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MUONIC X-RAY LASER ASSISTED BY CATALYZED FUSION OF DEUTERIUM & TRITIUM

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Abstract

The possibility of an x-ray laser by irradiation of muon beams on a thin rod of deuterium-tritium mixture is discussed. The excited state of $dt\mu$-mesomolecules $(J, v) = (1, 0)$ can be induced to make radiative chain transitions with photon energy of 90eV to the ground state $(0, 0)$, evacuated by fusion.

I. Introduction

The muonic (μ^-) mesomolecule physics of hydrogen isotopes (deuterium and tritium) has been well noted, particularly for catalyzed dt fusion (Alvarez et al. 1957; Zeldovich and Sakharov 1957; Jackson 1957). Interest in the subject was renewed by recent experiments (Jones et al. 1983). Jones et al.'s more recent experiment (1985) indicates that the sticking of muons by α-particles may may be smaller than theory predicts and that there may be additional sticking mechanism taking place at the dead end of the Auger process among rotationally and vibrationally excited states of the dt-molecule.

We propose in this paper a possible x-ray laser with photon energy of 90eV A muon beam is irradiated on a thin rod of a deuterium-tritium mixture. We utilize the stuck population of the excited state of the $dt\mu$-molecule for lasing. The main idea is based on the fact that as muonic level of energy in a mesomolecule is larger than the electronic one by the mass ratio of muon to electron m_μ/m_e, the corresponding muonic transitions have approximately 200 times shorter wavelength of photons than the electronic ones. It is, however, to be noted that different from electronic transitions in which multiple electrons consist of molecular configuration relevant to the excitation for lasing, we cannot expect to have more than one muon in a given molecule in the muonic transitions. This feature severely restricts possible useful muonic transitions for lasing. The molecule of hydrogen isotopes is a notable exception that can make up muonic molecular states by a single muon per molecule. We consider in particular the excited state of the $dt\mu$ mesomolecule $(J, v) = (1, 0)$, where J and v are the rotational and vibrational quantum numbers of the $(dt\mu)^+$ hydrogen-type nucleus. As shown schematically in Fig. 1, the initial mesomolecule is formed at $(J, v) = (1, 1)$, which is resonantly created (Gershtein & Ponomarev 1977; Ponomarev et al. 1978) at room temperature. The $(dt\mu)^+$ nucleus de-excites by the Auger electron emission with the selection rule $\Delta J = \pm 1$. The fast Auger processes are estimated to be of the order of 10^{-12} sec (Bogdanova et al. 1982). Possible processes are (i) $(1,1) \to (2,0) \to (1,0)$ and (ii) $(1,1) \to (0,1) \to (1,0)$. In terms of the second process,

however, it is possible that the fusion time at the level (0,1) may be substantially shorter than the decay time of the level (0,1), because the wave function with one vibrational mode at (0,1) could penetrate the potential wall more easily and a substantial amplitude of the wave function could reach the center $r = 0$. If this, in fact, turns out to be the case, the relevant process we can utilize is only the first process. With only two electrons available to emit, a substantial portion of muons should be stuck at the $(J, v) = (1, 0)$ level. The natural linewidth of this level is estimated to be $10^{+8}\,\mathrm{sec}^{-1}$. Some of the muons that decay to the ground state by the spontaneous emission process are quickly released due to the fast (10^{-12} sec) fusion (Bogdanov et al. 1981) of the d and t nuclei. This guarantees the vacated ground state and thus the population inversion of muons between $(1, 0)$ and $(0, 0)$. According to Bogdanova et al. (1981) the level $(0, 1)$ has also a fusion lifetime of $\sim 10^{-12}$ sec. The dipole transitions $(dt\mu)^+$ molecule exist while the $(dd\mu)^+$ do not exist in this case. The emitted photon of 90eV by $(1, 0) \rightarrow (0, 0)$ transition can trigger other muons at the stuck $(1, 0)$ level, thus causing chain reactions of radiative transition of $(1, 0) \rightarrow (0, 0)$. This process should initiate itself as long as the lasing medium sustains enough amplification and a small enough opacity of the medium is realized. Thus we expect that the dt mixture pumped by muons should radiate x-ray by itself whenever there is enough concentration of $dt\mu$-molecules.

II. X-ray Laser

The lasing conditions occur when (i) the population inversion is sufficient, (ii) the induced transition is much shorter than the spontaneous decay time, and (iii) the gain factor of the laser is sufficiently larger than the opacity of the medium. The gain factor is defined (Yariv 1967; Townes & Shawlow 1955) as

$$\gamma = \frac{\left(N_2 - \frac{g_2}{g_1}N_1\right)\lambda^2 g(\nu)}{8\pi n^2 \tau_{\rm sp}}, \tag{1}$$

where the intensity of laser as a function of the distance of propagation goes like $I(z) = I(0)e^{\gamma z}$, N_2 and N_1 are the number density of the excited state and the ground state, λ the wavelength of radiation, g_1 and g_2 are the numbers of states for the given energy, n the index of refraction, $g(\nu)$ is the line shape of the transition. Equation (1) reduces to

$$\gamma = \frac{N_\mu \lambda^2}{2\pi^2 \tau_{\rm sp} \Delta\nu}, \tag{2}$$

where we have used the relation $g(\nu) = 4/(\pi\Delta\nu)$ and we have assumed an index of refraction $n = 1$, and $N_2 - \frac{g_2}{g_1}N_1 = N_\mu$, where N_μ is the number density of the muons. The intensity I of the laser obeys

$$\frac{dI}{dz} = \left(\gamma - \frac{1}{\ell_{\rm op}}\right) I, \tag{3}$$

where the opacity length $\ell_{\rm op}$ is defined by the Born approximation as

$$\ell_{\rm op}^{-1} = N_{D_2}\sigma = \frac{2^8 \pi e^2 a_B^2 N_{D_2}}{3\hbar c}\left(\frac{Ry}{\hbar\omega}\right)^{7/2}, \tag{4}$$

where a_B is the Bohr radius and Ry is the Rydberg energy ($\simeq 13.6$eV), N_{D2} is the deuterium density and $\hbar\omega$ the emitted x-ray energy. From Eqs. (2) and (3) the threshold muon number density $N_\mu^{\rm cr}$ for lasing is obtained as

$$N_\mu^{\rm cr} \simeq 3\pi^2 \tau_{\rm sp} \Delta\nu / \lambda^2, \tag{5}$$

where $\ell_{\rm op}$ is taken to be 0.7 cm for the present medium. The spontaneous decay time $\tau_{\rm sp}$ is determined (Townes & Shawlow 1955) by

$$\tau_{\rm sp}^{-1} = \frac{32\pi^3\nu^3}{3hc^3}|M_{ij}|^2. \tag{6}$$

By comparing this formula with the appropriate electron transition, ν increases (m_μ/m_e) times, while $|M_{ij}|^2$ the dipole transition probability, decreases $(m_\mu/m_e)^2$ times, thus we obtain $\tau_{\rm sp}^{-1} \sim 10^8\,{\rm sec}^{-1}$, as we mentioned earlier. It is interesting to observe that the results of Jones et al.'s experiment (1985) showed the multiplicity of fusion catalysis to be about 100 (or larger). If the sticking of the $dt\mu$ excited state is among the main factors in determining the multiplicity, then the value in Eq. (6) is not inconsistent with the number obtained by Jones et al (1985).

Asserting that for the lasing amplification we take $\tilde{g}L = 5$ with $\tilde{g} = \gamma = \ell_{\rm op}^{-1}$, and taking Eq. (2) then the lasing medium L should be $L \simeq 10$cm. Since the formation of $[(dt\mu)d2e^-]$ molecule is (almost) in resonance, it is expected that the transition rate saturates at a certain low limit temperature due to the intrinsic quantum fluctuations. Thus it is reasonable to assume that the number of dt fusion catalyzed by one muon during its lifetime, X_μ, has approximately the value of 100 even at low temperatures such as $T \simeq 0.1°K$. The scale times for these processes are: the muon capture time $\sim 10^{-13} - 10^{-14}$ sec, (Gershtein & Ponomarev 1975) the fusion time 10^{-12} sec, the formation cycle time 10^{-8} sec (including the effective sticking time). We now have to determine the linewidth $\Delta\nu$, which is (Townes & Shawlow 1955)

$$\Delta\nu = \sum_i \Delta\nu_i, \tag{7}$$

where the main contributing widths are (i) the Doppler broadening, (ii) the collision between molecules, (iii) the pressure broadening, (iv) the saturation effects, (v) the fusion time, and (vi) the natural width. In our case, the first four are the main contributions, all of which is proportional to the temperature square root, $\sqrt{T}$, except for (iv). For example,

$$\Delta\nu_c \sim N_{D_2}\langle\sigma_e v\rangle,$$

$$\Delta\nu_{\rm Dopp} \sim \frac{\nu}{c}\sqrt{\frac{2kT\ell n2}{m}}. \tag{8}$$

N_{D_2} is the deuterium density, σ_e is the appropriate cross-section and v is the particle velocity, and ν is the collision frequency. At a temperature of $T \leqq 1000°\text{K}$ $\Delta\nu \simeq 10^{12}\,\text{sec}^{-1}$. From Eq. (5) we get the critical muon density $N_\mu^{\rm cr}$ is

$$N_\mu^{\rm cr} \simeq 6.10^{18}\text{cm}^{-3} \text{ at } T = 1000°K. \tag{9}$$

Here we have taken a typical temperature at 1000°K because the mesomolecule formation time is short around this temperature and yet the pressure may not be too exhorbitant.

III. Muon Beam and the dt Specimen

To determine the factors of the heating and thus the temperature of the lasing medium (dt mixture) the following stopping lengths (ranges) (Particle Data Group 1984; Von Ardenne 1962) must be taken into considerations: the muons, the α-particle at 3.6MeV due to the fusion of dt, and the electrons due to muon capture by the hydrogen isotope molecules and atoms. The muon capture, which is about 10^{-14} sec at $v_\mu \simeq \alpha c$ (i.e. at an energy of about 4 keV), corresponds to a range of 10^{-5} cm, and the range of α-particle at 3.6 MeV is about 10^{-4} cm. The range of electrons at energies $\varepsilon_e = p_e^2/2m_e$ which equal the muon energy $\varepsilon_\mu = p_\mu^2/2m_\mu + V_\mu \sim 6\text{keV}$ at $v_\mu \simeq \alpha c$ ($p_e \propto \sqrt{m_e/m_\mu}p_\mu$) is about 10^{-5} cm at liquid hydrogen density ($\phi = 1$). As long as the dimensions of the lasing medium are larger than the stopping length of μ^-, but thinner than the ranges of α and electrons, then the irradiated μ^- and the fusion product α do not cause the material to overheat. In this case the specimen of the lasing medium should have a thin rod or wafer type cross-section of the linear dimensions of 10^{-5} cm and the length of 10cm and thus a volume of about $10^{-9} \sim 10^{-6}$ cm^3. The necessary number of muons per pulse in the specimen is $n_\mu \simeq 6 \times 10^9 \sim 10^{12}$ at $T = 10^{3°}K$. However, the focusing of muon beam may restrict the possible size of the linear dimension of muon beam irradiated on the target. Accelerator physics (for example, Courant and Snyder 1958) tells us that the phase space

volume at focus is equal to that at injection: $\Delta r \Delta p_\perp \sim \Delta r_0 \Delta p_0$. This implies that $\Delta r \sim \Delta r_0/10$ assuming $\Delta p_0/\Delta p_\perp = 10^{-1}$. When we have $\Delta r_0 \sim 0.1$ cm, for example, then $\Delta r \sim 10^{-2}$ cm. In this case, the necessary total number of muons used (most of which are absorbed by the gateway material) should be $n_\mu^{\rm tot} = 2 \times 10^{14}$ at $T = 10^{3^\circ} K$. This corresponds to a muon current I_μ of 2×10^3 Amp. In this case of a thin rod, the number of dt fusion catalyzed by the one muon is taken to be $X_\mu \simeq 1$.

A method alternative to the above muon beam injection is a pulsed operation of direct tritium beam injection into the target rod. Synchronizing the pulsed magnetic field parallel to the axis of the rod, we inject the tritium beam. The tritium beam makes strong collisions with target tritium and deuterium, which create pions that eventually decay into muons. Once pions are created, they leave the target but always come back to the rod, executing the Larmor motion with the radius gradually decreasing due to the slowing-down collisions in the rod. It has the drift orbit circulating on the surface of the rod in the azimuthal direction (see Fig. 2), eventually ending up with a very small Larmor radius. This is because the muon collisions are primarily of the slow-down type of collisions (not the pitch angle-type) and the equation of motion can be simply put $d\mathbf{v}/dt = -e\mathbf{v} \times \mathbf{B}/m - \nu\mathbf{v}$. The calculation of orbits tells us the drift motion is parallel to the surface of the dt mixture (see Fig. 2). Thus, a certain substantial fraction of muons can end up in the target to participate in the lasing process. The particles heat up the target and rapidly expand it, with the confinement time being determined by the sound time. The cross-section of the target can and perhaps should be much larger than the previous example. We note that in recent years techniques of attaining very intense magnetic fields are advancing such as in Felber et al. (1985) that make the specimen smaller.

Once the induced radiation by a noise takes place, the rest of $dt\mu$-molecules are triggered by spontaneous radiation. This ensures the temporal coherence within 10^{-8} sec, while the spatial coherence is guaranteed because the photons are chain-triggered.

IV. Discussion

An alternative method of x-ray laser with muonic transitions is the following. It is known (Aristov et al. 1981) that the muonic charge exchange of a hydrogen isotope and ^3H_e involves a mesomolecule formation. For example, $t\mu + ^3 H_e \rightarrow \left[(^3H_e t\mu)e^-\right]^+ + e^-$. The mesomolecule has an excited state, which is a weak bound state. This excited state de-excites into the ground state $1s\sigma$ which is not a bound state. Thus, the mesomolecule quickly dissociates and emits photon. This dissociation process is very much reminiscent of the rare gas halide dissociation, such as $Kr^+ + F^- \rightarrow K_r + F$. The excimer lasers work based on such dissociation. One may look for a similar lasing mechanism based on muonic dissociation of $[^3H_e t\mu e^-]^+$. The formation rate of $^3H_e t\mu$ is very high at low temperature (0.004 eV).

The discussion and theoretical assertions in the present paper are only typical cases and a better understanding of the muon catalyzed processes might significantly modify the presented estimations. In any case, the special characteristics of the $dt\mu$-mesomolecule physics may be exploited not only for fusion processes but also for the creation of an x-ray laser with a coherent neutron pulse.

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Figure Captions

1. Schematic energy levels ε_{Jv} of the $dt\mu$-mesomolecule in bound states with various orbital angular momentum numbers J and vibrational quantum numbers v. The data are the energy levels in eV after Gerstein et al. (1977).
2. Negative pions created in the target rod execute damping Larmor motion, yielding drift orbits surrounding it.

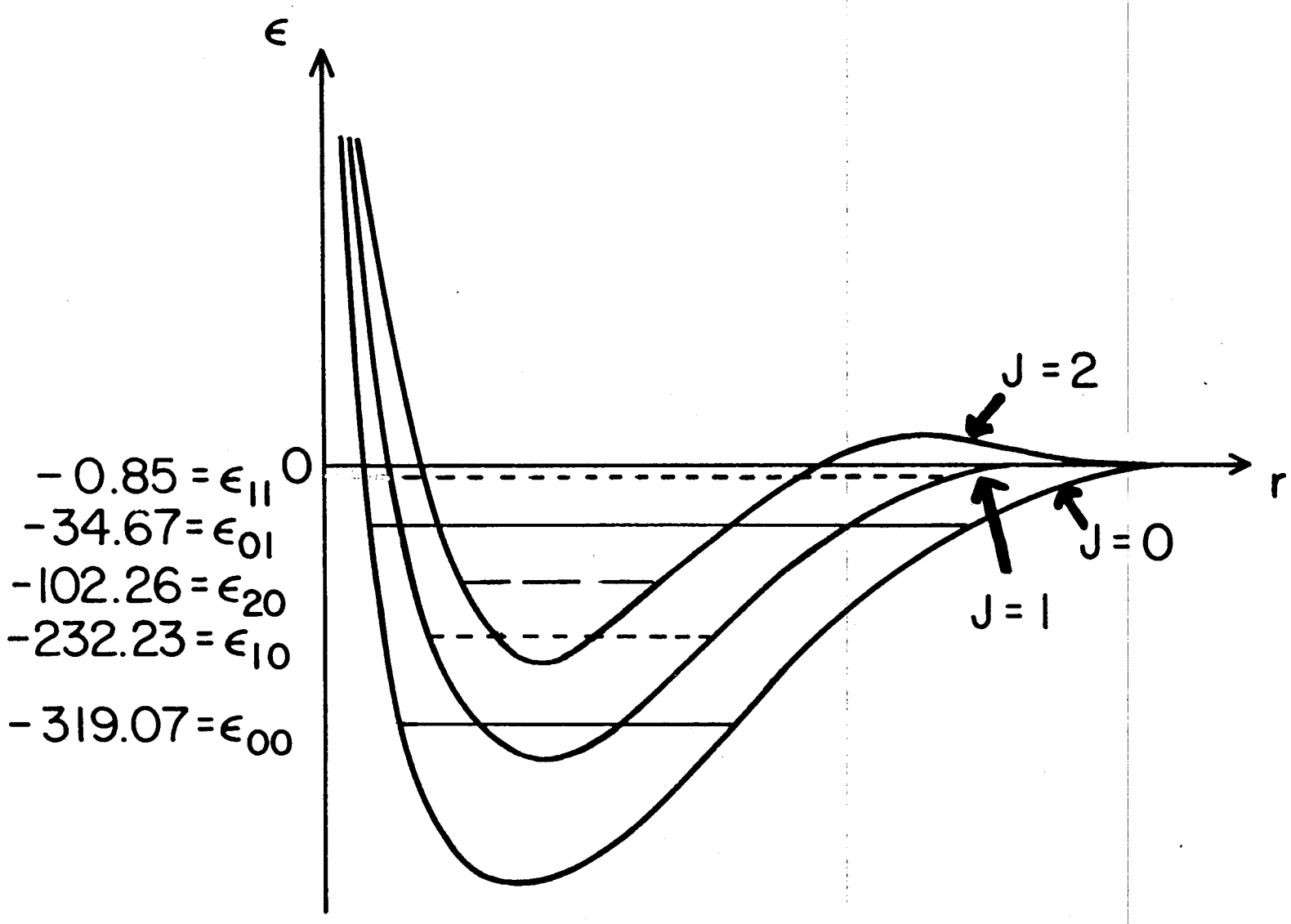

Fig. 1

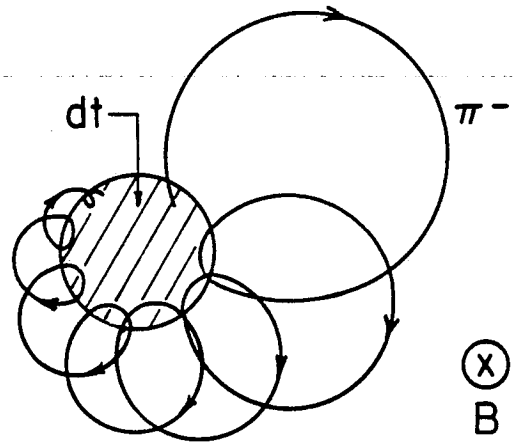

Fig. 2