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Neoclassical Electron Heat Conduction in Tokamaks Performed by the Ions

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Abstract

The increment to neoclassical ion heat conduction caused by electron collisions is shown to act like electron heat conduction since the energy is taken from and given back to the electrons at each diffusion step length. It can exceed electron neoclassical heat conduction by an order of magnitude.

A major problem universal to all tokamak experiments is the anomalous electron heat loss. If it is caused by an anomalous electron heat conduction, which most people have assumed but which has not been proved experimentally, it involves a thermal conductivity one or two orders of magnitude larger than the electron neoclassical value, the latter being the value which would apply to a stable plasma. Instabilities, and in particular microturbulence are observed in tokamaks and there are many theories aimed at explaining such turbulence and which predict anomalous electron thermal conductivity. (See the extensive review by Liewer[1].) But a definitive understanding of the anomalous electron energy loss is still lacking.

In a recent paper[2] the author derived the neoclassical ion heat conduction which is caused by electron-ion collisions. For conditions where $T_i \gtrsim T_e$, this increment to the ion heat conduction involves a higher order correction which is only a fraction of the ion neoclassical heat conduction caused by ion-ion collisions, although for $T_i \gg T_e$ it can become the largest component. The most important property of this part of the ion heat conduction was overlooked by the author. The dominant *ie*-collisions involved in the transport are the energy-scattering collisions[2] and the resulting heat conduction involves energy being taken from the electrons at one radius and being given back to the electrons a diffusion step length farther out, the step length being the radial width of a trapped ion's banana orbit. As a result, this part of the ion heat conduction acts like electron heat conduction.

The simplest example of such heat conduction can be seen with classical transport in a straight magnetic field. If $\mathbf{B} = B\mathbf{i_z}$ and there is an electron temperature gradient in the x-direction, when an ion is on the positive side of its cyclotron orbit it experiences an electron temperature $T_e + \delta x T_e'$, with $\delta x = v_\perp \sin\zeta/\Omega$, where Ω is the ion cyclotron frequency and the prime denotes the gradient in the x-direction. (The ion's velocity perpendicular to $\mathbf{B}$ has been taken to be $\mathbf{i}_x v_\perp \cos\zeta - \mathbf{i}_y v_\perp \sin\zeta$.) When the ion is on the negative side it sees the electron temperature $T_e - \delta x T_e'$ and the excess gain in energy on the positive side due to $\delta x T_e'$ is given back to the electrons on the negative side. In general the gradients of other parameters such as ν_e and T_i will be involved but it is seen that electron energy is being transported in the x-direction. Although the ions carry the energy, this particular part of the ion heat conduction acts essentially as **electron** heat conduction.

More precisely, the part of the ion distribution function which is first order in the ion Larmor radius and zero order in the collision frequency is given by

$$f_1^\circ = -v_\perp \sin\zeta \frac{f_o'}{\Omega} = -v_\perp \sin\zeta \frac{f_o}{\Omega}\frac{T_i'}{T_i}\left[\left(\frac{v}{v_T}\right)^2 - \frac{5}{2}\right], \tag{1}$$

taking a reference frame for which the mass motion in the y-direction is zero. The part of f_1 which is in phase with the x-component of velocity is given by

$$f_1^1 = \int \frac{d\zeta}{\Omega}\left[C_{ie}(f_1^\circ) + \delta x C_{ie}'(f_o)\right], \tag{2}$$

where $C_{ie}$is the collision operator for ie-collisions. The resultant ion heat conduction due to these particular collisions is then

$$\begin{aligned}(q_{ix})_{ie} &= \int \frac{m_i v^2}{2} v_\perp \cos\zeta f_1^1 d^3v \\ &= -3.75 n_i \nu_e \frac{m_e}{m_i}\rho_i^2 T_i' - \frac{10}{\Omega^2}\frac{\partial}{\partial x}\left[\frac{T_i}{m_i} n_i \nu_e \frac{m_e}{m_i}(T_e - T_i)\right] \end{aligned} \tag{3}$$

where $\rho_i^2 = 2m_i T_i/e^2B^2$ and ν_e is the electron collision frequency. Only the dominant and relevant energy scattering part of C_{ie} has been retained.

The time rate of change of f due to ie-collisions for ions whose orbit center is at x, when averaged over a cyclotron period is to second order in ρ_i

$$\left(\frac{\partial f}{\partial t}\right)_{ie} = \oint \frac{d\zeta}{\Omega} C_{ie}(f) / \oint \frac{d\zeta}{\Omega}$$
$$= C_{ie}(f_o + f_2^\circ) + \frac{1}{2\pi}\oint d\zeta \left\{ \delta x C'_{ie}(f_1^\circ) + \frac{\Omega^2 \delta x}{2}\left[\frac{\delta x}{\Omega^2} C'_{ie}(f_o)\right]' \right\}. \quad (4)$$

For an ion at position x whose y-component of velocity is $-v_\perp \sin\zeta$ and whose orbit center is therefore at $x - \delta x$, the term $C_{ie}(f_o)$ in Eq.(4) changes to $C_{ie}(f_o) + \frac{\delta x \delta' x}{2} C'_{ie}(f_o) + \frac{\delta x^2}{2} C''(f_o)$ and the other two terms remain unchanged to the required accuracy. Including this modification, the average rate of increase of ion energy at position x is given by

$$\int \frac{m_i v^2}{2}\left(\frac{\partial f}{dt}\right)_{ie} d^3v = Q_{ie} + Q_{ie2}$$
$$+ \int d^3v \left(\frac{1}{2} m_i v^2\right) \oint \frac{d\zeta}{2\pi}\left[\delta x C'_{ie}(f_1) + \Omega \delta x \frac{\partial}{\partial x}\frac{\delta x}{\Omega}\frac{\partial}{\partial x} C_{ie}(f_o)\right]$$
$$= Q_{ie} + Q_{ie2} - \frac{\partial}{\partial x}(q_{ix})_{ie} \quad (5)$$

where Q_{ie}, the lowest order part of the energy transfer, comes from the $C_{ie}(f_o)$ term and is the well known expression

$$Q_{ie} = 3 n_i \nu_e \frac{m_e}{m_i}(T_e - T_i). \quad (6)$$

Q_{ie2} is the second order correction coming from $C_{ie}(f_2^\circ)$.

Eq.(5) shows that the average rate at which the ions gain energy from the electrons, allowing for their cyclotron orbit excursions, is reduced by the divergence of the heat conduction $(q_{ix})_{ie}$. This result, plus the fact that the electrons at x lose energy at the rate $Q_{ie} + Q_{ie2}$, (ignoring the much smaller electron Larmor radius effects,) confirms the interpretation that $(q_{ix})_{ie}$ acts like electron heat conduction.

Turning to the case of a tokamak, a corresponding effect occurs with the trapped ions, their radial excursion from a magnetic surface being given by the larger quantity $\delta r = m_i v_\parallel h / eB_\theta (\sim r^{1/2}\rho_{i\theta}/R^{1/2})$ where B_θ is the poloidal magnetic field, $h \equiv 1 + (r/R)\cos\theta$ and $\rho_{i\theta} = (2 m_i T_i)^{1/2}/eB_\theta$. In this case, for the banana regime

$$f_i^\circ = \frac{-m v_\parallel h}{eB_\theta} f_o \left(\frac{T_i'}{T_i}\right)\left[\left(\frac{v}{v_T}\right)^2 - 1\right] \quad (7)$$

for large Z^i_{eff}, which will be assumed here (see reference 2). Considering only the contribution of the bulk of the trapped ions and ignoring the boundary layer effect associated with the thin layer of trapped ions adjacent to the passing ions (considered below), the ion heat conduction caused by C_{ie} is

$$
\begin{aligned}
(q_{ir})_{ie} &= -11.25\left(\frac{r}{R}\right)^{3/2} n_i\nu_e\frac{m_e}{m_i}\rho_{i\theta}^2 T_i'\left(1-0.48\frac{T_e}{T_i}\right)\\
&\quad -\frac{12}{\Omega_\theta^2}\left(\frac{r}{R}\right)^{3/2}\frac{\partial}{\partial r}\left[\frac{T_i}{m_i}n_i\nu_e\frac{m_e}{m_i}(T_e-T_i)\right]
\end{aligned}
\tag{8}
$$

But with energy-scattering collisions, the dominant neoclassical contribution comes from the boundary layer[3]. Taking $C_{ie}(f_1)$ as an example, because of f_1, the electrons give more energy to passing ions with $v_{||}<0$ and less to ions with $v_{||}>0$. In the steady state, the upward flow in energy of ions with $v_{||}<0$ and downward flow for $v_{||}>0$ must be balanced by flows in pitch angle, in the positive $v_{||}$ direction for high energy ions and in the negative direction for low energy ions. These flows involve ions entering and leaving the trapped particle region of velocity space. The high energy ions perform a banana orbit outwards in radius and become passing ions with $v_{||}>0$; the low energy ions perform a smaller banana orbit inwards, carrying less energy, and leave the trapped region with $v_{||}<0$. This is a typical boundary layer contribution to neoclassical transport and is the dominant part where energy scattering collisions are involved.

To determine this contribution consider first the solubility condition for f_2^1, namely[4]

$$
\frac{B_{\theta_o}}{2\pi B_o}\oint\frac{Bd\theta}{B_\theta q}\left[C_{ii}(f_2)+C_{ie}(f_o)-\frac{\partial f_o}{\partial t}\right]=\frac{1}{r}\frac{\partial}{\partial r}r\oint\frac{v_{dr}f_1^1 d\theta}{2\pi q}
\tag{9}
$$

where q is $v_{||}$ in ϵ, μ velocity space coordinates, v_{dr} is the radial component of the ∇B and curvature drift and , as usual, ν_e/ν_{ii} and $\partial/\partial t$ are taken to be of order $(\rho_{i\theta}/L)^2$. Since the collision operator is $-\boldsymbol{\nabla}_v\cdot\mathbf{J}$, the divergence in velocity space of the acceleration flow due to collisions, the pitch angle scattering part is

$$
\frac{-1}{v\sin\alpha}\frac{\partial J_\alpha\sin\alpha}{\partial\alpha}\qquad\text{or}\qquad\frac{-q}{B}\frac{\partial J_\alpha\sin\alpha}{\partial\mu}
$$

where $\cos\alpha=q/v$. Integrating Eq.(9) with respect to $Bd\mu$ from $\mu=0$ to the passing ion side of the boundary layer

$$
F_2\equiv\,<J_{2\alpha}\sin\alpha>\,\simeq\sigma v\left(C_{iiES}(\bar{f}_2)+C_{ieES}(f_o)-\frac{\partial f_o}{\partial t}\right)
\tag{10}
$$

$(\sigma v-q_B)$ has been approximated by σv and the small contribution to the $v_{dr}f_1^1$ term due to Phirsch-Schluter diffusion has been neglected so that this term is non-zero only in the trapped ion region. $\sigma = q/|q|$, $\bar{f}_2$ is the pitch angle average of f_2, C_{iiES} is the energy scattering part of C_{ii} and $<>$ denotes the usual volume average between adjacent magnetic surfaces.

F_2 is the acceleration flow in pitch angle, weighted by $\sin\alpha$, and since it is odd in q – it is towards higher μ for both signs of q–it must be continuous across the boundary layer. To third order accuracy the value of F just inside the trapped region must be the value of F_2 for a trapped ion whose mean radius is $r - (hq_B)/\Omega_\theta$. Thus $F_2 + F_3 = F_2 - (hq_B)F_2'/\Omega_\theta$ on the trapped particle side of the boundary layer. There is no contribution to F_3 from $C_{ii}(f_3) + C_{ie}(f_1) - \partial f_1/\partial t$ for trapped ions since the orbit integrals are zero. For passing ions near the boundary layer

$$(F_3)_p \simeq \sigma v \left[C_{iiES}(f_3) + C_{ieES}(f_1) - \frac{\partial f_1}{\partial t} \right] \tag{11}$$

since for Z^i_{eff} large, f_1 and f_3 are approximately constant in μ for passing ions.

If the width of the boundary layer is taken as $\Delta\mu$, the mean value of $\partial F_3/\partial\mu$ for this layer is $-\left[(F_3)_p + (hq_B)F_2'/\Omega_\theta\right]/\Delta\mu$. At the boundary $f_1 = -(hq_B)f_o'/\Omega_\theta$ and the $\partial/\partial t$ terms cancel. Retaining only the relevant C_{ieES} collision terms in F_3, the boundary layer contribution is

$$\begin{aligned}(q_{ir})_{ie} &= \int \frac{d\theta}{2\pi} \int \frac{m\epsilon}{\Omega_\theta} q^2 \frac{\partial F_3}{B\partial\mu} \frac{B d\mu d\epsilon}{|q|} \\ &= -26.7(\frac{r}{R}) n_i \nu_e \frac{m_e}{m_i} \rho_{i\theta}^2 T_i' \left(1 - 0.48\frac{T_e}{T_i}\right) - \frac{28.6r}{R\Omega_\theta^2} \frac{\partial}{\partial r} \left[\frac{T_i}{m_i} n_i \nu_e \frac{m_e}{m_i} (T_e - T_i) \right] \end{aligned} \tag{12}$$

The numerical coefficients in Eq.(12) are larger than in the classical case [Eq.(3)] because the factor $\left[(\frac{v}{v_T})^2 - \frac{5}{2}\right]$ in f_1 has changed to $\left[\left(\frac{v}{v_T}\right)^2 - 1\right]$ and there is less cancellation of the dominant $(v/v_T)^2$ part. The coefficients are larger than in Eq.(8), because the θ-average of q_B^2 is involved and not the average of $\int_0^{\mu_B} qB d\mu = q_B^3/3$. The numerical coefficients are all large because the ie-collisions are strongly weighted towards high ion energies. (Note in Eqs. (3), (8) and (12), total heat flows are involved; $5\Gamma_{ie}T_i/2$ has not been subtracted where Γ_{ie} is the corresponding ion diffusion caused by the ie-energy-scattering collisions. Also in Eqs.(8) and (12) no allowance is made for the increment in the electric field E_r required for ambipolarity as done in reference 2. The contribution to the ion heat conduction due to this increment in E_r as well as the contributions from $C_{ii}'(f_2)$ and $C_{ii}(f_3)$ are of the same order as Eqs. (8), (12) but do not involve energy being taken from and given back to the electrons. In general these contributions are small compared with the lowest order ion heat conduction coming from $C_{ii}(f_1)$.

As in the classical case, Eq.(5), the average rate on a magnetic surface at which electrons give energy to the ions is $Q_{ie} + Q_{ie2}$, where Q_{ie2} comes from $C_{ie}(f_2)$, with f_2 now the correction to f second order in $\rho_{i\theta}/L$. But the rate at which the ion energy increases on the same magnetic surface, after orbit averaging is

$$n_i \left\langle \frac{d\epsilon}{dt} \right\rangle_{ie} = Q_{ie} + Q_{ie2} - \frac{1}{r}\frac{\partial r(q_{ir})_{ie}}{\partial r} \tag{13}$$

so that the neoclassical $(q_{ir})_{ie}$ also acts like electron heat conduction. The formulas of Eqs.(12) and (13) will break down in an important way in two places. Firstly, in the loss-cone region near the wall where there are no trapped ions with $v_{\|} < 0$, the energy transported by $(q_{ir})_{ie}$ will be given back to the electrons for the last time. Secondly, near the magnetic axis the basic neoclassical assumption that f_1 is odd in $v_{\|}$ breaks down; above a low ion energy there are no trapped ions with $v_{\|} > 0$ sampling smaller radii, whereas the fraction with $v_{\|} < 0$ is greater than the usual $\sqrt{2r/R}$ formula because of ∇B drift effects. A term Q_{ie1} will appear in Eq.(13) which is first order in $\rho_{i\theta}/L$ and the divergence of $(q_{ir})_{ie}$ will be correspondingly larger. This type of heat conduction will therefore be an important loss mechanism for electrons in the central region.

To demonstrate the magnitude of $(q_{ir})_{ie}$, it is first noted that in Eq.(12) there is an approximate cancellation of the terms containing T_i'. If $(n_i\nu_e)'$ is assumed to be small ($n/T_e^{3/4}$ approximately constant) Eq.(12) reduces to

$$(q_{ir})_{ie} = -14.3\left(\frac{r}{R}\right) n_i \nu_e \frac{m_e}{m_i} \rho_{i\theta}^2 T_e' \tag{14}$$

Since the normal neoclassical heat conduction is $-1.8(r/R)^{1/2} n_e \nu_e \rho_{e\theta}^2 T_e'$ the value in Eq.(14) is larger by the factor $8(r/R)^{1/2}(n_i/n_e)(T_i/T_e)$ which in modern tokamaks where there is auxiliary ion heating can be an order of magnitude. If the distribution function f_o for the ions has an enhanced non-Maxwellian tail, which has been observed both with and without neutral beam heating[6], this factor will be substantially larger.

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