DOE-ET-53088-240 IFSR#240

AXISYMMETRIC MHD STABLE SLOSHING ION DISTRIBUTIONS

H.L. Berk, *N. Dominguez* and *G.V. Roslyakov**
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712-1060

* Institute of Nuclear Physics
Soviet Academy of Sciences, Novosibirsk, USSR

July 1986

Axisymmetric MHD Stable Sloshing Ion Distributions

Abstract

The MHD stability of a sloshing ion distribution is investigated in a symmetric mirror cell. Fokker-Planck calculations show that stable configurations are possible for ion injection energies that are at least 150 times greater than the electron temperature. Special axial magnetic field profiles are suggested to optimize the favorable MHD properties.

One method that has been proposed for many years to achieve a stable MHD configuration in an axisymmetric mirror machine, is to use a sloshing ion distribution, where the particle pressure can peak away from the mid-plane in a region of favorable curvature.[1,2] In such a system the pressure weighted curvature gives rise to a positive perturbed energy, δW, at least for sufficiently small beta. Hinton and Rosenbluth[2] have shown that appropriate ion distributions form from self-consistent collisional relaxation processes if ion beams are injected with sufficiently high energy. Ryutov[3] has emphasized that if a pressure distribution forms such that $P(B)/B^{3/2}(P(B) = P_\perp(B) + P_\parallel(B)$ with $P_\perp(B)$ and $P_\parallel(B)$ the perpendicular and parallel pressure and B the magnetic field) increases away from the midplane, a magnetic field configuration can be designed to yield arbitrarily good MHD stability properties to flute modes.

With MHD stability possible, it is prudent to quantitatively describe the conditions that achieve stability in an axisymmetric mirror machine. In this article we report results of solving a Fokker-Planck equation that is appropriate for neutral beam injected ions that collide with the background plasma. The collisional model includes the drag and angle scattering terms which are the main components in a collisional process. The bounce average Fokker-Planck equation used is appropriate for a square well model. Such an equation can then be solved in terms of explicit Legendre polynomials, from which the pressure factor $P(B)/B^{3/2}$ can be calculated.

We find that the beam injection energy needs to be greater than 150 times the electron temperature to produce sloshing ion distribution that can be MHD stable, i.e., one where $P(B)/B^{3/2}$ increases significantly away from the midplane. Such a condition appears compatible with existing neutral beam technology for testing this sloshing ion concept in present-day experiments as 20-100 keV ion beams are available and electron temperatures in present day mirror experiments are $\sim$ 100 eV. The stability condition is also compatible with reactor requirements where magnetic fields must be designed to confine 4 MeV alpha particles. However, extensive development of negative ion neutral beam sources is needed to supply the injected ion beam energy.

The flute interchange criterion for stability of an m=1 rigid-like displacement can be written in the paraxial approximation as,

$$\delta W = \int_{B_0}^{B_{\max}} \frac{dB}{B^{3/2}} \frac{dB}{ds} \frac{d}{dB} \left(\frac{P}{B^{3/2}} \right) > 0 \tag{1}$$

where B_0 is the midplane value of B, $B_{\max}$ the maximum value of B, and s the distance along the axis. If $P(B)/B^{3/2}$ can increase between $B_0 < B < B_{\text{eff}}$, then δW can be made arbitrarily large for magnetic field profiles of the form shown in Fig. 1. To be concrete, let us choose

$$B = \begin{cases} B_0, & |s| < s_1, \\ \dfrac{B_0}{\left[1 - \dfrac{(s - s_1)}{2L_1}\right]^2}, & s_1 < |s| < s_2 \\ \dfrac{R_{\text{eff}} B_0}{\left[1 - \dfrac{(s - s_2)}{2L_2}\right]^2}, & s_2 < |s| < s_{\max} \end{cases}$$

where

$$s_2 - s_1 = 2L_1 \left(1 - 1/R_{\text{eff}}^{1/2}\right)$$

$$s_{\max} - s_2 \gg L_2 \quad , \quad R_{\text{eff}} = B_{\text{eff}}/B_0 .$$

For this choice of B field,

$$\frac{1}{B^{3/2}} \frac{dB}{ds} = \frac{1}{B_0^{1/2}} \frac{1}{L_1} \quad , \quad s_1 < s < s_2$$

$$\frac{1}{B^{3/2}} \frac{dB}{ds} = \frac{1}{R_{\text{eff}}^{1/2}} \frac{1}{B_0^{1/2}} \frac{1}{L_2} \quad , \quad s > s_2 .$$

Assuming $P=0$ at $s = s_{\max}$, the condition that Eq. (1) be positive, yields the stability condition,

$$\left[\frac{P\left(B_{\text{eff}}\right)}{R_{\text{eff}}^{3/2}} - P(B_0)\right] \frac{L_2}{L_1} > \frac{P\left(B_{\text{eff}}\right)}{R_{\text{eff}}^{2}}. \tag{2}$$

Thus, if

$$\frac{P\left(B_{\text{eff}}\right)}{R_{\text{eff}}^{3/2}} - P(B_0) > 0. \tag{3}$$

Equation (2) can be satisfied arbitrarily well if L_2/L_1 is made arbitrarily large. The physical conditions that limit the ratio L_2/L_1 are that L_1 be large enough that particle adiabaticity be preserved (roughly this requires $L_1 > 10a_H$, with a_H the hot particle Larmor radius and the smoothing of the axial magnetic field profile at s_1 and s_2) and that L_2 not be too large to avoid excessive volume a high magnetic field.

To calculate the self-consistent pressure profile we consider the steady-state Fokker-Planck equation. In the region, $-s_1 < s < s_1$, the magnetic field is uniform and the square-well Fokker-Planck operator should be a good approximation for describing the steady-state distribution function. Only collisions with the background plasma is considered, and beam-beam collisions are ignored. The Fokker-Planck equation for the beam distribution F_b can then be written as[4]

$$-\frac{1}{U^2}\frac{\partial}{\partial U}\left[(1+U^3)F_b\right] - \frac{A}{2U^3}\frac{\partial}{\partial \xi}\left(1-\xi^2\right)\frac{\partial}{\partial \xi}F_b = \frac{S(U)}{\nu} \tag{4}$$

where

$$V \equiv \text{ particle speed}$$

$$U = V/V_I$$

$$V_I^3 = \frac{3\pi^{1/2}}{4} \frac{m_e}{m_b} V_e^3 \tilde{Z}$$

$$\xi = V_{\|}/V$$

$$\tilde{Z} = \frac{m_b}{n_0} \sum_{j\,(\text{ions})} \frac{Z_j^2 n_j}{m_j}$$

$$A = Z_{\text{eff}}/\tilde{Z} \,, \quad Z_{\text{eff}} = \sum_j Z_j^2 \; n_j/n_0$$

$$n_0 \equiv \text{ background electron density}$$

$$n_b \ll n_0 \text{ (is assumed)}$$

$$m_b \equiv \text{ beam particle mass, } m_j \equiv \text{ion mass of species } j$$

$$\frac{1}{2} \, mV_e^2 = T_e \equiv \text{ electron temperature}$$

$$\nu = 4\pi n_0 e^4 \tilde{Z} \ln \Lambda / m_b^2 V_I^3$$

$$S = \frac{S_0}{V_I^3 U_b^2} \delta(U - U_b) \sum_{\ell=0}^{\infty} (2\ell + 1/2) P_{2\ell}(\xi) P_{2\ell}(\xi_0) \left(\frac{1 + \frac{1}{U_{b\text{eff}}^3}}{1 + \frac{1}{U_b^3}} \right)^{\frac{A\ell(2\ell+1)}{3}}$$

$$\xrightarrow[\alpha_0 \gg 1]{} \frac{S_0}{V_I^3 U_b^2} \frac{\alpha_0}{\sqrt{\pi}} \delta(U - U_b) \left\{ \exp\left[-\alpha_0^2 (\xi - \xi_0)^2\right] + \exp\left[-\alpha_0^2 (\xi + \xi_0)^2\right] \right\}$$

$$\alpha_0^{-2} = \frac{2A\left(1 - \xi_0^2\right)}{3} \ln \left[\frac{1 + \frac{1}{U_b^3}}{1 + \frac{1}{U_{b\text{eff}}^3}} \right] . \tag{5}$$

(In our calculation we choose $U_{b\text{eff}}^3$ so that $\alpha_0^{-1} = .03$ and $A = \tilde{Z} = 1$; $n_j = n_i = n_0$). The solution for F_b is

$$F_b = \frac{S_0}{V_I^3 \nu} \sum_{\ell=0}^{\infty} \frac{(2\ell + 1/2) P_{2\ell}(\xi) P_{2\ell}(\xi_0)}{1 + U^3} \left(\frac{1 + 1/U_{b\text{eff}}^3}{1 + 1/U^3} \right)^{A\ell(2\ell+1)/3} . \tag{6}$$

To obtain the quantity $P(B) = P_\perp(B) + P_{\|}(B)$, we evaluate numerically the following integral

$$\hat{P}(B) \equiv \frac{P(B)}{\pi m_b V_I^2 S_0} = 2V_I^3 \frac{\nu}{S_0}\frac{B}{B_0}\int_0^{U_b} dU U^4 \int_{(1-B_0/B)^{1/2}}^{1} \frac{d\xi \xi \left[1 - \frac{1}{2}\frac{B}{B_0}(1-\xi^2)\right] F_b(U,\xi)}{\left[1 - \frac{B}{B_0}(1-\xi^2)\right]^{1/2}}. \tag{7}$$

The evaluation of this integral has been performed numerically. The critical expression for stability $\hat{P}(B)/B^{3/2}$ is shown in Figs. 2a and 2b below for various angles and energy of injection. In Table I we list the peak values in ratio $\frac{P(B_{\rm eff})}{R_{\rm eff}^{3/2} P(B_0)}$ that one extracts from the curves shown as well as for other curves we have examined.

We note that for $U_b \geq 3$ we can achieve MHD stability for if $\frac{L_2}{L_1} \gtrsim 5$. We consider this a practical conservative limit on the design of a stable configuration (although one should not rule out that one may be able to make L_2/L_1 even larger). For such a limit one requires, if $m_e/m_b = 1/3600$, an injection energy 150 times the electron temperature. In present day experiments, where $T_e \approx$100eV, we require injected ion beam energies of order 20keV, which is quite compatible with present-day technology. For a reactor, where $T_e \approx 10 - 20$keV, we would require injected beam energies of 1.5-3MeV.

Many additional theoretical factors need further consideration to assess the viability of the sloshing ion method proposed here. A set of possible problems that need further study, with brief comments are listed below.

1. Plug or Central Cell Sloshing Distributions

If this method of stabilization is used in the plugs of the tandem mirror, one is susceptible to trapped particle instabilities.[5] Criteria developed for stabilizing trapped particle modes would then have to be fulfilled. It is possible to use this sloshing particle stabilization method in the central cell. However, in a reactor this method has the limitation that the Q (fusion energy output/energy input) would then be comparable to unity as a significant fraction of the stored energy would be in the hot particles, which are slowing down in the background electron bath. For example, if the background fusion plasma has a nearly isotropic pressure, P_c, then MHD stability given in Eq. (2) requires

$$\frac{P(B_{\text{eff}}) + 2P_c}{R_{\text{eff}}^{3/2}} > P_0 + 2P_c$$

which requires

$$\frac{2P_c}{P(B_{\text{eff}})} < \frac{\left(\frac{1}{R_{\text{eff}}^{3/2}} - P_0/P_{\text{max}}\right)}{\left(1 - 1/R_{\text{eff}}^{3/2}\right)} .$$

Thus, $2P_c$ is comparable to $P(B_{\text{eff}})$, and the stored energy in the hot particles are appreciable. Hence, the loss rate due to drag of these particles can dominate the confinement time. The tandem mirror system would then lose the advantages of confinement that it is designed for. Hence, it seems probable that this stabilization method should be used to stabilize in a separate plug region, where the stored energy of the high energy component can be maintained at a small fraction of the plasma energy of the entire tandem mirror.

2. Decoupling of Beam Particle Dynamics

Our discussion assumed that the beam particles respond according to MHD criteria. However, for sufficiently energetic beams the hot particle dynamics decouple from the core plasma for sufficiently low core beta[6] $(\beta_c \lesssim r^2/L_p^2)$ where L_p is the overall length of the plug. If this happens it could be difficult to stabilize the core pressure component. To prevent such decoupling it is necessary that roughly[7]

$$\frac{n_H}{n_c + n_H} \frac{L_p L_t}{a_H^2} \gtrsim 1$$

where L_t is the overall length of the tandem mirror and a_H the hot particle Larmor radius.

3. Anisotropy Instability

Anisotropy instabilities such as the mirror mode,[8] Alfvén ion cyclotron instability,[9] two component plasma instabilities[10] and loss cone modes[11] need to be investigated further to determine if there is a compatability of all the requirements for satisfying the constraints imposed by adiabaticity, mirror mode and Alfvén ion cyclotron instability. With sloshing ions we expect a broader regime of compatability than found in Ref. 12, as in the present problem instability is guaranteed at low beta.

4. Magnetic Field Shifts

Shifts in the magnetic field profile due to finite beta may be important, especially since stability is sensitive to the magnetic field shape at the peak of the pressure profile.

5. Charge Exchange Effects

The sloshing pressure profile can be improved with charge exchange loss of the hot particles at a rate that is comparable or faster thant the slowing down time. Though this effect improves the MHD parameters, it produces more anisotropic distributions, thereby making the system more susceptible to microinstability. To determine the optimal state needs further study.

We see there are many factors that need further study. If these points are satisfactorily resolved, it appears that the formation of high energy sloshing ion distributions can be a viable basis for building an open-ended tandem mirror system.

We would like to acknowledge useful discussions with Prof. M.N. Rosenbluth, Dr. J.W. Van Dam and D.D. Ryutov.

H.L. BERK and N. DOMINGUEZ
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712-1060

G.V. ROSLYAKOV
Institute of Nuclear Physics
Soviet Academy of Sciences,
Siberian Branch
Novosibirsk 6030090 - U.S.S.R.

References

1. M.N. Rosenbluth and C. Longmire, Ann. Phys. **1**, 120 (1957).
2. F.L. Hinton and M.N. Rosenbluth, Nucl. Fusion **22**, 1547 (1982).
3. I.A. Kotlnikov, G.V. Roslyakov and D.D. Ryutov, "Stabilization of Flute Modes in Axisymmetric Open Traps with Sloshing Ions", Preprint - Institute of Nuclear Physics, Novosibirsk, 1986, Report#86-31 (to be published in *Fizika Plasmy*).
4. H.L. Berk, W. Horton, Jr., M.N. Rosenbluth, and P.H. Rutherford, Nuclear Fusion **15**, 819 (1975).
5. H.L. Berk, M.N. Rosenbluth, H.V. Wong, T.M. Antonsen, and D.E. Baldwin, Fiz. Plasmy **9**, 176 (1983); (Soviet J. Plasma Phys. **9**, 108 (1983).
6. J.W. Van Dam and Y.C. Lee in *Proc. of the EBT Ring Physics Workshop* (ORNL, Oak Ridge (1979) Conf. 791228, p. 471.
7. N.A. Krall, Phys. Fluids **9**, 820 (1966).
8. H. Grad, Phys. Fluids **10**, 137 (1967).
9. R.C. Davidson and J.M. Ogden, Phys. Fluids **18**, 1045 (1975).
10. L.D. Pearlstein and D.B. Cheng **9**, 953 (1966).
11. R.F. Post and M.N. Rosenbluth **9**, 730 (1966).
12. G.R. Smith, H.L. Berk and L.L. LoDestro, Phys. Fluids **29**, 798 (1986).

Figure Captions

1. Schematic magnetic field profile needed to optimize MHD-flute stability.
2. Curves of $\hat{p}(B)/B^{3/2}$ at different pitch angles, and speeds of injection.

Table I. Peak Values in Ratio $P(B_{\text{eff}})/R_{\text{eff}}^{3/2}P(B_0)$ at different injection speeds.

TABLE I

Peak Values of Ratio $\frac{P(B_{\text{eff}})}{R_{\text{eff}}^{3/2} P(B_0)}$

U_b	2	3	4	5	8
R_{eff}					
1.05	1.02	1.10	1.20	1.33	1.55
1.1	1.05	1.16	1.28	1.42	1.68
1.2	1.07	1.19	1.32	1.45	1.72
1.3	1.06	1.17	1.30	1.41	1.68
1.4	1.04	1.14	1.25	1.36	1.61
1.5	1.03	1.11	1.21	1.31	1.55

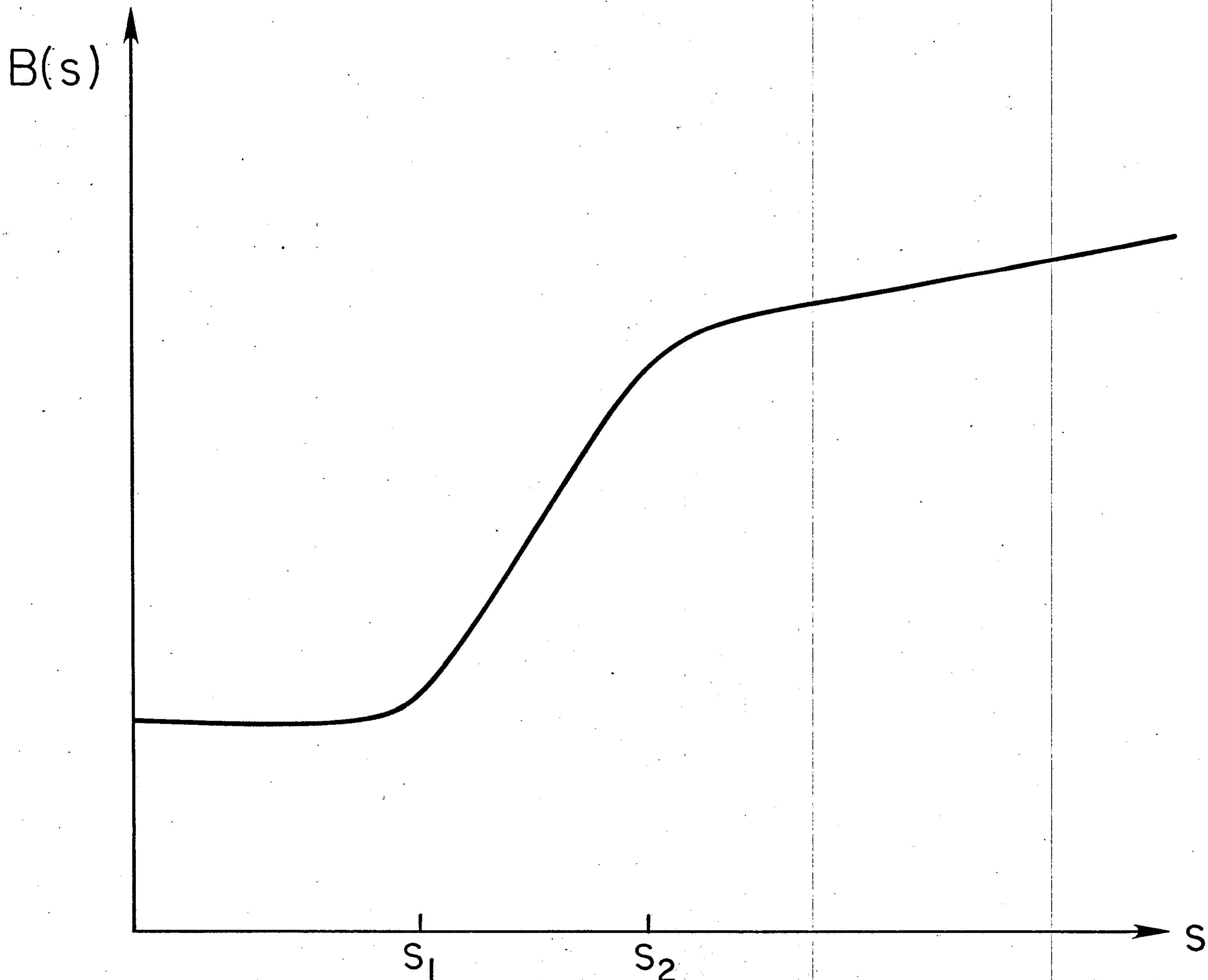
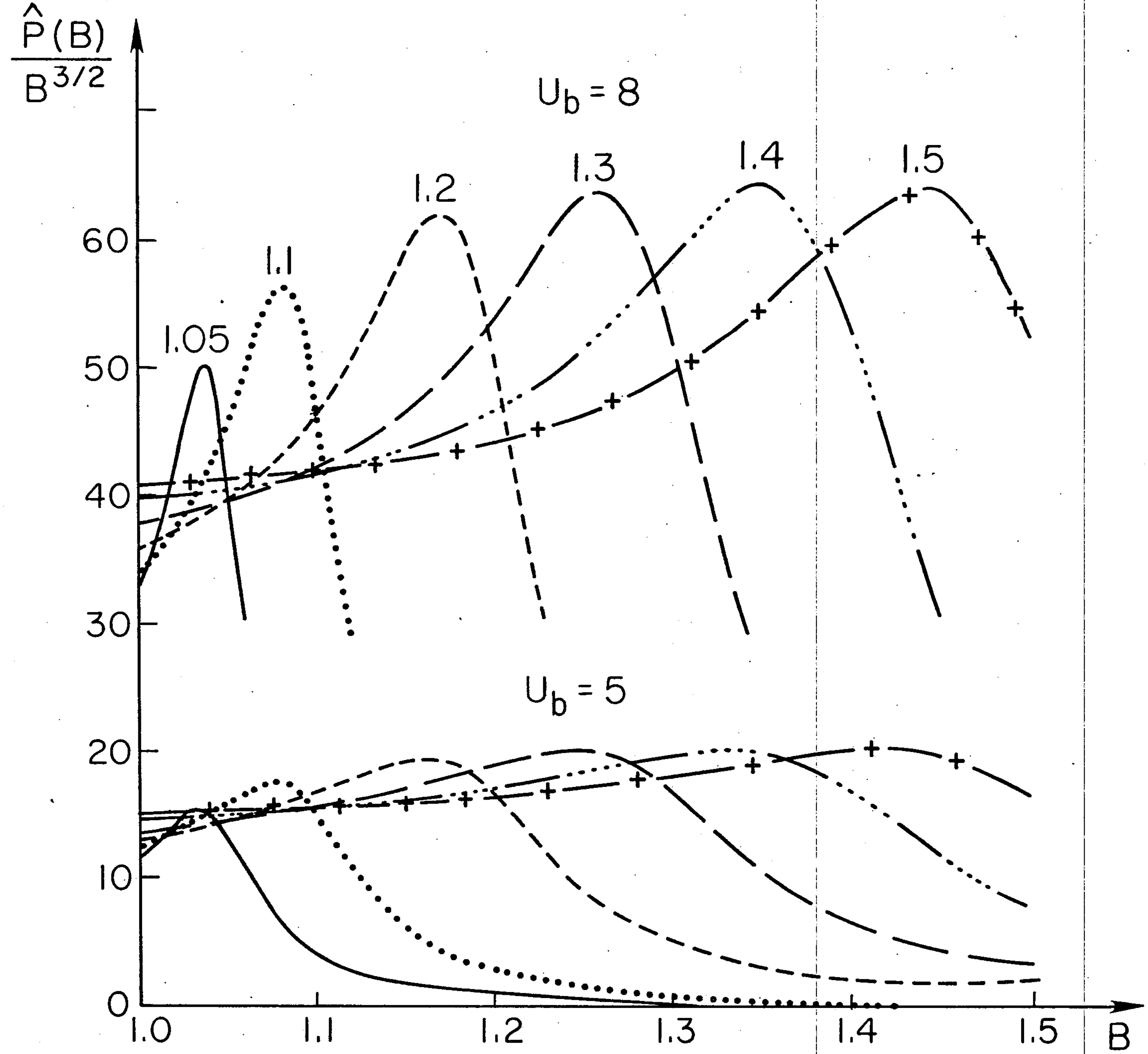


Fig. 1



P̂(B)/B^3/2
U_b = 8
1.05
1.1
1.2
1.3
1.4
1.5
U_b = 5
60
50
40
30
20
10
0
1.0
1.1
1.2
1.3
1.4
1.5
B

Fig. 2(a)

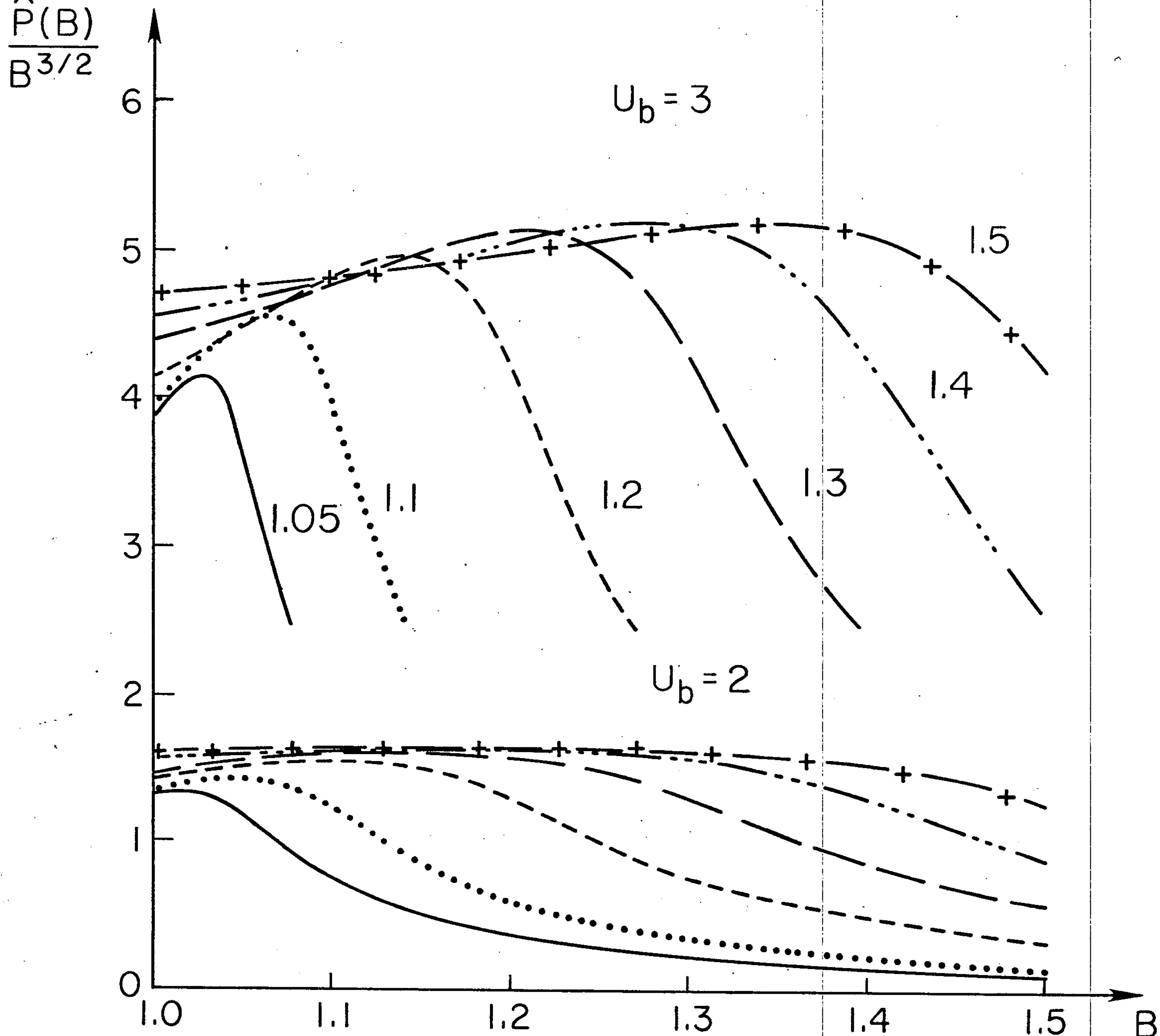

$\frac{\hat{P}(B)}{B^{3/2}}$
$U_b = 3$
$U_b = 2$
1.05
1.1
1.2
1.3
1.4
1.5
0
1
2
3
4
5
6
1.0
1.1
1.2
1.3
1.4
1.5
B

Fig. 2(b)