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CONFINEMENT OF A SELF-STABILIZED TOKAMAK UNDER AVERAGE MAGNETIC WELL CONDITIONS

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Abstract

It is well known that the average favorable magnetic curvature of a tokamak is stabilizing with respect to pressure-driven magnetohydrodynamic instabilities at low beta and that self-stabilization occurs at finite beta in the so-called second stability regime. Here we self-consistently investigate how these two effects, viz., the mean magnetic well and the self-stabilization, influence the energy confinement time in a tokamak, using the ballooning mode transport model proposed by Connor, Taylor, and Turner [Nucl. Fusion **24**, 642 (1984)].

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Recently, Connor, Taylor and Turner[1] have proposed an ideal MHD ballooning mode transport model to explain the experimentally observed degradation of confinement in tokamaks with auxiliary heating. This model provides a simple, self-consistent description of profile evolution that incorporates both stability and transport.

Subsequently, Fu et al.[2] employed this model to examine confinement in a tokamak plasma that can access the second stability regime by means of supplemental stabilization mechanisms such as energetic particles or cross-sectional shaping.

Here, we note that the average magnetic well of a tokamak is well known to be stabilizing against low-beta weakly ballooning instabilities on rational surfaces for which the value of the safety factor is larger than unity.[3,4] A necessary stability condition that incorporates both the effects of the mean magnetic well and the self-stabilization has been obtained by Mikhailovskii et al.[5,6] Employing this stability condition in conjunction with the ballooning mode transport model, we can investigate how the average magnetic well modifies energy confinement in a self-stabilized tokamak.

For a more detailed description of the ballooning mode transport equations, we refer to Refs. 1 and 2. Briefly stated, in this model the plasma is divided into a transport zone and a ballooning zone. In the transport zone, the plasma is theoretically stable against ideal ballooning modes and the thermal conductivity is assumed to have the form established for low-beta ohmic discharges. In this region, the pressure profile is determined by the thermal conduction equation,

$$\frac{1}{x}\frac{d}{dx}\left(x\frac{d\tilde{p}}{dx}\right)+\frac{1}{\lambda}h(x)=0 \tag{1}$$

and the poloidal magnetic field is determined by Ampère's equation and Ohm's law from

$$\frac{1}{x}\frac{d}{dx}(xb)=\tilde{p}^{3/2}(x)\Big/\int_0^1 dx\; x\tilde{p}^{3/2}(x). \tag{2}$$

Equations (1) and (2) are written in the large-aspect-ratio cylindrical approximation for equal ion and electron temperatures and constant density n, with the following normalized quantities: $x=r/a$ is the minor radius normalized to the plasma radius a; $\tilde{p}(x)=p(r)/p(0)$ is the normalized plasma pressure; $b(x)=B_\theta(r)/B_\theta(a)$ is the normalized poloidal magnetic field; and $h(x)$ is related to the heating power density, normalized so that $\int_0^1 dx\, xh(x)=1$.

The parameter λ is related to the energy confinement time τ by

$$\lambda = c\left(\frac{\tau}{\tau_I}\right) \Big/ \int_0^1 dx\; x\tilde{p}^{3/2}, \tag{3}$$

where $c = \frac{1}{4}\left[1 - \int_0^1 dx\; x^3 h(x)\right]$; also, $\tau_I = 3ca^2/2\chi_I$ is the INTOR confinement time associated with the thermal conductivity $\chi_I(m^2/\sec) = 5 \times 10^{19} n^{-1}$ for low-beta ohmic discharges.

In the ballooning zone where the plasma would be linearly unstable, the conductivity is considered to be so large that the pressure profile adjusts itself to remain marginally stable. Thus, the pressure gradient is determined by the condition for ideal MHD ballooning stability. In the large toroidal mode number limit, this stability condition involves only the shear $S = bd(x/b)/dx$ and the quantity α, which is related to the pressure gradient by

$$\alpha = -\frac{A\lambda x^2}{b^2(x)}\frac{dp}{dx}. \tag{4}$$

Note that $S = 2 - x\tilde{p}^{3/2} \Big/ b \int_0^1 dx\; x\tilde{p}^{3/2}$ approaches the value $S = 2$ at the plasma edge. The parameter A is the normalized total input power P:

$$A = \frac{4aP\tau_I}{3c\mu_0 I_p^2 R^2} = 3.2 \times 10^{-20}\frac{P(MW)a^3(m)n(m^{-3})}{I_p^2(MA)R^2(m)}, \tag{5}$$

with I_p the plasma current, n the density, and R the major radius.

Here, we propose to use the following necessary condition for marginal stability of ideal ballooning modes:[5,6]

$$\frac{S^2}{2} + \alpha\varepsilon\left(1 - \frac{1}{q^2}\right) - \frac{1}{2}S\alpha^2 + \frac{13}{128}\alpha^4 = 0. \tag{6}$$

The first term represents shear stabilization, the second term is the average magnetic well, the third term arises from pressure-gradient-driven destabilization, and the fourth term represents self-stabilization at finite beta. Here $\varepsilon = r/R$ is the inverse aspect ratio and q is the safety factor. Let us define $\varepsilon_* = \epsilon(1 - q^{-2})$. We see that Eq. (6) can be written as

$$S(\alpha) = \frac{\alpha^2}{2} \pm \left[\frac{2}{64}\alpha^4 - 2\alpha\varepsilon_*\right]^{1/2}. \tag{7}$$

There is no instability for $\alpha < \alpha_0 = 3.49\varepsilon_*^{1/3}$, at which point $S = S_0 = 6.10\varepsilon_*^{2/3}$, or for $S < S_m = 5.65\varepsilon_*^{2/3}$, at which point $\alpha = \alpha_m = 3.64\varepsilon_*^{1/3}$. For larger α values, the stability boundary becomes parabolic: $S \cong \frac{\alpha^2}{2}(1 \pm \sqrt{3}/4)$. [Note that Eqs. (4.4)-(4.6) of Ref. 5 contain minor typographical errors.] In the ballooning zone, the poloidal magnetic field is still determined by Eq. (2).

For fixed values of the power parameter A, the system of Eqs. (1), (2), and (7) constitutes an eigenvalue system with λ as the eigenvalue and $\tilde{p}(x)$ and $b(x)$ as the eigenfunctions. The boundary conditions are $d\tilde{p}/dx = 0$ at $x = 0$ and $\tilde{p}(x) = 0$ at $x = 1$; note that $b(0) = 0$ and $b(1) = 1$ by definition. This system can be numerically solved to obtain the trajectory of the plasma profile in (S, α) space and the corresponding confinement time, as the heating power is adiabatically increased from zero to large values.

The stability boundary of Eq. (7) has the features that it bifurcates as a function of the shear and that there is a minimum value of the shear, S_m, below which ballooning modes are stable for all α. Therefore, it qualitatively resembles the generic stability boundary that was used in Ref. 2 to represent enhanced stability. Since Ref. 2 considered the case for which $S_m = 1.0$, in this work we will take $\varepsilon_* = \varepsilon(1 - q^{-2}) = 0.074$. Strictly speaking, the stability boundary of Eq. (7) is valid in the low-beta, small-shear limit of a circular cross-section tokamak. In the present analysis, we transcend this limit for the sake of comparison at finite shear values.

Figure 1 shows the steady-state $S - \alpha$ profile trajectory for several increasing values of the power parameter A. For values of A greater than approximately 35, the trajectory enters the second stability region without encountering ballooning instability. Therefore, for these large powers, the confinement time, which had been decreasing as a function of A, as shown in Fig. 5, will revert to its favorable INTOR value. This is seen in Fig. 2, which is a plot of the confinement time as a function of the power. Figure 2 shows that the confinement time begins to degrade with auxiliary heating when the normalized power exceeds $A = 1.9$, but reverts to τ_I at high power ($A > 35.1$) when the plasma enters the second stability regime. We might also expect that if the power were now decreased, the confinement time would exhibit a hysteresis type of behavior.[2] That is to say, beginning from the high-beta regime and decreasing the power, we find that the confinement time first remains at τ_I and then drops to its degraded value, although this transition occurs at a lower value for A (viz., $A = 5.4$) than when the power is increased starting from the low-beta regime.

The large power required to achieve self-stabilization points to a possible area of improvement within our treatment. Large heating power ($A \gtrsim 7$) tends to depress the safety factor q on axis to a value below unity. Thus, we should perhaps take into account the effect of sawtoothing near the axis. Ref. 1 addressed this difficulty by flattening the p and q profiles within the $q = 1$ surface and matching the pressure gradient at $q = 1$ to the power deposited within it. Including a sawtooth region in this way would be expected to worsen the confinement.

In conclusion, we have pointed out that the average magnetic well in a tokamak can provide stability against ballooning modes until self-stabilization takes over. The marginal stability condition of Refs. 5 and 6, which contains both of these effects, thus provides an exact case study of a bifurcated $S - \alpha$ stability boundary, which had been previously examined by means of a generic model in Ref. 2.

The calculations in the present work indicate that steady-state high-beta self-stabilized equilibrium exist, albeit at rather high heating powers. Whether the plasma can actually evolve from a stable low-beta state to such a high-beta self-stabilized state requires the solution of the temporal evolution problem. Work on this problem is reported elsewhere.[7]

Acknowledgments

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Figure Captions

1. Plasma profiles of shear versus pressure gradient, for increasing values of the input power A, with $\epsilon\,(1 - q^{-2}) = 0.074$.

2. Hysteresis dependence of the confinement time on input power.

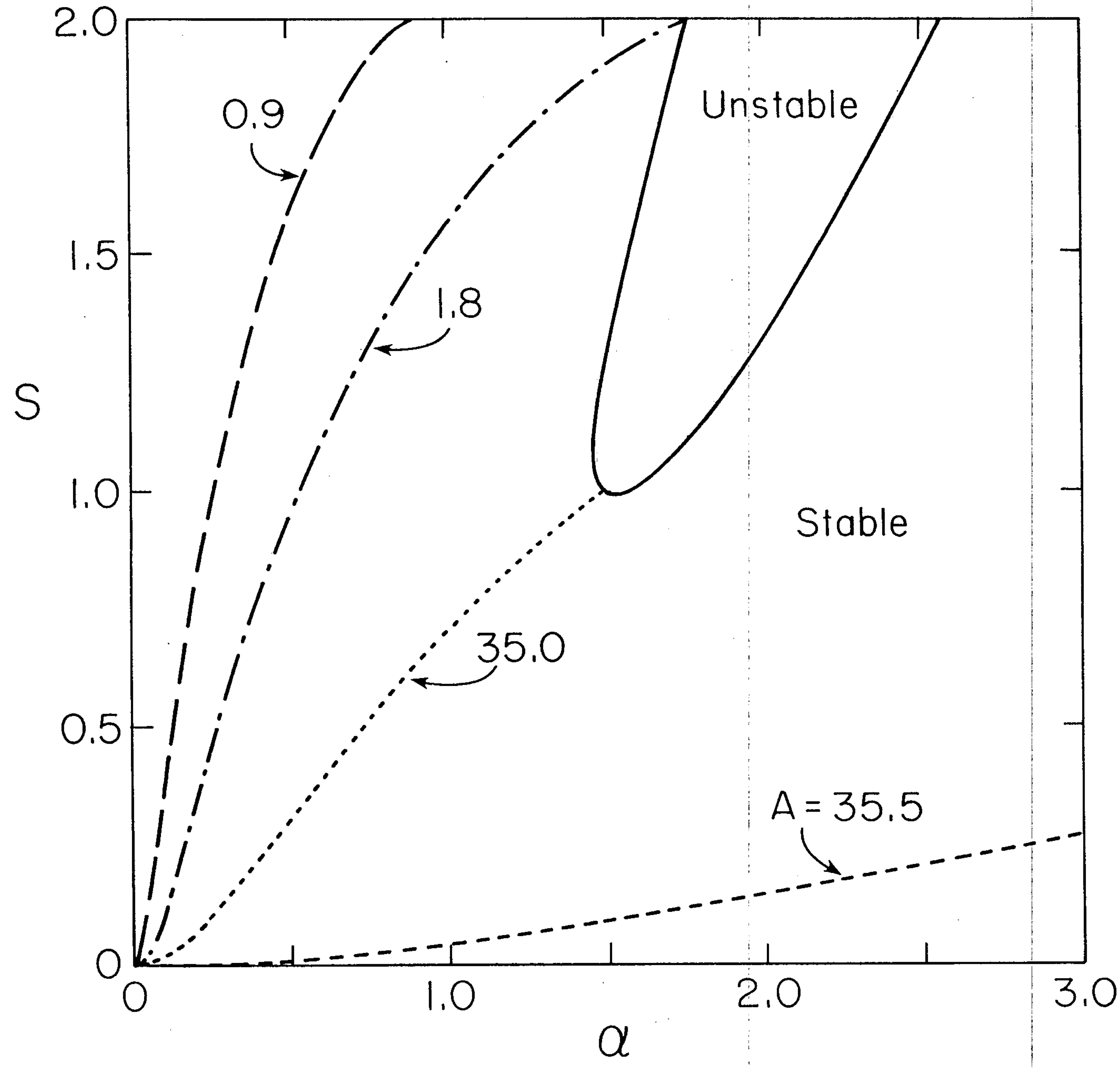

Fig. 1

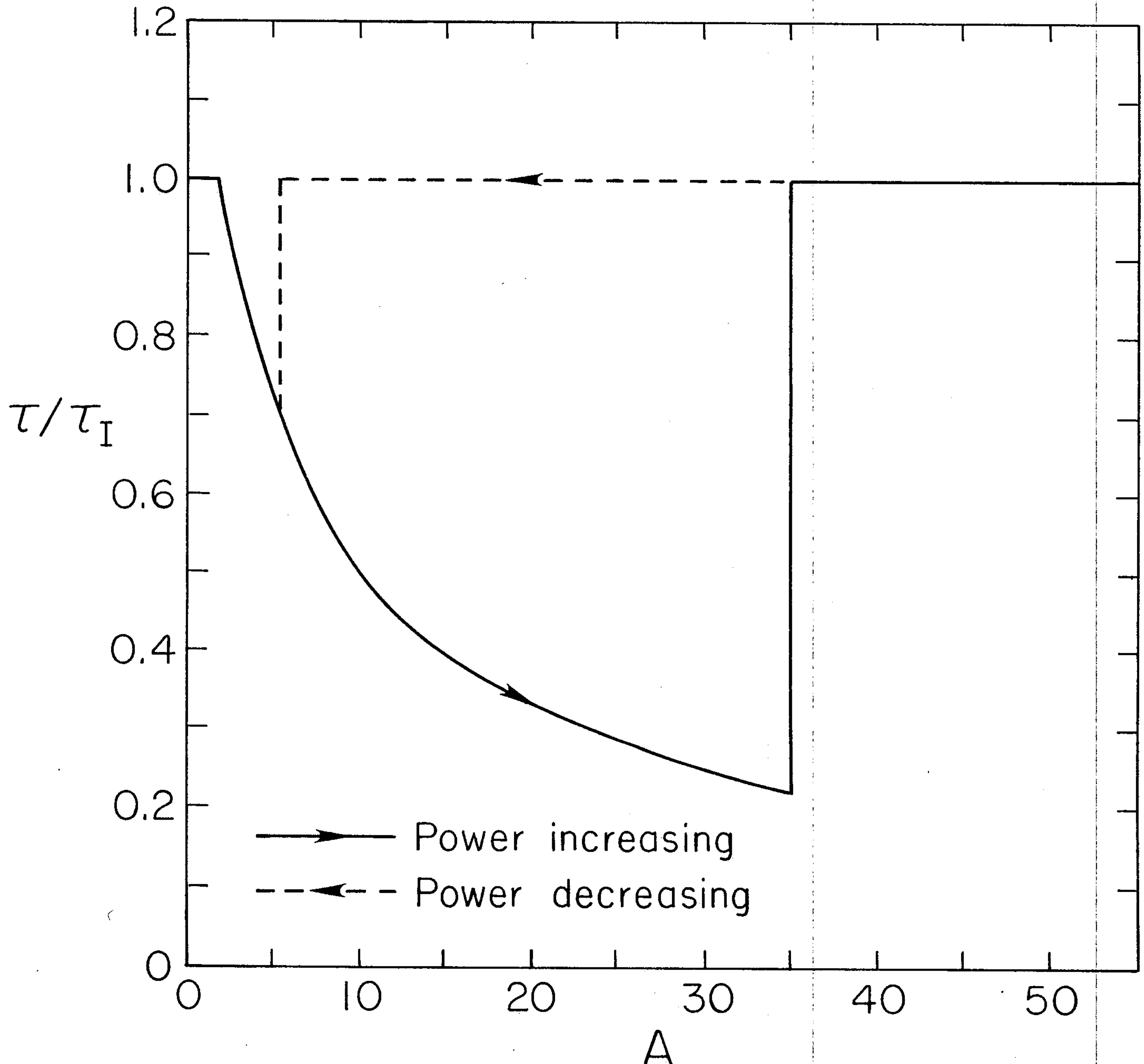
1.2
1.0
0.8
0.6
0.4
0.2
0
τ/τ_I
0
10
20
30
40
50
A
Power increasing
Power decreasing

Fig. 2