \sim L_\parallel^{-1}$. Here, Ω_i is the ion gyrofrequency, v_{ti} and v_{te} are the thermal velocities for ions and electrons, $\rho_s = C_s/\Omega_i = c(T_e M)^{1/2}/eB_0$ and $L_\parallel$ is the system size in the parallel direction. The hole consists of electrons trapped in a self-consistent potential well. Electrons which are not trapped by the potential and ions constitute passing species which together with the trapped electrons determine the potential through the quasineutrality condition.

The trapped electrons satisfy a drift kinetic equation,

$$\frac{\partial f_t}{\partial t} + v_\parallel \hat{z} \cdot \nabla f_t + \underset{\sim}{V}_E \cdot \nabla f_t - \frac{|e|}{m_e} E_\parallel \frac{\partial f_t}{\partial v_\parallel} = 0 \tag{1}$$

where $\underset{\sim}{V}_E = c\nabla\hat{\phi}\times\hat{z}/B_0$ is the crossfield E×B drift. Passing electrons are adiabatic, $f_p = (|e|/T_e)\hat{\phi}\, F_0(v)$, where F_0 is the equilibrium velocity distribution. The ion response may be treated as hydrodynamic. From the ion continuity equation, the ion density is

$$n_i(\hat{\phi}) = \left\{ \rho_s^2 \nabla_\perp^2 - \frac{iV_D}{\omega} \frac{\partial}{\partial y} \right\} \frac{|e|\hat{\phi}}{T_e} \tag{2}$$

where $V_D = C_s\rho_s/L_n$ and L_n is the density gradient scale length. This equation reflects the physics of crossfield ion motion obtained from the solution of the ion momentum equation to first order in (ω/Ω_i). The second term on the right hand side comes from the E×B drift and the first from the polarization drift $V_p = -\nabla_\perp\hat{\phi}\, c\omega/B\Omega_i$.

The hole is a steady state fluctuation of the equilibrium phase space density. Therefore, we seek a time independent solution of the drift kinetic equation for the trapped electrons. It is readily apparent that there exists a family of solutions

$$f_t(\underset{\sim}{x},\underset{\sim}{v}) = F\left(\frac{v_\parallel^2}{2} - \frac{|e|\hat{\phi}(\underset{\sim}{x})}{m_e}\right) \qquad (3)$$

where F is an arbitrary function of its argument. These solutions are Bernstein-Greene-Kruskal (BGK) equilibria[13] representing the parallel streaming of trapped electrons against the acceleration of the trapping electric field potential. This balance applies to the parallel motion only. Nevertheless, Eq. (3) is a solution to the three dimensional DKE because crossfield dynamics are contained solely in the E×B nonlinearity which is identically zero for any function whose spatial dependence enters only through the potential $\hat{\phi}(\underset{\sim}{x})$.

The function F in Eq. (3) is arbitrary. It is reasonable to seek the solution which represents the most probable fluctuation of the system. Specifically, we seek the BGK solution which maximizes the entropy of the system for a given mass, momentum and energy. Because of subtleties in the selection of the appropriate measure of entropy for kinetic systems[11], the issue of which entropy to use has never been completely resolved. For simplicity, we use the Maxwell-Boltzman entropy σ defined as

$$\sigma = n\int dv_\parallel \int d^3x \left(f_t \ell n f_t - F_o \ell n F_o \right) \qquad (4)$$

where $f_t = F_o$ on the hole boundary. Similarly, the mass M, momentum P, and energy T are defined as

$$\begin{bmatrix} M \\ P \\ T \end{bmatrix} = n\int dv_\parallel \int d^3x \,(f_t - F_o) \begin{bmatrix} m \\ mv_\parallel \\ mv_\parallel^2/2 - |e|\hat{\phi}(\underset{\sim}{x}) \end{bmatrix} \qquad (5)$$

Using Lagrange multipliers, it is straightforward to maximize σ while constraining M,P, and T with the result that

$$f_t = F_o(u_\parallel)\exp\left(\left\{\frac{mv_\parallel^2}{2} - |e|(\hat{\phi} - \hat{\phi}_m)\right\}/\tau\right) \tag{6}$$

inside the hole boundary and $f_t = F(u_\parallel)$ outside the hole boundary. $\hat{\phi}_m$ is the potential at the hole boundary where $E = mv_\parallel^2/2 - |e|\hat{\phi}$ = constant $= |e|\hat{\phi}_m$. Note that $mv_\parallel^2/2 > |e|(\hat{\phi} - \hat{\phi}_m)$ for particles trapped in the hole, implying that $\tau < 0$ for holes which are negative fluctuations ($f_t < F_o$).

To determine the spatial characteristics of the Maxwell-Boltzman hole, it is necessary to solve for the potential from the constituitive relation, which in this case is quasineutrality, $n_i = n_e$, or

$$\left(1 - \rho_s^2\nabla_\perp^2 + \frac{iV_D}{k_\parallel u_\parallel}\frac{\partial}{\partial y}\right)\frac{|e|\hat{\phi}}{T_e} = -\int_{trap} dv_\parallel\, F_o(u_\parallel)$$

$$\left\{\exp\left[\left(\frac{mv_\parallel^2}{2} - |e|(\hat{\phi} - \hat{\phi}_m)\right)/\tau\right] - 1\right\} \tag{7}$$

In Eq. (7), $u_\parallel$ is the center of mass velocity of the hole and the ion density has been written in the center of mass frame of the hole where the frequency ω is equal to the hole doppler frequency $k_\parallel u_\parallel$. This non-linear integro-differential equation is not tractable in generality. It is far more illuminating from the physics point of view to approximate the trapped particle distribution as a constant $\tilde{f}$ in the trapping region and zero outside. This is consistent with the shallow hole limit[11] where $(E + |e|\hat{\phi}_m)/\tau < 1$. Futher approximating the trapping region of phase space as a rectangle of dimensions Δx, Δy, $\Delta v_\parallel$, which at this stage are yet to be determined, yields the box-hole approximation:

$$f_t = \begin{cases} \tilde{f} & \begin{cases} -\Delta x \leqslant x \leqslant \Delta x \\ -\Delta y \leqslant y \leqslant \Delta y \\ -\Delta v_\parallel \leqslant v_\parallel + u_\parallel \leqslant \Delta v_\parallel \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

The third spatial dimension Δz, is effectively indeterminate since the shielding dynamics include crossfield motions only. The box hole is illustrated in Fig. 1.

The entropy of the box-hole is not as large as that of a Maxwell-Boltzman hole. The entropy of the box hole may still be maximized subject to fixed mass, momentum, and energy, and this leads to the specification of the constants $\tilde{f}$, Δx, Δy, and $\Delta v_\parallel$. In the 1-D Vlasov case, the maximum entropy of the box-hole was not significantly smaller than that of the Maxwell-Boltzman hole[11]. On the other hand, the box hole clearly and simply shows the relationship between hole depth, potential amplitude and hole width in velocity space which underlies the self-trapping phenomenon. Indeed, a major point in calculating the structure of the hole is to show that the maximum entropy hole satifies the self-trapping condition

$$\Delta V_{tr} \equiv \left(\frac{|e|\hat{\phi}}{m} \right)^{\frac{1}{2}} > \Delta v_\parallel . \tag{9}$$

Given the box-hole approximation, the potential equation becomes

$$\left[\rho_s^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - 1 - \frac{iV_D}{k_\parallel u_\parallel} \frac{\partial}{\partial y} \right] \hat{\phi} = \frac{T_e}{|e|} \tilde{f} \Delta v_\parallel \tag{10}$$

The solution to Eq. (10) is readily obtained by standard Green's function technique:

$$\hat{\phi} = -\int_{-\Delta x}^{\Delta x} dx' \int_{-\Delta y}^{\Delta y} dy' \; \frac{1}{L_y} \sum_{m=-\infty}^{\infty} G_m(x-x') \; \exp(ik_m(y-y')) \frac{T_e}{|e|} \frac{\Delta v_\parallel}{\rho_s^2} \tilde{f}$$

where the full Green's function $G(x-x', y-y')$ has been expanded in Fourier harmonics with $k_m = m\pi/L_y$ and

$$G(x-x',y-y') = \frac{1}{L_y} \sum_{m=-\infty}^{\infty} G_m(x-x') \exp(ik_m(y-y')) \quad .$$

The amplitude G_m satisfies

$$\left(\frac{\partial^2}{\partial x^2} - \lambda_m^{-2} \right) G_m(x-x') = \delta(x-x') \tag{11}$$

where

$$\lambda_m^{-2} = \frac{1}{\rho_s^2} \left(1 + k_m^2 \rho_s^2 - \frac{\omega_*}{k_\parallel u_\parallel} \right) \tag{12}$$

From Eq. (11) it is apparent that λ_m plays the role of a shielding length. When λ_m^{-2} is positive, the potential decays outside of the trapping region with an e-folding given by λ_m. For λ_m^{-2} negative, the potential oscillates outside of the trapping region with wavelengths $|\lambda_m|$. It can be seen from Eq. (12) that $\lambda_m^{-2} \gtrless 0$ implies $k_\parallel u_\parallel \gtrless \omega_*/(1 + k_m^2\rho_s^2) = \omega(k_m)$ where $\omega(k_m)$ is the real eigenmode frequency of electron drift waves. This is precisely the condition which determines whether resonant coupling of electrostatic drift holes to drift waves occurs. When $\lambda_m^{-2} > 0$ for all m, there is no hole-wave coupling, the shielding length is purely real, and the potential is localized about the trapping region. When $\lambda_m^{-2} < 0$ for any m, there is hole-wave coupling, the shielding length becomes imaginary and the potential has oscillatory character outside the trapping region. Hole-wave coupling affects both the structure and dynamics of the hole[14] and will not be considered further in this paper.

The solution of Eq. (11) yields

$$\hat{\phi} = \frac{-2T_e}{|e|} \frac{\tilde{f}\Delta v_\parallel}{\rho_s^2} \sum_{m=-\infty}^{\infty} \frac{\lambda_m^2}{L_y} \frac{\sin(k_m \Delta y)}{k_m} \exp(-k_m y) \{1 - \exp(-\Delta x/\lambda_m)\cosh(x/\lambda_m)\} \tag{13}$$

for $-\Delta x < x < \Delta x$. When $-\Delta x > x > \Delta x$, $\{1 - \exp(-\Delta x/\lambda_m)\cosh(x/\lambda_m)\} \rightarrow \exp(-|x-\Delta x|/\lambda_m)$. Note that the net shielding length is a composite of m-harmonics. Since $\lambda_m^2/k_m \simeq k_m^{-3}$, the sum over harmonics is heavily weighted to the lowest m's. From Eq. (13) it is clear that for $\lambda_m^{-2} > 0$, the potential is positive when $\tilde{f}$ is negative. Thus, the potential has the correct sign for trapping electrons, provided that the trapped particle distribution is negative.

From a knowledge of the potential the crossfield particle motion is given as $v_r = (c/B_o)\hat{E}_\theta = (c/B_o)\,\partial\hat{\phi}/\partial y$; $v_\theta = -(c/B_o)\hat{E}_r = -(c/B_o)\partial\hat{\phi}/\partial x$. Within the trapping region the perpendicular plane projection of particle motion consists of closed orbits about the center of the hole as shown in Fig. 2. As explained previously, the parallel electron motion consists of streaming in the potential well between bounce points.

The hole scales $\Delta x, \Delta y$, $\Delta v_\parallel$ and the hole depth $\tilde{f}$ are found by maximizing the entropy subject to fixed mass momentum and energy. For the box-hole, the entropy is simply given by

$$\sigma_o = \frac{M\tilde{f}}{2m_e F_o} \tag{14}$$

where M is the hole mass and $Mu_\parallel$ is the momemtum. The hole energy is

$$T = -\frac{N|e|}{2}\int_{-\Delta x}^{\Delta x} dx \int_{-\Delta y}^{\Delta y} dy \;\; \hat{\phi}(x,y)\;\tilde{f}\Delta v_\parallel + \frac{1}{6}\, mN(\Delta v_\parallel^2) \tag{15}$$

where the first term is the electrostatic self-energy associated with a potential in a dielectric medium and the second term is the kinetic energy. The maximization procedure yields two coupled equations for x and y. The solution of these equations is now summarized: the crossfield scales range from a few times ρ_s up to the system size L_y. Electrons are more deeply trapped in the small holes than in the large ones ($\Delta x, \Delta y \simeq \rho_s$) making small holes more probable. Probability considerations also indicate that the indeterminate scale Δz will tend to be the system size in the z direction. The hole depth $\tilde{f}$ is given by

$$-\tilde{f} \quad = \quad \frac{\Delta v_{\parallel}}{6v_{te}^2} \; g^{-1}(\Delta x/\rho_s) \tag{16}$$

where g(r) is a structure function which goes like $r^2/6$ for $r \to 0$ and increases monotonically to 1 as $r \to \infty$. Note that the hole is indeed a depression ($\tilde{f} < 0$) and that its depth is proportional to its width in velocity space. This effectively guarentees self-trapping since the potential amplitude $\hat{\phi}(x=o,\ y=o)$ is

$$\hat{\phi}(0) \quad = \quad \frac{-2T}{|e|}\,\tilde{f}\,\Delta v_{\parallel}\frac{\Delta y}{L_y}\;\left(\,1 - \exp(-\Delta x/\rho_s)\,\right) \tag{17}$$

Using Eq. (16), the width in velocity space satisfies

$$\Delta v_{\parallel} \quad = \quad \Delta V_{tr}\;\; g^{\frac{1}{2}}\;\; (1 - \exp(-\Delta x/\rho_s)\,)^{-\frac{1}{2}} \tag{18}$$

from which $\Delta v_{\parallel}/\Delta V_{tr} < 1$ follows. Aside from demonstrating the validity of self-trapping, it is important to note that this calculation yields fluctuations whose scales are based on maximal probability considerations rather than the properties of linear dispersion associated with collective owcillation.

Hole Dynamics

Incoherent fluctuations, including both clumps and holes, are nonlinearly unstable under fairly broad conditions. In the case of holes, growth occurs on a time scale which is longer that the bounce time ($\gamma/\omega_b < 1$) allowing the equilibrium structure calculated in the previous section to maintain itself as the amplitude $\tilde{f}$ increases. Conservation of phase space density along particle trajectories,

$$\frac{\partial}{\partial t}\int dx \int dv_{\parallel}\ \tilde{f}^2 \quad = \quad -2\frac{\partial}{\partial t}\int dx \int dv_{\parallel}\ f_o\tilde{f} \quad , \tag{19}$$

allows for hole growth in response to changes in the background plasma f_o induced by hole motion. The gradient in f_o is the source of free energy. It is thus radial motion of particles induced by the hole potential which is associated with hole growth. In particular, the potential causes passing ions to E×B drift radially with $v_r = c/B_o\ \partial\hat{\phi}/\partial y$. Because of the gradient in x, there is a net displacement of ions down the gradient. Passing electrons are similarly displaced in order to maintain quasineutrality. As a consequence of this motion, the background plasma f_o becomes larger in the vicinity of the hole. Conservation of total phase space density $f_o - |\tilde{f}|$ requires that the hole depth $|\tilde{f}|$ increase as the background density f_o increases. Hole growth is depicted in Fig. 3.

Eq. (19) is the basis for the analytic calculation of the growth rate. The average distribution is expanded about the hole center coordinates x_o, $u_{\parallel}$:

$$f_o \quad = \quad f_o(x_o,u_{\parallel}) \quad + \quad (x - x_o)\left.\frac{\partial f_o}{\partial x}\right|_{\substack{x=x_o \\ v_{\parallel}=u_{\parallel}}} \quad + \quad (v_{\parallel} - u_{\parallel})\left.\frac{\partial f_o}{\partial v}\right|_{\substack{x=x_o \\ v_{\parallel}=u_{\parallel}}}$$

The displacements $X = x-x_o$, $V_{\parallel} = v_{\parallel}-u_{\parallel}$ are governed by the drifts and acceleration of the parallel electric field, i.e.,

$$\frac{dX}{dt} \quad = \quad \frac{c}{B_o}\hat{E}_{\theta} \qquad\qquad \frac{dV_{\parallel}}{dt} \quad = \quad -\frac{|e|}{m_e}\hat{E}_{\parallel} \tag{20}$$

Substitution of Eqs.(20) into Eq. (19) then yields

$$\frac{\partial}{\partial t}\langle \tilde{f}^2\rangle_h = -2\left(\frac{c}{B_o}\langle \hat{E}_\theta \tilde{n}_e\rangle \frac{\partial f_o}{\partial x} - \frac{|e|}{m_e}\langle \hat{E}_\parallel \tilde{n}_e\rangle \frac{\partial f_o}{\partial v_\parallel}\right)\frac{f_o}{\Delta v_\parallel} \tag{21}$$

where $\langle\ \rangle_h = (\Delta v_\parallel)^{-1}\int dv_\parallel$ and $\langle\ \rangle = (\Delta v_\parallel)^{-1}\int dv_\parallel (\Delta x \Delta y)^{-1}\int dxdy$ and $\tilde{n}_e = \int dv_\parallel \tilde{f}$. At this point, quasineutrality is imposed, allowing the incoherent electron density to be expressed in terms of the ion density. Quasineutrality gives $\hat{n}_i = e\hat{\phi}/T_e + \tilde{n}_e$ where $e\hat{\phi}/T_e$ represents the passing electron response. The ion density can be written as a susceptibility χ_i times the potential $e\hat{\phi}/T_e$, so that

$$\frac{\partial}{\partial t}\langle \tilde{f}^2\rangle_h = 2n_o \frac{f_o(x_o,u_\parallel)}{\Delta v_\parallel}(\omega_* - k_\parallel u_\parallel)\left(\frac{e|\hat{\phi}|}{T_e}\right)^2 |\mathrm{Im}\chi_i| \tag{22}$$

where f_o has been assumed Maxwellian and only the real part of $\partial/\partial t\langle \tilde{f}^2\rangle_h$, i.e., the part giving secular evolution of $\langle \tilde{f}^2\rangle_h$, has been retained. Eq. (22) indicates that the hole grows when the rate of free energy released from the density gradient source exceeds the parallel heating rate, or $\omega_{*e} > k_\parallel u_\parallel$. The ion dissipation scaling is characteristic of nonlinear instabilities involving incoherent fluctuations and reflects the key role played by ions in the instability mechanism. The ion susceptibility is evaluated at the hole center of mass velocity, thus

$$|\mathrm{Im}\chi_i| = (\pi/2)^{\frac{1}{2}} \frac{T_e}{T_i}\frac{u_\parallel}{v_{ti}}\left(1 + \frac{|\omega_{*i}|}{k_\parallel u_\parallel}\right)\exp\left(-\tfrac{1}{2}(u_\parallel/v_{ti})^2\right).$$

For $u_\parallel > v_{ti}$, the ion susceptibility can be sizable.

Using Eq. (17) to express $|\hat{\phi}|^2$ in terms of $\tilde{f}^2\Delta v_\parallel^2$, it is possible to eliminate $\tilde{f}^2$ from Eq. (22). Then using Eq. (18) to express $\Delta v_\parallel$ in terms of $\hat{\phi}(0)$, the hole growth rate is given by

$$\gamma \equiv \frac{\partial}{\partial t}\ell n\langle \tilde{f}^2\rangle_h = (\omega_* - k_\parallel u_\parallel)\,|\mathrm{Im}\chi_i| \left(\frac{|e|\hat{\phi}(0)}{T_e}\right)^{\frac{1}{2}} H(\Delta x/\rho_s) \tag{23}$$

where H is a structure function. From this result it is possible to verify that the growth rate is smaller than a bounce frequency, i.e., $\gamma/\omega_b \approx \gamma/k_\parallel \Delta v_\parallel \approx \gamma/k_\parallel v_{te} \approx \omega_{*e}/k_\parallel v_{te} << 1$. Therefore the physics of electrostatic holes effectively involves two time scales. The structure or equilibrium properties are established on a rapid scale of electron bounce motion and the amplitude growth and gradient relaxation occur on a slow time scale relative to bounce motion.

It is instructive to contrast the electron drift-hole instability with the usual electron drift wave instability. The hole instability is manifestly nonlinear. Its growth rate is amplitude dependent, going as $\hat{\phi}^{\frac{1}{2}}$. Its fluctuations consist of localized structures as opposed to waves. Unlike the drift wave instability, no collective resonance need be satisfied. Thus the structures produced by the instability have scales governed by probabalistic considerations associated with entropy production in contrast to scales fixed by the equilibrium properties which determine linear dispersion. For holes ion dissipation is destabilizing rather than stabilizing. Consequently, holes can be unstable even in regimes where drift waves are stable.

There are a variety of mechanisms which cause saturation of the electron drift-hole instability. The simplest of these involves a mixing length argument based on the fact that as the hole grows the potential grows and the radial transit time of ions through the hole decreases. When this transit time becomes less than a growth time the ion interaction with the hole effectively ceases. Thus

$$\gamma\tilde{f} \simeq v\frac{\partial\tilde{f}}{\partial x} = \frac{d\langle x\rangle}{dt}\frac{\tilde{f}}{\Delta x} . \tag{24}$$

This has the canonical mixing length form expressing the mixing of energy inputed by the instability at rate γ by fluctuations of scale Δx. In contrast to the standard picture, the rate of energy input is nonlinear and the mixing scale Δx is a nonlinear fluctuation scale. From the growth rate calculation

$$\gamma\langle\tilde{f}^2\rangle_h \approx \gamma\tilde{f}^2 = -2\frac{\partial f_o}{\partial x}\frac{d}{dt}\langle x\tilde{f}\rangle \approx -2\frac{n_o}{L_n}\frac{d\langle x\rangle}{dt}\frac{\tilde{f}}{\Delta v_\parallel}$$

where $n_o = dv_\parallel f_o \approx \Delta v_\parallel f_o$. Substitution into Eq. (24) yields

$$\frac{\tilde{f}\Delta v_\parallel}{n_o} = \frac{\tilde{n}}{n_o} \simeq \frac{\Delta x}{L_n} \tag{25}$$

The replacement of a linear eigenmode width by the most probable fluctuation scale Δx provides an alternate view of plasma turbulence with the hole as an exciton, and in which nonlinear scales associated with statistical considerations characterize the turbulent flows.

Another saturation mechanism involves induced coupling of holes to waves by nonlinear resonance broadening and accompanying emission by holes into waves[14]. For a hole which statically has no wave coupling ($k_\parallel u_\parallel > \omega_*/(1+k_\perp^2\rho_s^2)$) nonlinear growth γ_{nl} broadens the ion response, i.e.,

$$\lambda_m^{-2} = \frac{1}{\rho_s^2}\left(1 + k_m^2\rho_s^2 - \frac{\omega_*}{k_\parallel u_\parallel - \gamma_{nl}}\right)$$

Wave coupling occurs when the potential reaches a level such that $\lambda^{-2} = 0$ or

$$\gamma_{nl}(\hat{\phi}) = k_\parallel u_\parallel - \frac{\omega_*}{1 + k^2\rho_s^2} \tag{26}$$

Substituting again from the growth rate calculation, the saturation level is

$$\frac{e\hat{\phi}(0)}{T_e} = |\mathrm{Im}\chi_i|^{-2} H(\Delta x/\rho_s)^{-2}\left(\frac{k_\parallel u_\parallel}{\omega_*} - \frac{1}{1 + k^2\rho_s^2}\right)^2. \tag{27}$$

Transport and Relaxation

Both electrostatic drift holes as well as clumps play an important role in transport. A striking example of this role and the physical considerations that underlie it is provided by the example of magnetic fluctuations in drift-Alfvén turbulence. Two related models are described. Both are simple extensions of the drift model, Eqs. (1) and (2), to include magnetic fluctuations through a flutter term $B_o^{-1} v_\parallel \nabla_y \hat{A}_\parallel \nabla_\perp f$ where $\hat{A}_\parallel$ is the parallel vector potential. The first model is concerned with the evolution of a isolated blob in a drift-Alfven system. A paradigm for such an object is a magnetostatic drift hole[15], which is a magnetic flutter analogue of the electrostatic drift hole just considered. In the second model, statistical averaging is used to construct a turbulent Lenard-Balescu collision integral for the relaxation of the average distribution due to fully developed turbulence. In both models, incoherent fluctuations of very general form are incorporated; the specific details of structure do not enter.

The relaxation of the average distribution in the presence of an isolated blob is proportional to the nonlinear growth of the blob. The description of this growth parallels that of the electrostatic drift hole which starts with Eq. (19). In the present case the evolution of the blob location includes a flutter term

$$\frac{dX}{dt} = \frac{c}{B_o} \hat{E}_\theta + v_\parallel \frac{\hat{B}_r}{B_o} .$$

It follows then that

$$\left\langle \frac{\partial \tilde{f}^2}{\partial t} \right\rangle_h = -2 \left. \frac{\partial f_o}{\partial x} \right|_{x_o, u_\parallel} \left(\frac{c}{B_o} \langle \hat{E}_\theta n_e \rangle - \frac{\langle \hat{B}_r \hat{J}_{\parallel e} \rangle}{|e| B_o} \right) \frac{f_o}{\Delta v_\parallel} \tag{27}$$

Quasineutrality is imposed and the first term on the right hand side becomes $c/B_0 \langle \hat{E}_\theta n_i \rangle = ck^2/B_0 \langle \hat{\phi}^2 \rangle \, \mathrm{Im}\chi_i$ as before. The flutter contribution is subject to Ampere's law ($\hat{J}_{\parallel e} = -\nabla_\perp^2 \hat{A}_\parallel$, for negligible ion current contribution). Since $B = \nabla \hat{A}_\parallel \times \hat{n}$ and $\nabla \cdot B = 0$, it follows that $\langle \hat{B}_r \nabla_\perp^2 \hat{A}_\parallel \rangle = -\langle \partial/\partial r (\hat{B}_r \hat{B}_\theta) \rangle$ which ultimately contributes only surface terms of $O(\Delta x/L_x) \ll 1$. Hence, in this simple system, magnetic fluctuations do not contribute to the growth of the blob, or to the relaxation of the average distribution. This result may be viewed as a consequence of the self-trapping fields which prevent the unbounded (in collisionless systems) parallel streaming necessary for radial steps. Alternatively, self-consistency constraints (Ampere's law) ensure that transport driven by blobs is ambipolar over blob scales.

Transport in fully developed drift-Alfvén turbulence is governed by an averaged Vlasov equation. The evolution of f_0 is driven by the radial phase space flux : $\partial f_0/\partial t = \partial \Gamma_r/\partial r$ where

$$\Gamma_r = \langle \frac{c}{B_0} \nabla_y (\hat{\phi} - \frac{v_\parallel}{c} \hat{A}_\parallel) \hat{h} \rangle = \sum_{k,\omega} \mathrm{Re} \left[i \frac{c}{B_0} k_y \langle (\hat{\phi} - \frac{v_\parallel}{c} \hat{A}_\parallel) \hat{h} \rangle_{k,\omega} \right] . \tag{28}$$

Here, $\hat{h}$ is the nonadiabatic part of the fluctuating electron distribution. Including fluctuations $h^{(c)}$ which are phase coherent with the potentials $\hat{\phi}$ and $\hat{A}_\parallel$ (collective modes) and incoherent fluctuations $\tilde{h}$ (clumps or holes), the right hand side of the average Vlasov equation can be represented as a turbulent Lenard-Balescu collision integral. The coherent fluctuations drive (quasilinear) diffusion and the incoherent fluctuations give rise to a collisionless drag. With the imposition of Ampere's law and quasineutrality a cancellation of the diffusion terms by the electron part of the drag occurs and only ion drag terms survive. This scaling with ion dissipation is consistent with the ion dissipation scaling in the growth rate of electron drift holes and electron clumps[3]. Since the ion part of the magnetic drag is proportional to negligible ion current contributions, the transport is governed by the ion part of the electrostatic drag:

$$\Gamma_\Gamma = \sum_{k,\omega} \frac{ck_y}{B_o|k_\parallel|} 2\pi\, \delta(\omega - k_\parallel u_\parallel)\, \mathrm{Re}\left(\mathcal{L}^{-1}_{k,\omega} \langle \hat{\phi}\tilde{h} \rangle\, d^{A,A}_{k,\omega}\, \mathrm{Im}\, d^{\phi,\phi}_{k,\omega(\mathrm{ion})} \right) \tag{29}$$

where $\mathcal{L}^{-1}$ is the inverse eigenfunction operator and the d's are dielectric tensor elements. This result is analogous to those obtained using the Lenard-Balescu equation for a 1-D electron-ion plasma. In that system, constraints of momentum and energy conservation on the interaction of discrete particles preclude momentum exchange in like-particle collisions, thus preventing relaxation of f_o.

In this application, incoherent ion fluctuations have been ignored. In analogy with the electrons, their contribution to transport will go as $\mathrm{Im}X_e$. Consequently, incoherent ion effects will be multiplied by a factor of the square root of the electron-ion mass ratio in comparison with electron effects. In tokamaks, this factor is dominant in the ratio of the susceptibilities since the argument of the exponent in the ion susceptibility, $u_\parallel^2/v_{ti}^2$, is of order unity. Therefore incoherent fluctuations in the ions can be ignored. There are several limitations which apply to the results obtained in the drift-Alfvén model. First, these results are applicable to collisionless systems only. Second, they are limited to regimes with weak to moderate spectral broadening ($\Delta\omega \leqslant \omega$). The role of incoherent fluctuations in strong broadening regimes is, if anything, enhanced. However, in that case the cancellation between diffusion and drag in the Lenard-Balescu turbulent collision integral is no longer exact and the integral takes a different form from that given here. Finally, stationary turbulence has been assumed throughout. Nonstationary turbulence (such as in the case of growing waves) permits the exchange of energy and momentum between waves and incoherent fluctuations, thus allowing different relaxation mechanisms.

In summary, electrostatic drift holes have been considered as a compliment to waves in the description of turbulence in a magnetized inhomogeneous plasma. As self-binding structures, maximum entropy drift holes represent an approach to the description and understanding of spatially intermittent plasma turbulence. In addition to providing natural scales of turbulent excitation which are not tied to the properties of linear collective modes, the instability of drift holes suggests that plasma stability, relaxation and transport may also be strongly affected by processes independent of collective oscillation. This assertion is borne out in the drift-Alfvén calculation where magnetic fluctuations are shown to make no contribution to transport, contrary to intuition based on descriptions of turbulence using wavelike excitations only.

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Figure Captions

Fig. 1 a) The box hole trapping region in configuration space; b) The box hole as a depression in the density profile; c) The box hole as a depression in the parallel velocity distribution.

Fig. 2 The contours of constant potential in the plane perpendicular to the magnetic field. Crossfield motion of trapped particles is indicated by the arrows.

Fig. 3 The process of nonlinear growth of the electron drift hole: a) Ions drift down the gradient under the action of E×B motion induced by the hole potential; b) Electrons follow to maintain quasineutrality; c) Displacement of electrons puts hole in region of greater average electron density. Hole depth increases in order to conserve total phase space density.

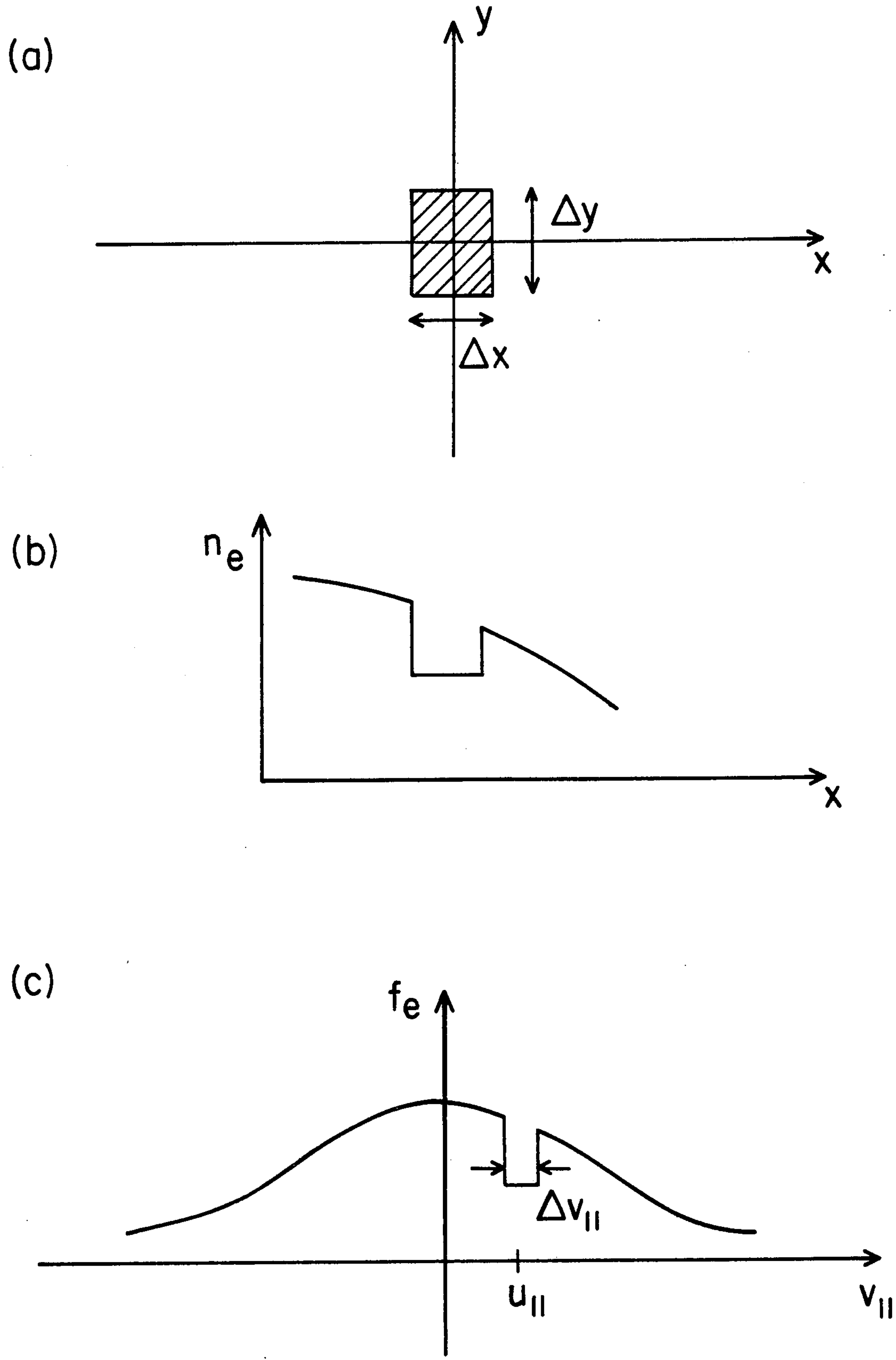

FIG. 1

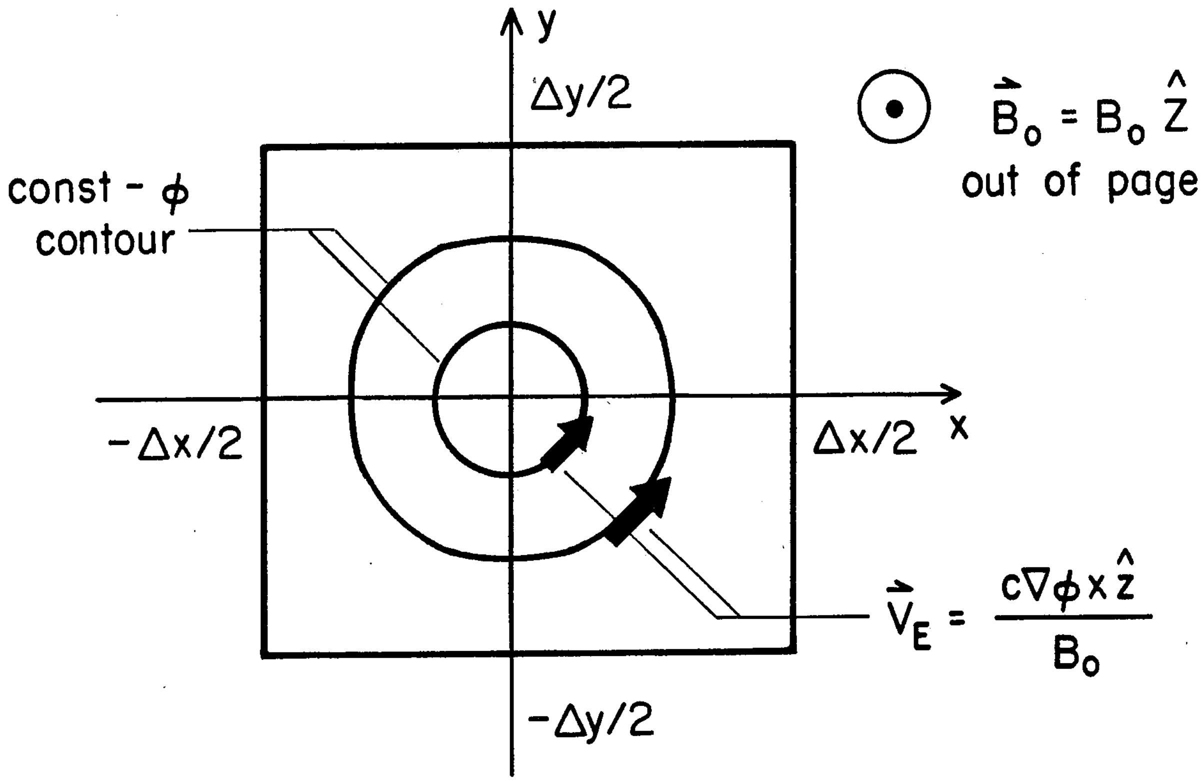
y
Δy/2
$\vec{B}_o = B_o\,\hat{Z}$
out of page
const − φ
contour
−Δx/2
Δx/2
x
$\vec{V}_E = \frac{c\nabla\phi \times \hat{z}}{B_o}$
−Δy/2

FIG. 2

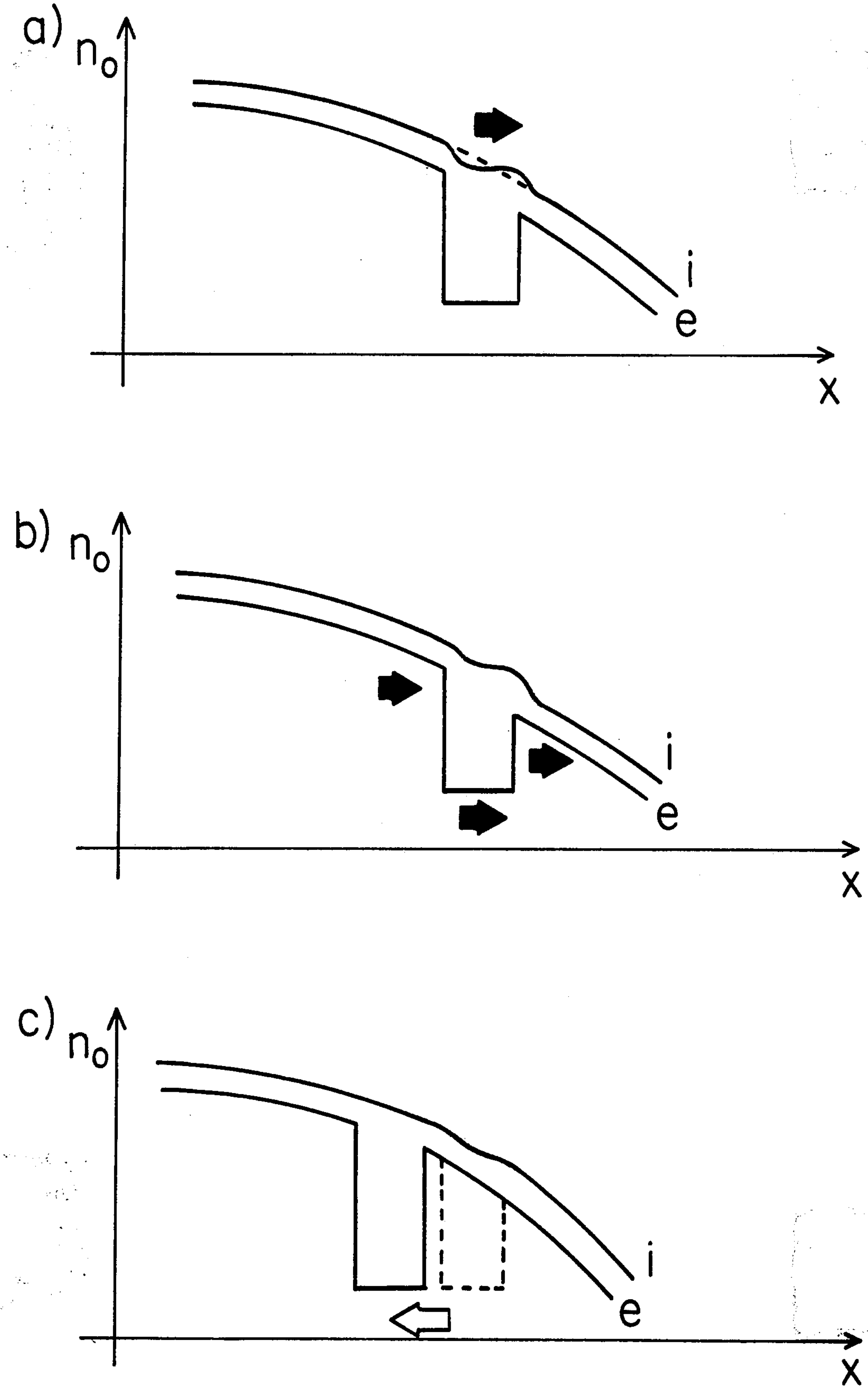

FIG. 3