DOE/ET-53088-244 IFSR #**244**

Limiter Effects on Scrape-off Layer Fluctuations and Transport

D. R. Thayer, P. H. Diamond, and *A. J. Wootton**
IFS and *FRC
The University of Texas at Austin
Austin, Texas 78712

August 1986

Limiter Effects on Scrape-off Layer Fluctuations and Transport

D. R. Thayer, P. H. Diamond, and *A. J. Wootton**

IFS and *FRC

The University of Texas at Austin

Austin, Texas 78712

Keywords: Limiter, Scrape-off Layer, Turbulence

Abstract

Edge turbulence experiments indicate that radial particle flux increases as a function of radius up to the scrape-off layer (SOL), and that the Boltzman relation is violated. Resistivity gradient driven turbulence (RGDT) theory has been shown to track the radial dependence of the particle flux in the plasma edge closer than dissipative density gradient driven turbulence (DDGDT) theory. Also, the Boltzman relation is not invoked for RGDT while it is usually assumed for DDGDT. Consequently, RGDT is a more likely candidate for an edge turbulence model. However, Langmuir probe experiments indicate that the particle flux is reduced by as much as 50% in the SOL. Thus, since basic turbulence theories do not account for limiter effects, the primary focus of this study is to include such effects in a RGDT theory of the SOL.

We present an analysis of SOL fluctuations using a rippling mode or RGDT calculation which incorporates the essential limiter boundary condition. The line-tying effects caused by a simple conducting poloidal limiter can lead to an order of magnitude reduction of the growth rate in the SOL region when parallel thermal conductivity is neglected. The basic effect of the limiter boundary condition can be understood by exploiting the conjugate relation between the parallel and radial mode widths, a consequence of magnetic shear. A reduction of the connection length, due to the limiter, implies an increase in the radial mode width. Assuming a conducting poloidal limiter with a large radial extent, the increased radial mode width causes an increase in thermal conduction damping which is

Paper presented at the Seventh International Conference on Plasma Surface Interaction, Princeton, NJ, 1986.

Accepted for publication in Journal of Nuclear Materials.

sufficiently large enough so that the rippling mode is rendered stable for typical parallel thermal conductivities. We have also included the destabilizing effect of impurity radiative cooling in the RGDT analysis which leads to an increase of the growth rate, the saturation potential, and the diffusion coefficient by approximately 30% for typical parameters. Additionally, DDGDT is reconsidered as a viable model of SOL turbulence since it exhibits shorter toroidal mode widths than RGDT; thus, it is less sensitive to the limiter line-tying effect. Saturation level estimates for DDGDT with limiter effects are presented.

1. Introduction

There are several pertinent reasons for the study of scrape-off layer (SOL) turbulence. First, a theoretical understanding of SOL turbulence is important since it can aid in an optimized design of limiters. Second, the SOL plasma is a boundary of the central plasma, so that its analysis is necessary for a global confinement study. Third, a challenging feature of the SOL fluctuations is that they are directly measured using Langmuir probes; hence, the good diagnostics in the plasma edge yield comparisons between theory and experiment.

The SOL turbulence is characterized by large density ($\tilde{n}$) and electrostatic potential ($\tilde{\phi}$) fluctuations (10–50%) [1-5] in a highly collisional regime exhibiting a large resistivity coefficient $\left(\eta_0/\mu_0 \approx 10^5 cm^2/\sec\right)$. One good theoretical candidate for the analysis of edge turbulence employs the resistive magnetohydrodynamic (MHD) equations for the study of resistivity gradient driven turbulence (RGDT). The linear part of this analysis, the rippling mode [6], results in fluctuations driven by the resistivity or temperature gradient. A nonlinear analysis [7], which produces a saturated potential amplitude on the order of experimental measurements, is used to balance the resistivity gradient drive with parallel thermal conduction damping. This nonlinear saturation calculation ignores the effect of the limiter; consequently, it is not valid in the SOL. In fact, by using this theory, the potential fluctuations are predicted to increase as the temperature decreases throughout the SOL. This is in direct contrast to the experimentally observed sharp reduction in fluctuations and radial particle diffusion beyond the limiter. Hence, it is important to include the limiter boundary condition effects on the SOL turbulence.

2. Resistivity Gradient Driven Turbulence Analysis

2.1. *Limiter Effects*

The linear rippling or resistivity gradient driven mode analysis [6,7,8], uses reduced resistive MHD equations in cylindrical geometry [9] and the resistivity evolution or heat conduction equation. In the electrostatic approximation, the potential ($\tilde{\phi}$), parallel current $\left(\tilde{J}_z\right)$, and resistivity ($\tilde{\eta}$) fluctuations are coupled through the parallel Ohm's law, parallel vorticity, and resistivity equations. The basic rippling mode instability is driven by the resistivity gradient, damped by the parallel thermal conduction, and radially localized by the magnetic shear. The reader is referred to Ref. 7 for the notations, parameter definitions, and basic equations assumed in the following analysis. The radial linear eigenmode equation for the potential fluctuation, with parallel thermal conduction neglected, results in the dominant growth rate $\gamma_0 = \left(\eta_0^3 J_{z0}^4 L_s^2 m^2 / 16 B_z^2 L_\eta^4 \rho_m r_s^2\right)^{1/5}$.

In this section, the basic rippling mode equation is solved with the limiter boundary conditions imposed. We consider a conducting poloidal ring limiter with a relatively small toroidal extent ($\Delta\phi \ll 2\pi$, where the half toroidal angle between opposite points on the limiter is ϕ_0 so that $\phi_0 \approx \pi$) and a moderate radial extent (Δr, which is typically 3 cm). It is most natural to impose the limiter boundary condition on the potential fluctuation in the coordinate along the field line (ζ) where the mode is tied to the limiter, $\tilde{\phi}(\zeta \approx \pm\phi_0) = 0$. To facilitate this analysis it is useful to employ an eikonal representation, $\tilde{\phi}(r, \theta, \phi) = \sum_{m,n} e^{\gamma t} \exp\left[i(n\phi - m\theta)\right] \int d\zeta \exp\left\{-i\left[n - m/q(r)\right]\zeta\right\} \tilde{\phi}(\zeta)$. Invoking the eikonal transformation of the second order radial rippling mode equation results in a fourth order equation along the field line,

$$0 = \left\{ -\frac{B_z^2}{\eta_0 \rho_m R_0^4} \chi_\parallel \frac{\partial^4}{\partial \zeta^4} + \left[\frac{B_z^2 \gamma}{\eta_0 \rho_m R_0^2} + \chi_\parallel \gamma k_\theta^2 \left(\frac{R_0}{L_s}\right)^2 \frac{1}{R_0^2} \zeta^2\right] \frac{\partial^2}{\partial \zeta^2} \right.$$
$$+ \left[-i \frac{B_z J_{z0}}{\rho_m L_\eta R_0} k_\theta + \chi_\parallel 4 k_\theta^2 \gamma \left(\frac{R_0}{L_s}\right)^2 \frac{1}{R_0^2} \zeta\right] \frac{\partial}{\partial \zeta} \tag{1}$$
$$\left. + \left[-\gamma^2 k_\theta^2 \left(\frac{R_0}{L_s}\right)^2 \zeta^2 + \chi_\parallel 2 k_\theta^2 \gamma \left(\frac{R_0}{L_s}\right)^2 \frac{1}{R_0^2}\right] \right\} \tilde{\phi}(\zeta).$$

Here, the following transformation relations were used: $d/dt \to \gamma$; $\nabla_\parallel \to R_0 \partial/\partial\zeta$; $\nabla_\perp^2 \to -k_\theta^2 \left(R_0/L_s\right)^2 \zeta^2$; $k_\theta = m/r$; and $\partial/\partial\theta \to -im$.

In the absence of parallel thermal conduction $(\chi_\parallel \to 0)$, Eq. (1) reduces to a second order equation, $\left[\partial^2/\partial\zeta^2 - iA\partial/\partial\zeta - B\zeta^2\right]\tilde{\phi}(\zeta) = 0$, where $A = \eta_0 R_0 J_{z0} k_\theta / (B_z L_\eta \gamma)$, and $B = \eta_0 \rho_m R_0^4 k_\theta^2 \gamma / \left(B_z^2 L_s^2\right)$. The free boundary condition solution (dominant Hermite mode, $H_0 = 1$) is $\tilde{\phi}_0(\zeta) = \phi_0 \exp\left[i(A/2)\zeta - \left(\sqrt{B}/2\right)\zeta^2\right]$, where the growth rate $\left(A^2 = 4\sqrt{B}\right)$ is identical to the radial eigenmode solution, $\gamma = \gamma_0$, as it should be.

The rippling mode solution including the limiter line-tying effect is expressed in terms of the dominant (even) mode, $\tilde{\phi}(\zeta) = \tilde{\phi}_0(\zeta) M\left[-\left(A^2/16\sqrt{B}\right) + .25, .5, \sqrt{B}\zeta^2\right]$, where M is a confluent hypergeometric Kummer's function [10], and the limiter boundary condition is expressed as

$$M\left[-\left(A^2/16\sqrt{B}\right) + .25, .5, \sqrt{B}\phi_0^2\right] = 0. \tag{2}$$

For typical parameters, $\sqrt{B}\phi_0^2 \ll 1$, a Bessel function approximation for the root of Eq. (2) can be utilized $(A = \pi/\phi_0)$. It follows that the growth rate with limiter effects is

$$\gamma = \left(\eta_0 R_0 J_{z0} k_\theta / B_z L_\eta\right)\left(\phi_0/\pi\right). \tag{3}$$

The rippling mode growth rate with and without the limiter effects can be compared by using typical Macrotor parameters [10]: $k_\theta \approx m/r_s$; $m \approx 5$; $r_s \approx 35$cm; $L_s \approx 103$cm; $R_0 \approx 90$cm; $a \approx 45$cm; $B_z = 2$kG; $L_\eta \approx 13.5$cm; $\eta_0/\mu_0 \approx 8.22 \times 10^4 cm^2/\sec$; $q(0) \approx 1.3$; $\mu_0 J_{z0}/B_z \approx 2/R_0 q(0)$; $\tau_R \approx 2.5\times10^{-2}$sec; $\chi_\parallel = \bar{\chi}_\parallel R_0^2/\tau_R$; $\bar{\chi}_\parallel = 2.5\times10^5$; $\rho_s \approx .22$cm; and $c_s \approx 4.15 \times 10^6$cm/sec. The free boundary condition growth rate is $\gamma_0 \approx 1.8 \times 10^4 \sec^{-1}$, while the limiter induced boundary condition growth rate is $\gamma \approx 1.3 \times 10^3 \sec^{-1} \approx \gamma_0/14$. Consequently, a single poloidal limiter results in a reduction of the rippling mode growth rate by over an order of magnitude.

The parallel thermal conduction correction of the rippling mode analysis with limiter boundary condition effects is contained in the fourth order parallel coordinate eigenmode equation, Eq. (1). Proceeding with a rigorous analytic calculation in the parallel coordinate is clearly a difficult task. To obtain a solution in the most expedient manner, it is useful to utilize the properties of the eikonal representation in order to transform the parallel limiter boundary condition to an effective radial boundary condition. The eikonal transformation expresses a Fourier transform relationship between the radial and parallel representation,

$\tilde{\phi}(x) = \int d\zeta \exp\left[-i\left(k'_{\parallel}x\right)R_0\zeta\right]\tilde{\phi}(\zeta)$, where $k'_{\parallel} = m/r_sL_s$. Hence, the conjugate parallel-poloidal width relationship is

$$\Delta x = 2L_sr_s/(R_0m\Delta\zeta)\,. \tag{4}$$

The growth rate, including thermal conduction effects, is obtained by utilizing a WKB connection or phase quantization condition. Using the radial eigenmode equation, the turning point, $\mathbf{X}_0 = 2\delta/\left(1+b\mathbf{X}_0^2\right)$, and the boundary condition, $\mathbf{X}_0/2 = \Delta\mathbf{X}$, $\left(\Delta\mathbf{X} = 2L_sr_s/R_0mx_R\phi_0\right)$ imply a solution if $G = 1$, where $G = \mathbf{X}_0/\left[2\delta/\left(1+b\mathbf{X}_0^2\right)\right]$. However, in the limit $b\mathbf{X}_0^2 \gg 1$, $\gamma \to 0$, and for typical parameters $G \to b\mathbf{X}_0^3/(2\delta) = 32q(0)r_sL_\eta\bar{\chi}_{\parallel}/\left(ma^2\phi_0^3\right) \approx 1.6\times10^4 \gg 1$, no solution exists. The rippling mode is found to be stable in the SOL since the large reduction in parallel connection length, due to the presence of the limiter, causes a (conjugate) large increase in radial width and resultant parallel conduction damping. The perpendicular wavenumber spectra will also alter due to the increased radial mode widths. There are two important restrictions on the applicability of this result. First, the instability can survive for a small parallel thermal conductivity case $\left(b\mathbf{X}_0^2 < 1\right)$ where $\bar{\chi}_{\parallel} < 10^3$. Second, the instability will also survive if the finite radial extent of the poloidal limiter is taken into account.

2.2. Radiation Effects

Recently, multifaceted asymmetric radiation from the edge (MARFE) [11] has been observed in tokamaks. In this section, the RGDT is extended to include a large source of radiation from the SOL region (which does not account for asymmetries). Previous calculations [12] have indicated the destabilizing effect of impurity radiation on the coupled set of temperature, fluid velocity, and density fluctuation equations. The following calculation focuses on destabilizing radiative effects on the rippling mode. The derivation proceeds by including an impurity radiative cooling term in the heat conduction equation, $(3/2)n_e dT_e/dt = \kappa_{\parallel}\nabla_{\parallel}^2T_e - (1/2)n_en_zL_z(T_e)$, where the impurity density is n_z, L_z is the radiation rate, and we assume a Spitzer resistivity relation, $T_e = A_0\eta^{-2/3}$. The modified resistivity evolution equation is $\left[d/dt - \chi_{\parallel}\nabla_{\parallel}^2 - \gamma_R\right]\tilde{\eta} = r^{-1}\left(\partial\tilde{\phi}/\partial\theta\right)\left(d\eta_0/dr\right)$, where $\gamma_R = \left(n_z/3\right)\left[5L_z/2T_0 - dL_z/dT_0\right]$, $\chi_{\parallel} = 2\kappa_{\parallel}/3n_e$, and an expansion of the radiation rate about the typical SOL temperature (T_0) has been performed. The additional radiation rate (γ_R) term is destabilizing for temperatures above a critical temperature (T_c), where the slope of the radiation rate is negative with respect to temperature. For typical SOL parameters (carbon impurity, $T_c \approx 5\text{eV}$, $n_z \approx 10^{10}\text{cm}^{-3}$, $n_0 \approx 10^{12}\text{cm}^{-3}$, $T_0 \approx 18\text{eV})\gamma_R \approx 6 \times 10^3\,\text{sec}^{-1} \approx \gamma_0/3$.

In order to calculate the linear growth rate, the radial rippling mode equation is simply altered by changing parameter definitions to include the radiative cooling effect, $\delta \to \delta' = L_s\eta_0 J_{z0}/\left(L_\eta B_z 2x_R\gamma'\right)$, $b \to b' = m^2\chi_\| x_R^2/\left(r_s^2 L_s^2\gamma'\right)$, and $\gamma \to \gamma' = \gamma - \gamma_R$. In the zero parallel thermal conductivity limit, the growth rate is

$$\gamma = \gamma_0/\left(1 - \gamma_R/\gamma\right)^{4/5}. \tag{5}$$

The radiative cooling produces a sizeable growth rate increase (for $\gamma_R/\gamma = 1/4$, or $\gamma_R \approx .32\gamma_0$, then $\gamma \approx 1.3\gamma_0$). For the case of large parallel conductivity $(b'X_0^2 \gg 1)$, the growth rate is only slightly increased due to radiation effects.

In the nonlinear saturated state, the vorticity equation decouples from the resistivity equation and Ohm's law since $\tilde{J}_z \approx 0$ [7]. After the convective derivative has been renormalized in the turbulent regime, the saturation eigenmode equation for the resistivity is $\left[\partial^2/\partial Y^2 + A^{4/3}\left(Y^{-1} + R - Y^2\right)\right]\tilde{\eta} = 0$, where $R = \gamma_R/\bar{\gamma}$, $\bar{\gamma} = \left(L_s\eta_0 J_{z0}/L_\eta B_z\right)^{2/3}\left(\chi_\| k_\|'^2\right)^{1/3} \approx 10^4\,\text{sec}^{-1}$, $R \approx 1/2$, $A = L_s\eta_0 J_{z0}\Delta_{\vec{k}}^c/L_\eta B_z D_{\vec{k}}$, and $\Delta_{\vec{k}}^c = \left(D_{\vec{k}}/\chi_\| k_\|'^2\right)^{1/4}$. The saturation diffusion coefficient is obtained by a phase integral quantization,

$$D_{\vec{k}} = \left(1.34/\kappa^2\right)\left(L_s\eta_0 J_{z0}/L_\eta B_z\right)^{4/3}\left(\chi_\| k_\|'^2\right)^{-1/3} = D_{\vec{k}}^0/\kappa^2, \tag{6}$$

where the κ term contains the radiation effect, $\kappa = (1.34)^{1/2}(\pi/2)/[\int_0^{Y_0} dY(Y^{-1}+R-Y^2)]$, and $Y_0^2 = Y_0^{-1} + R$. The corresponding saturation potential is

$$\left(e\tilde{\phi}/T_e\right) = \alpha 1.23\left(L_s\eta_0 J_{z0}/L_\eta B_z c_s\rho_s k_\theta\right) = \alpha\left(e\tilde{\phi}/T_e\right)_0, \tag{7}$$

where $\alpha = \kappa^{-3/2}$. For no radiation κ, $\alpha = 1$; however, for the typical parameters ($R = 1/2$), the diffusion coefficient and the saturation potential are increased by approximately 30% ($D_{\vec{k}} \approx 1.3D_{\vec{k}}^0 \approx 10^4\text{cm}^2/\text{sec}$, $\left(e\tilde{\phi}/T_e\right) \approx 1.3\left(e\tilde{\phi}/T_e\right)_0 \approx 10\%$).

3. Dissipative Density Gradient Driven Turbulence Analysis

The dissipative density gradient driven turbulence (DDGDT) analysis [13] is based on the density fluctuation equation, $\partial\tilde{n}/\partial t + n\nabla_{\|}v_{\|} + \vec{\nabla}\cdot(n\vec{v}_{\perp}) = 0$, and the parallel velocity equation, $dv_{\|}/dt + \nu_{ei}v_{\|} = v_{Te}^2\nabla_{\|}\left[\left(e\tilde{\phi}/T_e\right) - \tilde{n}\right]$. Here, $\vec{v}_{\perp}$ contains the $\vec{E}\times\vec{B}$ and curvature drift, ν_{ei} is the electron–ion collision frequency, and $\omega_{Te} = v_{Te}/R_0 q$ is the transit frequency.

The saturation potential from a nonlinear renormalized analysis is $\left(e\tilde{\phi}/T_e\right)_0 = \left(.25\rho_s/k_y\rho_s L_n\hat{s}^{3/2}\right)\left(\nu_{ei}c_s/\omega_{Te}^2 L_n\right)^{1/12} \approx .4$, and the turbulent radial diffusion coefficient is $D_0 \sim T_e^{1/6}n^{2/3} \approx 2.4\times10^4\text{cm}^2/\text{sec}$. Here, PreText parameters have been used, $\nu_{ei} \approx 10^6\,\text{sec}^{-1}$, $L_n \approx .5$ cm, $B \approx 8kG$, $c_s \approx 4.9\times10^6$cm/sec, $\rho_s \approx 6.4\times10^{-2}$cm, $R_0 \approx 53.3$ cm, $a \approx 14$ cm, $q(a) \approx 5$, $\hat{s} \approx 1$, $T_e \approx 25$eV, and $k_y\rho_s \approx .1$. The parallel mode-width obtained from this analysis is $\Delta\zeta \approx 2.5\pi$, which is much shorter than for the resistivity gradient driven mode. Consequently, it is expected that the limiter line-tying effect on the dissipative density gradient driven mode should not be as pronounced as for the resistivity gradient driven mode. To obtain a rough estimate of the altered saturated state including the limiter effect, it is useful to replace the connection length ($R_0 q$) free mode width ($\Delta\zeta \approx 2.5\pi$) with the limiter imposed parallel mode width ($\Delta\zeta \approx \pi$). The result is a 10% reduction in the potential fluctuation amplitude and a 30% reduction in radial particle diffusion,

$$\left(e\tilde{\phi}/T_e\right) \rightarrow \left(e\tilde{\phi}/T_e\right)_0(\pi/\Delta\zeta)^{1/6} \approx .34, \quad D \rightarrow D_0(\pi/\Delta\zeta)^{4/3} \approx 7.1\times10^3\text{cm}^2/\text{sec}. \quad (8)$$

4. Conclusions

The stabilizing effect of a conducting poloidal limiter on RGDT has been investigated. If the parallel thermal conduction damping is neglected, then the rippling mode growth rate is reduced by over an order of magnitude for typical parameters in the SOL. However, if parallel thermal conduction is included in the analysis, then the rippling mode is stable. Presently, experiments are being conducted on TEXT with a poloidal limiter and a moveable mushroom limiter (MML) to determine the relationship between connection length in the SOL and the turbulence level. The RGDT can be reduced by using a series of MML's which will further shorten the connection length. However, when a divertor is employed, the turbulence should be reduced near the separatrix since there is very strong shear and the temperature is relatively high. It should also be noted that the density doesn't track the potential for RGDT near the limiter. This is consistent with experiment, although it is not true for DDGDT since the Boltzman relation applies.

Calculations have also been performed which indicate that radiative cooling can lead to a 30% increase in potential fluctuation and diffusion saturation levels for typical parameters. Estimates for the DDGDT saturation levels including the limiter effect show that the particle diffusion and the potential fluctuation are reduced by 30% and 10%, respectively. These finite saturation levels indicate that DDGDT might be a good candidate for a study of SOL turbulence, so that further investigation is warranted.

Acknowledgements

We appreciate the numerous discussions with Ch. P. Ritz. This work is supported by the U. S. Department of Energy grant #DE-FG05-80ET-53088.

References

[1] C. M. Surko and R. E. Slusher, Science **221**, 817 (1983).

[2] S. J. Zweben and R. W. Gould, Nucl. Fusion **23**, 1625 (1983).

[3] S. J. Levinson, et al., Nucl. Fusion **24**, 527 (1984).

[4] Ch. P. Ritz, et al., Phys. Fluids **27**, 2956 (1984).

[5] A. J. Wootton, et al., submitted to Nucl. Fusion.

[6] H. P. Furth, et al., Phys. Fluids **6**, 459 (1963).

[7] L. Garcia, et al., Phys. Fluids **26**, 2147 (1985).

[8] B. A. Carreras, et al., Phys. Fluids **26**, 1231 (1982).

[9] H. R. Strauss, Phys. Fluids **19**, 134 (1976).

[10] **Handbook of Mathematical Functions**, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1972), p. 514.

[11] B. Lipschultz, et. al, Nucl. Fusion **24**, 977 (1984).

[12] T. E. Stringer, JET project JET-DN-T(85) **8**, Abingdon, Oxfordshire, Ox14 3EA, U. K.

[13] P. W. Terry and P. H. Diamond, Phys. Fluids **28**, 1419 (1985).