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Crystal X-ray Accelerator

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Abstract

An ultimate linac structure is realized by an appropriate crystal lattice (superlattice) that serves as a "soft" irised waveguide for X-rays. High energy ($\sim$ 40 keV) X-rays are injected into the crystal at the Bragg angle to cause the Bormann anomalous transmission, yielding the slow wave accelerating fields. Particles (e.g., muons) are channeled along the crystal axis.

A direction to attain ever higher energies by extrapolating the linac to higher accelerating fields, higher frequencies, and finer structures is prompted by several considerations, including the luminosity requirement which demands the radius of the colliding beam spot be proportionately small in high energies: $a_0 = \pi^{-1/2}\hbar c(f\mathcal{N})^{-1/2}P\varepsilon^{-2}$ with f, $\mathcal{N}$, P, and ε are the duty cycle, total number of events, beam power and beam energy respectively. This direction, however, encounters a physical barrier when the photon energy becomes of the order $\hbar\omega \sim \hbar\omega_p \sim mc^2\alpha^2 \sim 30$eV ($\alpha =$ the fine structure constant), corresponding to wavelength (scale length) $\lambda \sim 500\text{Å}$: the metallic wall begins to strongly absorb the photon, where ω_p is the plasma frequency corresponding to the crystal electron density. In addition, since the wall becomes not perfectly conducting for $\hbar\omega \geq mc^2\alpha^2$, the longitudinal component of fields becomes small and the photon goes almost straight into the wall (a soft wall regime). As the photon energy $\hbar\omega$ much exceeds $mc^2\alpha^2$ and becomes $\gtrsim mc^2\alpha$, however, the metal now ceases to be opaque. The mean free path of the photon is given by Bethe-Bloch as $\ell_i = (3/2^8\pi)a_B^{-2}\alpha^{-1}n^{-1}(\hbar\omega/Z_{\text{eff}}^2 R_y)^{7/2}$, where a_B is the Bohr radius, n the electron density, Z_{eff} the effective charge of the lattice ion, and R_y the Rydberg energy.

In the present concept the photon energy is taken at the hard X-ray range of $\hbar\omega \sim mc^2\alpha$ and the linac structure is replaced by a crystal structure, e.g., silicon or GaAs-AlAs. (A similar bold endeavor was apparently undertaken by R. Hofstadter already in 1968.[1]) Here the crystal axis provides the channel through which accelerated particles propagates with minimum scattering (channeling[2]) and the X-rays are transmitted via the Bormann effect (anomalous transmission[3,4]) when the X-rays (wavelength λ) are injected in the xz-plane with a polarization in their plane into a crystal at the Bragg angle θ_B

$$\frac{\lambda}{2b} = \sin\theta_B \tag{1}$$

where b is the transverse lattice constant and later a the longitudinal lattice constant ($a \sim b$) [see Fig. 1(a)]. The row of lattice ions (perhaps with inner shell electrons) constitutes the "waveguide" wall for X-rays, while they also act as periodic irises to generate slow waves. A superlattice[5] such as $Ge_cSi_{1-c}S_i$ (in which the relative concentration c ranges from 0 to 1 over 100Å or longer in the longitudinal z-direction) brings in an additional freedom in the crystal structure and provides a small Brillouin wavenumber $k_s = 2\pi/s$ with s being the periodicity length. We demand that the X-ray light in the crystal channel walls becomes a slow wave and satisfies the high energy acceleration condition

$$\frac{\omega}{(k_z + k_s)} = c \tag{2}$$

where ω and k_z are the light frequency and longitudinal wavenumber.

The energy loss of moving particles in matter is due to ionization, bremsstrahlung, and nuclear collisions. We can show[6] that a channeled high-energy particle moving fast in the z-direction oscillates in the xy-plane according to the Hamiltonian

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + V(x, y), \tag{3}$$

where V is the average of the electrostatic potential φ of the crystal. Therefore, the distance r from the nearest row of atoms is $\sim b/2$ for positive charge, while it is a fraction of b for negative charge. The energy loss due to the bremsstrahlung, taking into account the effect that channeled particles avoid close collisions with nuclei, is given by

$$\frac{\partial \ln E}{\partial z} = 4Z_{\text{eff}}^2 r_p^2 r_e^{-3} \alpha^7 \phi^{-3} K_0'(2\pi r/b) \ell n \left(\frac{2\gamma\alpha m_e}{m_p}\right), \tag{4}$$

where $r_p = e^2/m_pc^2$ is the classical radius of a particle with mass m_p (r_e for electron), $\phi = b/a_B$, and K_0 is the modified Bessel function of the second kind. We thus pick muons or protons to accelerate. Previously Neuffer[7] and Parkhomchuk and Skrinsky[8] have considered muon acceleration. As channeled particles are confined to the rows of atoms by electric fields of the order $1 - 10V/Å$ (equivalent 0.3-3M Gauss), crystals may be used to build a collider not only for acceleration sections but also for bending sections.

The present concept may also be looked upon as an outgrowth of the plasma fiber accelerator.[9] In the plasma beatwave accelerator[10] or the plasma fiber accelerator one uses a plasma as a medium to accelerate in or as a wall to confine and slow down photons with. The longitudinal (accelerating) electric field increases with the plasma density. In our pursuit of ultra-high energies, the ultimate high plasma density thus becomes that of the solid matter and the fiber ripple ultimately becomes the superlattice periodicity. In fact, the lattice ions and electrons in the solid behave not too differently from a plasma for high frequency hard X-ray photons. In what follows we analyze relevant physical processes of the present concept, particularly that of the crystal as a waveguide.

As the photon frequency ω is much greater than the atomic frequencies, the current is given by $\mathbf{J} = \left(\omega_{pe}^2/i\omega\right)\mathbf{E}$, where $\omega_{pe}^2(x) = 4\pi n(\mathbf{x})e^2/m$ is the local "plasma density" of the lattice ions and their inner shell electrons; conduction electrons, if any, may be looked upon as a uniform medium. Using fields of the form $A = \mathrm{Re}\,(e^{-i\omega t}A(\mathbf{x}))$ and the Lorentz guage ($A^4 = \mathrm{div}\mathbf{A}/i\omega$), we start from the Lagrangian $\mathcal{L} = \int d^3x(\omega^2 - \omega_{pe}^2)|\mathbf{A}|^2 - c^2|\nabla\mathbf{A}|^2 - c^2(\omega_{pe}^2/\omega^2)|\mathrm{div}\mathbf{A}|^2$, whose last term will be subsequently negligible. We seek for a solution of the vector of form $A_i = a_i(x,y)f_i(z)$, where the repeated index i does not mean summation ($i = 1 \sim 3$). Variations with respect to a_i and f_i give rise to two coupled equations

$$\left[\partial_z^2 + c^2k_z^2(z)\right]f_i(z) = 0, \tag{5}$$

$$\left[\nabla_\perp^2 + c^2k_\perp^2(x,y)\right]a_i(x,y) = 0, \tag{6}$$

where $c^2k_z^2(z) = \omega^2 - \left\langle\omega_{pe}^2\right\rangle_\perp - c^2\left\langle|\nabla_\perp a_i|^2\right\rangle_\perp$ and $c^2k_\perp^2(x,y) = \omega^2 - \left\langle\omega_{pe}^2\right\rangle_z - \left\langle|\partial_z f_i|^2\right\rangle_z$; the average $\langle\ \ \rangle_\perp$ in the xy-plane is weighted with $|a_i|^2(x,y)$ and $\langle\ \ \rangle_z$ is weighted with $|f_i|^2$. Since $c^2k_z^2 \sim \omega^2 \gg \omega_p^2$, Eq. (5) may be solved by the WBK method $f_i = k_z^{-1/2}\exp[i\psi(z)]$ with $\partial_z\psi = k_z$.

The general solution of Eq. (6) has the Bloch form:

$$a_i(x,y) = e^{ik_{Bx}x+ik_{By}y}u(x,y), \tag{7}$$

where $u(x,y)$ is periodic on the lattice. For the sake of simplicity we drop y-dependence hence. We note $k_\perp^2(x)$ is a periodic function with period b. The Fourier components of a_i, i.e., $a_0 = u_0 e^{ik_{Bx}x}$ and $a_N = u_N e^{i(k_{Bx}-Nk_b)x}$, are coupled, where N is the mode number in the x-direction. When $k_{Bx} \sim \frac{N}{2}k_b$, they have approximately the same frequency and we need to include both in a solution. Thus we obtain

$$\left.\begin{array}{ll} \left(V_0 - k_{Bx}^2\right) u_0 + V_N u_{-N} & = 0 \\ \left[V_0 - (k_{Bx} - Nk_b)^2\right] u_{-N} + V_{-N}u_0 & = 0 \end{array}\right\} \tag{8}$$

where $V_N = b^{-1}\int_0^b \exp(-iNk_b x)k_\perp^2(x)dx$. Averaging Eq. (5) over x and y, we obtain the dispersion relation $\omega^2 = \left\langle \omega_{pe}^2 \right\rangle_{\perp z} + c^2\bar{k}_z^2 + c^2V_0$, where $\langle \quad \rangle_{\perp z}$ is the average over both $\perp$ and z and $\bar{k}_z \equiv \lim_{z\to\infty} \psi(z)/z \simeq \left[\left\langle k_z^2(z)\right\rangle_z\right]^{1/2} \simeq \left[\left\langle |\partial_z f_i|^2\right\rangle_z\right]^{1/2}$. Solving the determinant equation for V_0 in Eq. (8), we obtain

$$\omega^2 - \left\langle \omega_{pe}^2\right\rangle_{\perp z} - c^2\bar{k}_z^2 = c^2\frac{N^2}{4}k_b^2 + c^2\left(k_{Bx} - \frac{N}{2}k_b\right)^2 - s\, c^2\sqrt{V_N V_{-N} + N^2k_b^2\left(k_{Bx} - \frac{N}{2}k_b\right)^2}, \tag{9}$$

where $s = -1$ corresponds to a standing wave with node near the atomic planes $a_i(x) \simeq a_i \sin\left(\frac{N}{2}k_b x\right)$. We choose $k_{Bx} = \frac{N}{2}k_b$ for our accelerator because then the group velocity $\partial\omega/\partial k_{Bx} = 0$ (the Bormann effect). The slow wave is generated from side bands: E_z (or A_x) has the main wavenumber $\bar{k}_z$, but $\psi(z)$ has a slight modulation. When there is a k-component in the plasma frequency, the phase is given by

$$\psi(z) = \bar{k}_z z + \frac{\omega_{pe}^2(k)}{2\bar{k}_z kc^2}\cos(kz) + \cdots,$$

where $\omega_{pe}^2(k) = 2a^{-1}\int_0^a \sin(kz)\left\langle \omega_{pe}^2\right\rangle_\perp dz$. The amplitude of the ℓ-th satellite $E_z(\ell)$ with $k_z = \bar{k}_z + \ell k$ is

$$E_z(\ell) \cong \frac{Nk_b\sqrt{\bar{k}_z}}{2\omega} a_x i^\ell J_\ell\left(\frac{\omega_{pe}^2(k)}{2\bar{k}_z kc^2}\right)\cos\left(\frac{N}{2}k_b x\right). \tag{10}$$

The amplitude for the carrier wave ($\ell = 0$) is $E_x(0) = i\omega a_x \bar{k}_z^{-1/2} \sin\left(\frac{N}{2}k_b x\right)$. Thus the coupling ratio of the ℓ-th satellite R_ℓ is given by

$$R_\ell = E_z(\ell)/E_z(0) = N k_b \bar{k}_z c^2 J_\ell\left(\omega_{pe}^2(k)/2\bar{k}_z k\right)/(2\omega^2).$$

For the largest satellite $\ell = 1$

$$\frac{E_z(\bar{k}_z + k)}{E_z(\bar{k}_z)} = \frac{N}{4}\frac{k_b}{k}\frac{\omega_{pe}^2(k)}{\omega^2}. \tag{11}$$

The phase velocity of this side band is

$$v_{ph} = \frac{\omega}{\bar{k}_z + k} = \frac{c\left[\bar{k}_z^2 + N^2 k_b^2/4 + \omega_{pe}^2(0)\right]^{1/2}}{\bar{k}_z + k}. \tag{12}$$

The condition for high energy acceleration is to choose $\bar{k}_z$ such that $v_{ph} = c$ in Eq. (12). The group velocity, on the other hand, is

$$v_{gz} = \frac{c^2}{v_{ph}}\left(1 - \frac{k}{\bar{k}_z + k}\right). \tag{13}$$

This should be positive and preferably near c for efficacious acceleration.

For $k \leqq 0$ no possibility of $v_{ph} = c$, while for $k > \frac{N}{2}k_b$, $v_{ph} = c$ leads to $v_g < 0$. Thus we consider the range $0 < k < \frac{N}{2}k_b$. It is evident that ω_{pe}^2 has a large component at $k = k_a \sim k_b$ due to the lattice periodicity. In order to exploit this longitudinal periodicity we need $N > 2$: from Eq. (13) for a negative charge (e.g., μ^-) confined away from the mid-plane of atoms N can be odd or even, while a positive charge (e.g., μ^+) channeled near the mid-plane N must be even. For $N = 3$ we find $\bar{k}_z = \frac{5}{8}k_b$, $\omega = \frac{13}{8}k_b c$, $R_1 = 0.56\omega_{pe}^2(k_b)/2k_b^2c^2$, and $v_{gz} = 0.38c$; for $N = 4$, $\bar{k}_z = \frac{3}{2}k_b$, $\omega = \frac{5}{2}k_b c$, $R_1 = 0.4\omega_{pe}^2(k_b)/2k_b^2c^2$, and $v_{gz} = 0.6c$.

Alternatively we may exploit the longitudinal superlattice periodicity $k = k_s$ (i.e. $\omega_{pe}^2(k_s) \neq 0$), where $k_s \lesssim 0.05k_b$. Introducing the parameter $\xi \equiv k_b/2k_s$, we find the high energy acceleration conditions:

$$\bar{k}_z = \frac{1}{4}\xi\left(N^2 - \xi^{-2}\right)k_b, \tag{14}$$

$$v_{gz} = c\left(N^2 - \xi^{-2}\right) / \left(N^2 + \xi^{-2}\right), \tag{15}$$

$$R_1 = 16n\xi^3 \left(N^2\xi^4 + 2N^2\xi^2 + 1\right)^{-1} \omega_{pe}^2(k_s)/2k_b^2c^2. \tag{16}$$

For this case we can use the modes $N = 1$ and 2. For $N = 1$ and $k_s = 0.05k_b$ we find $\bar{k}_z = 2.5k_b$, $v_{gz} = 0.98c$, $R_1 = 1.6\omega_{pe}^2(k_s)/2k_b^2c^2$; for $N = 2, \bar{k}_z = 10k_b$, $v_{gz} = 0.996c$, and $R_1 = 0.8\omega_{pe}^2(k_s)/2k_b^2c^2$, and $\omega = 10k_bc$ ($\hbar\omega = 40$keV). The latter example has a decent longitudinal coupling (modest value of R_1) and a good longitudinal group velocity (v_{gz}), for which reason we focus on this herewith.

We now discuss attainable fields and necessary power to overcome losses. In order for the Bormann effect to happen we need

$$L/b \gtrsim \alpha^{-1}, \tag{17}$$

where L is the transverse dimension of the X-ray beam. The ponderomotive force of intense X-rays is relatively modest because the crystal needs layers [Eq. (17)] of lattice to diffract light. The transverse force on an electron is

$$F_p = mc^2\nabla\left(\frac{eE}{mc\omega}\right)^2 \sim \frac{mc^2}{L}\left(\frac{eE}{mc\omega}\right)^2. \tag{18}$$

The electron is bound to the atom with a force $F_b \cong e^2Z_{\text{eff}}^2/a_B^2$. By demanding $F_p < F_b$, we obtain a condition

$$\nu^2 \equiv \left(\frac{eE}{mc\omega}\right)^2 < \frac{L}{a_B}\alpha^2. \tag{19}$$

If we assume that for $\nu < 10^{-2}$ the crystal is intact, this corresponds to the transverse electric field of 10^{13}V/cm for $\hbar\omega = 40$ keV, yielding the acceleration field of $\sim 10^9$V/cm. The conventional ionization loss is 2.4 MeV/g cm^2 or 5 MeV/cm. The radiation length for $Z = 30$, $n = 5\times10^{22}$ atoms/cm^3 for a muon is 10^6 cm that limits the length of acceleration. For example, a 10 TeV μ^- loses 10 MeV/cm due to bremsstrahlung. On the other hand, μ^+ channels in between atomic layers and the radiation length given by Eq. (4) amounts to 10^9 cm.

In order to overcome losses of μ^- energy of $\sim$ 10 MeV/cm with a typical value of $R_1 \sim 10^{-4}$, the X-ray field must exceed $E_x \sim 10^{11}$ V/cm, corresponding to the power density of 3×10^{19} W/cm^2. The required peak power is given at 3×10^9 W, since the minimal size is $L \sim 2 \times 10^3 a_B \sim 10^{-5}$ cm. In a uniform crystal the beam size $L(z)$ increases from the original size L_0 due to diffraction in the crystal according to Ref. 11 to

$$L(z) = \left[L_0^2 + \left(\partial^2\omega/\partial k_{Bx}^2\right)(z/L_0)^2\right]^{1/2}. \tag{20}$$

Since $\partial^2\omega/\partial k_{Bx}^2 \simeq N^2 b\alpha^{-2}/2\pi \sim 10^{-4}$ cm, it is evident that a confining structure of the X-ray beam ("optical" fiber of crystal) in the transverse direction is necessary. The pulse narrowing as discussed in optical fibers[12,13] may also take place in the present crystal fiber channels, enhancing the field strength to our favor.

Other questions, such as channeling and dechanneling and associated radiation, remain critically important subjects of investigation, treated to an extent in the existing literature.[14,15] The average quantity of dechanneling length may not be relevant to the high energy component, however. The origins of the kicks in the crystal are the emission of a bremsstrahlung photon and a channeling radiation photon, ionization of a crystal electron, and the scattering with a lattice phonon. Bremsstrahlung and synchrotron radiation are emitted along the channeling oscillation motion with power increasing with beam energy and thus exerts a friction on this motion, while the rest of the processes are not an increasing function of energy. This also leaves a possibility of using electrons as a channeling X-ray radiator. Our work on these subjects will be reported.

In conclusion intense X-rays (peak power $\geq 3 \times 10^9 W$; $\hbar\omega = 40$ keV) shone on a superlattice crystal at the Bragg angle can accelerate muons (or heavier particles such as protons) to high energies through the slow wave structure due to the localized periodic lattice charge distribution. Because of the channeling by the crystal structure the particle beams are very well collimated, leading to very high luminosity and brightness; the crosssection of accelerated particles is much less than $(1\AA)^2$. Although the analysis of the presently suggested concept is preliminary, this acceleration scheme seems to merit further investigations.

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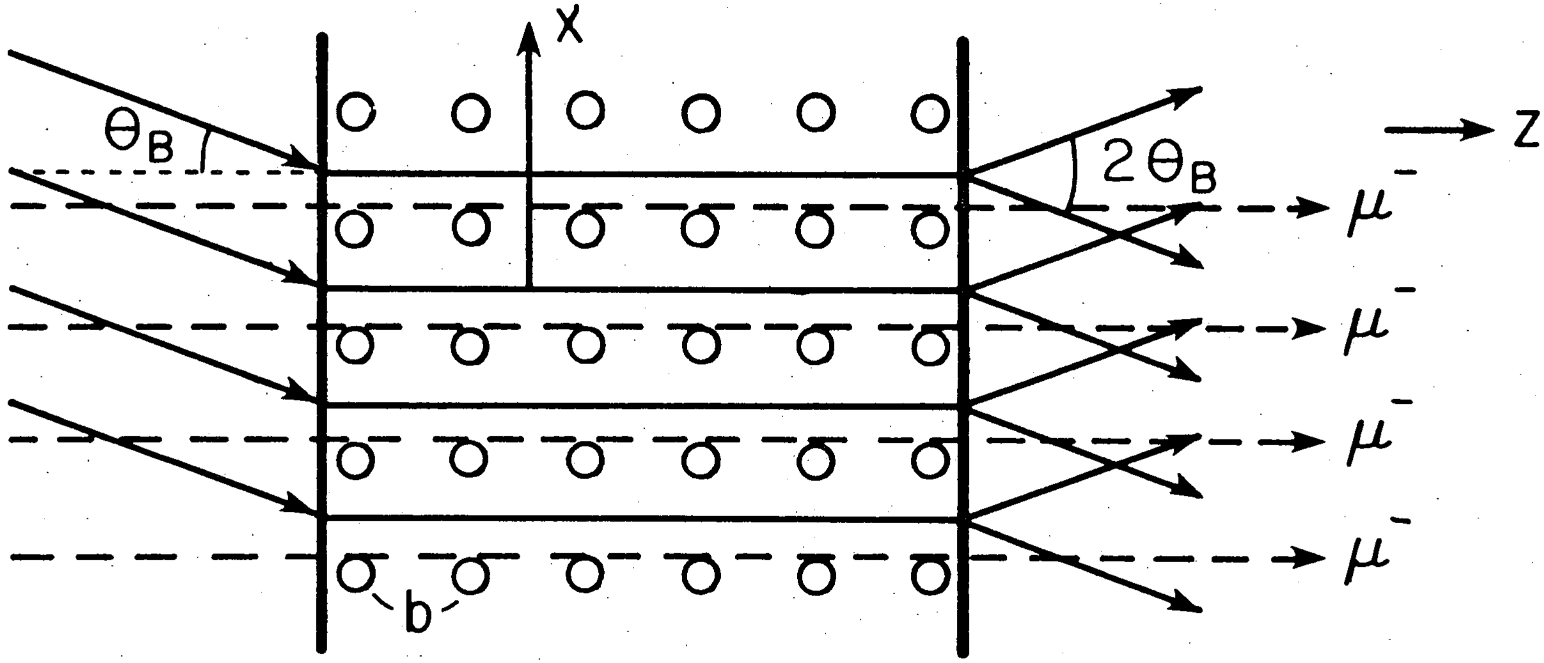
Figure Caption

1. The Bormann anomalous transmission. When the X-rays are injected at the Bragg angle, the Bormann effect takes place. Particle beams are injected along the crystal axis.


x
θ_B
$2\theta_B$
z
μ^-
μ^-
μ^-
μ^-
b