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LASER SELF-TRAPPING FOR PLASMA FIBER

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Abstract

A short-pulsed, intense laser is injected into an underdense plasma to sustain a self-trapped photon channel. With either high enough intensity or strong enough focusing the optical beam causes total electron evacuation on the beam axis. Under appropriate conditions this laser and plasma fiber system may provide a slow wave structure of the electromagnetic wave that is suitable for high energy acceleration.

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1 Introduction

The idea of a plasma fiber accelerator[1] driven by an intense laser has been discussed as an outgrowth of the beat wave accelerator.[2] Here we investigate the behavior and conditions for self-trapping of a single short laser pulse.

A short laser pulse is appropriate for the plasma fiber accelerator concept because in a very short time scale the ions can be considered stationary and only the electrons respond to the rapidly oscillating electric field. In a certain limit the photon (ponderomotive) pressure of the laser beam may blow out the electrons around the beam axis and thus form a 'vacuum' channel in plasma ('vacuum' in the sense that all the electrons are absent). The channel of low electron density that is created by the ponderomotive force of the laser pulse acts as optical fiber trapping the laser light.

2 Model for the Plasma Response

We consider a short laser pulse so that ions do not have enough time to respond to the laser field. The self-consistency of this assumption is examined later. Taking ions to be infinitely massive, only the electron density can fluctuate and the ion density remains constant:

$$\begin{aligned} n_e &= n_0 + \delta n_e \\ n_i &\approx n_0. \end{aligned} \tag{1}$$

Then Poisson's equation can be written as

$$\nabla \cdot \mathbf{E} \approx -4\pi e \delta n_e. \tag{2}$$

The electric field is non-uniform and induces the non-linear ponderomotive force, which, in the case of circular polarization, is given by[3]:

$$\mathbf{F}_p = -\boldsymbol{\nabla}\chi, \quad \text{where} \quad \begin{array}{l} \chi = mc^2\sqrt{1+(e/mc\omega)^2 I}, \\ I = |\mathbf{E}|^2. \end{array} \tag{3}$$

The consequent charge separation and the associated radial electrostatic field balance the ponderomotive force so that the net force vanishes:

$$\mathbf{F}_p - e\mathbf{E} = 0. \tag{4}$$

Thus Poisson's equation can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\chi + \frac{\partial^2}{\partial z^2}\chi + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\chi = 4\pi e^2 \delta n_e. \tag{5}$$

In the case of axisymmetric ($\partial/\partial\theta = 0$) and marginal ($\partial^2/\partial z^2 = 0$) variations this gives

$$\delta n_e/n_0 = (c/\omega_p)^2 \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\sqrt{1+I_n}, \tag{6}$$

where $I_n \equiv (eE/mc\omega)^2$ is the intensity normalized such that it equals the square of the quivering velocity divided by the speed of light. The self-consistency of the marginal approximation is discussed later. Under these assumptions the normalized electron density can be written as

$$N_e \equiv n_e/n_0 = 1 + \lambda_c^2 \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\sqrt{1+I_n}, \tag{7}$$

where $\lambda_c \equiv c/\omega_p$ is the collisionless skindepth. It should be noted that N_e should never be negative, since that would correspond to a situation where the fluctuations in the electron density were greater than the equilibrium density.

3 Hamiltonian Dynamics of Laser Light

Our starting point is Maxwell's equations and the equation of motion for relativistic electrons:

$$\begin{aligned}
&\nabla \cdot \mathbf{E} = 4\pi \sum e_j n_j && \nabla \cdot \mathbf{B} = 0 \\
&c\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t && c\nabla \times \mathbf{B} = 4\pi \mathbf{J} + \partial \mathbf{E}/\partial t. \\
&\partial \mathbf{p}/\partial t + \mathbf{v}\cdot\nabla\mathbf{p} = -e(\mathbf{E} + (1/c)\mathbf{v}\times\mathbf{B}) && \\
&\mathbf{p} = m\mathbf{v}(1 - v^2/c^2)^{-1/2}. &&
\end{aligned} \tag{8}$$

where $\mathbf{p}$ and $\mathbf{v}$ are the electron momentum and velocity, respectively. The ions being infinitely massive are at rest. The electron pressure gradient is neglected and the electrons are treated as a cold fluid. The assumption of immobile ions allows us to write

$$\begin{aligned}
\sum e_j n_j &\approx -e\delta n_e \quad \text{and} \\
\mathbf{J} &\approx -e(n_0 + \delta n_e)\mathbf{v}.
\end{aligned} \tag{9}$$

We express the electromagnetic fields in terms of the scalar and vector potentials:

$$\begin{aligned}
\mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\Phi \\
\mathbf{B} &= \nabla \times \mathbf{A},
\end{aligned} \tag{10}$$

accompanied with the Lorentz gauge:

$$c\nabla \cdot \mathbf{A} + \partial \Phi / \partial t = 0. \tag{11}$$

Combining these equations we get the wave equation for electromagnetic waves:

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = \frac{1}{\lambda_c^2}\frac{N_e}{\sqrt{1+I_n}}\mathbf{A}, \tag{12}$$

where N_e is given by Eq. (7). The right hand side represents all the relevant nonlinearities i.e., the ponderomotive force and electron relativistic mass effects. From here on we will use normalized vector potential: $\mathbf{A} \longrightarrow \mathbf{A}_n \equiv e\mathbf{A}/mc^2$.

To single out the rapid laser variations, we take a trial function of the form

$$\mathbf{A}_n = \mathbf{A}_{n0} \exp(ik_0 z - i\omega_0 t), \tag{13}$$

where we have chosen the coordinate system such that z-axis coincides with the direction of propagation and k_0 and ω_0 are the wavenumber and frequency of the laser wave in vacuum. Substituting this trial function into the wave equation we obtain

$$\left(\nabla^2 \mathbf{A}_{n0} + 2ik_0 \frac{\partial \mathbf{A}_{n0}}{\partial z}\right) = \frac{1}{\lambda_c^2}\frac{N_e}{\sqrt{1+\mathbf{A}_{n0}^2}}\mathbf{A}_{n0}. \tag{14}$$

We assume that the characteristic spatial length of the structure in our system is much greater than the wavelength of the wave, and that the characteristic time period involved is much longer than the laser oscillation period. This allows us to apply the approximations of geometrical optics. The vector potential can then be given by the eikonal expression:

$$\mathbf{A}_{n0} = \boldsymbol{\nu}(\mathbf{r}, t) e^{i\psi(\mathbf{r},t)}, \tag{15}$$

where the amplitude $\boldsymbol{\nu}$ and the phase ψ are real variables and it is understood that physical quantities are given by the real part. The local wavenumber and frequency are defined by

$$\begin{aligned} \mathbf{k} &\equiv \nabla\psi \ll k_0 \\ \omega &\equiv -\frac{\partial \psi}{\partial t} \ll \omega_0. \end{aligned} \tag{16}$$

This treatment is reasonable because we assume that no dissipation of laser energy takes place. Later this also enables us to resort to a Hamiltonian formalism. Substituting the eikonal expression into the wave equation, the real and imaginary parts of the wave equation yield, respectively,

$$2k_0\frac{\partial\psi}{\partial z}+\left(\frac{\partial\psi}{\partial r}\right)^2-\frac{1}{\nu r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\nu+\frac{1}{\lambda_c^2}\frac{N_e}{\sqrt{1+\nu^2}}=\frac{1}{\nu}\frac{\partial^2\nu}{\partial z^2}-\left(\frac{\partial\psi}{\partial z}\right)^2, \tag{17}$$

$$k_0\frac{\partial\nu^2}{\partial z}+\frac{\partial\nu^2}{\partial r}\frac{\partial\psi}{\partial r}+\frac{\nu^2}{r}\frac{\partial}{\partial r}r\frac{\partial\psi}{\partial r}=-\frac{\partial}{\partial z}\left(\nu^2\frac{\partial\psi}{\partial z}\right). \tag{18}$$

We proceed by following the approach taken by Felber:[4] Applying the slowly varying envelope approximation we neglect the right-hand sides (i.e., all higher order derivatives with respect to coordinate z) of Eqs. (17) and (18). Then Eq. (18) gives conservation of beam power, and Eq. (17) has the form of Hamilton-Jacobi equation. This suggests that we identify

$$\begin{aligned}&\psi\equiv\text{action}\\ &\frac{1}{2k_0}\left\{\left(\frac{\partial\psi}{\partial r}\right)^2-\frac{1}{\nu r}\frac{\partial}{\partial r}r\frac{\partial\nu}{\partial r}+\frac{N_e}{\lambda_c^2}\frac{1}{\sqrt{1+\nu^2}}\right\}\equiv H\\ &\equiv\text{Hamiltonian.}\end{aligned}$$

The canonical momentum is then given by

$$p_r=\frac{\partial\psi}{\partial r}, \tag{19}$$

and the Hamiltonian can be written as

$$H=\frac{1}{2k_0}\left\{p_r^2-\frac{1}{\nu r}\frac{\partial}{\partial r}r\frac{\partial\nu}{\partial r}+\frac{N_e}{\lambda_c^2}\frac{1}{\sqrt{1+\nu^2}}\right\}. \tag{20}$$

Applying Hamilton's equations, with z corresponding to 'time',

$$\begin{aligned}\frac{dr}{dz}&=\frac{\partial H}{\partial p_r}\\ \frac{dp_r}{dz}&=-\frac{\partial H}{\partial r},\end{aligned} \tag{21}$$

we get

$$\frac{d^2r}{dz^2}=\frac{1}{k_0}\frac{\partial p_r}{\partial z}=-\frac{1}{k_0}\frac{\partial H}{\partial r}=\frac{1}{k_0^2}\frac{\partial}{\partial r}\left\{\frac{N_e}{2\lambda_c^2}\frac{1}{\sqrt{1+\nu^2}}-\frac{1}{2r\nu}\frac{\partial}{\partial r}r\frac{\partial\nu}{\partial r}\right\}. \tag{22}$$

In order to discuss the global properties of the optical beam, we assume hereon the form of the solution near the beam axis to be Gaussian:

$$\nu=\nu_0e^{-r^2/2a^2}, \tag{23}$$

where ν_0 and a are slowly varying functions of z. This assumption allows us to carry out the calculations analytically, but at the expense of reduced accuracy: the solutions obtained using paraxial approximation are somewhat different from the solutions of the full slowly varying envelope approximation wave equation (see discussion in Ref. 4). This discrepancy arises because the paraxial approximation forces the beam into a Gaussian shape at all times.

Substituting the paraxial trial function into the equation and evaluating it at $r = \epsilon a$ with $\epsilon \ll 1$, we get (to the first order in ϵ):

$$\frac{d^2a}{dz^2} = -\frac{1}{2k_0^2}\left\{\frac{4}{a^3}\frac{\nu^2}{1+\nu^2}\left(1+\frac{1}{1+\nu^2}\right) - \frac{2}{a^3}\frac{\nu^4}{(1+\nu^2)^2} + \frac{1}{\lambda_c^2 a}\frac{\nu^2}{(1+\nu^2)^{3/2}} - \frac{2}{a^3}\right\}, \tag{24}$$

which is formally the equation of motion for the beam radius. The terms on the right hand side of the equation represent the forces acting on the beam radius. The last term is due to the photon pressure of the beam and it describes the Rayleigh spreading of laser beam in vacuum. The other terms represent plasma lens effects.

4 Self-focusing of Laser Light in Plasma

In the limit as the intensity goes to zero, the natural defocusing of the beam overcomes the plasma focusing and we recover pure Rayleigh spreading. In this limit the equation of motion (24) becomes

$$\frac{d^2a}{dz^2} = \frac{1}{k_0^2 a^2} \Longrightarrow a^2 = a_0^2\left\{1 + (z/R_L)^2\right\} \tag{25}$$

where $R_L \equiv k_0 a_0^2 \approx \pi a_0^2/\lambda$ is the Rayleigh length. As the intensity increases, the effect of plasma focusing becomes more important until it equals in magnitude the effect of the Rayleigh spreading term. This situation corresponds to zero force in the equation of motion, and thus, to a constant beam radius.

Self-focusing occurs when the "total force" acting on the beam is negative in Eq. (24). From Eq. (24) this happens when

$$\left(\frac{eE}{m\omega c}\right)^2\sqrt{1+\left(\frac{eE}{m\omega c}\right)^2} \geq 2\left(\frac{\lambda_c}{a}\right)^2\left\{1 - 2\left(\frac{eE}{m\omega c}\right)^2\right\}. \tag{26}$$

This equation defines the critical intensity as a function of plasma and laser parameters, $I_{\text{cr}}(\lambda_c/a)$, above which the self-focusing dominates defocusing. That is, if a laser has initially intensity higher than the critical intensity, it will start to focus as it enters the plasma.

Next we calculate what is the effective potential responsible for the defocusing/self-focusing of the beam. We let $P_n \equiv I_n a^2$ be the power at the entrance to the plasma, and we assume it to be practically constant: $P_n(z) \equiv P_n(0)$. Integrating the force term in Eq. (24), the equation of motion can be written in the Sagdeev form

$$\frac{d^2 a}{dz^2} = -\frac{\partial V}{\partial a}, \tag{27}$$

where

$$V(a) = \frac{1}{k_0^2}\left\{\frac{3}{2}\frac{1}{P_n + a^2} - \frac{1}{P_n}\ell n\left(1 + \frac{P_n}{a^2}\right) + \frac{1}{2\lambda_c^2}\frac{1}{\sqrt{1 + \frac{P_n}{a^2}}} + \text{constant}\right\}.$$

It is important to notice that this potential can not be applied as such for all values of beam radius – the potential diverges as the beam radius goes to zero, $V(a \to 0) = -\infty$. This feature can be traced back to the model for electron density:

$$N_e(r \to 0) = 1 - 2\frac{\lambda_c^2}{a^2}\frac{I_n}{\sqrt{1 + I_n}} + O(\epsilon^2). \tag{28}$$

It is seen that as the intensity of the beam is increased, the electron density eventually vanishes. We define the critical intensity I_v and corresponding beam radius a_v at which the depletion of electrons near the axis happens:

$$N_e = 0 \Longrightarrow \left(\frac{a_v^2}{\lambda_c^2}\right)^2 \sqrt{1 + I_v} = 2I_v, \quad I_v = P_n/a_v^2. \tag{29}$$

The beam radius corresponding to vacuum formation is plotted as a function of power in Figure 1. For $a < a_v$ we prevent the electron density from becoming negative by writing the equation of motion (24) in two parts:

$$\begin{array}{ll} \frac{d^2 a}{dz^2} = -\frac{1}{2k_0^2}\left\{\frac{4}{a^3}\frac{I_n}{1+I_n}\left(1 + \frac{1}{1+I_n}\right) - \frac{2}{a^3}\frac{I_n^2}{(1+I_n)^2} + \frac{1}{\lambda_c^2 a}\frac{I_n}{(1+I_n)^{3/2}} - \frac{2}{a^3}\right\} & \text{for} \;\; a > a_v, \\ \frac{d^2 a}{dz^2} = \frac{1}{k_0^2 a^3} & \text{for} \;\; a > a_v, \end{array} \tag{30}$$

where the plasma effect is absent once the electron density becomes zero. The second equation in Eq. (30) contains only the vacuum Rayleigh diffraction. Integrating the right-hand side of Eq. (30) with respect to a, the potential becomes

$$\begin{aligned} V(a) &= \frac{1}{k_0^2}\left\{\frac{3}{2}\frac{1}{P_n+a^2} - \frac{1}{P_n}\ell n\left(1+\frac{P_n}{a^2}\right) + \frac{1}{2\lambda_c^2}\frac{1}{\sqrt{1+\frac{P_n}{a^2}}} - \frac{1}{2\lambda_c^2}\right\} && \text{for } \ a > a_v, \\ V(a) &= \frac{1}{k_0^2}\left(\frac{1}{2a^2}+C\right) && \text{for } \ a < a_v, \end{aligned} \tag{31}$$

where $C = \dfrac{1}{2a_v^2}\dfrac{2-I_v}{1+I_v} - \dfrac{1}{P_n}\ell n(1+I_v) + \dfrac{1}{4\lambda_c^2}\left(\dfrac{1}{\sqrt{1+I_v}} - 1\right)$ and the constant of integration has been chosen such that the potential vanishes at infinity.

To see the qualitative nature of the potential, we look for its extrema by letting the first derivative equal to zero. This leads to equation for the extremum radius, a_e:

$$\left(P_n/\lambda_c^2\right)\sqrt{1+P_n/a_e^2} = 2\left(1 - P_n/a_e^2\right), \tag{32}$$

from which it is seen that an extremum exists only if the normalized power satisfies the condition

$$P_n \le P_{\rm cr} \equiv 2\lambda_c^2. \tag{33}$$

The extremum corresponds to a maximum and the potential behaves qualitatively as shown in Fig. 2. (It is important to note that the cusp in the potential at $a = a_v$ is an artifact arising from our insistence of a Gaussian profile even at the time when $N_r \to 0$). Thus the condition for self-focusing is

$$P_n \ge 2\lambda_c^2, \tag{34}$$

or, in terms of the quivering momentum of the electrons,

$$\left(p_{\rm os}/mc\right)^2 \ge 2\left(\lambda_c/a\right)^2. \tag{35}$$

Defining the beam power by

$$P = c\int\left\{E(r)^2/8\pi\right\}2\pi r\,dr = \frac{c}{8}a^2E_m^2, \tag{36}$$

where E_m is the field amplitude on the axis: $E_m \equiv E(r=0)$, the critical power is given by

$$P_{\rm cr} = \frac{c}{4}\left(\frac{mc^2}{e}\right)^2\left(\frac{\omega}{\omega_p}\right)^2 \approx 10^{10}(\omega/\omega_p)^2 W. \tag{37}$$

Therefore, provided that the initial beam power (or intensity - notice that for $P > P_{cr}$ also the condition (26) for intensity is satisfied) is high enough and that the initial divergence, da/dz, is small enough, the beam will self-trap. For other cases, i.e., when the beam power is too low and/or the divergence is too large, the beam will eventually defocus. The different cases are illustrated in Fig. 2. It is interesting to note that if either the power is high enough or the initial focusing is strong enough, electron evacuation on the axis takes place.

In the case of self-trapping the beam radius is expected to oscillate around an equilibrium radius: the beam focuses until the intensity is high enough to create a 'vacuum' channel. Then it starts Rayleigh spreading in the channel until the high density region around the channel reflects it back again.

In what follows we study the behavior of the system in the situation where the potential has an attractive part (i.e., $P > P_{crit}$) and the initial divergence of the beam is small enough to lead to self-focusing. The beam radius is expected to oscillate around an equilibrium value corresponding to the minimum of the potential. The oscillations are not, however, symmetric since the potential is not symmetric across the minimum. This asymmetry arises because the physical mechanism responsible for the self-focusing is not the same as the one that causes defocusing: self-focusing is due to the nonlinear interaction between the laser wave and the plasma, whereas defocusing is due to the absence of plasma. Thus the forces are necessarily different.

The equation of motion (27) in the neighborhood of the minimum of the potential, $a_0 \equiv a_v$, can be rewritten as

$$d^2a/dz^2 \approx -\left\{ \left.\frac{\partial V}{\partial a}\right|_{a_0} + \left.\frac{\partial^2 V}{\partial a^2}\right|_{a_0} (a - a_0) \right\} \Longrightarrow d^2\xi/dz^2 + \kappa^2 \xi = -\beta, \qquad (38)$$

where

$$\begin{aligned} \xi &\equiv a - a_0, \\ \beta &\equiv \left.\frac{\partial V}{\partial a}\right|_{a_0} \quad \text{and} \\ \kappa^2 &\equiv \left.\frac{\partial^2 V}{\partial a^2}\right|_{a_0}. \end{aligned}$$

The equation (38) has to be applied separately in the defocusing and self- focusing regions (see Fig. 3), because the quantities κ and β have different values in these regions. We shall denote the quantities related to self-focusing and defocusing phases by subindeces '1' and '2', respectively.

Consider now a laser beam with initial radius a_i and initial divergence a_i'. Assuming that the intensity of the beam is high enough and a_i' is weak enough, the beam starts self-focusing as it enters the plasma and the solution for this phase is given by

$$\xi_1(z) = \frac{\Delta_0'}{\kappa_1}\sin(\kappa_1 z) + \left(\Delta_0 + \frac{\beta_1}{\kappa_1^2}\right)\cos(\kappa_1 z) - \frac{\beta_1}{\kappa_1^2}, \tag{39}$$

where $\Delta_0 \equiv a_i - a_0$, $\Delta_0' \equiv a_i'$, $\kappa_1^2 \equiv \dfrac{3}{k_0^2 a_0^4}$ and $\beta_1 \equiv \dfrac{1}{k_0^2 a_0^3}$. After half of the period, at $z = z_0 = \pi/\kappa_1$, self-focusing gives way to defocusing and the beam starts to obey the second solution

$$\xi_2(z) = A\sin(\kappa_2 z) + B\cos(\kappa_2 z) - \beta_2/\kappa_2^2, \tag{40}$$

where

$$\kappa_2^2 \equiv \frac{1}{k_0^2 a_v^4}\frac{3 - 5I_v^2 - 11I_v}{(1+I_v)^3} \quad \text{and}$$

$$\beta_2 \equiv \frac{1}{k_0^2}\left\{\frac{1}{a_v^3}\frac{2I_v - 1}{(1+I_v)^2} + \frac{1}{2\lambda_c^2 a}\frac{I_v}{(1+I_v)^{3/2}}\right\}.$$

Matching the two solutions at $z = z_0$ the amplitudes A and B are obtained and the solution can be written in the form

$$\xi_2(z) = \left\{\frac{\beta_2}{\kappa_2^2} - 2\frac{\beta_1}{\kappa_1^2} - \Delta_0\right\}\cos\left(\frac{\kappa_2}{\kappa_1}\pi - z\right) + \frac{\Delta_0'}{\kappa_2}\sin\left(\frac{\kappa_2}{\kappa_1}\pi - z\right) - \frac{\beta_2}{\kappa_2^2}. \tag{41}$$

The behavior of the beam radius in the case of self-trapping is sketched in Figure 4.

The critical power expression is comparable to earlier work by Schmidt & Horton:[4] Schmidt & Horton predict self-focusing for systems satisfying

$$P_n > \lambda_c^2. \tag{42}$$

On the other hand, Felber[5] invoked quasineutrality to obtain the condition of self-focusing

$$P_n > 4\lambda_D^2, \tag{43}$$

where λ_D is the Debye length. Schmidt and Horton studied self-trapping considering only relativistic electron mass effects and neglecting the ponderomotive potential effects. Felber studied self-trapping assuming quasineutrality: the plasma response was given by

$$N_e \approx N_i = \exp\left\{-\left(\frac{1}{2\tau}\right)\left[\sqrt{1+I_n} - 1\right]\right\}, \tag{44}$$

where $\tau \equiv T/mc^2$. The strong temperature dependence of the plasma response is manifested in that the scale length in Felber's case is the Debye length, λ_D, coming in the exponential function. This is in contrast to the results obtained by Schmidt & Horton and us, where λ_c is the scale length. This strong temperature dependence may cause thermally unstable plasma profile in the quasineutral regime.

Another interesting comparison can be made with the filamentation mode. Felber and Chernin[6] have found that laser light in a plasma goes unstable with respect to filamentation when

$$\mathcal{C} \equiv 0.5\epsilon_1 k_0^2 a^2 > 0.5, \tag{45}$$

where $k_0 =$ unperturbed wavevector of the laser field, $\epsilon_1 = \left[\frac{I_n}{\epsilon_0}\frac{d\epsilon}{dI_n}\right]$ evaluated near the beam axis where the electric field can be approximated by a plane wave, $\epsilon =$ scalar dielectric constant of the plasma, and $\epsilon_0 =$ dielectric constant evaluated near the beam axis. In the short pulse regime (immobile ions) the plasma response may be described by the dielectric constant

$$\epsilon = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \frac{N_e}{\sqrt{1+I_n}}, \tag{46}$$

where N_e is defined by Eq. (7). This gives

$$\mathcal{C} = \frac{1}{2}\left(\frac{a}{\lambda_c}\right)^2 \left\{\frac{1}{2}\frac{I_n}{(1+I_n)^{3/2}} + 2\left(\frac{\lambda_c}{a}\right)^2 \frac{I_n}{(1+I_n)^2}\right\}. \tag{47}$$

The condition (45) for a plasma to go unstable against filamentation is now equivalent to:

$$I_n\sqrt{1+I_n} > 2\left(\frac{\lambda_c}{a}\right)^2 \left(1+I_n^2\right), \tag{48}$$

which resembles the condition for the self-focusing given by the inequality (26). Therefore the self-trapping of the laser light is probably closely related to the filamentation instability.

For very high powers it is not correct to take the ions as being infinitely massive because in reality they start following the electrons. For the present model to be self-consistent the characteristic time t_i for ions has to satisfy $t_i \gg t_{\text{pulse}}$, where t_{pulse} is the laser pulse length.

The equation of motion for ions gives the acceleration:

$$\dot{v}_i \sim eE_r/M, \tag{49}$$

where the radial electric field is given by Poisson's equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_r) = -4\pi e\delta n_e. \tag{50}$$

Evaluating the expression (6) for the fluctuations in the electron density near the axis we get

$$E_r = -4\pi e n_0 \left(\frac{\lambda_c}{a}\right)^2 r \frac{I_n}{\sqrt{1+I_n}}. \tag{51}$$

(The integration constant was set to zero in order to make the electric field vanish at infinity). The electric field is seen to vanish on the beam axis, and so, since we are looking for the maximum value of the electric field, we have to evaluate expression (51) away from the axis. Because of the exponential radial dependence of the intensity, we can approximate $\sqrt{1+I_n} \approx 1$ and obtain

$$E_r \approx \left(\frac{mc^2}{ea^2}\right) r I_0 \exp(-r^2/a^2), \tag{52}$$

where we have used the definition of the collisionless skindepth. The maximum electric field is found at $r^2 = 0.5a^2$:

$$E_{r,\max} \approx 0.4 I_0 (mc^2/ea), \tag{53}$$

or, equivalently, recalling that $I_0 = (v_{os}/c)^2$,

$$(eE_{r,\max}/m\omega_p c) = 0.4\frac{\lambda_c}{a}\left(\frac{v_{os}}{c}\right)^2.$$

Using this value for the electric field in expression (49) we find

$$\dot{v}_i \approx 0.4 c\omega_p \left(\frac{m}{M}\right)\left(\frac{\lambda_c}{a}\right)\left(\frac{v_{os}}{a}\right)^2. \tag{54}$$

Taking all the electrons to be displaced by the vacuum channel radius a_v, given by $(a_v/\lambda_c) \approx \sqrt{2I_v} = \sqrt{2}(v_{os}/c)$, we get a rough approximation for the ion oscillation time:

$$t_i \sim \sqrt{\frac{2a_v}{\dot{v}_i}} \sim \sqrt{\frac{M}{0.2m}\left(\frac{a_v}{c}\right)\left(\frac{c}{v_{os}}\right)}\frac{1}{\omega_p} \Longrightarrow t_i \approx \sqrt{\frac{10M}{m}}\frac{1}{\omega_p} \approx 140\omega_p^{-1}, \tag{55}$$

for a hydrogen plasma. The laser pulse length should be sufficiently shorter than t_i given by Eq. (55) .

5 Plasma Fiber Accelerator

The picture of laser self-focusing given in the previous section appears suitable for the plasma accelerator scheme:[1]

Assume we have produced a rarefied channel around the beam axis in a manner described in the previous chapter. This channel acts as an effective plasma wave guide thus providing a longitudinal component of the electric field. Letting $k_{||}$ denote the wavenumber of the wave parallel to the wave guide we have the familiar result for the phase velocity of an electromagnetic wave in a wave guide:

$$v_{ph} \equiv \omega / k_{||} > c. \tag{56}$$

Thus a standard wave guide cannot provide an appropriate field for particle acceleration.

If, however, the walls of the wave guide are rippled instead of straight (recall Fig. 4.) the system provides a slow wave structure of electromagnetic waves:

$$v_{ph} = \frac{\omega}{k_{||} + \kappa}, \tag{57}$$

where κ is the wavenumber of the ripples. (The ripples are called irises in accelerator physics, and a 'rippled' wave guide is called a loaded wave guide). Adjusting the parameters of the laser-plasma system we adjust the wavenumber of the ripples and can, at least in principle, provide the ideal coupling between the wave and the particles:

$$v_{ph} = v_{\text{particle}} \sim c. \tag{58}$$

The wavenumber of the ripples for this case is given by the dispersion relation:

$$c = \frac{\omega}{k_{||} + \kappa} = \frac{c\sqrt{k_{||}^2 + \left(\frac{\pi}{a_v}\right)^2}}{k_{||} + \kappa} \Longrightarrow \kappa \approx \frac{\pi}{4}\left(\frac{\lambda_0}{a_v^2}\right). \tag{59}$$

Using this expression we can check the self-consistency of both the marginal and the eikonal approximation. We have already discovered that the eikonal approximation is valid in the radial direction since $a_v/\lambda_0 \gg 1$. Therefore

$$L_{\text{ripple}} = 1/\kappa \approx (a_v/\lambda_0)a_v \gg \lambda_0, \tag{60}$$

and the eikonal approximation is valid also in the z-direction. Furthermore, since

$$L_{\text{ripple}} \gg a_v, \tag{61}$$

the marginal approximation is seen to be self-consistent.

The parallel electric field available to accelerating particles is roughly given by[7]

$$E_{\|}/E_T = \frac{1}{2}\left(\frac{\pi}{k_0 a}\right)\frac{\delta\omega_p^2}{\omega_p^2} \leq \frac{1}{2}\frac{\pi}{k_0 a}, \tag{62}$$

where $\delta\omega_p^2/\omega_p^2$ describes the fraction of plasma density that forms the ripples and E_T is the transverse laser field.

6 Discussion

We found that for an injected laser pulse short enough ($t \ll 140/\omega_p$) so that only electrons respond, a laser beam injected into the plasma is self-trapped, provided that the power of the beam exceeds a critical value $\left(P_{\text{crit}} = \frac{c}{4}\left(\frac{mc^2}{e}\right)^2\left(\frac{\omega}{\omega_p}\right)^2 \approx 10^{10}(\omega/\omega_p)^2 W\right)$. The beam waist oscillates around a (power dependent) equilibrium value and produces an effective plasma wave guide with rippled walls (irises). This condition is appropriate for the plasma fiber accelerator.[1] Note that in the present problem the pulse is so short that ions do not respond. In contrast to the long pulse problem[5] where quasineutrality determines the plasma response, a very weak dependence on the plasma temperature is expected. Also, since a fiber accelerator would operate on a timescale much shorter than the ion timescale, it should be free from ion instabilities, while a quasineutral fiber may be subject to many more plasma instabilities. The question of the stability of the acceleration structure needs, however, more detailed investigations.

In addition, the present self-trapping of laser light may be useful for efficient transport of the beam to the target in inertially confined laser fusion applications. It is also relevant to the laser heated long solenoid fusion concept.[8]

Acknowledgments

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Figure Captions

1. The beam radius corresponding to the electron evacuation on axis as a function of power $a_v^2(P)$.

2. The Sagdeev potential. Figure 2(a) shows the potential for the case where the initial beam power exceeds the critical value, and 2(b) for the case where the power is less than the critical value. Depending on the initial intensity and divergence of the beam at the entrance to the plasma, the beam will either self-focus or defocus. If the power is high enough, $P > P_{\rm cr}$, and the initial divergence is small enough, the beam will self-trap. This case is represented by point (1) in picture 2(a). If $P > P_{\rm cr}$, but the initial divergence is too strong, the beam will defocus. In the case of very strong initial convergence, the beam will first focus to a small radius and then defocus forever. These cases are represented by point (2) in picture 2(a). In the case of low beam power, $P < P_{\rm cr}$, if the beam is initially convergent, it will first focus like in the case above, and then defocus forever. Otherwise the beam defocuses as soon as it enters the plasma. These cases are represented by point (3) in picture 2(b).

3. Self-focusing and defocusing regions for the potential $V(a)$.

4. The behavior of the radius of the laser beam for $P > P_{\rm crit}$. The amplitude of the oscillations depends on the initial divergence of the beam (so that for large enough initial divergence the oscillations are destroyed giving way to defocusing).

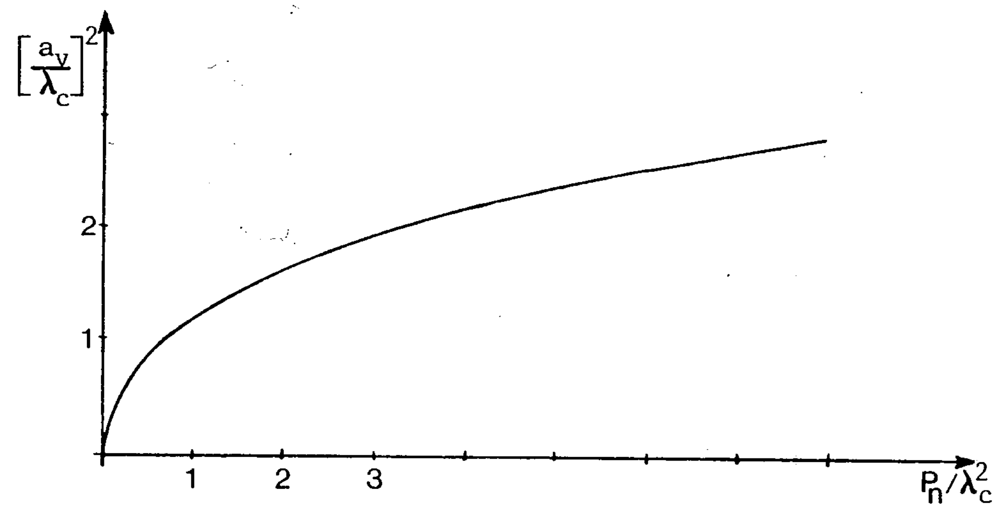

Fig. 1

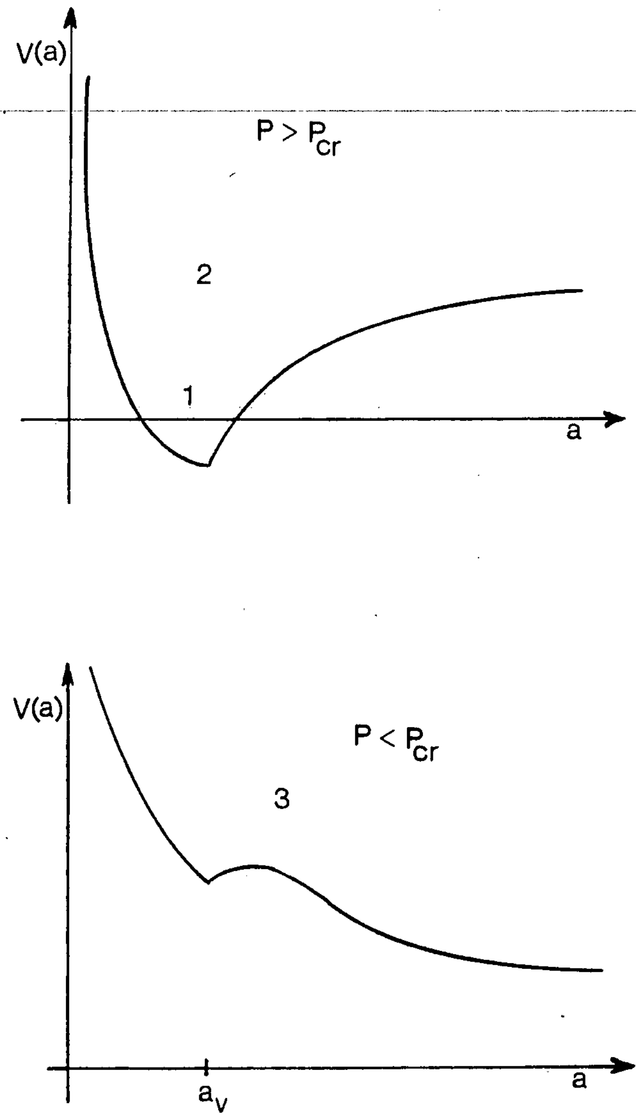

Fig. 2

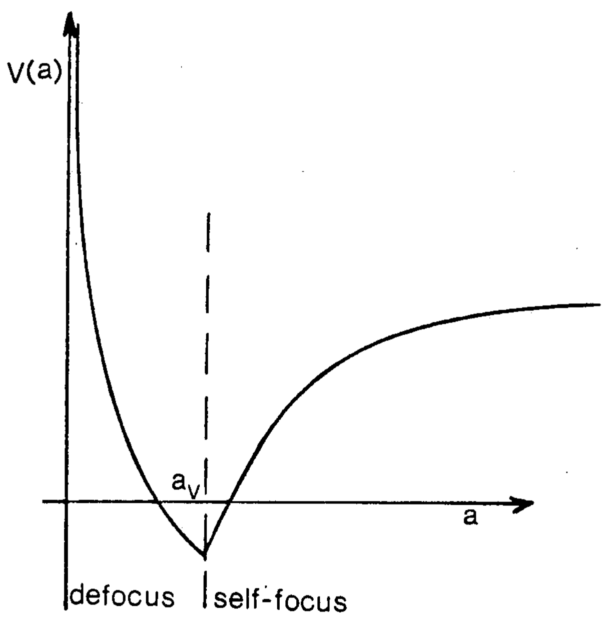

Fig. 3

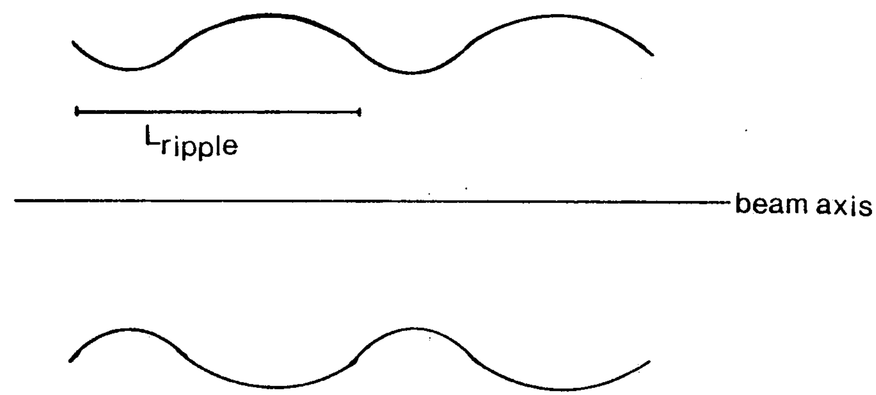

Fig. 4