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A Mechanism for Rapid Sawtooth Crashes in Tokamaks

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The standard picture of Kadomtsev reconnection process predicts sawtooth crash times that are longer than those observed in the present day large tokamaks. Ideal kink modes are investigated as a possible mechanism for these fast crashes, using fully toroidal, compressible, full magnetohydrodynamic equations. In systems with low shear, parallel-current and pressure-driven modes are identified well below the previously accepted poloidal beta limits. Linear and nonlinear calculations show good agreement with experiments and indicate that such modes may explain fast collapse times reported in the recent literature.

The sawtooth oscillations in the soft X-ray signals observed in Tokamaks are associated with periodic changes in the central electron temperature, T_e[1]. Typically, a slow phase during which the central temperature slowly rises is followed by a fast drop in T_e, associated with flattening of the central temperature. The time scale of the slow phase is determined by various transport processes such as ohmic heating. The resistive internal kink mode was invoked by Kadomtsev[2] to explain the crash phase of the oscillations. In this model, an $m = 1$ island (m is the poloidal mode number), associated with a safety factor, q, less than unity on axis, grows, forming a helical deformation of the internal plasma column. This kink structure subsequently relaxes to a symmetric state through complete reconnection of the helical flux inside the $q = 1$ surface with the flux from outside. The Kadomtsev model has been generally believed to explain the sawtooth oscillations in earlier tokamaks.

Recent observations, however, indicate that tokamaks exhibit various types of sawtooth oscillations that can not be fully explained by the Kadomtsev model. In addition to the simple sawtooth associated with this model, there are double or compound sawteeth[3,4] thought to be caused by the presence of two or more $q = 1$ surfaces in the plasma. Moreover, both TFTR and JET tokamaks report crash times that can not be easily reconciled with the Kadomtsev reconnection process[5].

Previous numerical studies of the sawteeth oscillations have concentrated on the $m = 1$ resistive tearing mode. Waddell[6] et al. performed the first nonlinear studies and found the current flattening times to be in agreement with the internal disruption times reported in the ST tokamak. Sykes and Wesson[7] included self-consistent evolution of the temperature and resistivity in order to follow periodic sawtooth oscillations. However, in their simulations, sawteeth decayed away after a few periods. Denton et al., by including parallel thermal conductivity, have produced repeating sawtooth oscillations[8]. They were able to reproduce qualitatively many of the experimentally observed features of sawteeth oscillations, including compound sawteeth, by adjusting the transport coefficients. However, their studies did not adequately address the quantitative features of sawteeth, in particular anomalously fast crash times observed in the present day large tokamaks. For instance, some crashes in JET are characterized by the absence of any discernible precursor oscillations, and a rapid collapse of the central temperature in about 100-microseconds[9]. During the crash phase, the hot core region rapidly moves outward and is replaced by colder plasma. Then, this highly asymmetric state relaxes (in $\sim 100\mu$sec) to a poloidally symmetric state in which a ring of hot plasma surrounds the colder core plasma, producing a hollow pressure profile. In Ohmic discharges, however, this symmetrization is sometimes observed to take up to $10ms$, leading to low-level successor oscillations[10]. These fast sawteeth crashes will be the subject of this letter.

Since the time scales involved appear to be too short for a resistive mode, Wesson[11] has suggested that a pressure-driven ideal kink mode could be the responsible instability mechanism. This mode is always unstable in a cylinder[12] for $q_0 < 1$, and $p' < 0$, where q_0 is the safety factor on axis, and p' is the radial pressure gradient, but is generally believed to be stabilized by toroidal effects. In particular, the analysis of Bussac et al.[13] for the $n = 1$ ideal kink mode predicts an instability threshold for the pressure gradient that is much larger than the values at which sawtooth oscillations are observed. [n is the toroidal mode number; because of toroidal mode coupling, the mode can no longer be identified with a single poloidal mode number.] However, this calculation is based on a low-β (β is the ratio of plasma pressure to magnetic field pressure), large aspect ratio ($\epsilon = a/R_0 \ll 1$) expansion of the ideal magentohydrodynamic (MHD) equations. In the usual tokamak ordering asumed in Ref. 13, $|1-q| \sim O(1)$, and the parallel wave vector, $k_{\|} = B_\theta/(rB)(1-q) \sim O(\epsilon)$ (for $m = n = 1$). As pointed out by Wesson[11], if the safety factor profile is such that $|1-q| \ll 1$ for a substantial portion of the plasma column, then this ordering breaks down. In this case, it is more reasonable to assume $|1-q| \sim O(\epsilon^n)$, and $k_{\|} \sim O(\epsilon^{n+1})$, with $n \geq 1$. A significant result of this change is that the stabilizing line bending energy is no longer the leading order term in the expansion of Ref. 13. [We still assume that $\int d\tau |\delta B_\phi|^2$ is minimized by letting $\nabla \cdot \xi_\perp = 0$.] Therefore, an analysis of ideal MHD modes for these systems needs to treat the line bending (shear) term, and the destabilizing pressure, parallel current gradients, and toroidal effects on an equal footing. Such an analysis, which assumes $k_{\|} \sim O(\epsilon^3)$, is given by Ware[14]. The remainder of this letter is devoted to linear and nonlinear studies of $n = 1$ internal kink mode. Unlike previous studies[15], we specialize to low shear systems with $q_0 \simeq 1$, and examine the questions of stability boundaries, instability growth rates, the nonlinear evolution of the mode, and its possible role in rapid sawtooth crashes.

For our numerical studies we use the MHD code CTD[16], which has recently been modified for toroidal geometry. CTD solves the full, non-reduced, compressible, nonlinear MHD equations without any expansions in the inverse aspect ratio, $\epsilon = a/R_0$. This work is an extension of our previous numerical study which made use of the full MHD equations in cylindrical geometry, and high-β reduced MHD equations with toroidal curvature[17].

The family of equilibria we consider have the safety factor profile $q(r) = q_0\{1 + r^{2\lambda}[(q_l/q_0)^\lambda - 1]\}^{1/\lambda} - q_1 \exp[-(r - r_{min})^2/\delta^2]$, where q_0, q_l are the on-axis and limiter values of $q(r)$, respectively. The radial coordinate r is normalized to the minor radius a. Finite q_1 introduces a minimum in the q-profile near $r = r_{min}$, while δ determines the width of the surface currents there. In this Letter, we will be concerned with $q-$profiles that approach unity from above so that $q_{min} \geq 0$. The pressure profile is given by $p(r) = p_0[1 - r^2]^\nu$. Note that for pressure driven modes, the relevant parameter is $\beta/\epsilon = \epsilon\beta_p/q_l^2$, rather than p_0 alone. Here we define the toroidal and poloidal beta's as $\beta = 2\mu_0 < p > /B_T(a)^2$, and $\beta_p = 2\mu_0 < p > /B_p(a)^2$, respectively. $< p >$ is the volume-averaged kinetic pressure, and $B_T(a)$, and $B_p(a)$ are the average toroidal and poloidal field strengths, respectively, measured at the limiter. The toroidal equilibria used in CTD are obtained by starting with the cylindrical equilibria described above and dissipating kinetic energy until an axisymmetric toroidal steady-state is reached.

The important features of the linear results are: i) Whereas the accepted toroidal theory of the $n = 1$ modes[13] predicts an instability only for $q_0 < 1$, for the low-shear systems considered here, unstable internal kink modes are found for $q_0 > 1, |1 - q| \ll 1$, both in cylindrical and toroidal geometries. ii) Although, in a cylinder, there is a critical poloidal beta below which the mode becomes stable, in toroidal geometry, we observe an unstable mode even for $\beta = 0$. This mode is further destabilized by finite pressure. iii) For the modes we consider, the stabilizing contributions to δW from the line bending energy, $\delta W_+ \sim \int k_\parallel^2 B^2 |d\xi/dr|^2 dr$, is minimized, not by a uniform displacement ξ vanishing at the rational surface, but by keeping $k_\parallel^2$ small in the region of interest. Thus, the mode is highly sensitive to the details of the q−profile, and an instability can be triggered by slight variations in $q(r)$. iv) These modes do not require the presence of a $q = 1$ surface in the plasma; they are non-resonantly unstable even when $q > 1$ at all radii.

Figure 1 shows the growth rate of the mode, normalized to τ_{Hp}^{-1}, where $\tau_{Hp} = R_0\sqrt{\mu_0\rho_0}/B_0$ is the poloidal Alfvén time, as a function of $\epsilon\beta_p$, for a sequence of flux conserving equilibria. The equilibrium parameters are $\epsilon = 1/4, q_0 = 1.01, q_1 = 0.01, \lambda = 4$, and $\nu = 3$. In cylindrical geometry (Fig. 1a), the mode is robustly unstable for $\epsilon\beta_p \geq 4 \times 10^{-3}$. The existence of a β_{crit} can be easily understood in terms of the cylindrical form of δW[18], where the stabilizing shear term becomes dominant for sufficiently small pressure gradients. Note, however, Ref. 18 predicts that $\gamma\tau_{Hp} \sim \epsilon\beta_p$, for $\epsilon\beta_p > \epsilon\beta_{crit}$, whereas in our low shear case, we have the scaling $\gamma\tau_{Hp} \sim \sqrt{\epsilon\beta_p}$. For low shear, it can be easily shown that inertia is important for all radii where $|1 - q(r)| \ll 1$, rather than only in a singular layer of width of $O(\epsilon^2)$, as it is assumed in Ref. 18. Then, it follows that $\gamma^2 \sim -\delta W$, rather than $\gamma \sim -\delta W$, leading to the observed scaling. Fig. (1b) shows the fully toroidal results. Here the mode is unstable for all $\beta \geq 0$. For $\beta \simeq 0$, it is a current-driven internal kink mode that is destabilized by toroidal effects[14] for $q \geq 1$. Note that this mode has no counterpart in cylindrical geometry, where $q_0 < 1$ is required for a $J_\parallel$−driven internal kink mode. Finite-β further destabilizes it, and eventually, for $\epsilon\beta_p \geq 0.01$, the pressure driving terms become dominant. As expected, because of the favorable average curvature in a torus, the growth rates in the pressure-driven regime is lower than in the purely cylindrical case. The equilibria used in these linear studies have an off-axis minimum at $r \simeq 0.35$. However, both the current-driven mode at zero β, and the pressure-driven modes at finite β are unstable even for flat $q-$ profiles ($q_1 = 0$), although $|1 - q|$ needs to be lowered to trigger the instability in the flat case.

Since there are unstable modes for all $\beta \geq 0$, modifications in the pressure profile due to, for instance, ohmic heating can not be a direct trigger mechanism for the sawtooth crash, and as pointed out by Wesson[11], a magnetic trigger is needed. In Fig. 2, we plot the growth rate of a pressure driven mode for $\epsilon\beta_p = 0.095$ as a function of small variations in the $q-$profile. In Fig. (2a), $q_0 = 1.02$, and the minimum at $r = 0.35$ is varied ($q_1 > 0$), while in Fig. (2b), the minimum is on axis ($q_1 = 0$), and q_0 is varied. Both profiles are stable at $q_{min} - 1 = 1.4 \times 10^{-2}$ and exhibit a sharp transition to the unstable regime, with a transition width of $\delta q \simeq O(10^{-3})$. Thus, either introducing an off-axis minimum in the $q-$profile, or a uniform reduction in $q(r)$ near the axis, can trigger an instability, leading to the crash. The former is probably more physical, since both our nonlinear calculations, and the periodic sawtooth calculations of Denton et al.[8] indicate that the crash replaces the hot core with colder plasma, with an annular region of hot plasma on the outside. Such an arrangement of plasma pressure would lead to surface current generation in the hot annulus during the rise phase of the sawtooth, introducing an off-axis minimum in the $q-$profile.

For the nonlinear studies with CTD, a q-profile with an off-axis minimum is chosen, with $q_0 = 1.01, q_1 = 0.014, q_l = 2.5, r_{min} = 0.35$, and $\lambda = 4$. The inverse aspect ratio, ϵ, is 1/3, with $\epsilon\beta_p = 0.094$, and $\beta = 0.65\%$. The linear growth rate of the mode for these parameters is $\gamma\tau_{Hp} = 7.40 \times 10^{-3}$.

The nonlinear evolution of the pressure field is depicted in Fig. 3. Both the contours of p, and the radial variation of p along the cord drawn on the contour plot are shown. Figure 3a shows the equilibrium pressure field at $t = 0$. By $t = 400$ (Fig.3b), the hot core is pushed outward appreciably. Note that this is an $n = 1$ helical distortion of the plasma column, and not an expansion in major radius. At $t = 617$ (Fig. 3c), a semi-circular ring of hot plasma has formed, while a cold "bubble" has invaded the center from the left. In cylindrical calculations with comparable values of $\epsilon\beta_p$, this bubble is eventually completely enclosed by the hot ring, forming a symmetric hollow pressure profile[17]. In toroidal calculations, however, this helical distortion seems to persist for experimentally relevant values of $\epsilon\beta_p$, as shown in Fig. 3d. It is not known at this time how long this structure survives, but it could possibly be the source of successor oscillations sometimes observed after the crash in JET[10]. An analysis of the cord-averaged pressure history indicates that this helical distortion of the plasma column would lead to a measured temperature drop of $\Delta T/T \simeq 12\%$. This figure is in agreement with experimental measurements which show a $5 - 10\%$ drop in the electron temperature with each sawtooth crash. In these nonlinear calculations, the crash phase is completed in $\sim 10^3$ poloidal Alfvén times. For large tokamaks such as JET and TFTR, $1\tau_{Hp} \sim 10^{-7}$ seconds, which gives an approximate crash time of $100\mu sec.$ for our simulations. This figure is in agreement with the rapid collapse time for the sawtooth oscillations reported for these machines[5,6].

In summary, we have studied the $m = 1, n = 1$ pressure driven ideal kink mode in low shear systems with $|1 - q_0| \ll 1$. The mode is found to be unstable for experimentally relevant values of $\epsilon\beta_p$. Its growth rate is much larger than that of the resistive tearing mode under comparable conditions. Nonlinearly, it leads to collapse of the central temperature in approximately $100\mu sec.$ Both the crash time, and the nonlinear evolution of the mode seem to be in agreement with those observed in the JET tokamak. Here we have concentrated on the crash phase of the sawtooth. A more self-consistent treatment would have to include the transport-dominated rise phase also. Such a study that couples a transport code with the initial value code used in this study is being contemplated.

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References

1. S. von Goeler, W. Stodiek, and N. Sauthoff, Phys. Rev. Lett. **33**, 1201 (1974).
2. B. B. Kadomtsev, Fiz. Plazmy **1**, 710 (1975) [Sov. J. Plasma Physics **1**, 389 (1975)].
3. J. Snipes, T. P. Kochanski, S. B. Kim, B. Richards, W. L. Rowan, and P. Phillips, Bull. Am. Phys. Soc. **28**, 1034 (1983).
4. W. Pfeiffer, Nucl. Fusion **25**, 673 (1985).
5. D. J. Campbell et al. in Proceedings of the Twelfth European Conference on Controlled Fusion and Plasma Physics, Budapest, Hungary (1985), Vol. 1 , p. 130, K. McGuire et al., ibid. Vol. 1, p. 134.
6. B. V. Waddell, M. N. Rosenbluth, D. A. Monticello, and R. B. White, Nuclear Fusion,**16**, 3 (1976).
7. A. Sykes, and J. A. Wesson, Phys. Rev. Lett. **37**, 140 (1976).
8. R. E. Denton, J. F. Drake, R. G. Kleva, and D. A. Boyd, Phys. Rev. Lett. **56**, 2477 (1986).
9. A. W. Edwards et al., Phys. Rev. Lett. **57**, 210 (1986).
10. D. J. Campbell, et al., Nucl. Fusion **26**, 1085 (1986).
11. J. A. Wesson, Plasma Phys. Controlled Fusion **28**, 243 (1986).
12. V. D. Shafranov, Zh. Tekh. Fiz. **40**, 241 (1970) [Sov. Phys. Tech. Phys. **15**, 175 (1970)].
13. M. N. Bussac, R. Pellat, D. Edery, and J. L. Soule, Phys. Rev. Lett. **35**, 1638 (1975).
14. A. A. Ware, Phys. Rev. Lett. **26**, 1304 (1971).
15. J. A. Holmes, B. A. Carreras, H. R. Hicks, V. E. Lynch, and K. E. Rothe, Phys. Fluids **25**, 800 (1982).
16. A. Y. Aydemir, and D. C. Barnes, J. Comp. Phys. **59**, 108 (1985).

17. A. Y. Aydemir, R. D. Hazeltine, Institute for Fusion Studies Report, IFSR# 250, September 1986.

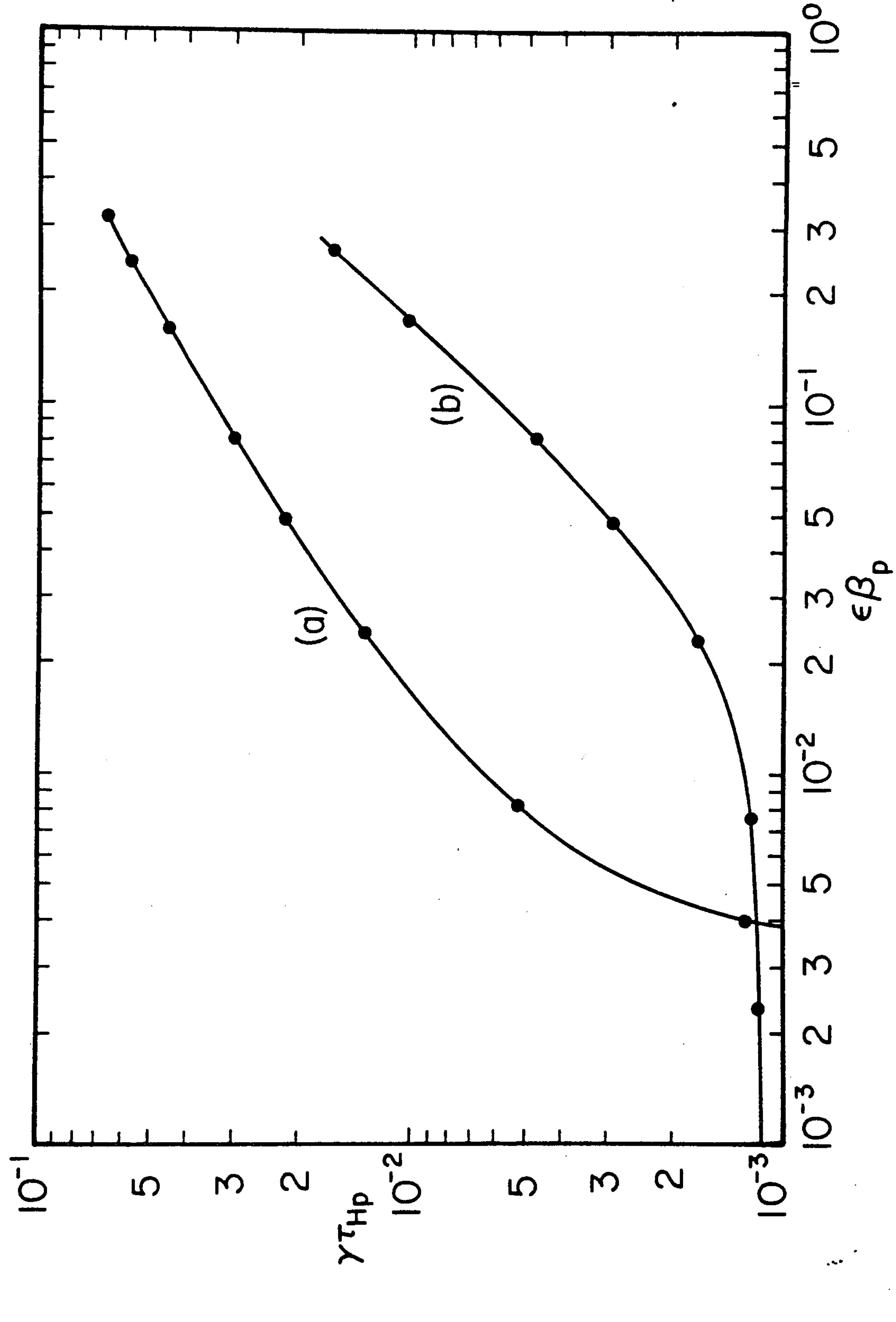
18. M. N. Rosenbluth, R. Y. Dagazian, and P. H. Rutherford, Phys. Fluids **16**, 1894 (1973).

Figure Captions

Figure 1. The growth rate of the mode as a function of $\epsilon\beta_p$. a) cylindrical, b) fully toroidal.

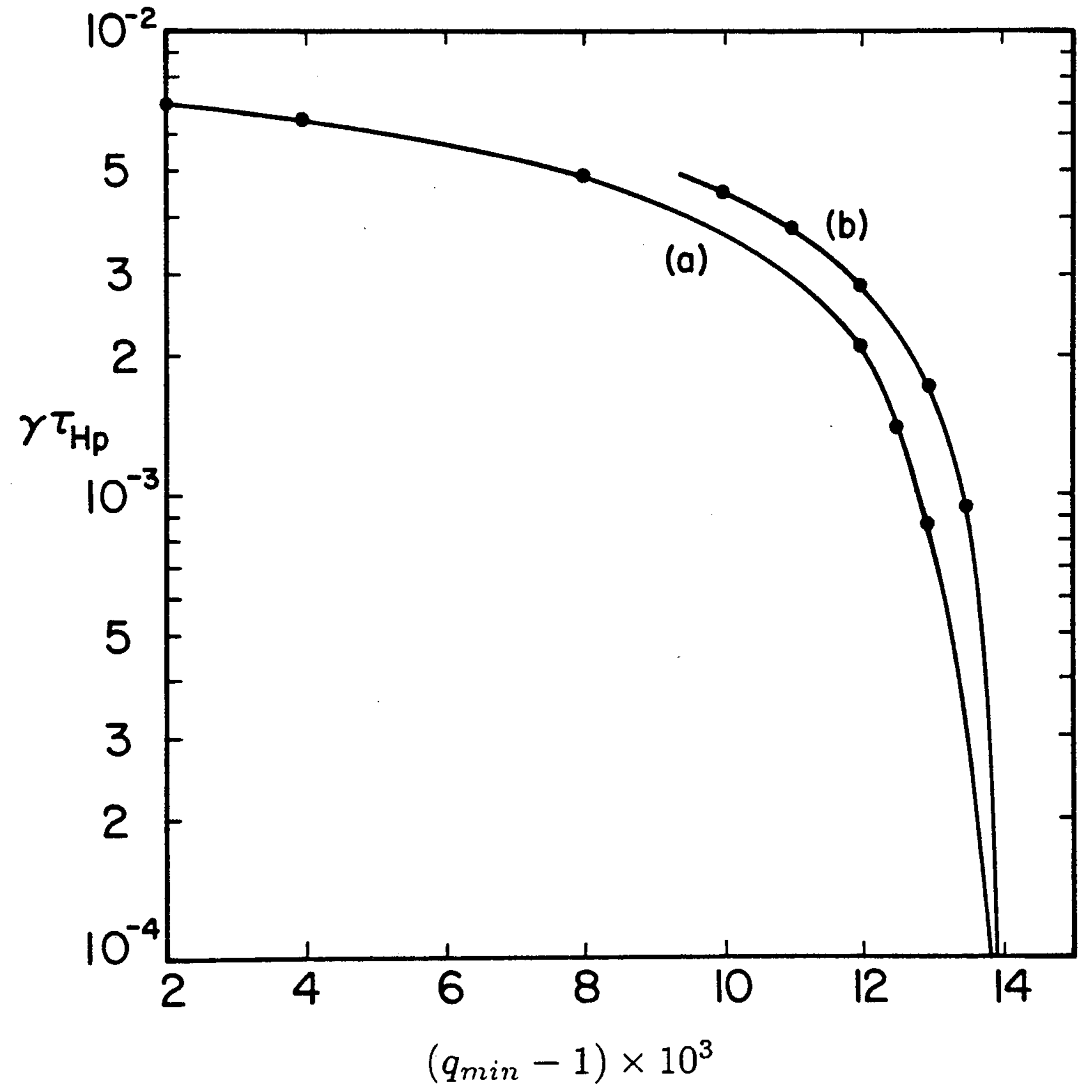
Figure 2. The growth rate of the mode as a function of $(q_{min} - 1)$ for $\epsilon\beta_p = 0.095$. a) $q_0 = 1.02$, and the depth of the off-axis minimum is varied, b) the minimum is on axis, and q_0 is varied.

Figure 3. The nonlinear evolution of the pressure field. a) t=0, b) t=400, c) t=617, d) t=1660. Times are normalized to the poloidal Alfvén time.



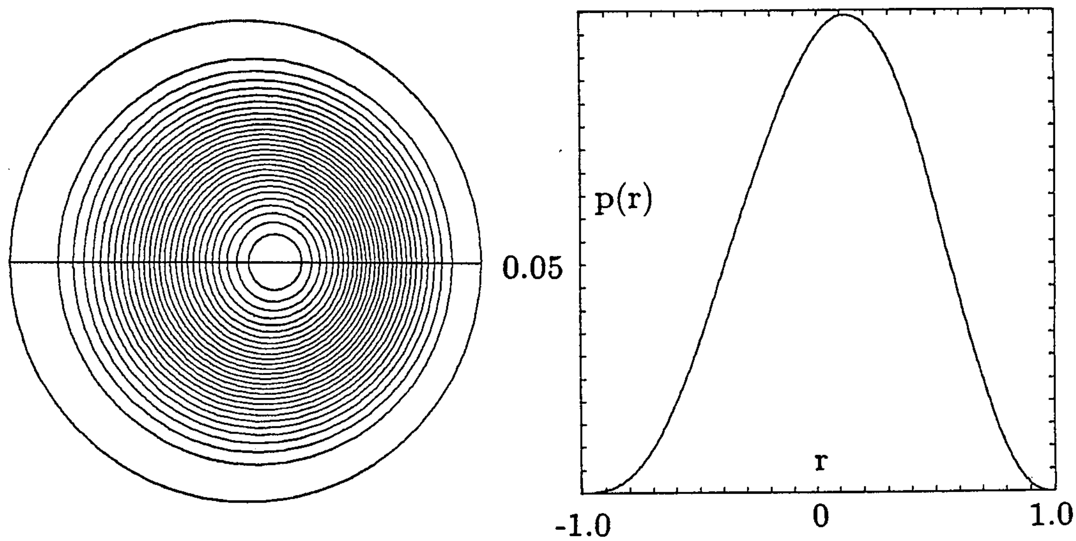
$\gamma\tau_{Hp}$
$\epsilon\beta_p$
(a)
(b)
10^{-1}
5
3
2
10^{-2}
5
3
2
10^{-3}
10^{-3}
2
3
5
10^{-2}
2
3
5
10^{-1}
2
3
5
10^{0}

Fig. 1

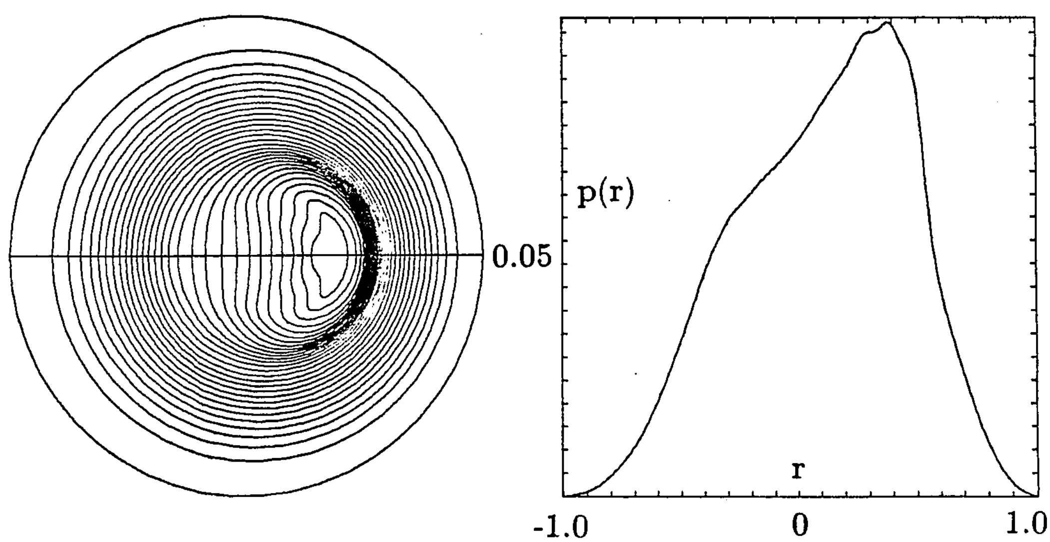


10^{-2}
5
3
2
$\gamma\tau_{Hp}$
10^{-3}
5
3
2
10^{-4}
(a)
(b)
2
4
6
8
10
12
14
$(q_{min}-1)\times 10^3$

Fig. 2

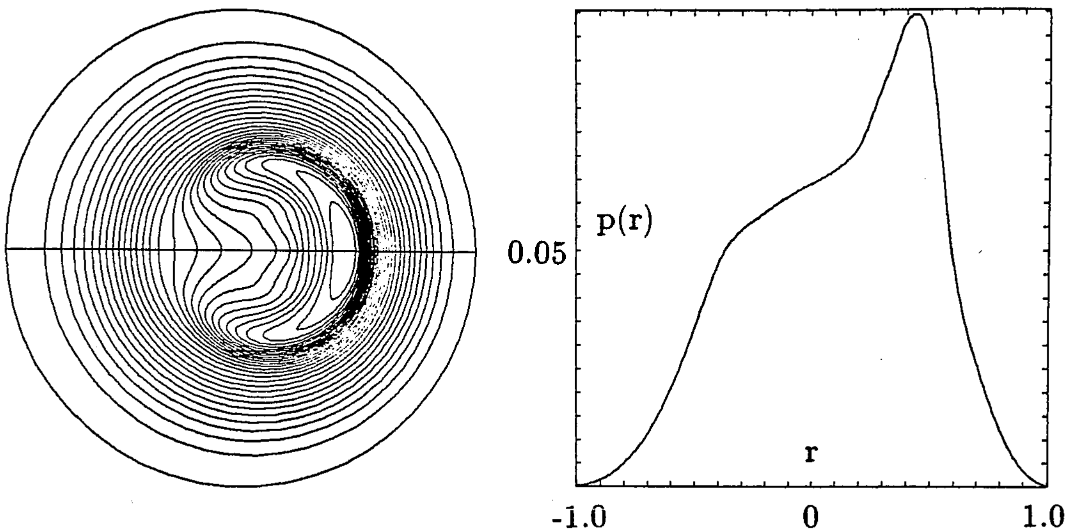


a) t=0

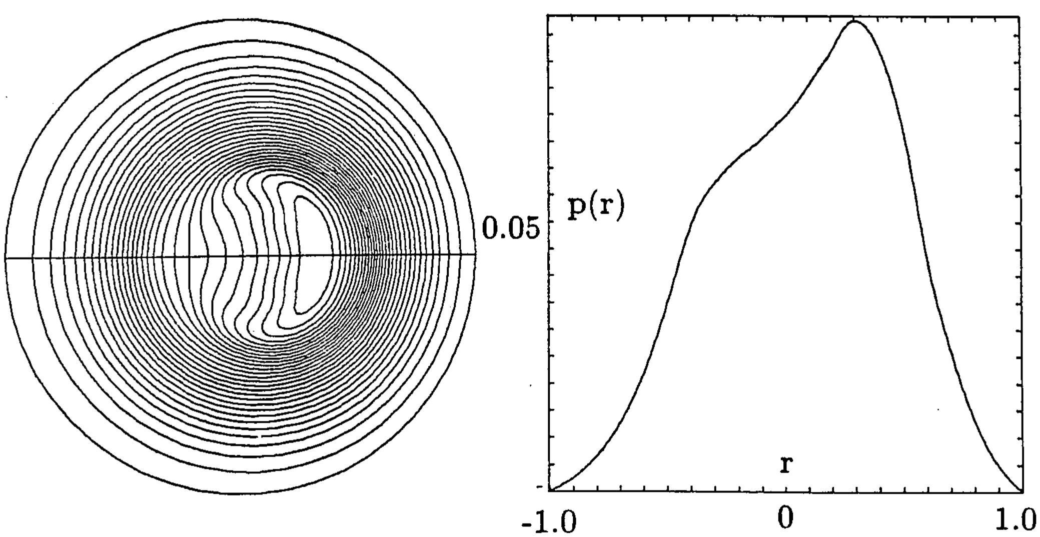


b) t=400

Fig 3



c) t=617



d) t=1660

Fig. 3

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April 1987

A Mechanism for Rapid Sawtooth Crashes in Tokamaks-II

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The remainder of this letter is devoted to linear and nonlinear studies of $n = 1$ internal kink mode. Unlike previous studies[13], we specialize to low shear systems with $q_0 \simeq 1$, and examine the questions of stability boundaries, instability growth rates, the nonlinear evolution of the mode, and its possible role in rapid sawtooth crashes.

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The important features of the linear results are: i) Whereas the accepted toroidal theory of the $n = 1$ modes[12] predicts an instability only for $q_0 < 1$, for the low-shear systems considered here, unstable internal kink modes are found for $q_0 > 1$, both in cylindrical and toroidal geometries. ii) Although, in a cylinder, there is a critical poloidal beta below which the mode becomes stable, in toroidal geometry, we observe an unstable mode even for $\beta = 0$. This mode is further destabilized by finite pressure. iii) For the modes we consider, the stabilizing contributions to δW from the line bending energy, $\delta W_+ \sim \int k_\parallel^2 |d\xi/dr|^2 dr$, is minimized, not by a uniform displacement ξ vanishing at the rational surface, but by keeping $k_\parallel^2$ small in the region of interest. Thus the mode is highly sensitive to the details of the $q-$profile, and an instability can be triggered by slight variations in $q(r)$. iv) These modes do not require the presence of a $q = 1$ surface in the plasma; they are non-resonantly unstable even when $q > 1$ at all radii.

Figure. 1 shows the growth rate of the mode as a function of $\epsilon\beta_p$ for a sequence of flux conserving equilibria. The equilibrium parameters are $\epsilon = 1/4, q_0 = 1.01, q_1 = 0.01, \lambda = 4$, and $\nu = 3$. In cylindrical geometry (Fig. 1a), the mode is robustly unstable for $\epsilon\beta_p \geq 4 \times 10^{-3}$. Fig. (1b) shows the fully toroidal results. Here the mode is unstable for all $\beta \geq 0$. For $\beta \simeq 0$, it is a current-driven internal kink mode that is destabilized by toroidal effects[16] for $q \geq 1$. The growth rate of this mode scales as $\gamma\tau_{Hp} \sim \epsilon^3$, where $\tau_{Hp} = R_0\sqrt{\mu_0\rho}/B_0$ is the poloidal Alfvén time (Fig. 2). Note that this mode has no counterpart in cylindrical geometry, where $q_0 < 1$ is required for a $J_{||}-$driven internal kink mode. Finite-β further destabilizes it, and eventually, for $\epsilon\beta_p \geq 0.01$, the pressure driving terms become dominant. As expected, because of the favorable average curvature in a torus, the growth rates in the pressure-driven regime is lower than in the purely cylindrical case. The equilibria used in these linear studies have an off-axis minimum at $r \simeq 0.35$. However, both the current-driven mode at zero β, and the pressure-driven modes at finite β are unstable even for flat $q-$ profiles ($q_1 = 0$), although $|1 - q|$ needs to be lowered to trigger the instability in the flat case.

Since there are unstable modes for all $\beta \geq 0$, modifications in the pressure profile due to, for instance, ohmic heating can not be a direct trigger mechanism for the sawtooth crash, and as pointed out by Wesson[10], a magnetic trigger is needed. In Fig. 3, we plot the growth rate of a pressure driven mode for $\epsilon\beta_p = 0.095$ as a function of small variations in the $q-$profile. In Fig. (3a), $q_0 = 1.02$, and the minimum at $r = 0.35$ is varied ($q_1 > 0$), while in Fig. (3b), the minimum is on axis ($q_1 = 0$), and q_0 is varied. Both profiles are stable at $q_{min} - 1 = 1.4 \times 10^{-2}$ and exhibit a sharp transition to the unstable regime, with a transition width of $\delta q \simeq O(10^{-3})$. Thus, either introducing an off-axis minimum in the $q-$profile, or a uniform reduction in $q(r)$ near the axis, can trigger an instability, leading to the crash. The former is probably more physical, since both our nonlinear calculations, and the periodic sawtooth calculations of Denton et al.[8] indicate that the crash replaces the hot core with colder plasma, with an annular region of hot plasma on the outside. Such an arrangement of plasma pressure would lead to surface current generation in the hot annulus during the rise phase of the sawtooth, introducing an off-axis minimum in the $q-$profile.

For the nonlinear studies with CTD, a q-profile with an off-axis minimum is chosen, with $q_0 = 1.01, q_1 = 0.014, q_l = 2.5, r_{mn} = 0.35$, and $\lambda = 4$. The inverse aspect ratio, ϵ, is 1/3, with $\epsilon\beta_p = 0.094$, and $\beta = 0.65\%$. The linear growth rate of the mode for these parameters is $\gamma\tau_{Hp} = 7.40 \times 10^{-3}$.

The nonlinear evolution of the pressure field is depicted in Fig. 4. Both the contours of p, and the radial variation of p along the cord drawn on the contour plot are shown. Figure 4a shows the equilibrium pressure field at $t = 0$. By $t = 400$ (Fig.4b), the hot core is pushed outward appreciably. Note that this is an $n = 1$ helical distortion of the plasma column, and not an expansion in major radius. At $t = 617$ (Fig. 4c), a semi-circular ring of hot plasma has formed, while a cold "bubble" has invaded the center from the left. In cylindrical calculations with comparable values of $\epsilon\beta_p$, this bubble is eventually completely enclosed by the hot ring, forming a symmetric hollow pressure profile[15]. In toroidal calculations, however, this helical distortion seems to persist for experimentally relevant values of $\epsilon\beta_p$, as shown in Fig. 4d. It is not known at this time how long this structure survives, but it could possibly be the source of successor oscillations observed after the crash in JET[17]. An analysis of the cord-averaged pressure history indicates that this helical distortion of the plasma column would lead to a measured temperature drop of $\Delta T/T \simeq 12\%$. This figure is in agreement with experimental measurements which show a $5-10\%$ drop in the electron temperature with each sawtooth crash. In these nonlinear calculations, the crash phase is completed in $\sim 10^3$ poloidal Alfvén times. For large tokamaks such as JET and TFTR, $1\tau_{Hp} \sim 10^{-7}$ seconds, which gives an approximate crash time of $100\mu sec.$ for our simulations. This figure is in agreement with the rapid collapse time for the sawtooth oscillations reported for these machines[5,6].

In summary, we have studied the $m = 1, n = 1$ pressure driven ideal kink mode in low shear systems with $|1 - q_0| \ll 1$. The mode is found to be unstable for experimentally relevant values of $\epsilon\beta_p$. Its growth rate is much larger than that of the resistive tearing mode under comparable conditions. Nonlinearly, it leads to collapse of the central temperature in approximately $100\mu sec$. Both the crash time, and the nonlinear evolution of the mode seem to be in agreement with those observed in the JET tokamak. Here we have concentrated on the crash phase of the sawtooth. A more self-consistent treatment would have to include the transport-dominated rise phase also. Such a study that couples a transport code with the initial value code used in this study is being contemplated.

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References

1. S. von Goeler, W. Stodiek, and N. Sauthoff, Phys. Rev. Lett. **33**, 1201 (1974).
2. B. B. Kadomtsev, Fiz. Plazmy **1**, 710 (1975) [Sov. J. Plasma Physics **1**, 389 (1975)].
3. J. Snipes, T. P. Kochanski, S. B. Kim, B. Richards, W. L. Rowan, and P. Phillips, Bull. Am. Phys. Soc. **28**, 1034 (1983).
4. W. Pfeiffer, Nucl. Fusion **25**, 673 (1985).
5. D. J. Campbell et al. in Proceedings of the Twelfth European Conference on Controlled Fusion and Plasma Physics, Budapest, Hungary (1985), Vol. 1 , p. 130, K. McGuire et al., ibid. Vol. 1, p. 134.
6. B. V. Waddell, M. N. Rosenbluth, D. A. Monticello, and R. B. White, Nuclear Fusion,**16**, 3 (1976).
7. A. Sykes, and J. A. Wesson, Phys. Rev. Lett. **37**, 140 (1976).
8. R. E. Denton, J. F. Drake, R. G. Kleva, and D. A. Boyd, Phys. Rev. Lett. **56**, 2477 (1986).
9. A. W. Edwards et al., Phys. Rev. Lett. **57**, 210 (1986).
10. J. A. Wesson, Plasma Phys. Controlled Fusion **28**, 243 (1986).
11. V. D. Shafranov, Zh. Tekh. Fiz. **40**, 241 (1970) [Sov. Phys. Tech. Phys. **15**, 175 (1970)].
12. M. N. Bussac, R. Pellat, D. Edery, and J. L. Soule, Phys. Rev. Lett. **35**, 1638 (1975).
13. J. A. Holmes, B. A. Carreras, H. R. Hicks, V. E. Lynch, and K. E. Rothe, Phys. Fluids **25**, 800 (1982).
14. A. Y. Aydemir, and D. C. Barnes, J. Comp. Phys. **59**, 108 (1985).
15. A. Y. Aydemir, R. D. Hazeltine, Institute for Fusion Studies Report, IFSR# 250, September 1986.
16. A. A. Ware, Phys. Rev. Lett. **26**, 1304 (1971).

17. D. J. Campbell, et al., Nucl. Fusion **26**, 1085 (1986).

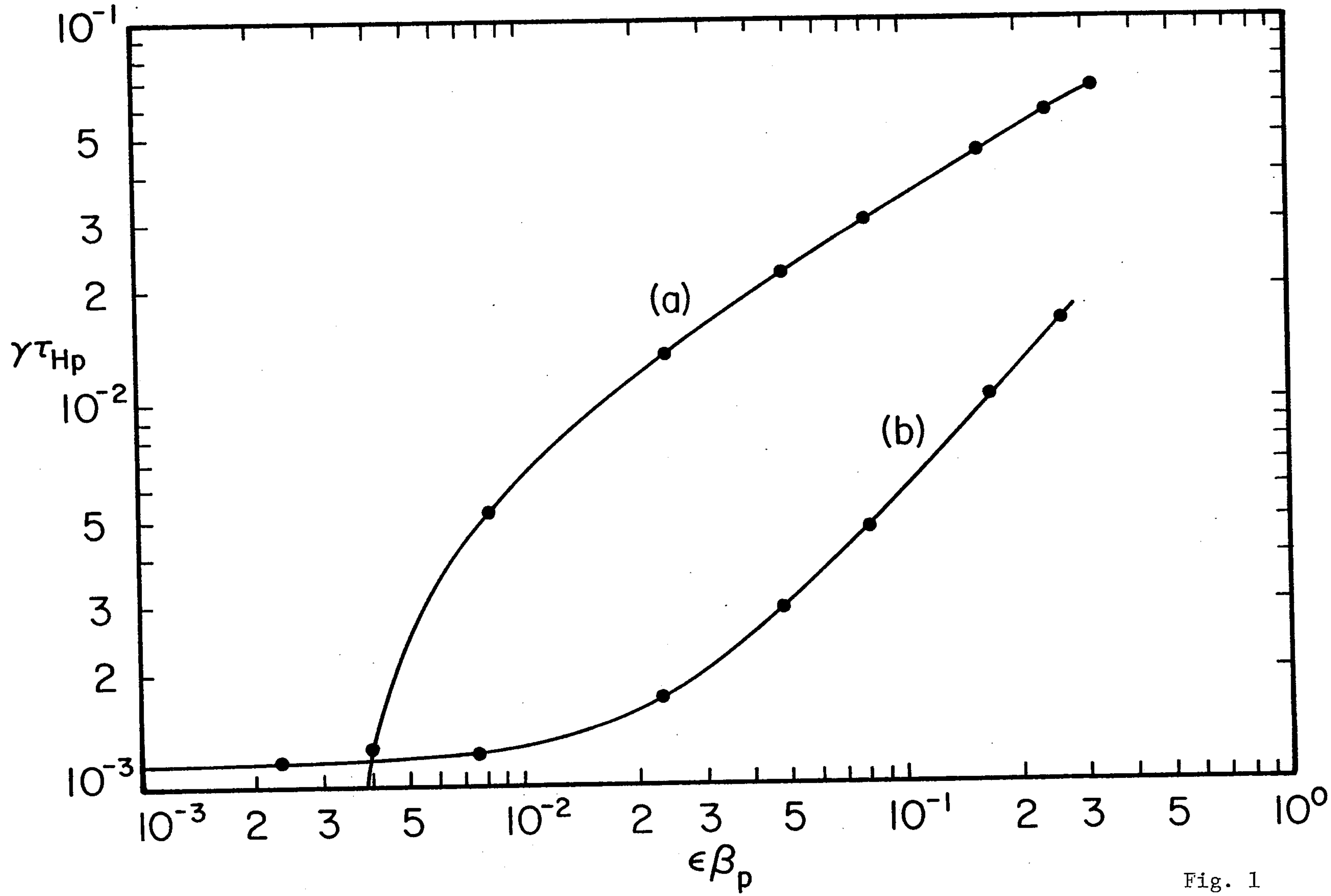
Figure Captions

Figure 1. The growth rate of the mode as a function of $\epsilon\beta_p$. a) cylindrical, b) fully toroidal.

Figure 2. The growth rate of the mode at zero β as a function of the inverse aspect ratio, $\epsilon = a/R_0$. The solid line has a slope of three.

Figure 3. The growth rate of the mode as a function of $(q_{min} - 1)$ for $\epsilon\beta_p = 0.095$. a) $q_0 = 1.02$, and the depth of the off-axis minimum is varied, b) the minimum is on axis, and q_0 is varied.

Figure 4. The nonlinear evolution of the pressure field. a) t=0, b) t=400, c) t=617, d) t=1660. Times are normalized to the poloidal Alfvén time.



10^{-1}
5
3
2
$\gamma\tau_{Hp}$
10^{-2}
5
3
2
10^{-3}
(a)
(b)
10^{-3} 2 3 5 10^{-2} 2 3 5 10^{-1} 2 3 5 10^{0}
$\epsilon\beta_p$

Fig. 1

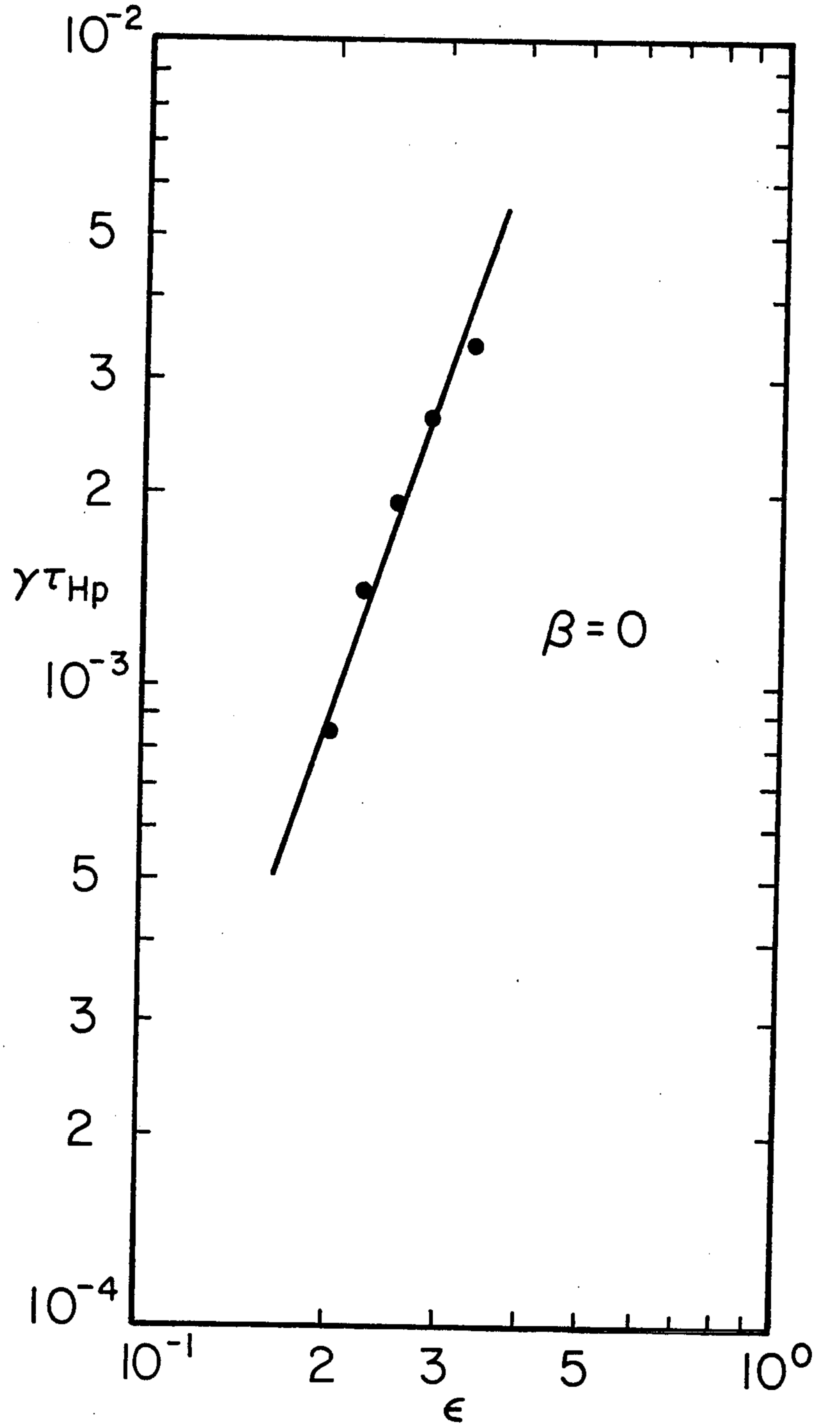


Fig. 2

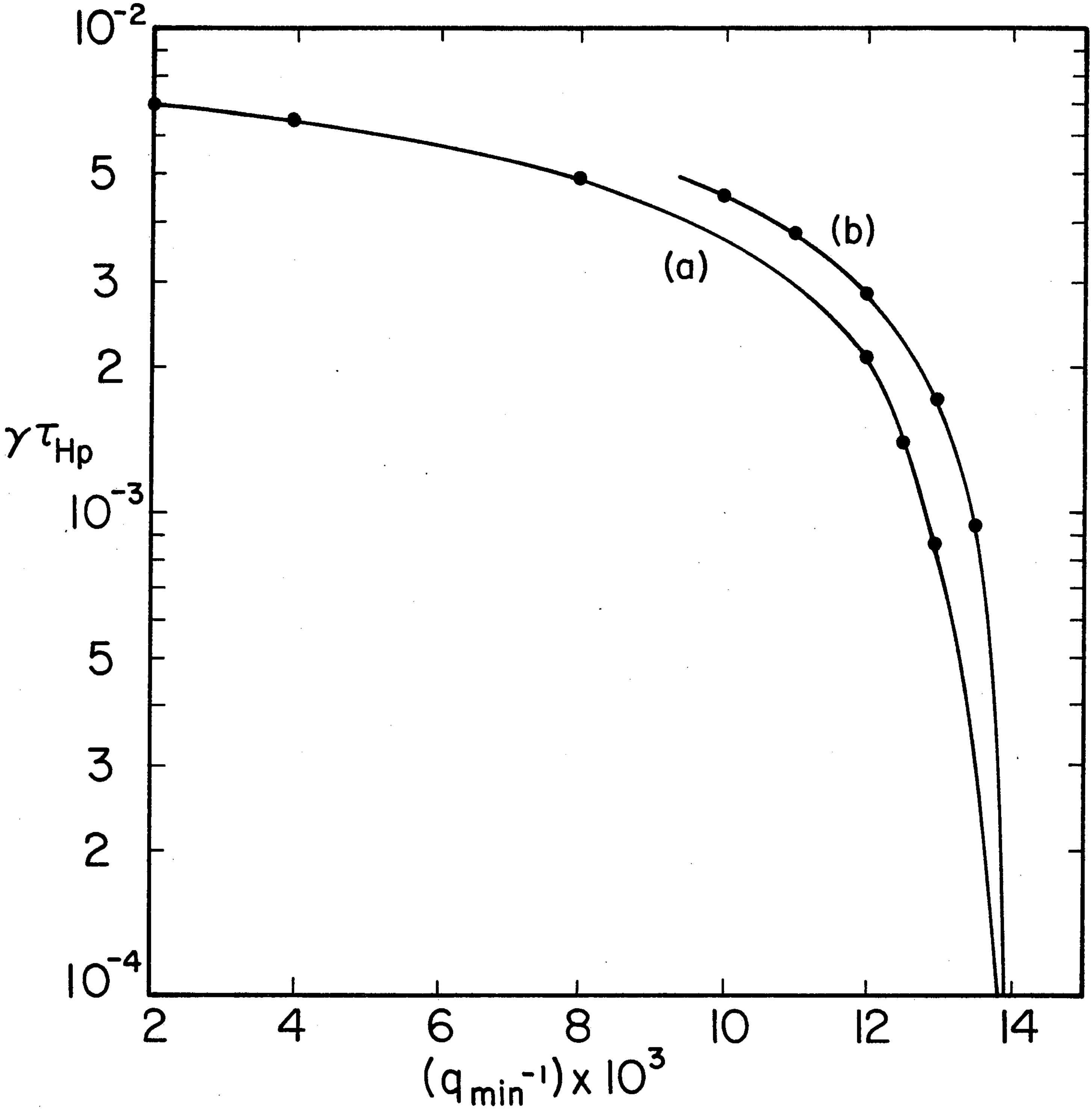
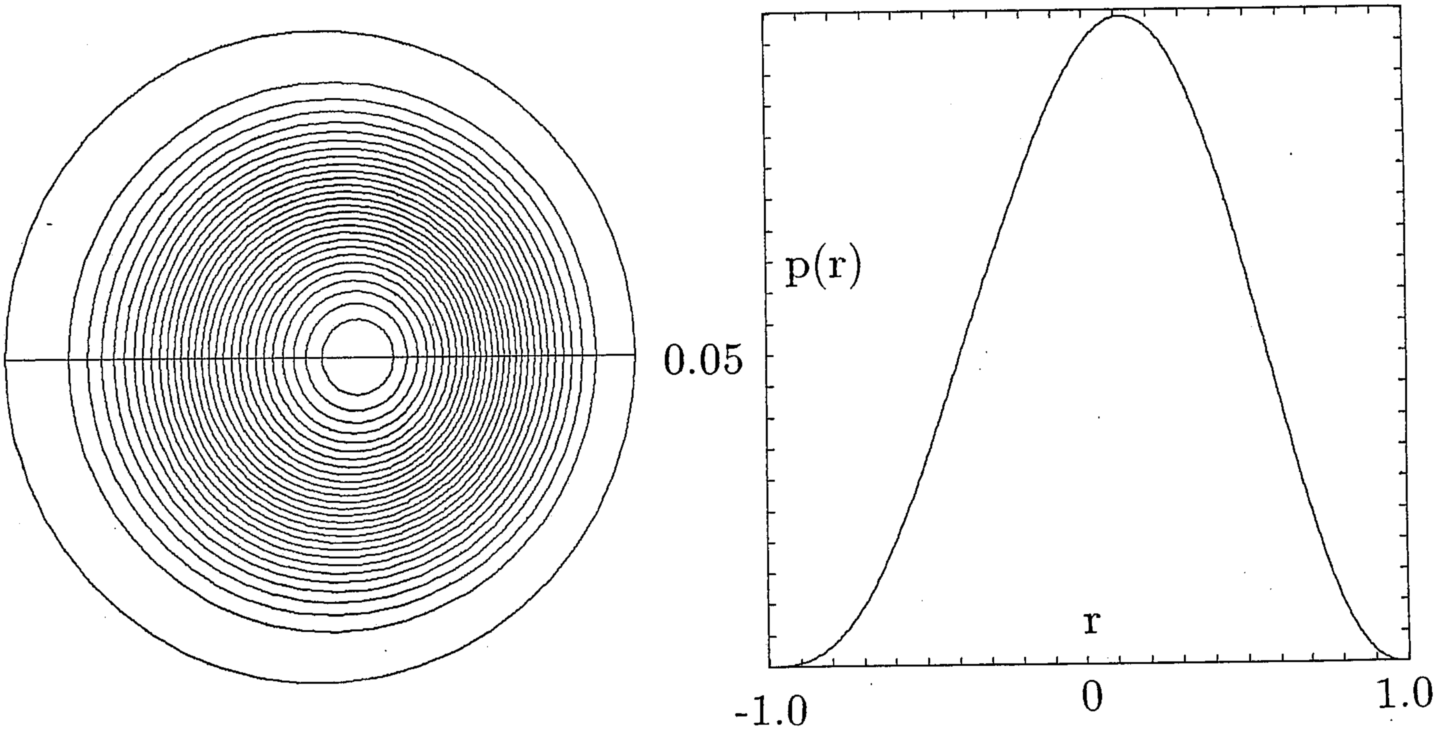
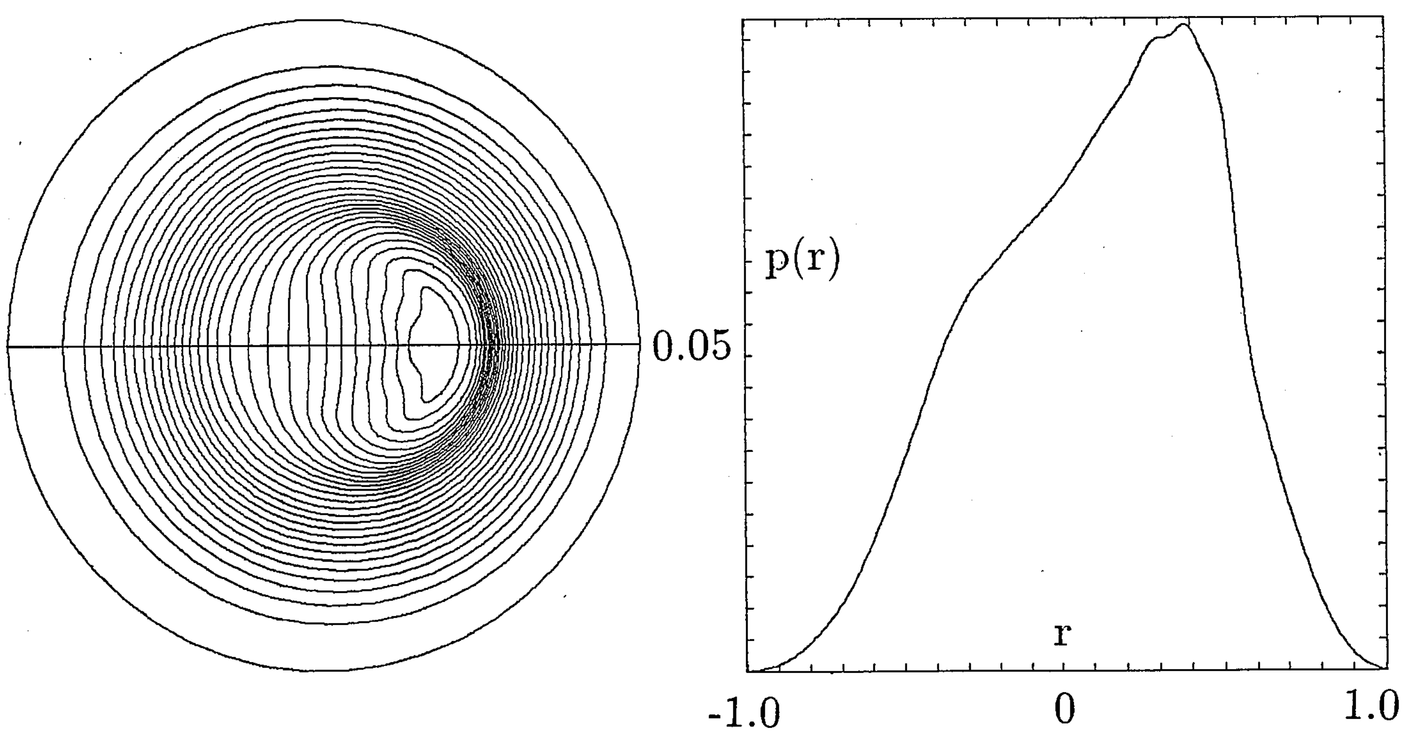

10^{-2}
5
3
2
$\gamma\tau_{Hp}$
10^{-3}
5
3
2
10^{-4}
(a)
(b)
2
4
6
8
10
12
14
$(q_{min}{}^{-1}) \times 10^3$

Fig. 3

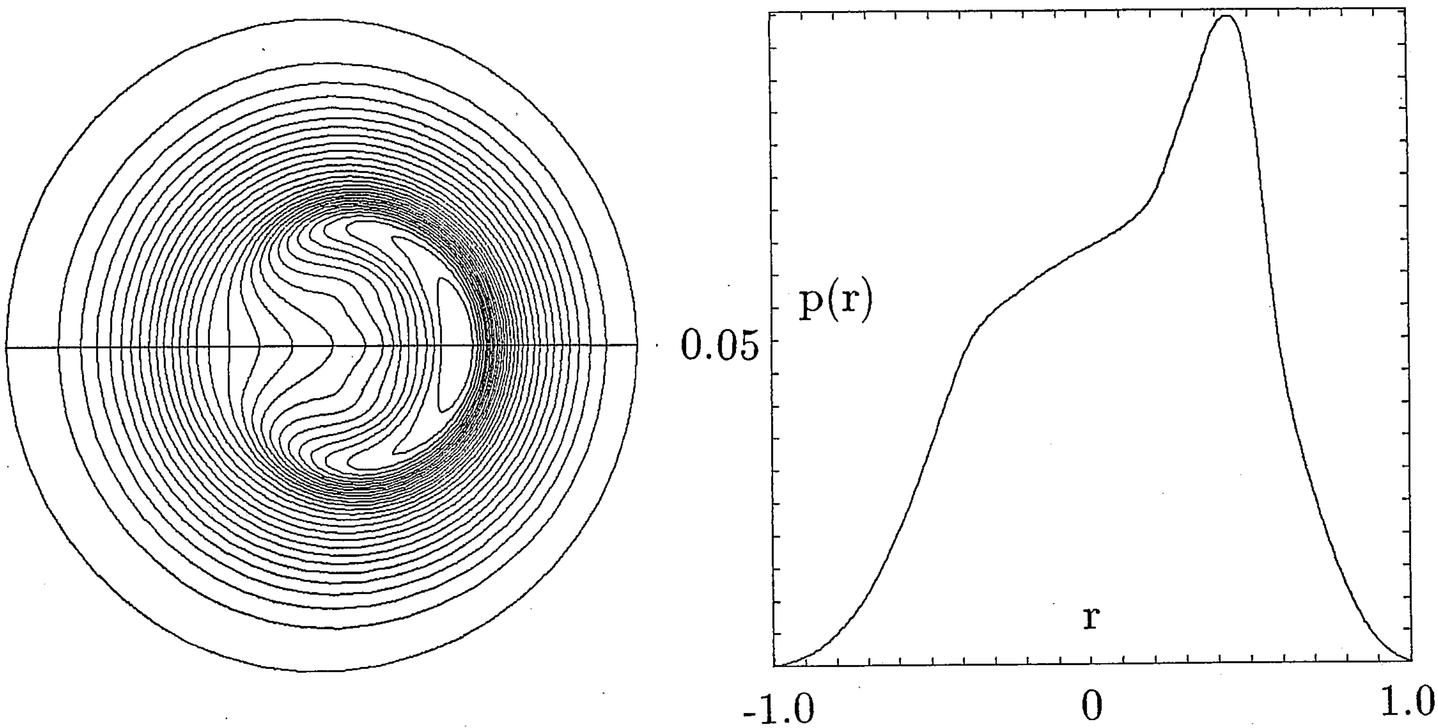


a) t=0

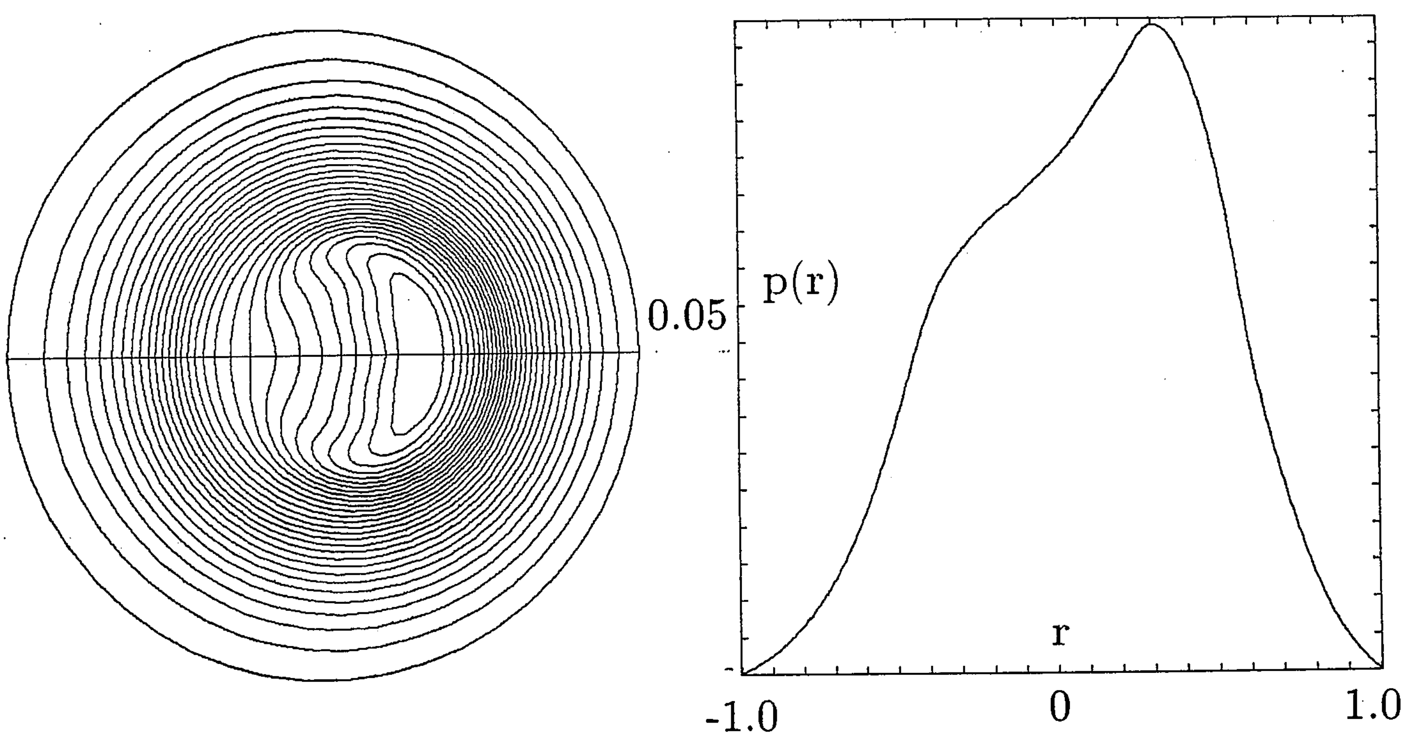


b) t=400

Fig. 4(a)-(b)



c) t=617



d) t=1660

Fig. 4(c)-(d)