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Tearing Mode Growth in a Regime of Weak Magnetic Shear

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The nonlinear growth for the $m/n \geq 2$ resistive tearing mode is studied in the case when the rational surface $q(r_0) = m/n$ falls in a regime of weak magnetic shear, $q'(r_0) \simeq 0$. The island width is determined self-consistently from the nonlinear, zero-helicity component of the perturbed magnetic flux that provides the local shear. It is found that the magnetic perturbation keeps growing exponentially in the nonlinear regime on a hybrid resistive-Alfvénic time scale, while the island width and the vorticity grow on a much slower time scale. Accordingly, much faster release of magnetic energy results for modes growing near minima of hollow q profiles.

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I Introduction

Resistive magnetohydrodynamic instabilities and the associated changes in the magnetic topology play an important role in magnetized plasmas.[1] The growth of tearing modes[2] in magnetically confined plasmas has been connected to anomalous transport, sawtooth oscillations and tokamak disruptions. Magnetic reconnection[3] may also occur in magnetospheric and astrophysical phenomena such as the formation of earth's magnetotail and the solar flares. The linear behavior of the resistive magnetohydrodynamic (MHD) modes, primarily their growth rates and the parameter regimes of instability, have been studied in a number of papers.[4] Of equal theoretical and practical interest have been studies of the nonlinear growth rates and the saturation levels. The single helicity approach has usually been involved in analyzing the situation.[5] Changes in the local magnetic shear caused by the perturbation have been traditionally neglected when the zero helicity component of the perturbation is retained.[6] Thus both the width of the island and the growth rates involve only equilibrium magnetic profiles. For the $m \geq 2$ modes it has been shown that the growth of the perturbation slows down to an algebraic time dependence when the island width exceeds the resistive layer width.[5]

In some cases of interest the unstable modes may grow in a regime of very weak external magnetic shear. For example this may occur if the rational surface, around which the perturbation is localized, falls near the bottom of a hollow q profile. In such cases it seems inevitable that the shear produced by the nonlinear, zero-helicity piece of the perturbation alters the local magnetic profile significantly. For sufficiently small external gradients the shear produced by the growing mode may soon prevail and the island width become self-determined. In this work we carry out a simplified nonlinear analysis of the mode evolution including the self-consistent change in the magnetic profile. We find that the magnetic perturbation keeps growing exponentially on a hybrid resistive Alfvénic time scale well inside the nonlinear regime. The width of the island is much larger than the width of the resistive layer and grows on a much slower time scale. Although our results were obtained in the limit of negligible background magnetic shear they nevertheless imply that nonlinear growth is accelerated whenever the parallel nonlinear currents are strong enough to affect the shaping of the magnetic islands.

II Mathematical Formulation

The vector fields $\mathbf{B}$ and $\mathbf{v}$ are related to the poloidal flux Ψ and the stream function Φ through

$$\mathbf{B} = B_T\hat{\zeta} - \hat{\zeta} \times \nabla\Psi \quad , \quad \mathbf{v} = \hat{\zeta} \times \nabla\Phi$$

with ζ the unit vector in the toroidal direction. The evolution of the system is described in the context of resistive incompressible MHD by the following set of coupled nonlinear equations involving the two scalar potentials Ψ and Φ

$$\frac{\partial\Psi}{\partial t} + \nabla_{||}\Phi = \eta J, \tag{1}$$

$$\frac{\partial\omega}{\partial t} + [\Phi, \omega] = -\nabla_{||}J, \tag{2}$$

$$J = \nabla_{\perp}^2\Psi, \tag{3}$$

$$\omega = \nabla_{\perp}^2\Phi, \tag{4}$$

where $[\Phi, \omega]$ is the Poisson bracket $\left[\frac{\partial\Phi}{\partial r}\frac{\partial\omega}{\partial\alpha} - \frac{\partial\Phi}{\partial\alpha}\frac{\partial\omega}{\partial r}\right]$. The magnetic field is normalized to the poloidal field B_P, length to the plasma characteristic length L, velocity to the poloidal Alfvén speed $v_A = \left(B_P^2/4\pi\rho\right)^{1/2}$, time to the transit time L/v_A and the normalized resistivity η is the inverse magnetic Reynolds number $S^{-1} = \eta_0 c^2/4\pi v_A L$. The field-aligned gradient $\nabla_{||}$ and the transverse gradient $\nabla_{\perp}$ are defined in terms of the unit vector along the total magnetic field $\mathbf{b} = \mathbf{B}/B$,

$$\nabla_{||} = \hat{b} \cdot \nabla \quad , \quad \nabla_{\perp} = \nabla - \hat{b} \cdot \nabla. \tag{5}$$

Equation (1) is the generalized Ohm's law combining the conductivity equation with the magnetoinductive limit of Maxwell's equations, and Eq. (2), the vorticity equation, is obtained by taking the $\mathbf{b} \cdot \nabla\times$ projection of the momentum equation.

We will consider coherent helically symmetric perturbations with the helical angle α defined by

$$\alpha = n\zeta - m\theta, \tag{6}$$

where ζ is the toroidal and θ the poloidal angle. Helical symmetry is expressed by

$$f(r,\theta,\zeta) \equiv f(r,\alpha), \tag{7}$$

for any perturbed quantity f. Accordingly, the nonlinear operator $\nabla_{\|}$ is expressed as

$$\nabla_{\|} f = -\frac{m}{r}\left[\left(\frac{n}{m}r + \frac{\partial\Psi}{\partial r}\right)\frac{\partial f}{\partial\alpha} - \frac{\partial\Psi}{\partial\alpha}\frac{\partial f}{\partial r}\right] = -\left[\Psi_H, f\right], \tag{8}$$

where Ψ_H is the total helical flux $\Psi_H = \Psi + (n/2m)r^2$. The unperturbed potentials are

$$\Psi_{H_0}(r) = \Psi_0(r) + \frac{n}{2m}r^2 \quad , \quad \Phi_0 = 0, \tag{9}$$

while the perturbed flux ψ and stream function ϕ are given by

$$\begin{aligned} \psi(r,\alpha,t) &= \psi_0(r,t) + \sum_{n\geq 1}\psi_n(r,t)\cos n\alpha \\ \phi(r,\alpha,t) &= \sum_{n\geq 1}\phi_n(r,t)\sin n\alpha. \end{aligned} \tag{10}$$

Substituting Eqs. (8) and (10) into Eqs. (1)-(2) and retaining only the fundamental ($n = 1$) one obtains

$$\frac{\partial\psi_1}{\partial t} - k\frac{\partial\bar{\psi}}{\partial r}\phi_1 = \eta\nabla_\perp^2\psi_1 \tag{11}$$

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi_1 - \frac{k^2}{\eta}\left(\frac{\partial\bar{\psi}}{\partial r}\right)^2\phi_1 = \frac{k}{\eta}\frac{\partial\bar{\psi}}{\partial r}\frac{\partial\psi_1}{\partial t}, \tag{12}$$

with

$$k = \frac{m}{r} \quad , \quad \bar{\psi} = \oint\frac{d\alpha}{2\pi}\Psi_H = \Psi_{H_0}(r) + \psi_0(r,t). \tag{13}$$

For mathematical tractability only the $\psi_0\phi_1$ terms were kept among the nonlinear products $\psi_i\phi_j$ with $i + j = 1$, assuming that existing higher harmonics remain at small amplitude. We note in passing that, under certain circumstances, nonlinear cascades could be taken into account by the use of a phenomenological nonlinear resistivity. Similarly we neglect coherent contribution from nonlinear convection. Equations (11)-(12) are similar to the usual linearized tearing mode equations with the addition of the zero-helicity piece of the perturbation $\psi_0(r)$ in the local magnetic profiles.

We are interested in modes localized near a rational surface r_0 defined by

$$\mathbf{B}_0 \cdot \nabla\alpha = \frac{\partial \Psi_{H_0}}{\partial r} = \frac{nB_\theta}{r}\left(q - \frac{m}{n}\right) = 0 \quad \text{for} \quad r = r_0, \tag{14}$$

where q is the safety factor $q = rB_P/RB_T$. As in the general case of the $m/n \geq 2$ tearing mode there are two well-separated spatial scales, parametrized by the plasma characteristic length L and the island width W with $L \ll W$. Viewed from the macroscopic scale the ideal ($\eta = 0$) solutions of Eqs. (1)-(2) develop a singularity in the vicinity of $r = r_0$, resolved by the inclusion of finite resistivity and asymptotic matching across the magnetic island. The connection between the inner solutions and the outer plasma layers enters through the (macroscopic) discontinuity in the logarithmic derivative

$$\Delta' = \left[\frac{\partial}{\partial r}\ell n\psi\right]_{r_0-\epsilon}^{r_0+\epsilon} . \tag{15}$$

We will consider Δ' as given and focus on the structure of the solutions inside an island.

Since $W \ll L$ we expand locally perturbed and unperturbed quantities in $x = r - r_0$ around the resonant layer

$$\begin{aligned}
\Psi_{H_0}(x) &= \frac{1}{2}\Psi''_{H_0}(0)x^2, \\
\psi(x,\alpha,t) &= \frac{1}{2}\psi''_0(0,t)x^2 + \sum_{n\geq 1}\psi_n(t)\cos n\alpha, \\
\phi(x,\alpha,t) &= \sum_{n\geq 1}\phi_n(x,t)\sin n\alpha.
\end{aligned} \tag{16}$$

Here $\psi_n(t)$ is x-independent according to the "constant-ψ" approximation, while $\phi(x,t)$ and the merging velocity $v_x \simeq k\partial\phi/\partial\alpha$ are odd in x with $v_x = 0$ at the resonant surface $x = 0$. Our primary interest is in the case when the mode rational surface r_0 falls very close to a point of zero shear $q'(r_0) = 0$. This situation is relevant to hollow q profiles associated with skin currents. Location of the resonant surface in the neighborhood of weak shear $q' \simeq 0$ also implies $\Psi''_0(r_0) \simeq 0$ according to Eq. (14). Then it may occur that the zero-helicity piece of the perturbed flux $\psi_0(x,t)$ becomes much larger than the background $\Psi_0(x)$ within the island separatrix. To further clarify this statement we consider the corresponding island half-width,

$$W = 2\left\{\frac{\psi_1(t)}{|\Psi''_0(0) + \psi''_0(0,t)|}\right\}^{1/2}, \tag{17}$$

where from now on we drop the subscript H for helical flux. Even for small $\psi \sim \epsilon$, the ratio Ψ_0''/ψ_0'' may become small if Ψ_0'' is sufficiently small. The island width in this case is determined by the shear produced by the perturbation itself. The subsequent analysis applies to cases of sufficiently small background shear in the above sense.

One other assumption concerning the expansion is that the perturbed nonlinear piece $\psi_0(x)$ is an even function of x with $(\partial\psi_0/\partial x)_{x=0}$, implying that the location of the rational surface is not altered by the growth of the mode. This is justified by noting that the zero-helicity component of the perturbation evolves from the interaction $\psi_0 \simeq \langle k\psi_1\phi_1'\rangle\, \gamma^{-1}$ where both ϕ_1' and ψ_1 are even in x during the initial quasilinear stage. The original symmetry is preserved through the nonlinear evolution by the similarity transformations used in solving Eqs. (1)-(2). Asymmetric islands and possible motion of the rational surface, although of some interest, are beyond the scope of this work.

We will show that a class of approximate solutions to Eqs. (1)-(2) exists with the following salient features: (a) The growth of the magnetic flux remains exponential well inside the nonlinear regime. (b) The kinetic energy and the island width grow on a much slower time scale. Thus saturation in the ordinary sense[5] does not occur in this case and slowing down will eventually start when the mode alters significantly the equilibrium profile to change Δ'.

III Derivation of the Growth Rate

We start out with the generalized Ohm's law in order to make the connection between inner and outer regimes. During the nonlinear stage the perturbed energy is mostly magnetic with $\omega \ll \Psi$; thus the right-hand side of (2) must also be small,

$$(\mathbf{B}\cdot\nabla)J \simeq 0 \quad , \quad J = J(\Psi). \tag{18}$$

The flux function prescription for the sheet current J implies that the skin time $\tau_s \sim W^2/\eta$ is much shorter than the characteristic time $\tau_c \sim 1/\gamma$. The validity of this condition will be checked *a posteriori* from the computed values for γ and W. Since both ω and ψ evolve from the original linear stage of equal magnitudes, relative smallness of ω at later times indicates much slower growth for the velocity fields than the magnetic fields. This separation of time scales has some important consequences in case of weak shear, as will be soon clear. Consistency among (3), (8) and (18) requires that Ψ be of the form $\Psi = \hat{\Psi} + \delta\hat{\Psi}$ with $\hat{\Psi}$ satisfying $(\mathbf{b}\cdot\boldsymbol{\nabla})J = k(\partial\hat{\Psi}/\partial r)\nabla_\perp^2\hat{\Psi} = 0$. Although $\hat{\Psi}$ is unknown, it is checked by substitution that Ψ in Eq. (16) is of the required form as

$$(\hat{b}\cdot\nabla)\Psi \simeq Wk\,[\psi_0''(0,t)]^2 \sim 0(\epsilon^2). \tag{19}$$

This small departure $\delta\Psi \sim \epsilon^2$ from an exact flux function provides a small driving term for the vorticity equation to which we will return later on.

To compute the total current inside the magnetic island we first eliminate the vorticity from Eq. (1) by taking the average over the helical angle α at constant Ψ. For simplicity we consider only one harmonic in the perturbed flux and use the expansion (16) in x for the zero-helicity piece, keeping for the moment the contribution from the background magnetic shear. Integrating across the island, by switching variables from (x,α) to (Ψ,α), and matching the macroscopic logarithmic singularity with the total current we obtain, after Rutherford,

$$\frac{\partial\psi_1}{\partial t} = \frac{\eta}{4}\frac{\Delta_1'}{A}\left[\frac{\psi_1(t)}{2\,|\Psi_0''+\psi_0''(0,t)|}\right]^{-1/2}\psi_1(t), \tag{20}$$

with

$$A = \int_b^c ds \left\langle \frac{\cos\alpha}{|s-\cos\alpha|^{1/2}}\right\rangle^2 \div \left\langle |s-\cos\alpha|^{-1/2}\right\rangle \simeq 0.7$$

where (b,c) is $(-1,\infty)$ or $(-\infty,1)$ for $\Psi_0''+\psi_0''$ positive or negative respectively and $\langle f\rangle = (1/2\pi)\int d\alpha\; f$ is the helical angle average. When Ψ_0'' remains much larger than ψ_0'' at all times the well known algebraic growth in time is obtained for ψ_1

$$\frac{\partial}{\partial t}\psi_1^{1/2} = \frac{\eta\Delta_1'\,|2\Psi_0''(x=0)|^{1/2}}{8A}. \tag{21}$$

A completely different picture emerges in the case under consideration with Ψ_0'' small enough to be dominated by ψ_0'' after a short time from the triggering of the instability. The equation becomes

$$\begin{aligned}\frac{\partial}{\partial t}\psi_1(t) &= \frac{\eta\Delta_1'\sqrt{2}}{2WA}\psi_1(t),\\ W &= 2\left[\frac{\psi_1(t)}{|\psi_0''(0,t)|}\right]^{1/2},\end{aligned} \tag{22}$$

where the island width W is now determined by the shear generated by the perturbation itself. It will be seen that W evolves on the same slow time scale with the velocity and thus it can be considered as constant on the fast time scale growth of the magnetic perturbation q. The solutions of Eq. (22) with constant W grow exponentially well inside the nonlinear regime with a growth rate given by

$$\gamma = \frac{\eta\Delta_1'}{4W}. \tag{23}$$

The gradual transition from algebraic to exponential growth is connected to the relative contribution between perturbed and unperturbed flux in Eq. (17), albeit only the two limiting cases Eqs. (21) and (23) seem amenable to analytic methods.

The boundary condition needed for the island width in Eq. (23) is obtained by taking the helical angle average of Eq. (11) at the rational surface $x = 0$. Noting that $v_x = 0$ there, one finds

$$\frac{1}{2}k\psi_1(t)\frac{\partial\phi_1(0,t)}{\partial x} = \eta\psi_0''(0,t). \tag{24}$$

The term $(\partial\psi_0/\partial t)$ has been dropped in the above equation, being much smaller than the right-hand side, since for arbitrary growth rate of the form $\gamma \sim \tau_R^{-\zeta}\tau_A^{-(1-\zeta)}$, and inside the magnetic island $x < W$ we have according to Eqs. (16) and (23): $(\partial\psi_0/\partial t)/(\eta\psi_0'') < \gamma W^2/\eta \sim (\tau_A/\tau_R)^{1-\zeta} \ll 1$ for any $\zeta < 1$. According to Eq. (24) nonlinear convection near the center is balanced mainly by dissipation while the growth of the island width is tied to the velocity growth through

$$\sigma W^2 = \frac{8\eta}{k}\left[\frac{\partial\phi_1(0,t)}{\partial x}\right]^{-1}, \tag{25}$$

where $\sigma = \mathrm{sgn}\,[\psi''(0,t)]$. Thus the assumption of slow evolution for the vorticity establishes through (23) and (25) that the island width also grows slowly while both the zero and unit helicity components of the perturbed flux grow on the same fast time scale. It will be shown that the remaining vorticity equation, required to close the set of equations, indeed admits a class of solutions consistent with the perturbed magnetic profile and growing slowly on time, $d\log\phi_1/dt \ll \gamma$. Thus the solutions found agree with the assumption of time scale separation.

We note that for a flow ϕ_1 similar to the linear flow with $\partial\phi_1/\partial x < 0$ we have from (25) that $\psi''(0,t) < 0$. The nonlinear current flows against the linear current[5] and creates negative shear, opposite to the shear of the background field. A change of sign for $\psi''(0)$ in Eq. (16) converts x-points in the magnetic topology to 0-points and vice versa.

In order to find $d\phi_1/dx$ we combine Eqs. (11) and (12) into

$$\eta\frac{\partial}{\partial t}\phi_1'' - k^2\left[\psi_0'(x,t)\right]^2\phi_1 = k\psi_0'(x,t)\frac{\partial\psi_1}{\partial t}. \tag{26}$$

Equation (26) has the form of a linear tearing mode with time varying coefficients, since the local magnetic profile is determined by the perturbed flux ψ_0 that evolves in time. The small deviation of the current J from an exact flux function provides the driving term on the right-hand side.

We now introduce the ansatz

$$\phi_1(x,t) = V(x)\exp\left\{\int_0^t dt'\gamma\left[\psi_0''(0,t')\right]^2\right\}, \tag{27}$$

for the solutions of Eq. (26). The exponential remains finite for $\psi_0(x,t)$ tending to 0 as $|x|\to\infty$ ($\psi_0(x,t)$ scales as x^2 only inside the current layer) and for $\psi_0(x,t)$ staying finite as $t\to\infty$ (implying that $\gamma(\tau)\to 0$ on the slow time scale $\tau\to\infty$). Solution (27) evolves on the slow time scale, $d\log\psi_1/dt \sim \gamma(\psi_0'')^2 \ll \gamma$. Substitution of (27) and use of the expression (23) for the island width factors the fast time dependence out of (26) yielding

$$\frac{d^2V}{dx^2} - \frac{k^2x^2}{\gamma\eta}V = xk\left(\frac{\sqrt{2}\Delta_1'}{4A}\right)^2\frac{\eta}{\gamma^2}\sigma. \tag{28}$$

Within the island interior $x < W$ and after rescaling to $\xi = kV/\gamma$ and $X = x/\ell$, Eq. (28) is recast into the familiar form,

$$\frac{d^2\xi}{dX^2} - \frac{X^2}{Q}\xi = \frac{X}{Q}Z \tag{29}$$

first obtained for the standard linear tearing mode. Here the coefficients are given by

$$\begin{aligned}\ell &= (\eta/k)^{1/3} \quad , \quad Q = \gamma(\eta k^2)^{-1/3}, \\ Z &= \frac{1}{\ell}\frac{\eta^2}{\gamma^2}\left(\frac{\sqrt{2}\Delta_1'}{4A}\right)^2\sigma.\end{aligned} \tag{30}$$

The solution of the inhomogeneous Eq. (29) is considerably simplified[7] by taking the Fourier transform of ξ, $\hat{\xi}(\chi) = \int_{-\infty}^{\infty} dX\ \xi(X)\ \exp(iX\chi)$, transforming (29) into

$$-\frac{1}{Q}\frac{d^2\hat{\xi}}{d\chi^2} + \chi^2\hat{\xi} = \frac{2\pi i}{Q}Z\delta'(\chi). \tag{31}$$

The solution of the above equation is expressed in terms of the parabolic cylinder functions U, solutions of the homogeneous equation[8] with $Z = 0$,

$$\begin{aligned}\hat{\xi}(X) &= \Lambda U\left(0, \pm\sqrt{2}Q^{1/4}X\right) \quad , \quad X \gtrless 0 \\ \Lambda &= -\frac{i\pi Z\sigma}{U(0,0)},\end{aligned} \tag{32}$$

from which it follows that

$$\frac{d}{dx}\phi_1(0,t) = C\frac{\gamma}{k\ell}\frac{dV}{dx} = -\frac{C\Gamma}{\sqrt{2Q}}\frac{\eta^2}{\gamma k\ell^2}\left(\frac{\sqrt{2}\Delta_1'}{4A}\right)^2\sigma, \tag{33}$$

where $\Gamma = \int_0^\infty dX\ XU(0,X)/U(0,0) \sim 1$ and $C(\tau)$ is the slowly varying factor of Eq. (27). We may set $C(\tau)$ equal to a constant $C_0 \sim 1$ over time periods of order $1/\gamma$. Equations (23), (25) and (33) then form a closed set involving γ, W and $d\phi_1/dx$. Solving, undoing the normalizations and approximating $(C\Gamma)^{2/7} \simeq 1$ we find an exponential hybrid resistive-Alfvénic nonlinear growth rate

$$\gamma \simeq \frac{1}{4}\tau_R^{-5/7}\tau_A^{-2/7}(kL)^{2/7}(\Delta_1'L)^{8/7}, \tag{34}$$

and an island width

$$2W \simeq 2\sqrt{8}LS^{-2/7}(kL)^{-2/7}(\Delta_1'L)^{-1/7}, \tag{35}$$

much larger than the resistive layer

$$w \simeq LS^{-3/7}(kL)^{-3/7}(\Delta_1'L)^{2/7}. \tag{36}$$

Using $\Delta_1'L \sim 10^2$ and $kL \sim 1$ as typical values in Eq. (35) one obtains $2W \sim 3LS^{-2/7}$. Typical large tokamak parameters as for the recent JET discharges[9] are $R \simeq 3m$, $a/R \simeq 0.4$, $B_T \simeq 3.4T$ at the center, $n\tau_E T_i = 10^{20}m^{-3}$.sec.keV with $\tau_E = 0.8\,\text{sec}$, $T_i \sim 12.5$ keV, and $T_e \sim 5$ keV. Assuming that the $q = 2$ rational surface is at $r = a/2 \simeq L$ and employing the Spitzer parallel resistivity at half the central electron temperature we obtain $S \simeq 10^7$ and $2W \simeq 1.8$ cm.

We note that both the island width and the growth rate do evolve on the slow time scale, as seen by reviving the slow time dependence $C(\tau)$ in Eq. (34) and (35). Equation (23) also shows that the growth rate $\gamma(\tau)$ tends to decrease with increasing island width $W(\tau)$. Finally the skin time can be evaluated from (34) and (35). It is found that $\gamma\tau_s \sim S^{-2/7} \ll 1$; thus the requirement for the validity of Eq. (18) is satisfied.

IV Conclusions

In this paper we investigated the nonlinear evolution of tearing modes $m \geq 2$ localized in regimes of very weak magnetic shear. Fast growth of the magnetic perturbation was found to persist well within the nonlinear regime, while the nonlinear velocities and island width evolve on a slower time scale. The change in the equilibrium profiles away from the island, caused by the nonlinear current, will change the associated value for Δ', eventually leading to saturation. Few numerical simulations relevant to the discussed situation have appeared so far, although the situation may change in the future because of the reviving interest in hollow q profile plasmas. However qualitative agreement appears with the numerical results of Ref. 10 for runs with large shear length L. Fast evolution deeply into the nonlinear regime persists in these results for magnetic Reynolds number $S \geq 10^4$ becoming almost exponential at $S = 10^6$ over a time period of 10^4 Alfvén transit times.

The magnetic Reynolds number $S = Lv_A/c^2\eta$ in these runs was defined in terms of the plasma characteristic length L at the mode rational surface. The amount of reconnected flux $\Delta\Psi = \int_0^{2\pi} d\alpha\, |\psi_1(\alpha)|$ increased 10^2 times while the separatrix width increased by a factor of 10. The final magnetic perturbation energy was 10^2 times larger than the kinetic energy starting from an initial state of equipartition. This time scale separation tends to be stronger with increasing L. It seems however that in these runs L was never large enough for the background shear to cease being a factor. Our analysis, although strictly valid in the limit of infinitesimal equilibrium shear, nevertheless implies that acceleration of the nonlinear evolution occurs whenever the nonlinear current interferes with the shaping of the magnetic island. If nonlinear effects compare with the effect of the unperturbed profile the nonlinear evolution is expected to be faster than algebraic and the island width is to be saturated at much larger width than the resistive layer. Much faster release of magnetic energy should result from tearing modes growing inside hollow q profiles.

References

1. R.B. White, D.A. Monticello, M.N. Rosenbluth and B.V. Waddel in *Proc. of the Sixth International Conference*, **Plasma Physics and Controlled Nuclear Fusion Research**, 1976, Berchtesgaden (IAEA, Vienna, 1977), vol. 1, p. 569.

2. H.P. Furth, J. Killeen and M.N. Rosenbluth, Phys. Fluids **6**, 459 (1963).

3. V. Vasyliunas, Rev. Geophys. Space Phys. **13**, 303 (1975).

4. See for example R.D. Hazeltine and J.D. Meiss, Phys. Reports **121**, 1-164 (1985) and references therein.

5. P.H. Rutherford, Phys. Fluids **16**, 1903 (1973).

6. R.D. Hazeltine, J.D. Meiss and P.J. Morrison, Phys. Fluids **29**, 1633 (1986).

7. W.M. Manheimer and C. Lashmore-Davis in M.H.D. Instabilities in Simple Plasma Configurations (Naval Research Lab., Edit., Washington DC) p. 93-94 (1984).

8. M. Abramowitz and I.A. Stegun, in **Handbook of Mathematical Functions**, (Dover, New York, 1972) Ch. 19.

9. P.E. Stott, Bull. Am. Phys. Soc. **32**, 1103 (1987).

10. R.S. Steinolfson and G. Van Hoven, Phys. Fluids **27**, 1207 (1984).