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A Mechanism for Rapid Sawtooth Crashes in Tokamaks

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The standard picture of Kadomtsev reconnection process predicts sawtooth crash times that are longer than those observed in the present day large tokamaks. The role of pressure-driven ideal internal kink modes is investigated as a possible mechanism for these fast crashes. Linear and nonlinear calculations show considerable agreement with experiment and indicate that such modes may explain fast collapse times reported in the recent literature.

The sawtooth oscillations in the soft X-ray signals observed in Tokamaks are associated with periodic changes in the central electron temperature,T_e[1]. Typically, a slow phase during which the central temperature slowly rises is followed by a fast drop in T_e, associated with flattening of the central temperature. The time scale of the slow phase is determined by various transport processes such as ohmic heating. The resistive internal kink mode was invoked by Kadomtsev[2] to explain the crash phase of the oscillations. In this model, an $m = 1$ island, associated with a safety factor, q, less than unity on axis, grows, forming a helical deformation of the internal plasma column. This kink structure subsequently relaxes to a symmetric state through complete reconnection of the helical flux inside the $q = 1$ surface with the flux from outside. The Kadomtsev model has been generally believed to explain the sawtooth oscillations in earlier tokamaks.

Recent observations, however, indicate that tokamaks exhibit various types of sawtooth oscillations that can not be fully explained by the Kadomtsev model. In addition to the simple sawtooth associated with this model, there are double or compound sawteeth[3,4] thought to be caused by the presence of two or more $q = 1$ surfaces in the plasma. Moreover, both TFTR and JET tokamaks report crash times that can not be easily reconciled with the Kadomtsev reconnection process[5].

In this letter, we will focus on the fast crash times observed in the large tokamaks, especially the fast crashes observed in JET[6]. These sawtooth oscillations are characterized by the absence of any discernible precursor oscillations, and a rapid collapse of the central temperature in about 100 microseconds. During the crash phase, the hot core region rapidly moves outward and is replaced by colder plasma. Then, this highly asymmetric state relaxes (in $\sim 100\mu$sec) to a poloidally symmetric state in which a ring of hot plasma surrounds the colder core plasma, producing a hollow pressure profile.

Since the time scales involved appear to be too short for a resistive mode, Wesson[7] has suggested that a pressure-driven ideal kink mode could be the responsible instability mechanism. This mode is always unstable in a cylinder[8] for $q_0 < 1$, and $p' < 0$, where

q_0 is the safety factor on axis, and p$'$ is the radial pressure gradient, but is generally believed to be stabilized by toroidal effects. In particular, the analysis of Bussac et al.[9] for the $n = 1$ ideal kink mode predicts an instability threshold for the pressure gradient that is much larger than the values at which sawtooth oscillations are observed. However, this calculation is based on a low-β (β is the ratio of plasma pressure to magnetic field pressure), large aspect ratio expansion of the ideal magnetohydrodynamic equations. As pointed out by Wesson, the expansion procedure used in the analytic calculations breaks down for $|1 - q_0| \ll 1$, and a modified calculation removes the threshold as $q_0 \rightarrow 1$, and the magnetic shear is reduced[7]. Physically, interchanging flux tubes at finite shear leads to an increase in the magnetic field energy due to field-line bending, which has a stabilizing influence. As shear is reduced, however, flux tubes can be interchanged without appreciable bending in the fields, and an instability results for any value of p$' < 0$. Equivalently, for any value of p$' < 0$, there is a threshold in magnetic shear below which the pressure driven mode is expected to be unstable.

The remainder of this letter is devoted to linear and nonlinear studies of pressure driven $m = 1, n = 1$ internal kink mode, where m and n are the poloidal and toroidal mode numbers, respectively. Unlike previous studies[10], we specialize to low shear systems with $q_0 \simeq 1$, and examine the questions of stability boundaries, instability growth rates, the nonlinear evolution of the mode, and its possible role in rapid sawtooth crashes.

We study this pressure driven mode using a generalized reduced magnetohydrodynamic (RMHD) code (FOUR) that solves the four-field model of Hazeltine et al.[11], and also a full MHD code (CTD)[12]. For this study, parallel compressibility and finite Larmor radius terms are neglected in FOUR, thus reducing the system to high-beta reduced MHD equations of Strauss[13]. In FOUR, the interchange term has only the toroidal field curvature, whereas CTD, which solves the full (non-reduced), compressible, nonlinear MHD equations, has no toroidal curvature effects. Magnetosonic waves in this system are treated implicitly[12], thus removing a severe time-step constraint imposed by the fast waves. Despite the fact that FOUR and CTD have different components of the field line curvature, nonlinear evolution of the mode is quite similar in both systems. Only the nonlinear results from CTD is presented here. For linear studies, however, FOUR with toroidal curvature effects is used.

The family of equilibria considered here have the safety factor profile $q(r) = q_0\{1+r^{2\lambda}[(q_l/q_0)^\lambda - 1]\}^{1/\lambda} - q_1 \exp[-(r-r_{mn})^2/\delta^2]$, where q_0, q_l are the on-axis and limiter values of $q(r)$, respectively. The radial coordinate r is normalized to the minor radius a. Finite q_1 introduces a minimum in the q-profile near $r = r_{mn}$, while δ determines the width of the surface currents there. The pressure profile is given by $p(r) = p_0[1-r^2]^\nu$. Note that for pressure driven modes, the relevant parameter is $\beta/\epsilon = \epsilon\beta_p/q_l^2$, rather than p_0 alone. Here $\epsilon = a/R_0$ is the inverse aspect ratio, and we define the toroidal and poloidal beta as $\beta = 2\mu_0 < p > /B_T(a)^2$, and $\beta_p = 2\mu_0 < p > /B_p(a)^2$, respectively. $< p >$ is the volume-averaged kinetic pressure, and $B_T(a)$, and $B_p(a)$ are the average toroidal and poloidal field strengths, respectively, measured at the limiter. The toroidal equilibria used in FOUR are obtained by starting with the cylindrical equilibria described above and dissipating kinetic energy until an axisymmetric toroidal steady-state is reached.

The linear results are in qualitative agreement with Wesson's analysis. Figure 1 shows the growth rate of the mode as a function of q_0 for $q_1 = 0, \lambda = 4, \nu = 3$, and $\epsilon\beta_p = 0.078$. Note that the growth rate is a strong function of q_0 for $q_0 \simeq 1$. For the particular equilibrium chosen here, $q_0 = 1.01$ is a stable point. As q_0 decreases, the growth rate rapidly increases to $\gamma\tau_{Hp} \sim O(10^{-2})$. (Times are normalized to the poloidal Alfvén time, $\tau_{Hp} = R_0\sqrt{\mu_0\rho_m}/B_{T0}$.) For comparison, at zero beta, the corresponding resistive mode has a growth rate of $\gamma\tau_{Hp} = 6.68 \times 10^{-4}$ for $q_0 = 0.99$, and $S = 10^8$, where S is the ratio of the resistive time to poloidal Alfvén time. The growth rate is also a strong function of $\epsilon\beta_p$ near $\epsilon\beta_p \simeq 0$. Fig. 2 shows $\gamma\tau_{Hp}$ as a function of $\epsilon\beta_p$ for $q_0 = 1.01, q_1 = 0.01, \lambda = 4$, and $\nu = 3$. At zero-β, this equilibrium with an off-axis minimum in the q-profile is stable. However, because of very low shear, even very small pressure gradients destabilize the $n = 1$

mode. Again, the growth rate is substantial ($\gamma\tau_{Hp} \sim O(10^{-2})$) for values of $\epsilon\beta_p \gtrsim 0.07$. For a typical JET discharge with ion cyclotron resonance heating (ICRH), we estimate $\epsilon\beta_p \sim 0.1$, where this mode is expected to be unstable with a growth rate of $\gamma\tau_{Hp} > 10^{-2}$.

For the nonlinear studies with CTD, a q-profile with an off-axis minimum is chosen, with $q_0 = 1.01, q_1 = 0.01, q_l = 2.5, r_{mn} = 0.35$, and $\lambda = 4$. The inverse aspect ratio, ϵ, is 1/3, with $\epsilon\beta_p = 0.031$, and $\beta = 0.164\%$. This modest value of $\epsilon\beta_p$ is chosen so that the linear growth rate of the mode predicted by CTD agrees with the growth rate calculated by FOUR for more typical values of $\epsilon\beta_p$. In general, full MHD equations with cylindrical curvature predicts very similar behavior with somewhat higher growth rates than the reduced MHD model with toroidal curvature.

The nonlinear evolution of the pressure field is depicted in Fig. 3. Both the contours of p, and the radial variation of p along the cord drawn on the contour plot are shown. Figure 3a shows the poloidally symmetric pressure field at $t = 0$ before the linear mode has reached an appreciable amplitude. By $t = 290$ (Fig.3b), the hot core is pushed outward, forming a semi-circular ring of hot plasma, while a "bubble" of colder plasma moves into the central region. By $t = 399$ (Fig.3c), this cold bubble is completely enclosed by the hot ring. Subsequent evolution of the mode symmetrizes the core, and Fig. 3d shows the nearly symmetric plasma column with a hollow pressure profile at $t = 954$. At this point, $q(r) > 0$ everywhere. Note that the radius of the hot ring in the final state is approximately 0.30, suggesting that the mixing radius is determined by the location of the minimum in the original q-profile. In these nonlinear calculations, the crash phase is completed in $\sim 10^3$ poloidal Alfvén times. For large tokamaks such as JET and TFTR, $1\tau_{Hp} \sim 10^{-7}$ seconds, which gives an approximate crash time of $100\mu sec.$ for our simulations. This figure is in agreement with the rapid collapse time for the sawtooth oscillations reported for these machines[5,6]. The nonlinear evolution of the mode is similar for the high-β RMHD model up to the point where a hot ring encloses the cold bubble (Fig.3c). From this point on, the full MHD model exhibits a sloshing motion of the core which was reported earlier in a different context[14]. This oscillatory motion, which is absent in FOUR, are damped by viscosity in the simulations with CTD.

Wesson's suggestion of a "magnetic trigger" for the start of the crash[7] is born out by our calculations. For $\epsilon\beta_p = 0.078$, an equilibrium with $q_0 = 1.01$, and $q_1 = 0.0$, i.e. a flat q-profile with $q > 1$ everywhere, is stable, whereas a similar profile with an off-axis minimum ($q_1 = 0.01, r_{mn} = 0.35$) is unstable with a growth rate of $\gamma\tau_{Hp} = 7.4 \times 10^{-3}$. Thus, after a sawtooth crash that flattens the q-profile and raises $q > 1$ everywhere, the next crash is triggered when a) the hollow pressure profile becomes peaked again due to, for instance, ohmic heating, and b) q drops to $q = 1$ at some radius $r = r_{mn} > 0$, thus destabilizing the pressure driven mode again.

Because of the initial value approach used, we can conclude that the hollow pressure profile obtained at the end of our simulations is stable to further $m = 1, n = 1$ activity, which suggests an obvious stabilization mechanism for these modes: The deliberate introduction and maintainance of hollow pressure profiles by auxiliary heating mechanisms with carefully tailored energy deposition profiles may lead to complete stabilization of these modes and the prevention of saw-tooth oscillations.

In summary, we have studied the $m = 1, n = 1$ pressure driven ideal kink mode in low shear systems with $|1 - q_0| \ll 1$. The mode is found to be unstable for experimentally relevant values of $\epsilon\beta_p$. Its growth rate is much larger than that of the resistive tearing mode under comparable conditions. Nonlinearly, it leads to collapse of the central temperature in approximately $100\mu sec$. Both the crash time, and the nonlinear evolution of the mode seem to be in agreement with those observed in the JET tokamak.

For the present study, we used a high-β reduced MHD code (FOUR) with only toroidal curvature, and a full MHD code (CTD) with only cylindrical curvature. Both models give similar results nonlinearly. For more accurate calculations of the linear growth rates, CTD is being modified to include toroidal curvature. We will report on these fully toroidal, full MHD calculations in the near future.

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Figure Captions

Figure 1. The growth rate of the mode as a function of q_0 for $q_l = 2.5, q_1 = 0, \lambda = 4, \epsilon\beta_p = 0.078$.

Figure 2. The growth rate of the mode as a function of $\epsilon\beta_p$ for $q_0 = 1.01, q_1 = 0.01, \lambda = 4$.

Figure 3. The nonlinear evolution of the pressure field, shown at four different times: a) t=0, b) t=290, c)t=399, d) t=954. Times are normalized to the poloidal Alfvén time.

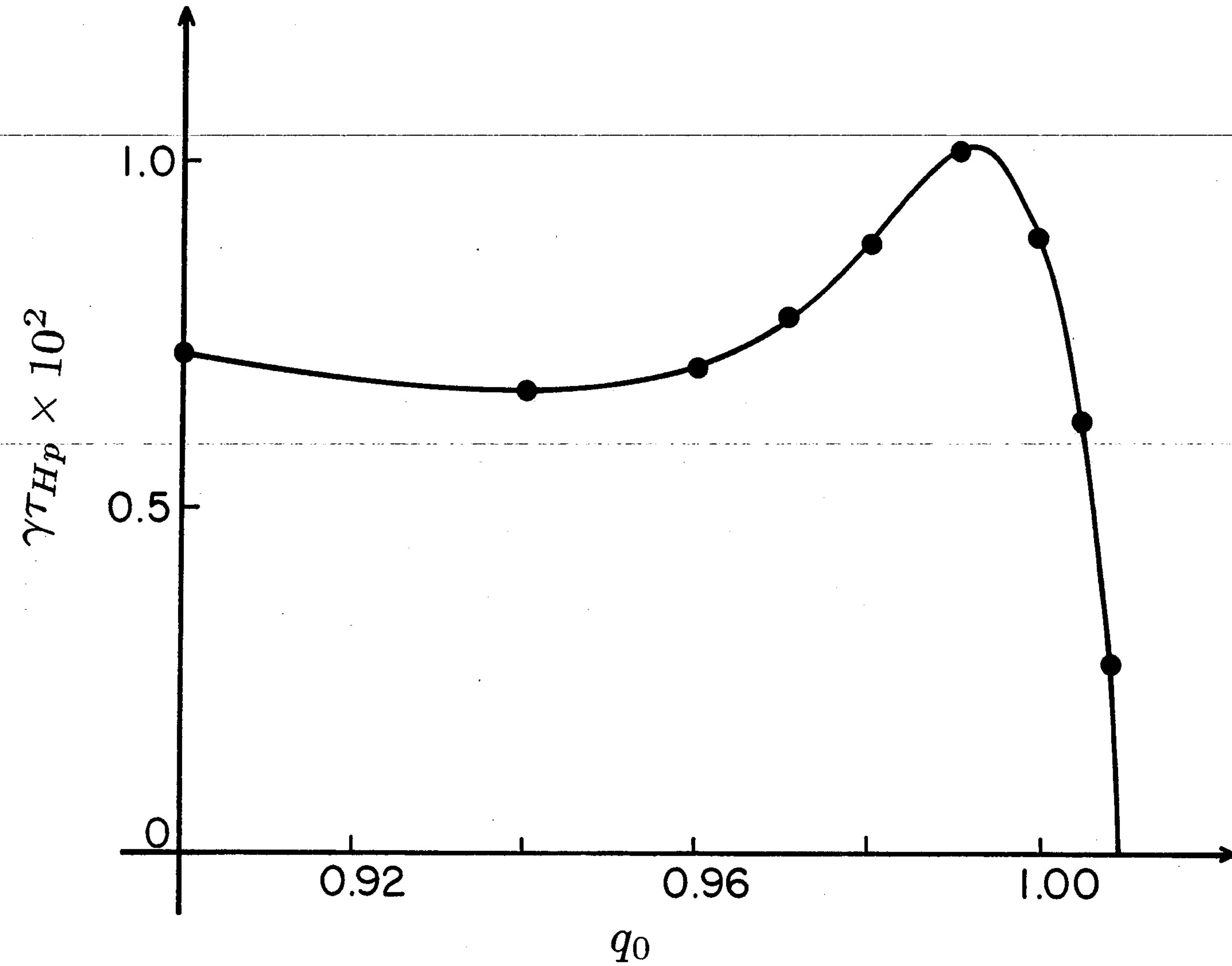
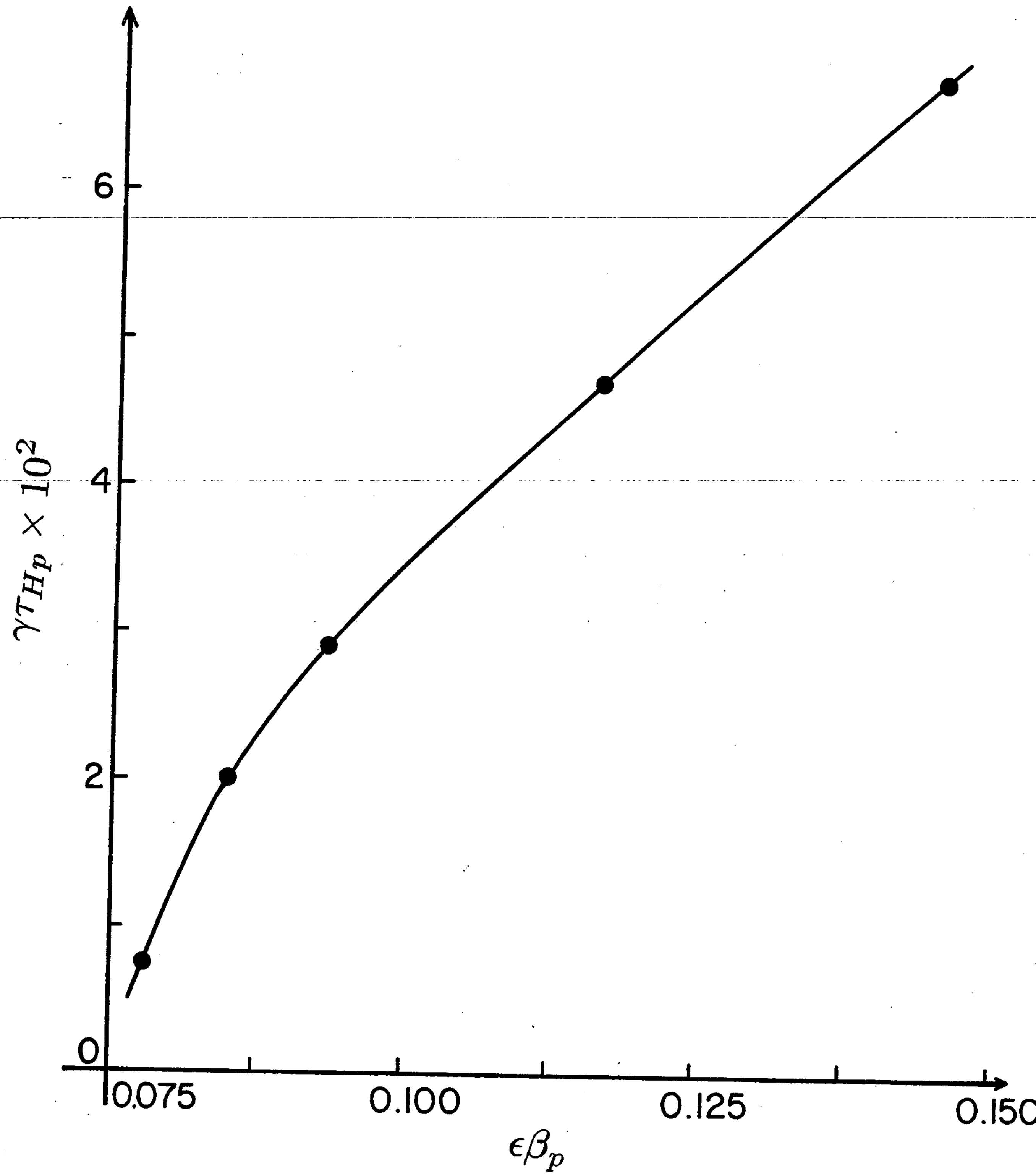


Fig. 1


6
4
2
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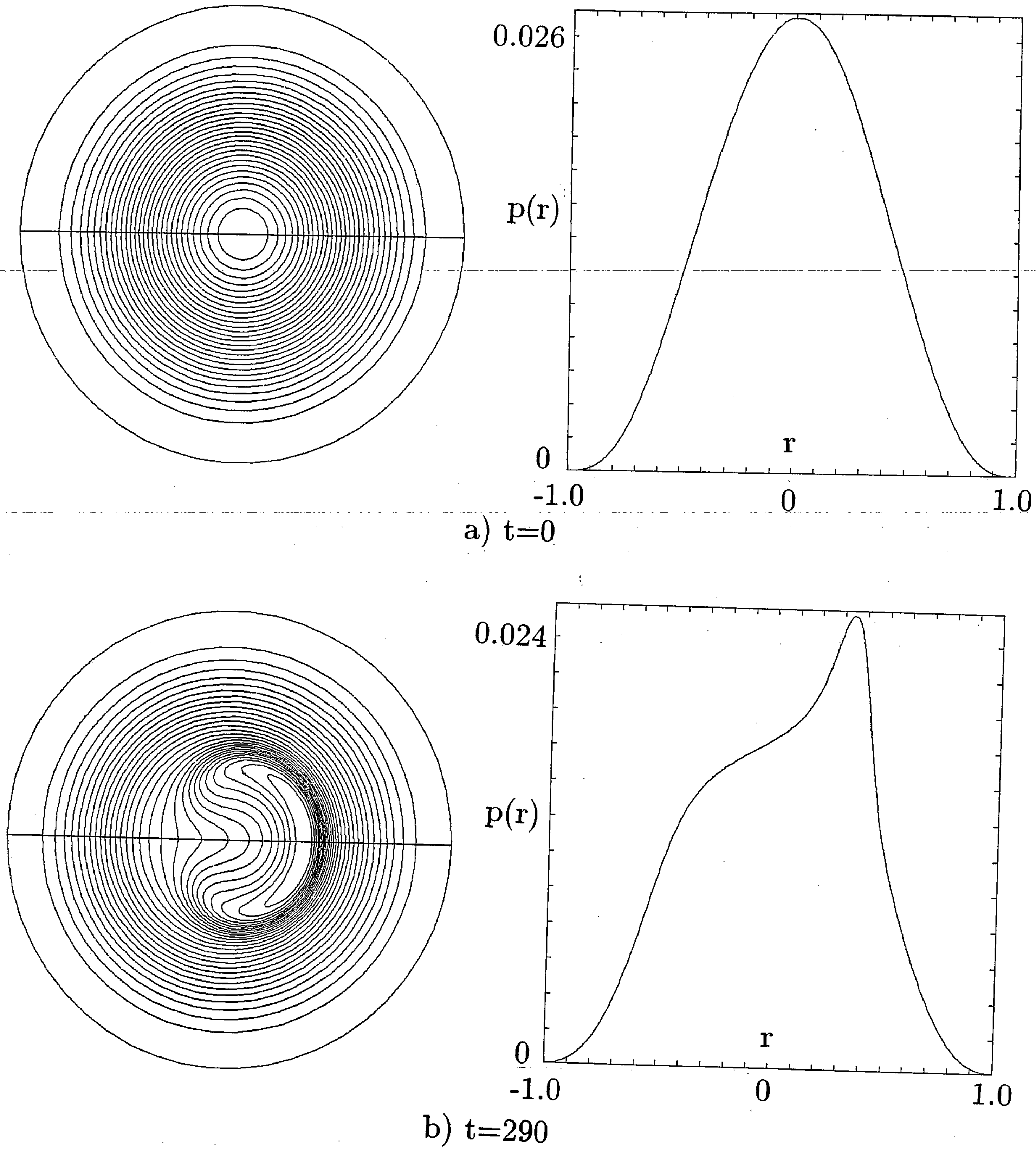
0.125
0.150
$\epsilon \beta_p$

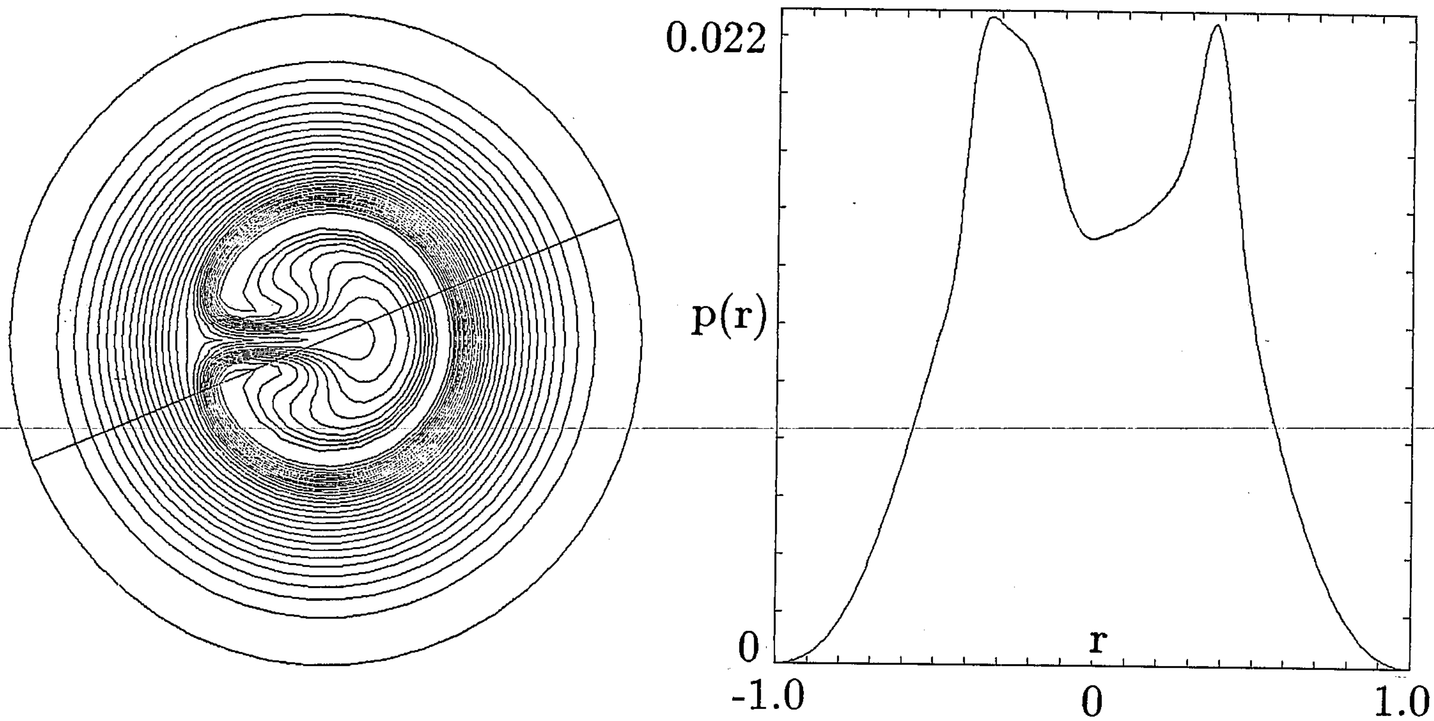
Fig. 2



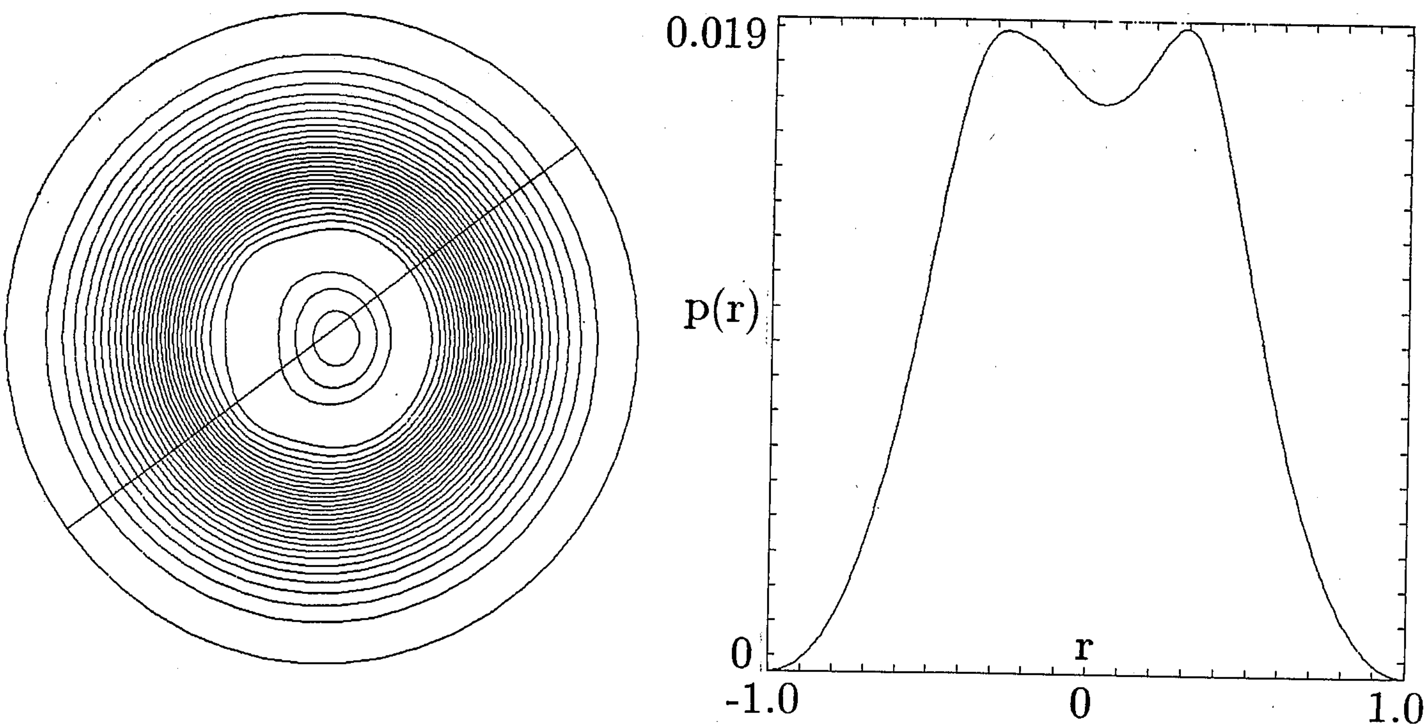
a) t=0

b) t=290

Fig. 3



c) t=399



d) t=954

Fig. 3