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Superdense Muonic Matter

T. Tajima
Department of Physics
and
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

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The University of Texas at Austin
Austin, Texas 78712, U.S.A.

Abstract

A possible method of creation of superdense matter with approximate atomic density $4 \times 10^{29}\text{cm}^{-3}$ is suggested. A pulsed beam of 10^8 muons, with duration 3×10^{-6}sec is shone on liquid hydrogen of volume $\sim (300\text{Å})^3$. A muon charge-exchanges with an electron in a hydrogen atom: with enough muonic hydrogen atoms, the compressibility tends to diverge and condensation into a much higher density state begins. The muon beam should be cooled by the ionization process and channeled through crystal axes before irradiation on the hydrogen specimen. When magnetic fields are present upon irradiation, the fields may be enhanced up to 10^9 Gauss. A possible state of this matter is speculated.

I. Introduction

Hydrogen molecules take a liquid state when the temperature and pressure are appropriate (the density 4.25×10^{22} cm^{-3}) due to the attractive nature of the van der Waals force. When negative muons (μ^-) are shone on the liquid hydrogen, absorption of muons by hydrogen molecules involves formation of mesohydrogen atoms through charge exchange $H + \mu^- \rightarrow p\mu + e^-$. The meso-hydrogen atom $p\mu$ is electrically neutral and smaller than the hydrogen atom (a_B: the Bohr radius) by the mass ratio of an electron to a muon $m/m_\mu \simeq 1/207$, where m is the electron mass and m_μ the muon mass. At the liquid hydrogen density the time scale to exchange charge is approximately 2×10^{-13} sec.[1,2] If a sufficiently large number of hydrogen atoms become mesohydrogen atoms, they begin to attract each other by the van der Waals force. Since muonic mesohydrogen atoms are smaller than hydrogen atoms by a factor of 207, the minimum position of the van der Waals potential for mesohydrogen is reduced by 207 and the depth of the potential is enhanced by 207. Thus the aggregate of mesohydrogen atoms could form a very dense (liquid) state, whose density is enhanced by a factor of $(207)^3$ over the value of the liquid hydrogen. Since the binding electric field of hydrogen atoms of ~1 eV/Å (~300 k Gauss) is enhanced by a factor of $(207)^2$ for meso-hydrogen atoms, the equivalent binding electric field is of the order of ~4 keV/Å. Under such a strong binding field, magnetic fields that were originally embedded in the medium are easily enhanced to a value exceeding 10^9 Gauss.

In the following we consider possible physical ways to realize superdense mesohydrogen matter.

II. Irradiation of a Muon Beam and Condensation

In order to cause an enough number of mesohydrogen atoms to form a condensed state, we have to irradiate a muon beam of enough density with slow enough beam velocity ($v_\mu \sim \alpha c$ with α being the fine structure constant) within a life time of muons (τ_μ). The muon capture time suddenly becomes quite short ($\sim 10^{-13}$ - 10^{-14} sec) as muons slow down to the velocity $v_\mu \sim \alpha c$,[1,2] which corresponds to the mean free path ℓ_μ of 10^{-4} - 10^{-5} cm. If the muon velocity is larger than αc, the mean free path becomes much longer. Thus the necessary condition for the muon beam density at $v_\mu \sim \alpha c$ may be derived by demanding that the number of muons deposited within the mean free path ℓ_μ within the life time of muons is equal to the number of hydrogen atoms within that volume:

$$n_\mu v_\mu a^2 \tau_\mu = n \ell_\mu a^2 \quad , \tag{1}$$

where n is the liquid density of hydrogen atoms and n_μ the muon density in the beam. The necessary muon density $n_\mu = n\ell_\mu/(v_\mu \tau_\mu)$ is a quite substantial number 10^{15} - 10^{16} cm^{-3}.

We suggest the following method to create a dense muon beam and to transport it to the target hydrogen. Figure 1 shows a schematic description of this. We focus a tritium beam of the optimum energy of 3 GeV (1 GeV/nucleon) with density n_t on area b^2 of deuterium at the liquid hydrogen density. The thickness of the deuterium specimen should be larger than the mean free path for the strong interaction with the tritium beam ℓ_{mfp}, which is 97 cm.[3] In addition to the "useful" π^- creating inelastic collisions the beam will see the "useless" elastic collisions as well as the "useless" inelastic collisions, creating π^0 or π^+. Furthermore, the beam suffers the electromagnetic collisions whose mean free path may be calculated by the Bethe-Bloch formula.[4] The energy loss E_π during the creation of one negative pion is

$$E_\pi = \frac{1}{y_\pi} \left(\frac{\varepsilon_{el}}{y_{el}} + \frac{\varepsilon_{in}}{y_{in}} + \frac{\varepsilon_I}{A} \right) \quad , \tag{2}$$

where the yield y^π is equal to the reciprocity of the number of nucleonic projectiles necessary to create one pion, $y^\pi \sim \frac{1}{1.3} Y^\pi_{AB}$ with Y^π_{AB} defined in Ref. 3, the elastic collision energy loss $\varepsilon_{el} \sim 100$ MeV, the inelastic loss $\varepsilon_{in} \sim 300$ MeV, $y_{el} \sim 0.5$, $y_{in} \sim 0.5$, $\varepsilon_I = (\partial\varepsilon_I/\partial x)\ell_s \sim 0.66\ \ell_s$ MeV (ℓ_s being the mean free path for entire strong processes), and $A \sim 3$. This number E_π in Eq. (2) is typically 3.7 GeV for d - t collisions at 1 GeV/ nucleon energy.

After enough pions (π^-) are created with majority of pions forward scattered, they either get absorbed by the surrounding moderator or decay into muon (μ^-). Typical pion energy-angle spectra were experimentally[5] measured and theoretically calculated.[6] Decay muons may be primarily forward scattered if they decay from high energy pions, while muons may distribute in a wide angle if they decay from low energy pions. For this reason we need acceleration of pions.

The next important step is to cool down the muon beam so that when we slow down muons we do so simultaneously. Neuffer[7] and Skrinsky et al.[8] proposed to cool muon beams by ionization processes through condensed matter. The cooling occurs for muon energy E_μ larger than ~0.5 GeV. After cooling we have to decelerate the beam to the velocity $v_\mu \sim \alpha c$.

When we shine the tritium beam on deuterium atoms, the d - t collision rate λ is given by

$$\lambda = n_d \sigma v_t \quad , \tag{3}$$

where n_d is the deuterium atom density, $\sigma \simeq 243$ mb, and v_t is the tritium beam velocity. The rate for π^- creation is $\lambda_\pi \sim 0.2\ \lambda$. The number of pions created is given by

$$N_\pi = b^2 \ell_{mfp}\ n_t \cong \frac{5b^2 n_t}{\sigma n_d} \quad , \tag{4}$$

where b^2 is the area of the tritium beam cross section. Assuming 100% π^- decay into usable μ^-, $N_\mu \cong N_\pi$. The available number of muons N_μ is, on the other hand, determined by Eq. (1) because

$$N_\mu = n_\mu v_\mu \tau_\mu a^2 \quad . \tag{5}$$

Equating (5) with (4), we obtain

$$\begin{aligned} n_t &= \frac{1}{5}\ (n_d \sigma \ell_\mu) \left(\frac{a^2}{b^2}\right)\ n_d \\ &\simeq 8 \times 10^{14} \left(\frac{a^2}{b^2}\right) \quad , \end{aligned} \tag{6}$$

where we assumed $n \sim n_d$.

Although $a < b$, Equation (6) indicates that the density of the tritium beam is substantially high in order to satisfy the necessary condition for condensation of muonic mesohydrogen atoms.

A possible method to create and/or accelerate such a high density beam is by the crystal X-ray accelerator method proposed by the present author and M. Cavenago (submitted to Phys. Rev. Lett.). In this the tritium ions are channeled through the crystal axis (channeling[9]) so that the tritium ions are confined by the crystal electrostatic field at an appropriate distance away from the crystal axis and propagate without excessive collisions with lattice ions. While tritium ions are channeled, we shine X-rays at the Bragg angle to effect the Bormann anomalous transmission in the crystal.[10] Under the appropriate conditions by Tajima and Cavenago the X-rays can accelerate ions in a relatively short distance. Similarly we put pions through crystal channels to confine and accelerate them. For deceleration of muons we again need crystal channels. (D. Taqqu also discussed a method of phase space compression of muon beams in a different method.[11]) One may estimate the emittance through the channeling. The hamiltonian of a muon is $H = (p_x^2 + p_y^2)/2m_\mu - e\phi$, where $\phi \sim 100$ eV in a crystal. Therefore, the typical momentum spread of muons would be

$$\Delta p \simeq (m_\mu e \Delta\phi)^{\frac{1}{2}} \simeq 10^5 \text{ eV/c} \quad . \tag{7}$$

Thus the transverse emittance can be estimated by using $\Delta p/p \sim 5 \times 10^{-4}$. The number of channels at the vicinity of the H_2 target is approximately 100×100, which is in part determined by the X-ray deceleration of muons by the crystal X-ray method (Tajima and Cavenago) and is in part determined by the electron mean free path (see below). Since the length of the target specimen is $\sim \ell_\mu$, the number of muons absorbed by the target should be $N_\mu \sim 10^8$.

Once muons are absorbed by hydrogen atoms through charge exchange, electrons will be ejected. Their mean free path is of the order of

$10^{-4} \sim 10^{-5}$ cm.[12] The above dimension of the target specimen is small enough to wash out most of the electrons. The dimensions of the specimen (i.e. cross section vs. depth) are flexible to a certain degree (by a factor of 3-5) to optimize electron washout and muon absorption. Quickly (excited) mesoatoms pμ will be formed. These atoms are smaller than the corresponding electronic atom pe by a factor of 207 and their binding energy is larger by the same factor. Their atomic [and molecular (pμpμ)] attractive force is to exert condensation of these atoms (and molecules). The phase transition from more sparse electronic structure to the very much denser muonic one takes place through the nucleation process. The nucleation process is essentially driven by the interatomic and intermolecular attractive force and the surface tension: the larger the droplet is, the easier it can absorb smaller clusters of atoms. The nucleation may begin at such a location as a boundary of bubbles. The speed of nucleation may be determined by

$$t_{N_m} = \int_0^{N_m} d\tau_N = \int_1^{N_m} \frac{dN}{n\sigma_\mu vN} \quad , \tag{8}$$

where N_m is the number of atoms in the nucleating droplet, n is the density of the liquid hydrogen, v the velocity of atoms, and σ_μ is the mesoatomic cross section. We demand that the time for nucleation be limited by the life time of muons:

$$t_{N_m} = \frac{1}{n\sigma_\mu v} \ell_n N_m \leqq \tau_\mu \quad . \tag{9}$$

Equivalently we obtain the number of muonic mesoatoms upper bound by

$$N_m \leqq e^{n\sigma_\mu v\tau_\mu} \sim e^{100} >> 1 \quad , \tag{10}$$

where we estimated $v \sim c_s$ (the sound speed). We indeed expect a condensed state of matter, as long as we inject enough number of muons.

III. Crystal X-ray acceleration and cooler

In the crystal X-ray accelerator concept (Tajima and Cavenago) the photon energy is taken at the hard X-ray range of $\hbar\omega \sim mc^2\alpha$ and the linac structure is replaced by a crystal structure, e.g., silicon or GaAs-AlAs. Here the crystal axis provides the channel through which accelerated particles propagates with minimum scattering (channeling[13]) and the X-rays are transmitted via the Bormann effect (anomalous transmission[11]) when the X-rays (wavelength λ) are injected in the xz-plane with a polarization in their plane into a crystal at the Bragg angle θ_B

$$\frac{\lambda}{2b} = \sin\theta_B \tag{11}$$

where b is the transverse lattice constant and later a the longitudinal lattice constant ($a \sim b$). The row of lattice ions (perhaps with inner shell electrons) constitutes the "waveguide" wall for X-rays, while they also act as periodic irises to generate slow waves. A superlattice[14] such as $Ge_cSi_{1-c}S_i$ (in which the relative concentration c ranges from 0 to 1 over 100Å or longer in the longitudinal z-direction) brings in an additional freedom in the crystal structure and provides a small Brillouin wavenumber $k_s = 2\pi/s$ with s being the periodicity length. We demand that the X-ray light in the crystal channel walls becomes a slow wave and satisfies the high energy acceleration condition

$$\frac{\omega}{(k_z + k_s)} = c \tag{12}$$

where ω and k_z are the light frequency and longitudinal wavenumber.

The energy loss of moving particles in matter is due to ionization, bremsstrahlung, and nuclear collisions. We can show that a channeled high-energy particle moving fast in the z-direction oscillates in the xy-plane according to the Hamiltonian

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + V(x, y), \tag{13}$$

where V is the average of the electrostatic potential φ of the crystal. Therefore, the distance r from the nearest row of atoms is $\sim b/2$ for positive charge, while it is a fraction of b for negative charge. The energy loss due to the bremsstrahlung, taking into account the effect that channeled particles avoid close collisions with nuclei, is given by

$$\frac{\partial \ln E}{\partial z} = 4Z_{\rm eff}^2 r_p^2 r_e^{-3} \alpha^7 \phi^{-3} K_0'(2\pi r/b), \tag{14}$$

where $r_p = e^2/m_p c^2$ is the classical radius of a particle with mass m_p (r_e for electron), $\phi = b/a_B$, and K_0 is the modified Bessel function of the second kind. The transverse cooling takes place independent of Eq. (14), as a muon oscillates in the transverse potential in Eq. (13). As channeled particles are confined to the rows of atoms by electric fields of the order $1 - 10V/\AA$ (equivalent 0.3-3M Gauss), crystals are used not only for acceleration sections but also for cooling and bending[15] sections. The confinement of charged particles in the channel facilitates us to have a dense muon beam required for the present purpose.

IV. Possible State

The state of the mixture of electronic hydrogen atoms and muonic ones is an interesting subject of investigation. Obviously too many things are unknown or undetermined at this time and strongly dependent on the exact way that this experiment may be carried out. Therefore, it is extremely dangerous to speculate such a question. It is, nevertheless, worth mentioning a few things here. The hydrogen atoms (or molecules) generally possess the attractive force at a large distance primarily due to the polarization (the dipole–dipole interaction) of atoms. This is manifested, for example, in the attractive term in the Lenard-Jones potential $\propto -1/r^6$. This force makes it possible for the gas to condense at low temperatures. On the other hand, when two atoms approach too close to each other (approximately less than the size of the atom), they repulse each other due to the strong electrostatic force of the nuclear charge, etc., which is manifested in the repulsive term in the Lenard-Jones potential $\propto 1/r^p$ with $p = 11.4 \sim 12 \sim \infty$. The effects of these forces are epitomized in the phenomenological equation of state of van der Waals:

$$P = \frac{\rho T}{1 - \rho b} - a\rho^2, \tag{15}$$

where ρ is the density, T and P are the temperature and pressure, b is approximately four times the volume of the atom $4\pi r_0^3/3$ and

$$a = -2\pi \int_{2r_0}^{\infty} U(r) r^2 dr, \tag{16}$$

with r_0 and $U(r)$ being the atomic radius and the atomic potential. The compressibility of the matter is defined by

$$\kappa_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T. \tag{17}$$

The compressibility is finite for the gas phase, while it is very small for the liquid phase. However, it becomes infinite at the so-called critical temperature T_c

$$T_c = \frac{8a}{27b}, \tag{18}$$

with critical pressure $P_c = a/27b^2$.

It is apparent that when a sufficient amount of muonic hydrogens is created, the compressibility Eq. (17) diverges, because there is a void to be filled and the surface tension will crush such a void, if it becomes sufficiently large. This can be considered by the following argument. The thermodynamical potential Ω under constant volume is expressed as

$$d\Omega = -SdT - Nd\mu + \alpha ds, \tag{19}$$

where S is entropy, N is the number of atoms, μ the chemical potential, α the surface tension, and s the area of the surface. We assume the void is a sphere. The thermodynamical potential is given by

$$\Omega = -P_l V_l - P_g V_g + \alpha s, \tag{20}$$

where $\mu_l = \mu_g = \mu$ and $T =$ const with the suffix l and g being the liquid and gas. From $d\Omega = 0$ we obtain

$$P_l - P_g = \frac{2\alpha}{r}. \tag{21}$$

When the void is small, however, instead of Eq. (21), we have

$$\ln \frac{P_g}{P_l} = -\frac{2\alpha v_g}{rT}, \tag{22}$$

where v_g is the volume of the void.

The tension effect may be considered. The liquid surface tension is described by the free energy of the void or droplet

$$F = \alpha_0 s + s\frac{T}{2\pi}\int \ln\left(1 - e^{-\hbar\omega/T}\right) k\, dk, \tag{23}$$

where ω is the frequency of liquid surface oscillations and α_0 is the surface tension at $T = 0$. Equation (23) is rewritten as

$$F = \alpha_0 s - s\frac{\hbar}{4\pi}\int \frac{k^2 d\omega}{e^{\hbar\omega/T} - 1}. \tag{24}$$

If we give the dispersion of the disturbance, i.e., ω vs. k, Eq. (24) can be explicitly integrated.

It is worth mentioning that the ratio of the van der Waals force between electronic hydrogens to that between muonic ones are $(m_e/m_\mu)^2$. The force between electronic one and muonic one should be in between. Otherwise the physics applies in parallel.

V. Applications

We briefly discuss possible applications of the present formation of superdense matter. If we apply a strong magnetic field that is allowed to be present without destroying the material (~ 1 eV/Å, equivalent to ~ 300 k Gauss), this magnetic field will be trapped in the matter and will be enhanced by a factor of $(207)^2$ when the condensation of matter takes place as a result of muon irradiation. It may be possible to obtain a magnetic field of 10^9 - 10^{10} Gauss by this method.

One may be able to study a unique matter that is so dense that nuclei of hydrogens are as close as 300 fm (3×10^{-11} cm) apart. The wavefunctions of nuclei that go like exp(-r/a) begin to interfere with each other. In this way one may be able to observe collective phenomena of nuclei.

Certainly one of these effects can be realized by putting in deuterium molecules instead of hydrogen molecules. Their nuclei begin to interfere and fuse. Of course, once this happens, the material will come apart. I believe that further investigations of this or other methods to create superdense muonic matter are necessary, considering the possible immense prospect as discussed in this preliminary investigation.

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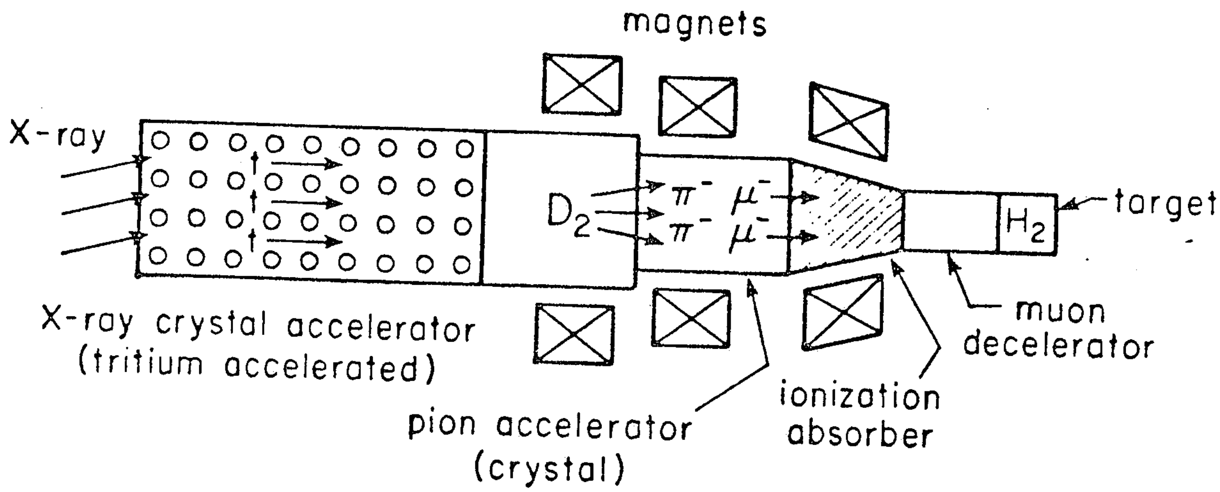
Figure Caption

Fig. 1: Schematic description of the method of creating a superdense matter. A tritium beam is injected on a crystal from the left. So are X-rays at the Bragg angle. The target hydrogen is in the liquid state.


magnets
X-ray
D_2
π^-
μ^-
H_2
target
X-ray crystal accelerator
(tritium accelerated)
pion accelerator
(crystal)
ionization
absorber
muon
decelerator