

MINIMUM SCALING LAWS IN TOKAMAKS

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Abstract

Scaling laws governing anomalous electron transport in tokamaks with ohmic and/or auxiliary heating are derived using renormalized Vlasov-Ampere equations for low frequency electromagnetic microturbulence. It is also shown that for pure auxiliary heating (or when auxiliary heating power far exceeds the ohmic power), the energy confinement time scales as $\tau_E \sim P_{inj}^{-1/3}$, where P_{inj} is the injected power.

Recent experiments on tokamaks have shown a universal deterioration of energy confinement in the presence of auxiliary heating. When the injected auxiliary heating power (neutral beam or radio frequency heating) highly exceeds ohmic power, the energy confinement time exhibits the scaling $\tau_E \sim P_{inj}^{-1/3} - P_{inj}^{-2/3}$.^{1,2}

The observed deterioration in energy confinement may well be due to altogether new instabilities unleashed by the auxiliary heating source, for example, there is a plethora of modes like the *H*-mode and the *L*-mode which have been associated with these instabilities.³⁻⁵ The universality of the loss of confinement, however, does strongly suggest that a dominant universal mechanism, independent of the specificity of the heating source, must be operative. This mechanism is likely to be just the mechanism which causes anomalous electron transport in ohmic discharges. It seems reasonable, therefore, to attempt to explain the observed experimental results in terms of instabilities pertaining to the ohmic plasmas (which are expected to persist when beam or r.f. heating is turned

on); no new instabilities need be invoked. In a sense, this will lead to a minimum scaling law as if the auxiliary heating simply added to the energy content of the plasma without seriously effecting its dynamics.

In this letter, we investigate the nonlinear process in a low frequency electromagnetic micro-turbulence driven by electron temperature gradients; this instability is taken to be the mechanism responsible for the anomalous thermal conductivity of electrons. The effects of the nonlinear process must be incorporated into the equilibrium moment equations to self-consistently derive an expression for the anomalous thermal conductivity; it is the self-consistency that distinguishes our work from Ohkawa's who has put forward heuristic arguments to obtain the same scaling law.^{6,7}

Our starting point is the nonlinear drift-kinetic equation. The analysis is greatly simplified by neglecting the electrostatic potential $\delta\varphi$, and the ion contribution to parallel current ($\delta J_{\parallel i}$). Although both $\delta\varphi$ and $\delta J_{\parallel i}$ could contribute importantly to particle transport as well as to the details of linear instability, their effect on electron energy transport is expected to be insignificant in current tokamak conditions, where ion transport is essentially neo-classical.

The nonlinear processes investigated in this letter are characteristic of strong turbulence, in which neither the radial structure of the linear mode (strong mode coupling would smear out the radial behavior of a particular mode about a specific mode rational surface) nor the linear growth rate play a crucial role in determining the saturation level of turbulence. We do specify, however, that the turbulence is driven by electron temperature gradients; a specific driving mechanism is needed for self-consistency. The assumptions of short perpendicular wavelength and smeared out radial structure allows us to use a local slab model and consider the turbulence to be spatially homogeneous. This model is clearly not valid for edge turbulence, for plasmas with divertors or magnetic limiters, in fact, in any situation where either the electrostatic potential or the radial structure of turbulence may be important.

Within the context of preceding discussion, our nonlinear model for a low- β tokamak

plasma consists of the electron drift-kinetic equation and the parallel component of Ampere's law:

$$\left\{ \partial_t + v_{\parallel} \nabla_{\parallel} + (v_{\parallel}/B)(\nabla \delta A_{\parallel} \times \mathbf{b}) \cdot \nabla + (e/mc)(\partial_t \delta A_{\parallel}) \partial_{\parallel} \right\} f = 0 \quad (1)$$

$$\nabla_{\perp} \delta A_{\parallel} = \frac{4\pi e}{c} \int dv_{\parallel} v_{\parallel} \delta f \quad (2)$$

where $f = f_0 + \delta f$ with $f_0 \equiv \langle f \rangle$ satisfying $(\partial_t + v_{\parallel} \nabla_{\parallel}) f_0 = 0$, and $\langle \dots \rangle$ denoting ensemble average. In Eqs. (1), (2), $-e$ is the electron charge ($e > 0$), m is the electron mass, c is the speed of light, $\mathbf{b} = \mathbf{B}/|B|$ is the unit vector along the magnetic field \mathbf{B} , $v_{\parallel} = \mathbf{b} \cdot \mathbf{v}$, \mathbf{v} is the electron velocity, $\nabla_{\parallel} \equiv \mathbf{b} \cdot \nabla$, $\partial_{\parallel} \equiv \mathbf{b} \cdot (\partial/\partial \mathbf{v})$, $\nabla_{\perp} \equiv \nabla - \mathbf{b}(\mathbf{b} \cdot \nabla)$, and δf and δA respectively denote the fluctuating distribution function and the field.

For typical tokamak parameters, most electrons are expected to be non-resonant, in the sense that the electron thermal speed v_e is larger than the parallel phase velocity of a representative wave of the turbulence. For a turbulence driven by electron thermal gradients, the inequality translates as $k_{\parallel} v_e \sim v_e/qR > \omega_{*}$, where q is the safety factor, R is the major radius of the tokamak, $\omega_{*} = (cT_e k_{\perp}/eBL_T)$ is the electron diamagnetic frequency with T_e as the electron temperature, k_{\perp} as the perpendicular wave number and $L_T = (d/dr)\ell n T_e$ as the scale length of temperature gradients. Nonetheless, the resonant particles are still very important as to determine the energy dissipation mechanism. Thus, we keep both the resonant as well as the non-resonant contributions in our analysis.

For non-resonant particles the solution of Eq. (1) is straightforward. For resonant particles we use wave-particle renormalization following the procedure given in Ref. 8. We find that the non-resonant electrons make a coherent contribution to the linear current, and an incoherent (but, not intrinsically incoherent) contribution to the nonlinear current. The resonant electrons, on the other hand, lead to a coherent nonlinear current and an intrinsically incoherent nonlinear current. After manipulating Eqs. (1) and (2), we obtain

the nonlinear wave equation

$$\begin{aligned} [\omega - \omega_{\mathbf{k}} + i\gamma_{\mathbf{k}}^{(n)}] \delta A_{\mathbf{k}} = & \sum_{\substack{k_1+k_2=k \\ (k_1 \neq k)}} \mathcal{E}^{(2)}(k_1, k_2) \delta A_{k_1} \delta A_{k_2} \\ & + \sum_{\substack{k_1+k_2+k_3=k \\ (k_1 \neq k)}} \mathcal{E}^{(3)}(k_1, k_2, k_3) \delta A_{k_1} \delta A_{k_2} \delta A_{k_3} \end{aligned} \quad (3)$$

with

$$\omega_{\mathbf{k}} = \omega_{\star} / (1 + \mathbf{k}^2 c^2 / \omega_{pe}^2) \quad (4)$$

$$\gamma_{\mathbf{k}}^{(n)} = (\omega_{\star} - \omega)(\omega / \Gamma)(\omega_{pe}^2 / c^2 k_{\perp}^2) / (1 + \omega_{pe}^2 / c^2 k_{\perp}^2) \quad (5)$$

where $k = (\omega, \mathbf{k})$, $\mathbf{k} = \mathbf{k}_{\perp}$ is the perpendicular wave number, $\omega_{pe} = (4\pi n e^2 / m_e)^{1/2}$ is the electron plasma frequency, Γ is the frequency broadening of the resonant electron propagator, and

$$\begin{aligned} \text{Re}\mathcal{E}^{(3)}(k_1, k_2, k_3) & \equiv \left\{ (3/4)(\omega_{pe} v_e / c B)^2 (\mathbf{k}_1 \times \mathbf{k}) \cdot \mathbf{b} \right. \\ & \quad \left. (\mathbf{k}_2 \times \mathbf{k}_3) \cdot \mathbf{b} [(\omega_2 - \omega_{\star}(\mathbf{k}_2)) / \omega_2 - (\omega_3 - \omega_{\star}(\mathbf{k}_3)) / \omega_3] \right\} \cdot \\ & \quad \cdot [(\omega_2 + \omega_3)(\mathbf{k}^2 + \omega_{pe}^2 / c^2)]^{-1} \\ \text{Im}\mathcal{E}^{(3)}(k_1, k_2, k_3) & \equiv - [(\omega_{ce} / B^2) / (\omega_2 + \omega_3)] \\ & \quad \cdot \left[(\omega_2 \mathbf{k}_3^2 + \omega_3 \mathbf{k}_2^2) (\mathbf{k}_1 \times \mathbf{k}) \cdot \mathbf{b} + \frac{1}{2} \omega_1 (\mathbf{k}_2 \times \mathbf{k}_3) \cdot \mathbf{b} (\mathbf{k}_3^2 - \mathbf{k}_2^2) \right] \\ & \quad \cdot [\mathbf{k}^2 + \omega_{pe}^2 / c^2]^{-1}. \end{aligned} \quad (6)$$

The coherent part of the resonant particles contributes only the imaginary, the $i\gamma_{\mathbf{k}}^{(n)}$ term to Eq. (3). The intrinsically incoherent term from resonant particles $\mathcal{E}^{(2)}(k_1, k_2)$ is very small, because the equilibrium distribution function is even in v_{\parallel} . When v_{\parallel} -dependence of frequency broadening is neglected, $\mathcal{E}^{(2)}(k_1, k_2)$ goes to zero for resonant particles. Although it is small, we still retain it to avert an artificial difficulty in dealing with nonlinear wave equation. The manipulation of $\mathcal{E}^{(2)}(k_1, k_2)$ term is very subtle and lengthy. For the purpose of this letter we do not give its explicit expression.

We emphasize that the nonlinear wave equation should be analyzed within the framework of a renormalized turbulence theory, because we are dealing with strong turbulence

of which a primary characteristic is the (experimentally observed) existence of a wide frequency band spectrum for a specific wave number. The proper renormalization procedure for this type of interaction can be inferred from Ref. 8, and it essentially consists of dividing the non-intrinsically incoherent term into coherent and intrinsically incoherent parts. The renormalization of the wave equation is essential to distinguish between the different saturation mechanisms operative in weak turbulence and strong turbulence. The conventional nonlinear spectrum equation of weak turbulence is obtained by balancing the linear growth and the nonlinear damping rates. This procedure is plausible only if the observed spectrum is basically a delta function $\delta(\omega - \omega_{\mathbf{k}})$. For strong turbulence, the frequency spectrum cannot be predetermined, and is one of objects that the theory seeks to derive. Clearly, the saturation level will come from the balance of some nonlinear processes as will be seen immediately.

After renormalizing Eq. (3), and using quasi-Gaussian approximation to evaluate the intrinsically incoherent part, we obtain the spectrum equation

$$I_k = \frac{6 \sum_{k_1+k_2+k_3=k} |\mathcal{E}^{(3)}(k_1, k_2, k_3)|^2 I_{k_1} I_{k_2} I_{k_3} + 2 \sum_{k_1+k_2=k} |\mathcal{E}^{(2)}(k_1, k_2)|^2 I_{k_1} I_{k_2}}{\left(\omega - \omega_{\mathbf{k}} - 2 \sum_{k_1} \text{Re} \mathcal{E}^{(3)}(k_1, -k, k) I_{k_1} \right)^2 + \left(\gamma_k^{(n)} - 2 \sum_{k_1} \text{Im} \mathcal{E}^{(3)}(k_1, -k, k) I_{k_1} \right)^2} \quad (7)$$

where $I_k = \langle \delta A_k \delta A_k^* \rangle$. In principal, Eq. (7) can be solved to obtain the turbulence spectrum. To obtain the scaling law, however, we shall use the dominant balance argument. After multiplying by δA_k^* , the real part of Eq. (3) can be written as

$$\left\{ \omega - \omega_{\mathbf{k}} - 2 \sum_{k_1} \text{Re} \mathcal{E}^{(3)}(k_1, -k, k) I_{k_1} \right\} I_k = \text{Re} \sum_{\substack{k_1+k_2=k \\ (k_1 \neq k)}} \mathcal{E}^{(2)}(k_1, k_2) \langle \delta A_{k_1} \delta A_{k_2} \delta A_k^* \rangle. \quad (8)$$

In a quasi-Gaussian process the three wave correlation term does contribute to the nonlinear scattering and three wave interaction. However, its magnitude is small compared to the remaining terms in Eq. (8). The obvious balance is

$$\omega \sim \omega_{\mathbf{k}} \sim 2 \sum_{k_1} \text{Re} \mathcal{E}^{(3)}(k_1, -k, k) I_{k_1} \quad (9)$$

which, coupled with Eqs. (4) and (6), yields a scaling for the saturation level of the perturbed field

$$\frac{\delta B}{B} \sim \frac{\omega_*}{v_e k_\perp} \left(1 + \frac{c^2 k_\perp^2}{\omega_{pe}^2} \right)^{-1/2}, \quad (10)$$

where k_\perp is a typical perpendicular wavenumber. Equation (10) allows a very simple physical interpretation. For $c^2 k_\perp^2 / \omega_{pe}^2 \sim O(1)$ [this will be shown later], the balance is obtained when the electron drift velocity in the fluctuating magnetic field ($v_d \sim v_\parallel \delta B / B \sim v_e \delta B / B$) resonates with the perpendicular phase velocity of the wave.

In a random walk model, the electron turbulent diffusion coefficient in a fluctuating magnetic field (δB) is

$$D_\perp^{(T)} \sim (\delta B / B)(v_e / k_\perp) \quad (11)$$

which, when combined with Eq. (11), yields

$$D_\perp^{(T)} \sim (\omega_* / k_\perp^2) \left(1 + \frac{c^2 k_\perp^2}{\omega_{pe}^2} \right)^{-1/2}. \quad (12)$$

In order to obtain a genuine diffusion coefficient we must eliminate k_\perp in terms of plasma parameters. We begin with a standard result of the renormalized theories of wave-particle interaction,

$$\Gamma = k_\perp^2 D_\perp^{(T)} \quad (13)$$

and obtain the relationship between Γ and k_\perp [using Eq. (13)],

$$\Gamma = \omega_* \left(1 + \frac{c^2 k_\perp^2}{\omega_{pe}^2} \right)^{-1/2}. \quad (14)$$

The main dissipation process in the wave-particle system is the work done by the parallel electric field. Using resonant electron descriptions, scaling of anomalous conductivity is seen to be

$$\sigma \sim \frac{n e^2}{m_e \Gamma}. \quad (15)$$

Equation (15) is now used to obtain a renormalized quasilinear current which is substituted into the right-hand side of Ampere's law [Eq. (2)] to yield

$$k_\perp^2 = \frac{\omega_{pe}^2}{c^2} \cdot \frac{\omega}{\Gamma}. \quad (16)$$

Internal consistency of Eqs. (16), (14) and (4) demands the scaling

$$\frac{k_{\perp}c}{\omega_{pe}} \sim O(1). \quad (17)$$

The physical process leading to Eq. (17) could be summarized as follows. The average lifetime of a test wave $\tau = \text{Wave Energy}/\text{Dissipation Rate} = [\delta B]^2/8\pi]/\sigma(\delta E_{\parallel})^2$ because the only dissipation process is the work done by the parallel electric field $\delta W = \delta J_{\parallel}\delta E_{\parallel} = \sigma(\delta E_{\parallel})^2$. During this time, the wave travels (in the perpendicular direction) a typical distance $\Delta \sim \tau(\omega/k_{\perp})$. The attenuation length Δ must also determine the perpendicular wavenumber $k_{\perp} = \Delta^{-1}$, giving $\tau\omega \sim 1$, which leads to $k_{\perp}c \sim \omega_{pe}$ within the framework of Eqs. (15) (16) and the Faraday's law. This argument elucidates the nonlinear origin, and the turbulent dissipative process connected with the well-known scaling law embodied in Eq. (17).

We now proceed to solve $D_{\perp}^{(T)}$ self-consistently and begin with a purely ohmic plasma; the equations are written in a scaling form. In addition to the scalings derived above from microturbulence we also need macroscopic moment equation describing the transport process.

The energy balance equation in ohmic heating (J is the equilibrium current density)

$$J^2/\sigma = D_{\perp}^{(T)} nT_e/L_T^2, \quad (18)$$

and the equilibrium Ampere's law

$$J = Bc/4\pi qR, \quad (19)$$

in conjunction with Eqs. (14)-(16) leads to

$$D_{\perp}^{(T)} = \frac{1}{\sqrt{2}} \left(1 + \frac{c^2 k_{\perp}^2}{\omega_{pe}^2} \right)^{-1/2} \left(\frac{c}{\omega_{pe}} \right)^2 \frac{v_e}{qR}. \quad (20)$$

When use is made of Eq. (17), we obtain

$$D_{\perp}^{(T)} \sim \frac{1}{2} \left(\frac{c}{\omega_{pe}} \right)^2 \frac{v_e}{qR}, \quad (21)$$

the self-consistent scaling law for ohmic heated tokamak plasmas. This is essentially the same scaling law as was given by Ohkawa in 1978.⁶ Notice that for ohmic discharges, the k_{\perp} scaling given by Eq. (17) is not quite essential for the derivation of Eq. (20). We have indicated before, and we emphasize now that the details of the turbulent energy dissipation process is not very crucial to the determination of the anomalous thermal conductivity in ohmic heating plasma. We would like to state here the fact that tokamak experiments do not strongly suggest anomalous resistivity in their stable operation stage, but neither do they reject the form we used in Eq. (15). The difference between the two mainly lies in their profiles. The classical conductivity is proportional to $T_e^{3/2}(r)$, while the turbulent conductivity given by Eq. (15) is proportional to $n(r) / \sqrt{T_e(r)}$.

The above procedure is easily extended to deal with plasma with an auxiliary heating source, the energy balance equation [Eq. (18)] should be replaced by

$$P_e + J^2/\sigma = D_{\perp}^{(T)} n T_e / L_T^2 \quad (22)$$

where P_e is the power density of auxiliary heating absorbed by electron. Non-consistently the scaling law can be cast in the form

$$D_{\perp}^{(T)} \sim \frac{1}{2} \left(\frac{v_e}{qR} \right) \sqrt{1 + \frac{P_e}{\eta J^2}} \quad (23)$$

where $\eta = \sigma^{-1}$. Equation (24) is again basically the same as that heuristically derived by Ohkawa in 1985.⁷ In case, the auxiliary power far exceeds the ohmic power ($P_e \gg \eta J^2$); the energy balance equation becomes

$$P_e = D_{\perp}^{(T)} n T_e / L_T^2, \quad (24)$$

leading to a self-consistent solution of the anomalous electron thermal conductivity (use of Eq. (17) is crucial for this case)

$$D_{\perp}^{(T)} \sim \pi^{1/3} \left(\frac{c}{\omega_{pe}} \right)^{4/3} \left(\frac{v_e}{B} \right)^{2/3} P_e^{1/3} \quad (25)$$

for plasmas with strong auxiliary heating. Notice that the energy confinement time

$$\tau_E \sim P_{inj}^{-1/3} \quad (26)$$

where P_{inj} is the injected power. It is interesting that our result for a formal solution of diffusion coefficient D is the same as Ohkawa's [Eq. (24)], but the conclusion for τ_E -scaling is somewhat different from his [see Eq. (25) and Eq. (26)]. Equation (26) seems to be the best P_{inj} -scaling of the energy confinement time τ_E , obtained experimentally so far. The significance of our conclusion is that the deterioration of confinement caused by auxiliary heating is an overall phenomenon independent of the auxiliary heating source; the scaling is not expected to be better than $\tau_E \sim P_{\text{inj}}^{-1/3}$.

Finally, we end this letter by displaying the derived scaling laws which are valid when (i) classical collisions are present, and (ii) when current driven by auxiliary source is taken into account. Classical collisions modify Eq. (22) to the form

$$D_{\perp}^{(T)} = \frac{c^2}{2\omega_{pe}^2} \left\{ \sqrt{\left(\frac{v_e}{qR}\right)^2 + \nu_c^2} - \nu_c \right\}, \quad (27)$$

where $\nu_c < \Gamma$ is the classical collision frequency. Notice that our result has an additional term ν_c , originating from the frequency shift, which is missing in Ohkawa's formula. [In fact, Ohkawa had remarked about the emission of frequency shift]. Due to this change, the effect of classical collisions will improve confinement instead of deteriorating it further.

For lower hybrid current drive, the effective power density is $P_{\text{eff}} = \epsilon\mu Jn$, where $\epsilon = 1.06 \times 10^6$ in Gaussian unit, and $\mu (> 1)$ is defined by the equality $\mu P_{\text{eff}} = P_{\text{r.f.}}$, where $P_{\text{r.f.}}$ is the total injected r.f. power. The appropriate $D_{\perp}^{(T)}$ comes out to be

$$D_{\perp}^{(T)} = \left(\frac{c}{\omega_{pe}}\right)^{4/3} \left(\frac{\mu\epsilon n c v_e^2}{8B}\right)^{1/2}. \quad (28)$$

If $\hat{P}_{\text{r.f.}}$ and \hat{P} respectively denote the r.f. and ohmic power to drive the same amount of current, then for the lower hybrid scheme, they are related by

$$\frac{\hat{P}_{\text{r.f.}}}{\hat{P}} = \frac{R_{[m]} N_{20}}{0.15 V_{[v]}}$$

which implies that $\mu\hat{P}_{\text{r.f.}}/\hat{P}$ is usually greater than unity for large tokamaks like PLT and Alcator-C. Thus current-drive schemes are not likely to improve energy confinement in large tokamaks.

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Figure Captions

Fig. 1 (a) $\tilde{T}_e(x)$ versus $x = r/a$. Solid curves 1, 2, 3 in (a) are the normalized electron temperature profiles corresponding to the electron power density profiles represented, respectively, by solid curves 1, 2, 3 in Fig. 1(b). The dots with error bars represent the experimental results in TFTR "edge heating,"¹⁶ corresponding to power density profile given by dashed lines in Fig. 1(b).

(b) Electron power profile $\tilde{P}_e(x)$ versus $x = r/a$.

Fig. 2 Λ versus $T_e(0) \big|_{\text{exp}}$, Λ —the characteristic central electron temperature given by Eq. (8), and $T_e(0) \big|_{\text{exp}}$ the experimental result.

Fig. 3 $\tau_{Ee}(0)$ versus \hat{n}_e . Dots represent the experimental results on ALCATOR-A.²³ The dashed line represents $\hat{n}^{9/14}$ curve fitting to the data. ($\hat{n}_e = 6 \times 10^{14} \text{cm}^{-3}$ is taken as reference point.)

Fig. 4 $T_e(0)$ versus P_{total} . Dots are experimental results on Doublet-III.²⁴ The dashed line is the theoretical curve $T_e(0) \sim P_{\text{total}}^{1/2}$ ($P_{\text{total}} = 3 \text{MW}$ is taken as reference point).

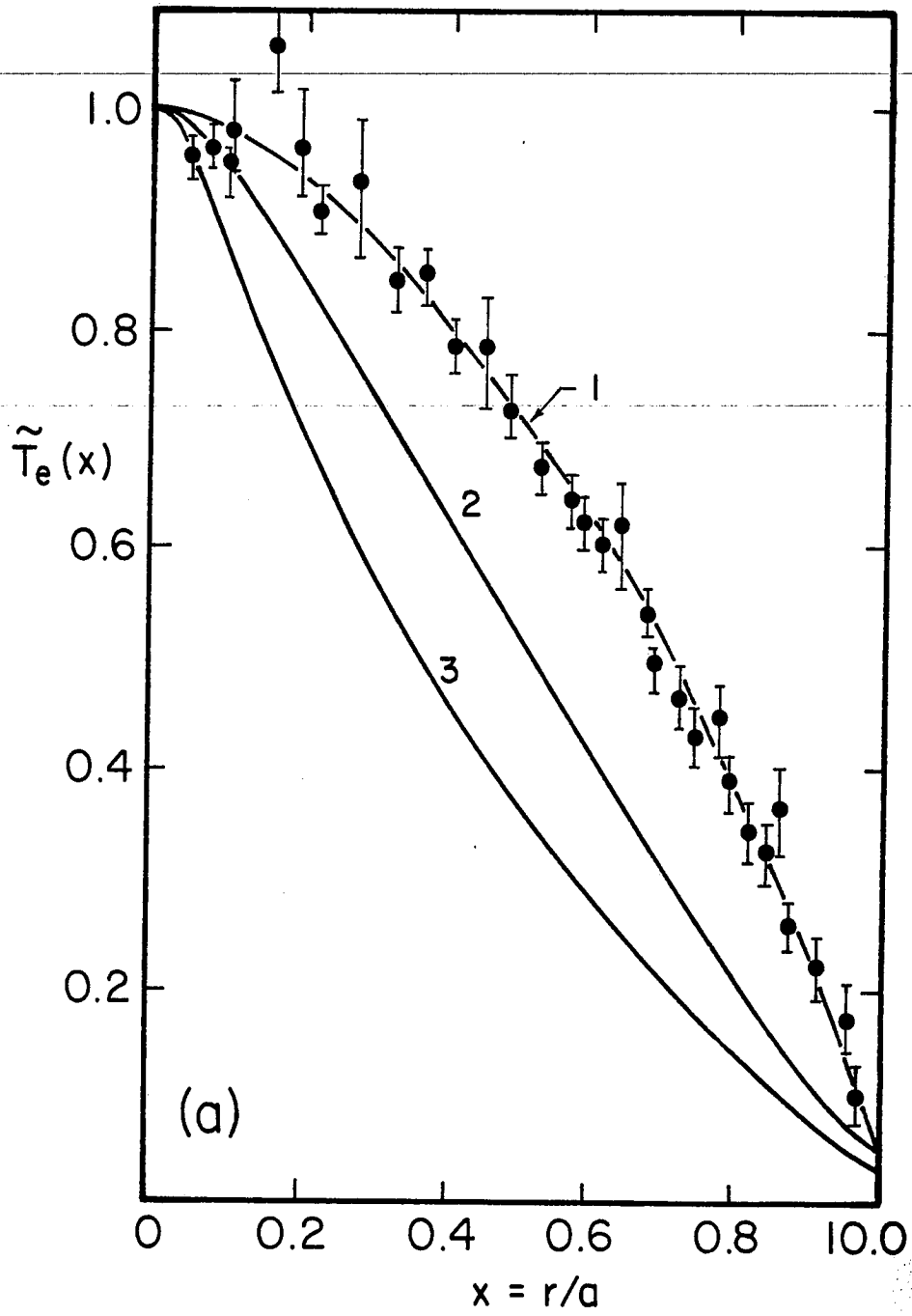


Fig. 1a

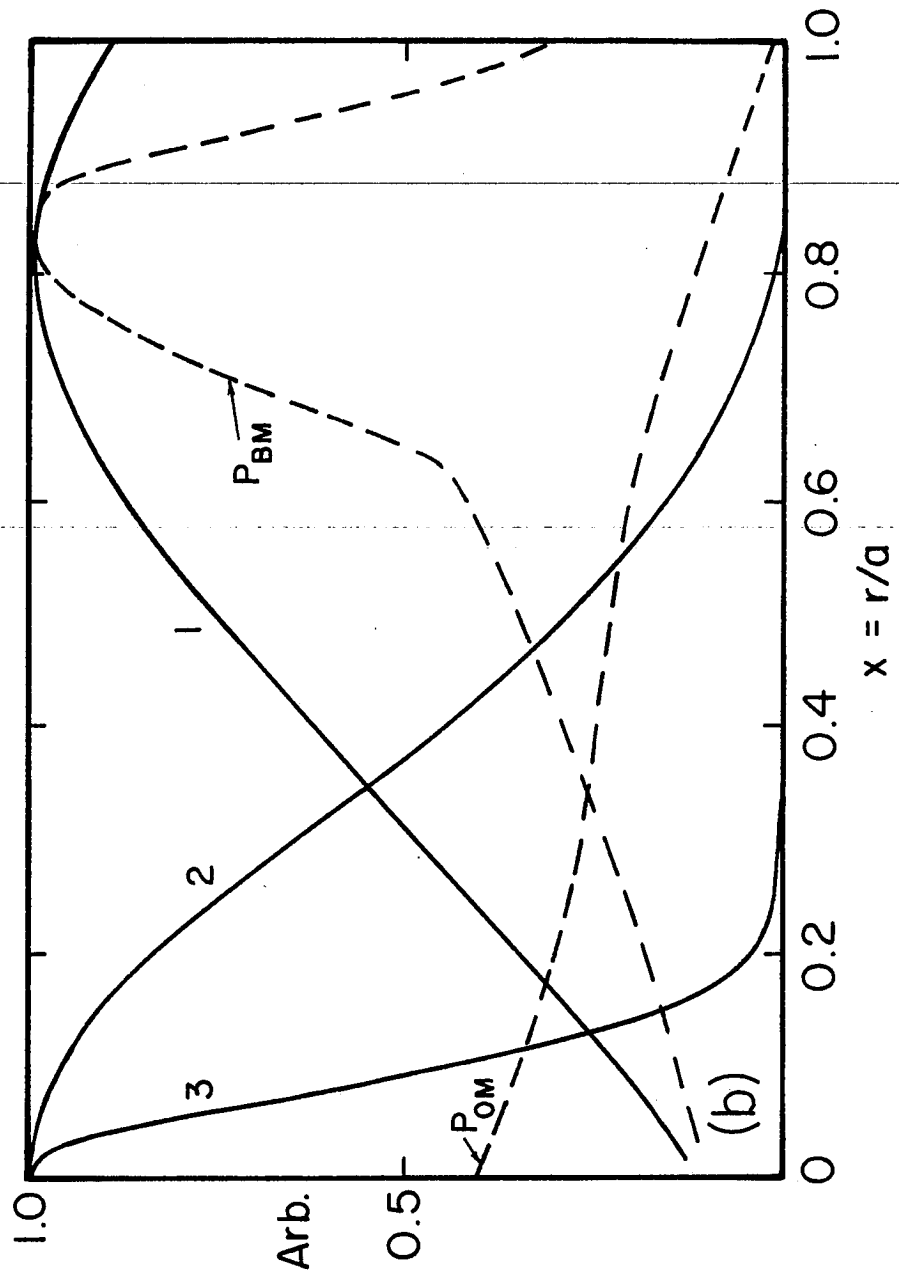


Fig. 1b

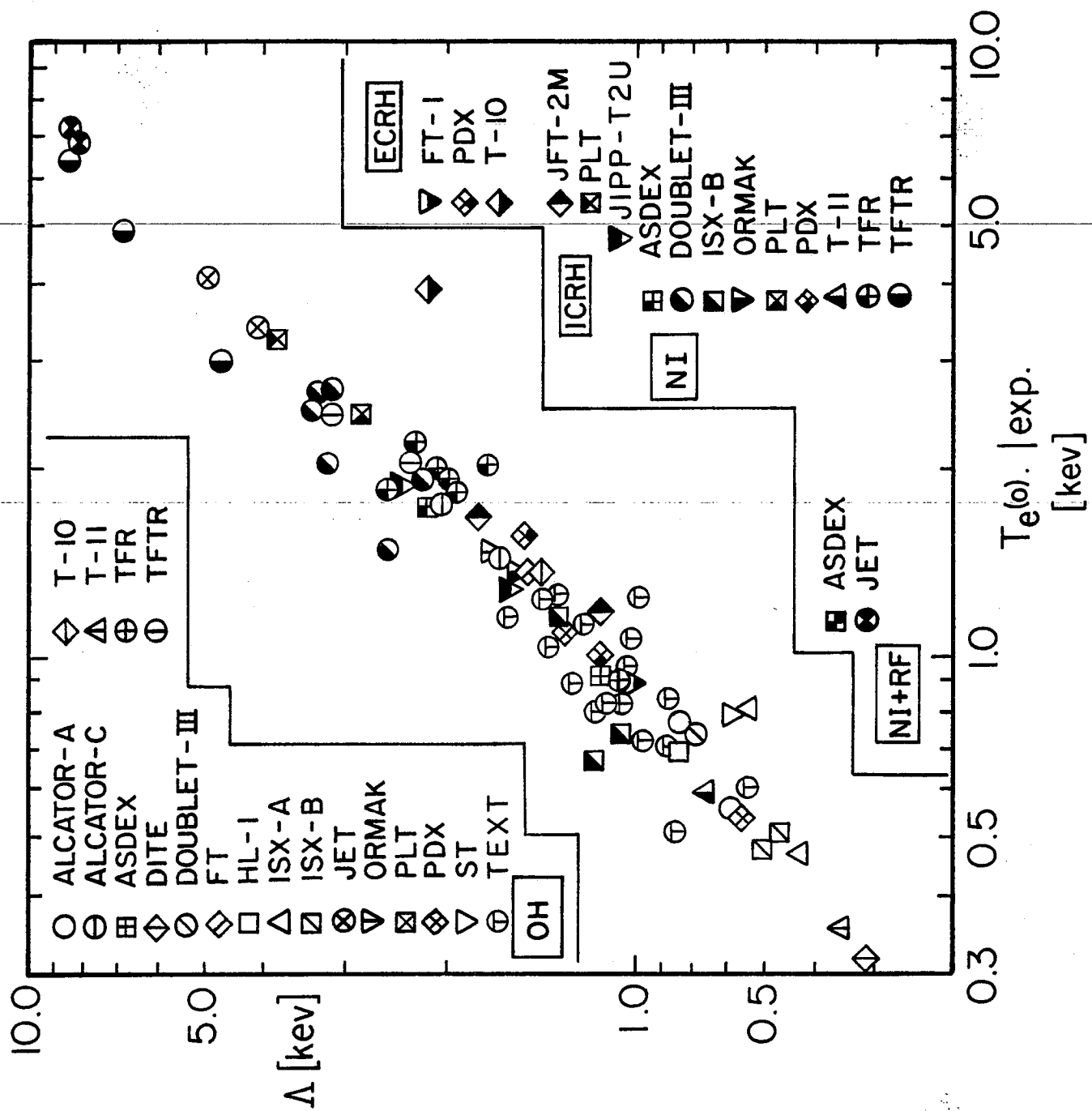


Fig. 2

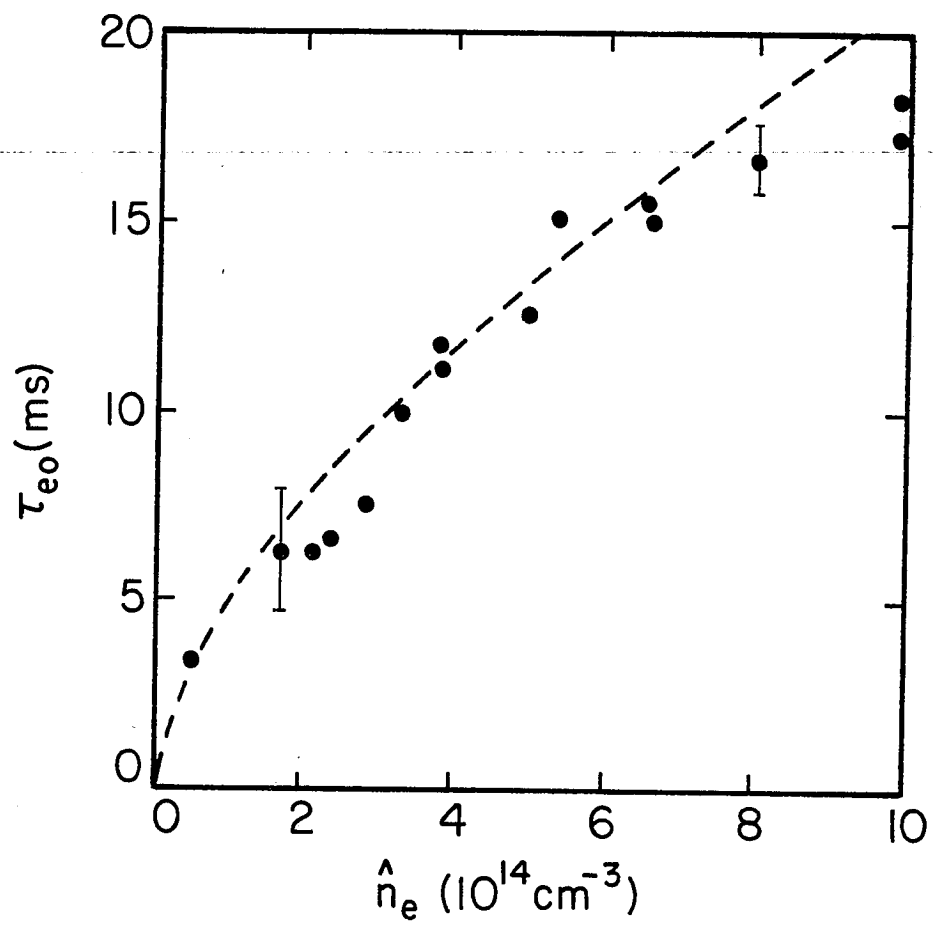


Fig. 3

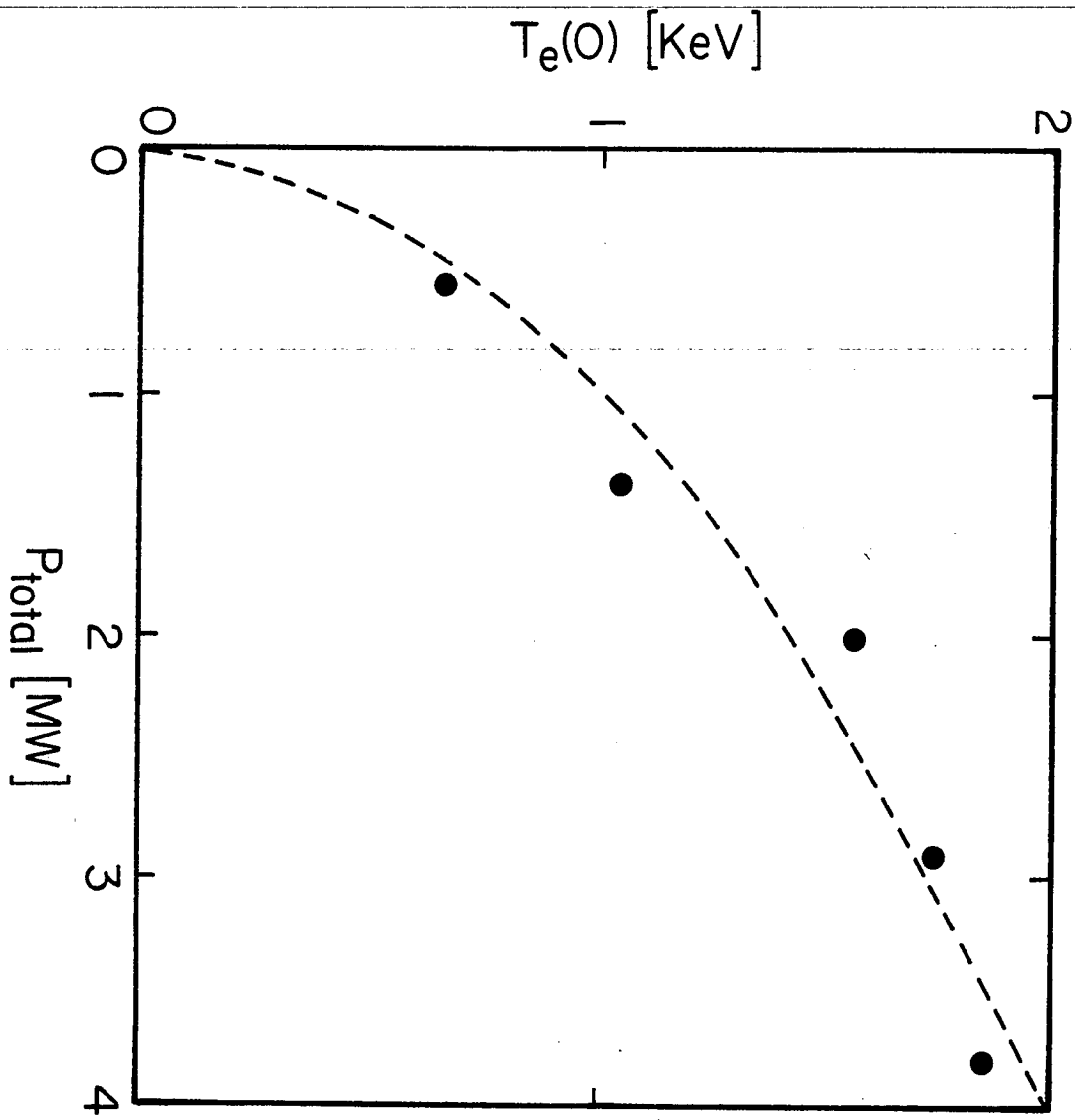


Fig. 4