PROCEEDINGS OF
US-JAPAN WORKSHOP
MAGNETIC RECONNECTION
DECEMBER 10-13, 1984
AUSTIN, TEXAS
FORWARD

The Magnetic Reconnection Workshop held at the University of Texas at Austin, December 10–13, 1984 was organized by the Institute for Fusion Studies, in connection with the Joint Institute for Fusion Theory, to exchange information in theoretical plasma physics. There were approximately thirty-five scientists participating which included six visitors from Japan. There were eight sessions with formal presentations on key issues related to magnetic reconnection.

The main goal of the workshop was to exchange recent results and ideas between theorists, computer simulationists, and experimentalists. Leading scientists representing these three categories made presentations in their respective areas. Each session included stimulated interaction between the speaker and his audience.
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T. Sato – 3D Simulations of Energy Relaxation Processes

R. Nebel – Flux Regeneration in ZT-40M at High Theta and the MHD Model

D. Spicer – Reconnection and Its Role in Solar Flares

J. Holmes – Nonlinear Coupling of Tearing Modes In The Reversed Field Pinch And The Tokamak Disruption

H. Strauss – RFP: Mean Field MHD Model

T. Amano – Nonlinear Evolution of Tearing Modes in Reversed Field Pinches

W. Park – Magnetic Reconnection in Tokamaks

R. Pellat – Half Coalescence of Nonlinear M=1 Mode

T. Jensen – A Sawtooth Oscillation Model

S. Kim – Soft X-Ray Imaging of Transient and Double Sawteeth on TEXT

A. Aydemir – Half Coalescence of the M=1 Island in Tokamaks

R. Kleva – Are Vacuum Bubbles a Cause of Major Disruptions?

Y. Kondoh – Energy Principle for Toroidal Plasma with Boundary

T. Hayashi – Spheromak and Magnetic Reconnection

R. Moses – Kinetic Theory of Driven Reconnection at an X-Point

G. Kurita – Bubble Formation Due to Surface Tearing Modes

R. Steinolfson – Reverse Flow Vortices and Current Sheets in Nonlinear Tearing

B. McNamara – Lie Transform Approach to Drift-Resonant Interactions in Fluid and Plasma Turbulence

M. Kotschenreuther – Resistive Dynamics of Magnetic Islands with Curvature and Pressure

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MAGNETIC RECONNECTION

December 10-13, 1984
Institute for Fusion Studies
Austin, Texas

Sponsored By
US-Japan Joint Institute for Fusion Theory

Agenda

Monday, December 10, 1984

8:20 AM Registration

8:50 AM Opening Remarks: P. Diamond

Morning Session Chairman: A. Bhattacharjee

9:00-9:45 AM T. Sato, "3D Simulations of Energy Relaxation Processes"

9:55 AM Break

10:15-11:00 AM R. Nebel, "Flux Regeneration in ZT-40M at High θ and the MHD Model"

11:10-11:55 AM D. Spicer, "Reconnection and Its Role in Solar Flares"

12:05 PM Lunch

Afternoon Session Chairman: T. Sato

2:00-2:40 PM J. Holmes, "Nonlinear Coupling of Tearing Modes in the Reversed Field Pinch and the Tokamak Disruption"

2:50-3:30 PM H. Strauss, "RFP: Mean Field MHD Model"

3:40 PM Break

4:00-4:40 PM T. Amano, "Nonlinear Evolution of Tearing Modes In Reversed Field Pinches"
Tuesday, December 11, 1984

Morning Session  
Chairman: P. Diamond

9:00-9:45 AM  
W. Park, "Magnetic Reconnection in Tokamaks"

9:55 AM  
Break

10:15-11:00 AM  
R. Pellat, "Half Coalescence of Non-linear m=1 Mode"

11:10-11:50 AM  
T. Jensen, "A Sawtooth Oscillation Model"

12:00 noon  
Lunch

Afternoon Session  
Chairman: R. Kulsrud

2:00-2:30 PM  
S. Kim, "Soft X-ray Imaging of Transient and Double Sawteeth on TEXT"

2:40-3:10 PM  
A. Aydemir, "Half Coalescence of the m=1 Island in Tokamaks"

3:20 PM  
Break

3:30-4:10 PM  
R. Kleva, "Are Vacuum Bubbles a Cause of Major Disruptions?"

4:20-5:00 PM  
Y. Kondoh, "Energy Principle for Toroidal Plasma With Boundary"
Wednesday 12, 1984

Morning Session  
9:00-9:45 AM  T. Hayashi, "Spheromak and Magnetic Reconnection"
9:55 AM  Break
10:15-11:00 AM  R. Moses, "Kinetic Theory of Driven Reconnection At An X-Point"
11:10-11:50 AM  G. Kurita, "Bubble Formation Due to Surface Tearing Modes"
12:00 noon  Lunch

Afternoon Session  
2:00-2:40 PM  R. Steinolfson, "Reverse Flow Vortices and Current Sheets in Nonlinear Tearing"
2:50-3:20 PM  B. McNamara, "Lie Transform Approach To Drift-Resonant Interactions in Fluid and Plasma Turbulence"
3:30 PM  Break
3:50-4:20 PM  M. Kotschenreuther, "Resistive Dynamics of Magnetic Islands With Curvature and Pressure"
4:30-5:00 PM  A. Bhattacharjee, "Relaxation of Toroidal Plasmas"

7:00 PM  BANQUET DINNER  
The Old Pecan Street Cafe  
(Transportation provided)
Thursday, December 13, 1984

Morning Session  
Chairman:  

9:00-9:45 AM  
J. Mizushima, "Statistical Properties of MHD Turbulence"

9:55 AM  
Break

10:15-11:00 AM  
D. Schnack, "Relaxation, Turbulence, and Dynamo Action In Driven Systems"

11:10-11:50 AM  
S. Cowley, "The Dynamics of Tearing Modes"

12:00 noon  
Lunch

Afternoon Session  
Chairman:  R. Moses

1:30-2:00 PM  
J.N. Leboeuf, "Computer Modeling of Fast Collisionless Reconnection"

2:10-2:40 PM  
T. Tajima, "Explosive Coalescence"

2:50 PM  
M.N. Rosenbluth, Concluding Remarks
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3D SIMULATIONS OF ENERGY RELAXATION PROCESSES

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\[
\frac{E \times B}{\mu_0} \quad \text{magnetic energy}
\]

\[
\nabla p \cdot v \quad \text{thermal energy}
\]

\[
\frac{1}{2} \rho v^2 \quad \text{kinetic energy}
\]

\[
\eta J^2
\]

\[
\frac{\gamma p v}{\gamma - 1}
\]
Energy Relaxation

\[ \Delta W = \text{thermal energy} \]

\[ W_1 \quad W_2 \]

\[ \tau_R \quad \tau_D \]

time

\[ \tau_R \ll \tau_D \]

What can cause rapid energy dissipation?

1. Turbulence \( \rightarrow \tau_R = R^2 \Omega \) (large \( R \))

2. Driven Reconnection \( \rightarrow \tau_R = \eta J^2 \) (large \( J \))
Driven Reconnection

- Initially no anti-parallel configuration is required.
- Plasma convection creates anti-parallel configuration.

\[ \text{Cause: } \text{Converging plasma flows } V_0 \]

or convection electric field \( E_D \)

\( E_D = -V_0 \times B \)

Effect: Generation of neutral sheet current \( J_{NS} \)

\( J_{NS} = \frac{E_D}{\eta} \)

Enhancement of local dissipation

\[ \eta J_{NS}^2 = \frac{E_D^2}{\eta} \]

( inversely proportional to \( \eta \) )
Dissipation Rate

\[ \tau_R^{-1} = \int \eta J_{ns}^2 dV \propto \eta J_{ns}^2 d \]

\[ \eta J_{ns} = E_d \]

\[ \mu_0 J_{ns} = \frac{B}{d} \]

\[ \therefore \tau_R^{-1} \propto \frac{E_d B}{\mu_0} = \text{indep. of } B, \eta \]
**Single helicity kink instability**

\[ \psi(r, \theta, z) = \psi_0(r) + \psi_1(r) \sin(\theta - \frac{2\pi}{L} z) \]

\[ B_{10} \propto \frac{\partial \psi_1(r)}{\partial r} \sin(\theta - \frac{2\pi}{L} z) \]

\[ B_{1r} \propto \psi_1(r) \cos(\theta - \frac{2\pi}{L} z) \]

**Toroidal flux converted from** \( B_{1r} \)** through driven reconnection**

\[ \Delta \chi^- = 2\pi \int_0^1 a B_{1r} \, dz = -\frac{\psi_1(a)}{\pi} \]

\[ \Delta \chi^+ = 2\pi \int_0^1 a B_{1r} \, dz = \frac{\psi_1(L)}{\pi} \]

\[ (\Delta \chi^- + \Delta \chi^+ = 0) \]
m = 1 helical kink instability

axial plasma flows
nonlinear distortion
non-linear driven reconnection
generation of \( B_r \)
self-reversal

Basic flow pattern induced by a helical kink instability on a toroidal plane

Helically distorted magnetic field configuration
Radial fields induced by nonlinear development of a helical kink mode

Top view of nonlinearly deformed poloidal fields
SIMULATION STUDY OF
REVERSED FIELD GENERATION
AND MAINTENANCE IN THE RFP

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Initial Conditions

Force-free equilibrium

\[ \mu B = \text{rot } B, \quad B_r = 0, \]

where

\[ \mu = \begin{cases} 
\lambda \text{ (const.)} & 0 < r < \xi \\
\frac{\lambda}{2} \left[ \cos \left( \pi \frac{r-\xi}{a-\xi} \right) + 1 \right] & \xi < r < a 
\end{cases} \]

\[ F = 0 \]

\[ \Theta = 1.73 \]
Imposed Perturbations at $t=0$

$$\delta v_{m,n} = \delta v_0 \exp i \left( m\theta - 2\pi nz/L \right)$$

<table>
<thead>
<tr>
<th></th>
<th>m</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>case 1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>case 2</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>case 3</td>
<td>1</td>
<td>4~7</td>
</tr>
</tbody>
</table>

![Graph showing values of q for different n values]
Simulation Model

3-D Cylindrical MHD Code

Dimensions

\[ a = 0.1 \text{ m} \]
\[ L = 1.5 \text{ m} \]

Grid points

\[ r \quad 44 \]
\[ \theta \quad 16 \]
\[ z \quad 160 \]

Boundary conditions

Equations (zero-\( \beta \) model)

\[ \frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \mathbf{v}) \]

\[ \frac{\partial \rho \mathbf{v}}{\partial t} = - \nabla \cdot [\rho \mathbf{v} \mathbf{v} - \frac{1}{\mu_0} (\mathbf{B} \mathbf{B} - \frac{B^2}{2} \mathbf{I})] \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \mathbf{J}) \]

\[ \mu_0 \mathbf{J} = \nabla \times \mathbf{B} \]

\( \eta \): resistivity (constant and uniform)
Radial \( q \)-profile,
Axial profiles of the radial field
and the axial plasma flow
(a) log_{10}K

(b) F

(c) |Φ_N/Φ|

Time

Non-resonant case
Resonant case
Multi-helicity case
temporal evolutions of Fourier amplitudes
of kinetic energy

non-resonant case

multi-helicity case
contours of negative toroidal field

in poloidal section for non-resonant case

- Time: 12.5
- Time: 22.5
- Time: 32.5
- Time: 42.5
- Time: 52.5
- Time: 62.5
- Time: 72.5
- Time: 82.5
contours of negative toroidal field

in poloidal section for multi-helicity case

time: 12.5  22.5  32.5  42.5  52.5  62.5  72.5  82.5
temporal evolutions of generated reversed flux for several single helicity cases

Current is not sustained.

\[ \frac{|\Phi_N|}{\Phi} \]

\( S=5000 \)
\( S=1000 \)

Current is sustained.

\[ \frac{|\Phi_N|}{\Phi} \]

\( S=1000 \)
\( S=5000 \)
Simulation of Self-Reversal in RFP
Simulation of Spheromak Merging

\[ W = W_1 + W_2 \]  \quad \text{Magnetic Energy}

\[ K = K_1 + K_2 \]  \quad \text{Helicity}

\[ V = V_1 + V_2 \]  \quad \text{Kinetic energy}

Different
scenario of setting-up process in the RFP

- Current rising
  - Growth of $m = 1$
    - $n \approx n_0 \pm 1$
  - Growth of $m = 1$
    - $n = n_0$

  *Mode coupling*

  - Growth of $m = 0$
    - $n \approx 1$

- Non-reversal pinch configuration
  - Generation of reversed field (self-reversal)
  - Axisymmetrization

- Axisymmetric RFP configuration

Single Helicity Case

Helical RFP configuration
MAGNETOHYDRODYNAMIC SELF-ORGANIZATION

IN THREE DIMENSIONS

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Initial Field Pattern in 3D

NO. H08

T = 0

MAX = 1.439

BXY (J = 20)

MAX = 1.069

BXZ (I = 25)

BZY (L = 20)

MAX = 1.335

MAX = 1.376
FLUX REGENERATION IN ZT-40M AT HIGH $\Theta$

AND THE MHD MODEL

R. NEBEL

LOS ALAMOS NATIONAL LABORATORY
Flux Regeneration in ZT-40M at High $\theta$ and the MHD Model

R. Nebel

K. Werley
Conclusions

1. Any toroidally asymmetric flux regeneration mechanism must arise from the $m=0$ component of the dynamics.
   - torus → periodic cylinder
   - quadratically nonlinear
   - Ohm's law

2. The flux regeneration mechanism in ZT-40M at high $\Theta$ is toroidally asymmetric.

3. $m=0$ single mode MHD instabilities always oppose flux regeneration. (anti-dynamo)
Conclusions cont.

4. $m=0$ magnetic perturbations driven nonlinearly by multi-helicity $m=1$ tearing modes can qualitatively produce toroidally asymmetric flux regeneration, but it usually is smaller than the symmetric component.
   - Algebraically: lowest order interactions
   - Numerically: 3D simulations.

5. Hall effects may couple the spectrum in such a way as to enhance the asymmetric component of the toroidal flux regeneration mechanism.
   - Speculative, only phasings have been computed, signs and magnitude are unknown.
   - Island distortions and toroidal phasings of $m=1$ and $m=0$ peaks arise from these terms
1. Multiple helicity $m=1$ modes driven unstable by diffusion.

2. Nonlinear beatings of the $m=1$ modes cause the $m=0, n=1$ mode's "O" point to become a flux source.

3. Magnetic pressure from the "O" point causes $v_T$ to carry flux from the "O" point to the "X" point where it reconnects to form equal and opposite amounts of positive and negative flux.

4. Hall terms may enhance this process as well as distort the $m=0$ island.
Toroidal Behavior of Flux Regeneration

Assume (for convenience) a resistive MHD ohm's Law:

\[ \vec{E} + \nabla \times \vec{B} = \eta \vec{J} \]  \hspace{1cm} (1)

Expand in Fourier series (periodic cylinder):

\[ \vec{E}(r, \theta, z) = \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \left[ E_m^0 \sin(m \theta + k_n z) + E_m^0 \cos(m \theta + k_n z) \right] \] \hspace{1cm} (2)

\[ \vec{V}(r, \theta, z) = \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \left[ V_m^0 \sin(m \theta + k_n z) + V_m^0 \cos(m \theta + k_n z) \right] \] \hspace{1cm} (3)

\[ \vec{B}(r, \theta, z) = \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \left[ B_m^0 \sin(m \theta + k_n z) + B_m^0 \cos(m \theta + k_n z) \right] \] \hspace{1cm} (4)

\[ \vec{J}(r, \theta, z) = \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \left[ J_m^0 \sin(m \theta + k_n z) + J_m^0 \cos(m \theta + k_n z) \right] \] \hspace{1cm} (5)

Where \( k_n = n/R \) and \( R \) is major radius.
\[ \langle \dot{\Phi}_z(r_0) \rangle = -\frac{r_v}{2\pi R} \int_0^{2\pi} \int_0^{2\pi} E_0(r_0, \theta, z) d\theta d\zeta \]

\[ = \langle \dot{\Phi}_z(r_0) \rangle_{\text{visc}} + \langle \dot{\Phi}_z(r_0) \rangle_{\text{visc}} \mathbf{v} \times \mathbf{B} \]

(6)

where

\[ \langle \dot{\Phi}_z(r_0) \rangle_{\text{visc}} = -2\pi r_v \eta \int_0^{\theta_0} J_0(r_0) \]

(7)

\[ \langle \dot{\Phi}_z(r_0) \rangle_{\text{visc}} \mathbf{v} \times \mathbf{B} = r_v \int_0^{2\pi} \int_0^{2\pi} \mathbf{v} \times \mathbf{B} d\theta d\zeta \]

(8)

Cowling's theorem:

\[ \langle \dot{\Phi}_z(r_0) \rangle_{\text{visc}} \mathbf{v} \times \mathbf{B} = 0 \Rightarrow \langle \dot{\Phi}_z(r_0) \rangle_{\text{visc}} \leq 0 \]

(9)

Necessary condition for flux regeneration:

\[ \langle \dot{\Phi}_z(r_0) \rangle_{\text{visc}} \mathbf{v} \times \mathbf{B} \neq 0 \]

(10)

Note that numerator of eq. (8) is the norm for Fourier series. Consequently only terms that contribute have:

\[ m_1 = m_2 \]

\[ n_1 = n_2 \]

(11)
Consider toroidal dependence of \( \Phi_{n}(r) \). Keeping only terms which contribute to \( (\Phi_{n}(r) \cdot \nabla \times \vec{B})_{r} \) (i.e. \( m_{1} = m_{2} \), \( n_{1} = n_{2} \))

\[
\dot{\Phi}_{2}(r, \theta) = r_{r} \int_{0}^{2\pi} (\nabla \times \vec{B})_{r} \, d\theta
\]

\[
= \pi r_{r} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left[ V_{k_{r},m_{r}}^{m_{n}}(r_{r}) \, B_{r_{r}}^{m_{r}}(\theta) + V_{k_{r},m_{r}}^{m_{n}}(r_{r}) \, B_{r_{r}}^{m_{r}}(\theta) - V_{k_{r},m_{r}}^{m_{n}}(r_{r}) \, B_{r_{r}}^{m_{r}}(\theta) - V_{k_{r},m_{r}}^{m_{n}}(r_{r}) \, B_{r_{r}}^{m_{r}}(\theta) \right]
\]

\[
+ 2\pi r_{r} \sum_{n=\infty}^{\infty} \left[ (V_{k_{r},m_{r}}^{n}(r_{r}) \, B_{r_{r}}^{n}(\theta) - V_{k_{r},m_{r}}^{n}(r_{r}) \, B_{r_{r}}^{n}(\theta)) \cdot \sin k_{n} z + (V_{k_{r},m_{r}}^{n}(r_{r}) \, B_{r_{r}}^{n}(\theta) + \right.
\]

\[
(V_{k_{r},m_{r}}^{n}(r_{r}) \, B_{r_{r}}^{n}(\theta) - V_{k_{r},m_{r}}^{n}(r_{r}) \, B_{r_{r}}^{n}(\theta)) \cdot \sin k_{n} z \cdot \cos k_{n} z +
\]

\[
(V_{k_{r},m_{r}}^{n}(r_{r}) \, B_{r_{r}}^{n}(\theta) - V_{k_{r},m_{r}}^{n}(r_{r}) \, B_{r_{r}}^{n}(\theta)) \cdot \cos^{2} k_{n} z \]

- Only terms with \( z \) dependence are \( m=0 \) terms
- Procedure works for any quadratically nonlinear term in Ohm's law, not just \( \nabla \times \vec{B} \).
Section 10
TOP: USXR
SIDE: Grazing Incidence
BOTTOM: PI Loop + PGF

Section 9
TOP: FIR Interf. (1) + SB (1)
BOTTOM: FIR Interf. (1) + [Dn Monitor] (1)

Section 8
TOP: SB Diodes (2)
   B.O.
SIDE: Unused
   PGF (GA)
BOTTOM: B.O.

Section 7
TOP: 2-color Interf. + PGF
SIDE: Flash Lamp
BOTTOM: 2-color Interferometer

Section 6
TOP: Multichord Spectrom. (6)
BOTTOM: SBD (8)

Section 5
TOP: Window
   Spectrometer (McPherson)
SIDE: Window
   Window + PGF (GA)
BOTTOM: Spectr. cal., [Dn Monitor]

Section 11
TOP: IR Interferometer
   [Dn Monitor]
SIDE: UV Spectrometer
   PGF (GA)
BOTTOM: IR Interferometer

Section 12
TOP: IR Interferometer (8 ports)
BOTTOM: IR Interferometer (8 ports)

Section 1
TOP: PGF + PI Electrode
SIDE: Pump + [Dn Monitor]
BOTTOM: Fast IG + Flash Lamp

Section 2
TOP: SP Thomson scattering
   SP Thomson scattering
SIDE: SP Thomson scattering
   Window + PGF (GA)
BOTTOM: SP Thomson scattering

Section 3
TOP: Bolometer
SIDE: J.H. VUV Spectrometer
BOTTOM: B.O.

Section 4
TOP: PGF + Spectrometer (Hilger)
   + Gas Supply
SIDE: Pump
BOTTOM: Si(Li) Detector
Status: March 2, 1984
[] means temporary diagnostic
Figure 4

ZT40 MDP  shot 17446 11-Sep-84 1814 mode 0 type 3 7167
Bank-kV IP 3.0 IT 11.3 VF 7.6 PI 4.8 CP 3.6 CT 5.5
P-mtorr PD 1.92 P2-0.59 P3 0.00
Time-ms BI 0.70 BC 1.45 BV 1.40 IC 0.95 INTEGRAL

\(\Phi_\phi\)

\(\Phi_\phi\)

\(\Phi_\phi\)

\(\Phi_\phi\)

\(B_\phi\)

\(B_\phi\)

\(I_\phi\)

USXR

USXR
Figure 5
F-θ diagram
Experimental Results

- Relaxation occurs when traveling magnetic disturbance reaches toroidal observation point.

- Positive flux (inside reversal surface) increases ~20% during passage of disturbance.

Conclusion:

- Traveling disturbance is the source of the flux regeneration, and it is toroidally asymmetric.
Standard $m=0$, $n=1$ reconnection

$$B_r^{0,1}(r,\tau) = B_r^{0,1}(r) \sin \frac{\tau}{R}$$  \hspace{1cm} (13)

$$V_r^{0,1}(r,\tau) = V_r^{0,1}(r) \cos \frac{\tau}{R}$$  \hspace{1cm} (14)

$$V_\tau^{0,1}(r,\tau) = V_\tau^{0,1}(r) \sin \frac{\tau}{R}$$  \hspace{1cm} (15)

$$B_\tau^{0,1}(r,\tau) = B_\tau^{0,1}(r) \cos \frac{\tau}{R}$$  \hspace{1cm} (16)

$$E_\theta^{0,1}(r,\tau) = E_\theta^{0,1}(r) \cos \frac{\tau}{R}$$  \hspace{1cm} (17)

$$J_\theta^{0,1}(r,\tau) = J_\theta^{0,1}(r) \cos \frac{\tau}{R}$$  \hspace{1cm} (18)

and $V_r^{0,1}(r) = 0$, $V_\tau^{0,1}(r) < 0$, $B_r^{0,1}(r) > 0$, $B_\tau^{0,1}(r) = 0$, $E_\theta^{0,1}(r) > 0$, $J_\theta^{0,1}(r) > 0$.  \hspace{1cm} (19)
Now

\[ \langle \dot{\Phi}_z (rv) \rangle = -2\pi rv \left[ \eta J_0^0 (rv) - \frac{1}{2} V_z (rv) B_r^0 (rv) \right] \]

\[ \leq 0 \]  

(20)

since \( J_0^0 (rv) > 0 \), \( V_z (rv) < 0 \), \( B_r^0 (rv) > 0 \).

\[ \therefore m = 0 \text{ instabilities oppose flux regeneration.} \]

"Kerstian Interpreation"

Flux reconnects through "\( r \)" point and then flows to the "\( 0 \)" point where it is annihilated (i.e. "\( 0 \)" point is a flux sink)

Consider flux enclosed by "\( 0 \)" point

\[ \dot{\Phi}_z (rv, 0) = -\int_0^{\pi} E_0 (rv, 0) rv \text{ d} \theta \]

\[ = -2\pi rv \eta (J_0^0 (rv, 0) + J_0^r (rv, 0)) \]

\[ \leq 0 \]

\[ \therefore \text{ Flux always decreases inside the "} 0 \text{" point for a single mode, hence "} 0 \text{" point is a flux sink.} \]
Multi helicity Instabilities

- Can multi helicity $m=1$ tearing mode interactions turn $m=0$ "0" point from a flux sink into a flux source?

- Consider only lowest order interactions.

Principal modes: $m=1, n=-10$
$\quad m=1, n=-11$
$\quad m=1, n=-10$

Lowest order nonlinearly driven modes
$\quad m=0, n=1$
$\quad m=2, n=-21 \& ignore$

\begin{align*}
V_r^{1-10} &= V_r^{1-10} \left[ \cos \left( \theta - \frac{11}{10} \right) + \sin \left( \theta - \frac{11}{10} \right) \right] \\
V_r^{2-10} &= V_r^{2-10} \left[ \cos \left( \theta - \frac{11}{10} \right) - \sin \left( \theta - \frac{11}{10} \right) \right] \\
B_r^{1-10} &= B_r^{1-10} \left[ \cos \left( \theta - \frac{11}{10} \right) - \sin \left( \theta - \frac{11}{10} \right) \right] \\
B_r^{2-10} &= B_r^{2-10} \left[ \cos \left( \theta - \frac{11}{10} \right) + \sin \left( \theta - \frac{11}{10} \right) \right] \\
V_r^{1-10} &= V_r^{1-10} \left[ \cos \left( \theta - \frac{10}{10} \right) + \sin \left( \theta - \frac{10}{10} \right) \right] \\
V_r^{2-10} &= V_r^{2-10} \left[ \cos \left( \theta - \frac{10}{10} \right) - \sin \left( \theta - \frac{10}{10} \right) \right] \\
B_r^{1-10} &= B_r^{1-10} \left[ \cos \left( \theta - \frac{10}{10} \right) - \sin \left( \theta - \frac{10}{10} \right) \right] \\
B_r^{2-10} &= B_r^{2-10} \left[ \cos \left( \theta - \frac{10}{10} \right) + \sin \left( \theta - \frac{10}{10} \right) \right]
\end{align*}
Considering these modes along with the (0,1) mode described in eqs (13)-(18) one finds:

\[
\langle \dot{\phi}_e (r, \theta) \rangle = -2\pi r \nu \left[ \eta \left( J_0^{0,0} (r, \theta) + J_0^{0,1} (r, \theta) \cos \frac{\pi}{R} \right) + V_\nu^{1,-10} B_e^{1,-10} (r, \theta) - V_\nu^{1,-11} B_e^{1,-11} (r, \theta) - V_\nu^{1,-10} B_e^{1,-10} (r, \theta) - V_\nu^{1,-11} B_e^{1,-11} (r, \theta) \right]
\]

and

\[
\dot{\phi}_e (r, \theta) = -2\pi r \nu \left[ \eta \left( J_0^{0,0} (r, \theta) + J_0^{0,1} (r, \theta) \cos \frac{\pi}{R} \right) + V_\nu^{1,-10} B_e^{1,-10} (r, \theta) + V_\nu^{1,-11} B_e^{1,-11} (r, \theta) \cos \frac{\pi}{R} - V_\nu^{1,-10} B_e^{1,-11} (r, \theta) - V_\nu^{1,-11} B_e^{1,-10} (r, \theta) \right] + \left( V_\nu^{1,0} B_e^{1,0} (r, \theta) + V_\nu^{1,1} B_e^{1,1} (r, \theta) - V_\nu^{1,0} B_e^{1,1} (r, \theta) - V_\nu^{1,1} B_e^{1,0} (r, \theta) \right)
\]

Assume that the toroidally symmetric components of eq. (31) are small (in rough agreement with \(2\pi - 40\))

\[
V_\nu^{1,0} B_e^{1,0} (r, \theta) + V_\nu^{1,1} B_e^{1,1} (r, \theta) - V_\nu^{1,0} B_e^{1,1} (r, \theta) - V_\nu^{1,1} B_e^{1,0} (r, \theta) = 0
\]

Then \(\dot{\phi}_e (r, \theta)\) (i.e. \(\dot{\phi}_e\) at the "0" point) becomes:

\[
\dot{\phi} (r, 0) = -2\pi r \nu \left\{ \eta \left( J_0^{0,0} (r, \theta) + J_0^{0,1} (r, \theta) \right) + V_\nu^{1,0} B_e^{1,0} (r, \theta) + V_\nu^{1,1} B_e^{1,1} (r, \theta) - V_\nu^{1,0} B_e^{1,1} (r, \theta) - V_\nu^{1,1} B_e^{1,0} (r, \theta) \right\}
\]
In this case the "0" point becomes a flux source if:

\[
\begin{bmatrix}
V_v^{1,1-10}(r) B_z^{1,10}(r) + V_v^{1,1-10}(r) B_z^{1,1-11}(r) \\
-V_v^{1,1-10}(r) B_z^{1,11}(r) - V_v^{1,1-10}(r) B_z^{1,1-10}(r)
\end{bmatrix} \leq \eta (J_r^{0,0} + J_\theta^{1,0})
\]

\[ (24) \]

\[ \therefore \text{Non linear beatings of } m=1 \text{ tearing modes may cause } \n= 0, \text{ } n=1 \text{ "0" point to become a flux source. } \]

\[ \text{Question: Do the nonlinear beatings of } m=1 \text{ tearing modes provide the correct flows?} \]

\[ \text{Recall that } V_z^{0,1}(r) \sim \sin \frac{\pi r}{R} \text{ from eq. (15). Check to see that proper phasing is obtained from momentum equation's } z \text{ component:} \]

\[
\rho \frac{\partial V_z(r, \theta, z)}{\partial t} + \rho V_r(r, \theta, z) \frac{\partial V_z(r, \theta, z)}{\partial \theta} + \rho V_\theta(r, \theta, z) \frac{\partial V_z(r, \theta, z)}{\partial r}
\]

\[ + \rho V_z(r, \theta, z) \frac{\partial V_z(r, \theta, z)}{\partial t} = J_r(r, \theta, z) B_\theta(r, \theta, z) - J_\theta(r, \theta, z) B_r(r, \theta, z) \]

\[ (35) \]
Equations (22)-(29) also have:

\[ V_0^{l_{11}}(r, \theta, z) = V_0^{l_{11}}(r) \left[ \cos \left( \theta - \frac{11 \pi}{R} \right) - \sin \left( \theta - \frac{11 \pi}{R} \right) \right] \]  

\[ V_0^{l_{10}}(r, \theta, z) = V_0^{l_{10}}(r) \left[ \cos \left( \theta - \frac{10 \pi}{R} \right) - \sin \left( \theta - \frac{10 \pi}{R} \right) \right] \]  

\[ B_0^{l_{11}}(r, \theta, z) = B_0^{l_{11}}(r) \left[ \cos \left( \theta - \frac{11 \pi}{R} \right) + \sin \left( \theta - \frac{11 \pi}{R} \right) \right] \]  

\[ B_0^{l_{10}}(r, \theta, z) = B_0^{l_{10}}(r) \left[ \cos \left( \theta - \frac{10 \pi}{R} \right) + \sin \left( \theta - \frac{10 \pi}{R} \right) \right] \]  

\[ J_r^{l_{11}}(r, \theta, z) = J_r^{l_{11}}(r) \left[ \cos \left( \theta - \frac{11 \pi}{R} \right) - \sin \left( \theta - \frac{11 \pi}{R} \right) \right] \]  

\[ J_r^{l_{10}}(r, \theta, z) = J_r^{l_{10}}(r) \left[ \cos \left( \theta - \frac{10 \pi}{R} \right) - \sin \left( \theta - \frac{10 \pi}{R} \right) \right] \]  

\[ J_\theta^{l_{11}}(r, \theta, z) = J_\theta^{l_{11}}(r) \left[ \cos \left( \theta - \frac{11 \pi}{R} \right) + \sin \left( \theta - \frac{11 \pi}{R} \right) \right] \]  

\[ J_\theta^{l_{10}}(r, \theta, z) = J_\theta^{l_{10}}(r) \left[ \cos \left( \theta - \frac{10 \pi}{R} \right) + \sin \left( \theta - \frac{10 \pi}{R} \right) \right] \]  

\[ J_z^{l_{11}}(r, \theta, z) = J_z^{l_{11}}(r) \left[ \cos \left( \theta - \frac{11 \pi}{R} \right) + \sin \left( \theta - \frac{11 \pi}{R} \right) \right] \]  

\[ J_z^{l_{10}}(r, \theta, z) = J_z^{l_{10}}(r) \left[ \cos \left( \theta - \frac{10 \pi}{R} \right) + \sin \left( \theta - \frac{10 \pi}{R} \right) \right] \]  

Substituting eqns. (22-29) and (36)-(45) into the (0, 1) component of equation (45) yields:
\[ \rho \frac{dV_{e,1}^{0,1}}{dt} \cos^2 R + \rho \frac{dV_{e,s}^{0,1}(r)}{dt} \sin \frac{2}{R} = \left[ \rho \frac{V_{e}^{1,11} V_{e}^{1,11} - \rho V_{e}^{1,10} V_{e}^{1,11}}{r} \right. \\
+ \rho \frac{V_{e}^{1,11} V_{e}^{1,11} - \rho V_{e}^{1,10} V_{e}^{1,11}}{2r} - \rho \frac{V_{e}^{1,10} V_{e}^{1,11}}{2r} \\
- \frac{\rho}{R} V_{e}^{1,10} V_{e}^{1,11} + J_{\phi}^{0,0} B_{\phi}^{0,0} \right] \sin \frac{2}{R}. \]

Note that all of the nonlinear convolutions beat into the \( \sin^2 R \) velocity component, consistent with the reconnection picture. A net flux regeneration will occur when:

\[ V_{e}^{0,1}(r) B_{r}^{0,1}(r) > 2 \eta J_{\phi}^{0,0}(r) \]

(47)

Question: Does this scenario work out with the correct signs and magnitudes numerically?
Fig. 19. (a) axial magnetic flux contained within the field reversal surface $r_v$; (b) $m = 0$, $n = 0$ component of the poloidal electric field at the field reversal surface; and (c) poloidal mode contributions to the mean poloidal electric field at the field reversal surface, as functions of time for the three-dimensional high-$B$ case.
Result: \( \nabla \times \vec{B} \approx \nabla \cdot \vec{B} \) for instabilities implies that nonlinear beating induced electric fields are always smaller than the self-beating induced electric fields. Hence, symmetric regeneration \( \Rightarrow \) asymmetric regeneration.

MHD Conclusions:

1. \( m=0 \) instabilities do not produce flux regeneration.

2. \( m \neq 1 \) nonlinear couplings can produce asymmetric flux regeneration but it is usually smaller than the symmetric component.

3. MHD may lead to an asymmetric dynamo if:
   a. There are several \( m=1 \) modes of comparable magnitude present
   b. They are phase locked to beat into the same \( m=0, n=1 \) mode.

4. An essential part of this picture is the \( m=0, n=1 \) "0" point becoming a toroidal flux source.
Hall Effect

Ohm's law now becomes:

$$\bar{E} + \nabla \times \bar{B} + \frac{\bar{j} \times \bar{B}}{\hbar} = \eta \bar{J}$$

Combining eqs (48), (6), (13), (16), (24), (25), (28), (29), (40), (41), (44) and (45) one finds (assuming $n = \hbar c^n$):

$$\langle \Phi_{\text{Hall}}^{(n)} \rangle = -\frac{\hbar c^n}{2\pi R} \int_0^{2\pi} \int_0^{\pi} \left( \mathcal{J}_x(n) \mathcal{B}_x(n) \left[ \cos^2 (\theta - \frac{11\pi}{n}) - \sin^2 (\theta - \frac{11\pi}{n}) \right] + \mathcal{J}_z(n) \mathcal{B}_z(n) \right) d\theta d\psi$$

$$= -\frac{\hbar c^n}{2\pi R} \int_0^{2\pi} \int_0^{\pi} \left\{ \mathcal{J}_z(n) \mathcal{B}_x(n) \left[ \cos^2 (\theta - \frac{11\pi}{n}) - \sin^2 (\theta - \frac{11\pi}{n}) \right] + \mathcal{J}_z(n) \mathcal{B}_z(n) \right\} d\theta d\psi$$

$$= 0$$

\(\therefore\) Hall effect produces no net flux regeneration but it does couple the spectrum.
We now have

\[ \Phi_e (r, \varphi) = -2 \pi \Gamma \ Associated \ \{ \eta \left[ J^{q,0}_\varphi (r) + J^{q,1}_\varphi (r) \right] \cos \frac{2 \pi \varphi}{R} + J^{q,1}_\varphi (r) \sin \frac{2 \pi \varphi}{R} \]

\[ + V_{e}^{1-10} (r) B^{1-10}_e (r) + V_{e}^{1-11} (r) B^{1-11}_e (r) + V_{e}^{0,0} (r) B^{0,0}_e (r) \cos \frac{2 \pi \varphi}{R} \]

\[ + V_{e,1}^{0,1} (r) B^{0,1}_e (r) \sin \frac{2 \pi \varphi}{R} - V_{e,1}^{1-0} (r) B^{1-0}_e (r) - V_{e,1}^{1-11} (r) B^{1-11}_e (r) \]

\[ - V_{e,1}^{0,1} (r) B^{0,1}_e (r) \sin \frac{2 \pi \varphi}{R} - V_{e,1}^{0,1} (r) B^{0,1}_e (r) \cos \frac{2 \pi \varphi}{R} + V_{e,1}^{1-10} (r) B^{1-10}_e (r) \cos \frac{2 \pi \varphi}{R} \]

\[ + V_{e,1}^{1-11} (r) B^{1-11}_e (r) \cos \frac{2 \pi \varphi}{R} - V_{e,1}^{1-10} (r) B^{1-10}_e (r) \cos \frac{2 \pi \varphi}{R} - V_{e,1}^{1-11} (r) B^{1-11}_e (r) \cos \frac{2 \pi \varphi}{R} \]

\[ \cos \frac{2 \pi \varphi}{R} - \frac{J^{1-11}_e (r) B^{1-11}_e (r)}{n} \sin \frac{2 \pi \varphi}{R} + \frac{J^{1-10}_e (r) B^{1-10}_e (r)}{n} \sin \frac{2 \pi \varphi}{R} \]

\[ - \frac{J^{1-11}_e (r) B^{1-11}_e (r)}{n} \sin \frac{2 \pi \varphi}{R} + \frac{J^{1-10}_e (r) B^{1-10}_e (r)}{n} \sin \frac{2 \pi \varphi}{R} \right\} \]

Note that the \( m=0, n=1 \) configuration described in eqs (13)-(19) is no longer valid, but instead there are both sine and cosine terms in the equations (i.e. the "0" point is no longer at \( z=0 \)).
The "X" and "O" points are located at

$$ z = R \arctan \left( -\frac{B_{r,c}^{0,0}(n)}{B_{r,c}^{0,1}(n)} \right) $$

(51)

From eqs. (50) and (51) it is obvious that both the $\nabla \times B$ terms and the Hall terms contribute at the "O" point unless

$$ B_{r,c}^{0,1}(n) = 0 $$

(52)

which is unlikely since

$$ \frac{dB_{r,c}^{0,1}(n)}{dt} = \frac{J_{x}^{0,1}(n) B_{r,c}^{1,0}(n) - J_{r,c}^{0,0}(n) B_{r,c}^{0,1}(n) - J_{e}^{1,0}(n) B_{r,c}^{0,0}(n) - J_{x}^{1,1}(n) B_{r,c}^{1,1}(n)}{n} $$

(53)

Note that the Hall terms do not enter directly into the momentum equation, but rather indirectly:

$$ \rho \frac{dv_{r,c}^{0,1}(n)}{dt} = J_{r,c}^{0,1}(n) B_{r,c}^{0,0}(n) - J_{e}^{0,0}(n) B_{r,c}^{0,1}(n) $$

(54)

whereas the cosine component was zero in the MHD model.
Experimental Implications

- Narrowing in $\Theta$ space at high current operation
  - $m=0, n=1$ reconnection is not proceeding fast enough.
  - Cure: Keep resistivity at reversal surface high by keeping plasma cold.
    i.e. gas refuel

- Quiescence
  - Proper balance of different nonlinearities to minimize $\delta B/B$. 
RECONNECTION AND ITS ROLE IN SOLAR FLARES

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SOME MAJOR QUESTIONS CONCERNING SOLAR ACTIVITY

- THE SOURCE(S) OF SOLAR MAGNETIC FIELDS.
- THE EMERGENCE AND DISAPPEARANCE OF MAGNETIC FLUX.
- THE FORMATION OF ACTIVE REGIONS.
- THE CAUSE(S) OF SOLAR FLARES.
- THE CAUSE(S) OF CORONAL TRANSIENTS.
- THE CAUSE(S) OF CORONAL HEATING.
SOURCES OF SOLAR FLARES: MAJOR ISSUES

- SOURCE OF FREE ENERGY.

- MECHANISM(S) THAT DISSIPATE THE AVAILABLE FREE ENERGY.
SOURCE OF FREE ENERGY: MAGNETIC

- How is this free magnetic energy stored?
  (a) Remotely (subphotospherically).
  (b) Insitu coronal storage (orthodox view).

- There are serious questions with both possibilities.
INSITU CORONAL MAGNETIC ENERGY STORAGE

- Constraints imposed by all known magnetic dissipation mechanisms that the available insitu coronal free magnetic energy and its corresponding energy density be high cause serious difficulties for insitu storage (e.g., insitu storage requires free magnetic energy with energy densities of order $10^4$ ergs/cm$^3$ which is approximately $10^4$ greater than the ambient gas pressures).

- Solar magnetic fields are not force-free since there exist finite forces (global curvature, gravity and pressure gradients, both gas and magnetic) which must balance one another.

- The usual assumption of constant $\alpha$ for force-free fields is dubious since such an assumption implicitly neglects coronal potential fields due to sub-photospheric currents.

- Inductive insitu coronal storage of magnetic energy imposes the constraint that at most approximately $10^{32}$ electrons/s can be directly accelerated by any flare mechanism that dissipates the inductively stored energy.
FLARE MECHANISMS

- PRINCIPAL MECHANISMS:
  (a) RECONNECTION
  (b) DOUBLE LAYERS
  (c) ANOMALOUS RESISTIVITY

- MECHANISM MUST DISSIPATE APPROXIMATELY $10^{32}$ OF MAGNETIC ENERGY IN APPROXIMATELY 10-100 SECS.

- MECHANISM MUST RESULT IN TEMPORAL STRUCTURE WITH AN HIERARCHY OF TIME $10^{-3}$ TO $10^{2}$ SEC.

- THE RELEASED ENERGY MUST SPREAD THROUGHOUT A VOLUME OF APPROXIMATELY $10^{30}$ CM$^3$ IN APPROXIMATELY 10-100 SECS.

- THE MECHANISMS(S) MUST BE CAPABLE OF PRODUCING $10^{35}$-$10^{38}$ 20 KEV ELECTRONS/S IN SIMILAR TIMES.
MAGNETIC STRUCTURE AT THE SOLAR SURFACE

- Flux emerges in concentrated "active" regions which occupy only a small fraction of the solar surface.

- These regions are made up of numerous flux "bundles" (also sometimes called "ropes" or "tubes"). These bundles are a few hundred kilometers in size with field strengths approaching 1500 gauss.

- Bundles appear with a broad spectrum of sizes.

- The largest active regions contain up to $10^{23} \text{ Mx (Mx} = 1 \text{ gauss cm}^2\)).$

- Smallest ephemeral regions have only $10^{18}$-to-$10^{19} \text{ Mx, but appear in great numbers.}$

- Unipolar flux is observed. The explanation for this apparent contradiction with physical law appears to be due to our inability to resolve finer structure bundles of the opposite polarity at the periphery of the observable active region.

- Emerging flux carries dense plasma from the convection zone into the photosphere.
• AS A LOOP OF MAGNETIC FLUX EMERGES, THE FOOTPRINTS OF THE LOOP MOVE STEADILY APART.

• THE USUAL VIEW OF EMERGENCE IS OF A TWISTED SUBSURFACE ROPE OF MAGNETIC FLUX WHICH DEVELOPS A RISING LOOP OR KINK THAT BREAKS THROUGH THE SURFACE.

• TURBULENCE AND FLOWS EXCLUDE MAGNETIC FLUX FROM THE CENTER OF THE CELLS AND VORTICES WHICH FILL THE CONVECTION ZONE AND CONCENTRATE IT IN ROPES AT THE PERIPHERY OF CELLS.

• OBSERVATIONS SHOW THAT FLUX, PARTICULARLY IN THE FORM OF SUNSPOTS, ROTATES ACROSS THE VISIBLE SURFACE AT A FASTER RATE THAN THE SURFACE PLASMA. THIS SUGGESTS THAT THIS FLUX IS CONNECTED MAGNETICALLY TO THE MORE RAPIDLY ROTATING, DEEPER LAYERS. DRAG AND THUS HEATING SHOULD RESULT.

• LITTLE IS KNOWN ABOUT THE RATE OF EMERGENCE OF FLUX.
NONLINEAR COUPLING OF TEARING MODES IN THE
REVERSED FIELD PINCH AND THE TOKAMAK DISRUPTION

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NONLINEAR COUPLING
OF
TEARING MODES
IN THE
REVERSED FIELD PINCH
AND THE
TOKAMAK DISRUPTION

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For presentation at the US-Japan Joint Institute for Fusion Theory Workshop on Magnetic Reconnection, Austin, TX, Dec. 1984.
**Tokamak**

- Two tearing modes unstable (2/1 and 3/2)
- Island overlap
- Stochasticization of field lines
  + Explosive growth
  - Loss of confinement
  - Disruption (bad!)

**RFP**

- Many m=1 tearing modes unstable with singular surfaces close together
- Island overlap
- Stochasticization of field lines
- Dynamo (good!)
Motivation: To study how the nonlinear coupling of resistive tearing modes can lead to such different phenomena as disruptions in tokamaks and sustainment of profiles against resistive diffusion (dynamo) in RFP's.
OUTLINE OF TALK

• Computational Models

• Tokamak Disruptions
  • Description
  • Nonlinear Coupling Mechanism

• RFP Tearing Modes
  • Description
  • Nonlinear Coupling Behavior
  • Consistency with Tokamak Disruption Mechanism

• Conclusions and Future Direction
Computational Model

• Resistive MHD

• Cylindrical Geometry Adequate for Tearing Modes

• Full Set of Equations with Constant Mass Density Valid for RFP and Tokamak
  • Compressible or Incompressible
  • With or Without Equilibrium Resistive Diffusion
  • Options on Equation for Pressure Evolution

• Reduced Set of (Stauss) Equations Valid for Tokamak Only.

• Solution by Initial Value Approach
  • Cylindrical Coordinates \((r, \theta, \phi)\)
  • Finite Difference in \(r\) and time \(t\)
  • Fourier Expansion in \(\theta\) and \(\phi\) using Periodic Boundary Conditions.
Multiple Helicity Tearing Mode Evolution
Tokamak Disruption

3/2 Magnetic Energy Growth Rate

$\frac{\mathcal{E}}{\rho}$ vs $t \times 10^{-3}$

Linear regime
Rutherford regime
Nonlinear destabilization
Linear Regime

Rutherford Regime

Nonlinear Interaction + Destabilization

Major Disruption
NONLINEAR MODE INTERACTION MODEL
(Carreras, Rosenbluth, Hicks), (An, Diamond)

- Assume: 
  - Sheared Slab Model \( (q = q(x)) \).
  - Low \( \beta \) Reduced Resistive MHD Equations.
- Modes: 
  - \( \vec{k} \) a test mode (unstable)
  - \( \vec{k}' \) a passive background
  - \( \vec{k} + \vec{k}' \) a nonlinearly driven mode.

- Results: 
  - Driven field associated with localized current \( \vec{\tilde{J}}_{k+k'} \).
  - Nonlinear beating of \( \vec{J}_{k+k'} \) and \( \vec{B}_k \) generates vorticity change for the \( k \) mode.
  - If driven mode unstable \( (\Delta'_{kk'} > 0) \), test mode \( k \) further destabilized by nonlinear coupling to new free energy source.
  - If driven mode stable \( (\Delta'_{kk'} < 0) \), nonlinear coupling opens new energy sink for the mode \( k \), which is stabilized.
NONLINEAR COUPLING TOKAMAK DISRUPTION MECHANISM
(Carreras, Rosenbluth, Hicks)

- Modes Involved: (0,0) Equilibrium.
  (2,1) and (3,2) Linearly Unstable.
  (5,3) and (1,1) Lowest Order Couplings.
  Can neglect nonresonant (1,1).

- (2,1) and (3,2) Magnetic Islands Alter Current Profile Quasi-
  Linearly, Destabilizing (5,3) \( q = \frac{3}{2} \)
  \( f = \frac{5}{3} \) \( q = 2 \)

- Nonlinear Coupling of Destabilized (5,3) and (2,1) through \( \mathbf{J} \times \mathbf{B} \) force drives rapid increase in kinetic energy of (3,2)

- \[ \gamma_{(3,2)}^2 \approx | B_r^{(2,1)} | \Delta_1 \frac{a^2}{B_0 W_{(2,1)} \tau_{H}^2} \text{ for } \Delta_{(5,3)}' > 0 \]

\( \Delta_{(5,3)}' < 0 \) tends to stabilize the (3,2)
Linear Destabilization of $(5;3)$ Nonlinearly Destabilizes the $(3;2)$.
NONLINEAR COUPLING IN THE RFP

- Modes Involved: (0;0) Equilibrium, (1;1n), (1;1n+1), etc. Linearly Unstable (nn=10).
  (2;2n+1) and (0;1) Lowest Order Couplings.
  (0;1) Not resonant between (1;1n) and (1;1n+1).

- Contrast with Tokamak: Close spacing of (1;1n) and (1;1n+1) => Strong Interaction and Island Overlap "Earlier" than in Tokamak. Quasilinear current profile modification will be small.

- To compare with Tokamak, let us regard (1;1n) as (2;1), (1;1n+1) as (3;2), and (2;2n+1) as (5;3).
The (1;12) mode is nonlinearly stabilized. The (2;23) is always stable.
Saturated Island Width is Smaller in the Presence of Nonlinear Couplings.
The stabilization carries over to many mode cases.
Removing the $m=2$ modes reduces the nonlinear stabilizing effect.
Though the driven modes play a crucial role, they are not easy to detect.
In spite of the stabilization effects, large regions of stochastic field lines are generated.
Conclusions + Future Direction

- Nonlinear couplings of two unstable tearing modes are
  - Destabilizing in tokamak disruptions.
  - Stabilizing in the RFP.
  - Understandable in terms of an analytic model.

- The nonlinear stabilization observed in the RFP carries over to cases for which many modes are included.

- Future work should attempt to couple analytic and numerical efforts to understand steady state turbulence in RFP's.
RFP: MEAN FIELD MHD MODEL

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RFP : Mean Field MHD Model

1. MHD "explains" RFP
   a) $\alpha$ effect
   b) transport: $\beta \approx 20\%$

Tokamak: $q \approx 1, q' > 0$
RFP: $q \approx 0/R, q' < 0$

2. Transport-like MHD model
   a) simpler
   b) good agreement (better than incomp MHD)
   c) generalized quasi/nonlinear

III. $\beta = 0$ derivative
IV. $\alpha$ effect
V. $\beta = 0$
VI. numerical
\[ B = \hat{B} + \vec{B} \]

\[ 2\vec{B} = \int \frac{\partial \theta dz}{2\pi R} \vec{B} \]

\[ 1 |\vec{B}| << 1 |\vec{B}| \]

Mean Field \( \vec{B} = D \hat{B} \)

\[ \hat{A} = \frac{\partial}{\partial z} (\vec{B} \cdot \vec{v}) = \nabla \times \vec{B} + E_z \hat{z} - \eta \hat{J} \]

\[ \frac{\partial}{\partial z} (\nabla \times \vec{B} \cdot \vec{v}) = 0 \]

\[ \frac{\partial}{\partial t} (\nabla \times \vec{B} \cdot \vec{v}) = 0 \]

\[ \frac{\partial}{\partial t} (\nabla \times \vec{B} \cdot \vec{v}) = 0 \]

\[ \hat{v} = \alpha + \left( E_z \hat{z} - \eta \hat{J}, \vec{B} \right) / \vec{B}^2 \]

\[ \left( \frac{1}{r} \frac{\partial}{\partial r} \frac{r \vec{B}^2}{\partial r} - \frac{\vec{B}^2}{r^2} \right) \hat{x} = \frac{2}{r} \left( \frac{\vec{B} \cdot \hat{z}}{r} \right) - \frac{2}{r} \frac{\partial}{\partial r} \left( \nabla \times \vec{B} \cdot \vec{v} \right) \]

\[ \alpha = \frac{\nabla \times \vec{B} \cdot \vec{B}}{\vec{B}^2} \]

Steady \( \frac{J_0 \vec{B}_z}{\vec{B}_\theta} \]

State \( \gamma \frac{\nabla \cdot \vec{B}}{\vec{B}} = \alpha \vec{B}^2 + E_z \vec{B}_z \)
Fluctuations

1. Small amplitude
   \[ |\hat{B}| / |\bar{B}| \sim \lambda \ll 1 \]
   exp. overlap

2. All linear terms

3. \( \bar{B} \cdot \nabla \sim \hat{B} \cdot \nabla \sim \lambda \) at islands

4. \[ \frac{2}{dr} \sim \frac{1}{\lambda} \]
   boundary layer

5. \[ \frac{\partial}{\partial t} |\hat{B}| \sim \lambda |\bar{B}| \]
   small manta

6. \( \eta \sim \lambda^3 \)
   nonlinear tearing
\[
\frac{\partial}{\partial t} \hat{A} = \frac{\partial}{\partial t} \bar{B} + \frac{\partial}{\partial t} \rho \nu r
\]
\[
= \nabla \times \bar{B} + \nabla \hat{u} - \eta \hat{J}
\]
\[
\hat{B} \quad \Rightarrow \quad \dot{\hat{B}}
\]
\[
\nabla \times (\bar{B} \times \bar{B}) = 0 \quad \Rightarrow \quad \hat{\phi}
\]
\[
(\bar{B} \cdot \hat{B} \approx 0)
\]
\[
\hat{v} = (\nabla \hat{u} - \hat{\phi} \nu r) \times \bar{B} / \bar{B}^2
\]
\[
\hat{\nabla} \cdot \left( \rho \frac{\partial}{\partial t} \times \bar{B} / \bar{B}^2 \right) = \bar{B} \cdot \nabla \frac{\nabla \times \bar{B}}{\bar{B}^2} \Rightarrow \dot{\hat{u}}
\]
\[
\left[ \frac{\partial}{\partial t} \bar{B} \sim \eta \hat{J} \ll \frac{\partial}{\partial t} \hat{B} \right]
\]
\[
\hat{B} = \nabla \times (\hat{\psi} \bar{B} + \hat{\phi} \nu r)
\]
\[ RFP \text{ eqs. } (\beta = 0) \]

\[ \frac{\partial}{\partial t} \phi = \frac{\mathbf{B} \cdot \nabla \mathbf{u}}{B^2} + \gamma \hat{\sigma} \]

\[ \frac{d}{dt} \nabla \cdot \left( \frac{\mathbf{B}}{B^2} \nabla \mathbf{u} \right) = \mathbf{B} \cdot \nabla (\hat{\sigma} + \bar{\sigma}) \]

\[ \left( \frac{1}{r^2} \frac{\partial}{\partial r} + \frac{d^2}{d\theta^2} \right) \phi = \frac{\bar{\mathbf{B}} \cdot \nabla}{\bar{B}} \frac{\partial \hat{\phi}}{\partial \theta} \]

\[ \sigma = \bar{\mathbf{B}} / B^2 \]

\[ \hat{\sigma} = \frac{1}{B} \nabla \cdot (\hat{\mathbf{B}} \times \bar{\mathbf{B}}) \]

\[ \frac{\partial}{\partial t} \bar{\phi} = \alpha + \left( E^2 \mathbf{E}_2 - \eta \bar{\mathbf{J}} \cdot \bar{\mathbf{B}} \right) / B^2 \]

\[ \mathcal{L}(\dot{\mathbf{x}}) = \mathcal{F}(\dot{\phi}) \]

\[ \mathbf{B} = \nabla \times (\hat{\phi} \bar{\mathbf{B}} + \dot{\mathbf{x}} \sigma \cdot \bar{\mathbf{B}} + \dot{\phi} \sigma r) \]

\[ \alpha = \frac{\mathbf{B} \cdot \nabla \mathbf{u} - \mathbf{B} \cdot \frac{\partial \phi}{\partial t}}{\mathbf{B} \cdot \nabla \mathbf{u} - \mathbf{B} \cdot \frac{\partial \phi}{\partial t}} \]
Energy:
\[ E = \frac{1}{2} \left\langle \bar{\rho} (\nabla u / \bar{\mathbf{b}})^2 + \bar{\mathbf{b}}^2 + \bar{\mathbf{v}}^2 \right\rangle \]

\[ \frac{\partial}{\partial t} E = \left\langle (E_0^2 \mathbf{v}^2 - \eta \mathbf{J}) \cdot \mathbf{J} \right\rangle + O(\frac{\mathbf{v}^2 \partial \mathbf{b}}{\partial t}, \frac{\nu^2 \partial \mathbf{b}}{\partial t}) \]

* effect

1. No current drive in steady state

\[ \alpha \mathbf{b}^2 = \mathbf{B} \cdot \nabla \mathbf{u} - \mathbf{B} \cdot \frac{\partial \mathbf{b}}{\partial t} \gtrsim 0 \]

\[ \left\langle \alpha \mathbf{b}^2 \right\rangle = 0 \]

\[ \left\langle \bar{\mathbf{b}}^2 \mathbf{v}^2 \right\rangle = 0 = \left\langle \eta \mathbf{J} \cdot \mathbf{b} \right\rangle = E_0 \left\langle B_0 \right\rangle \]
2. Quasilinear: \( U \sim U(r,t) \exp \left( i m \theta - i n z / R \right) \)

\[ \alpha \overline{B}^2 = \frac{1}{r} \frac{d}{dr} \left( r \overline{U} \overline{B}_r \right) \]

a) Steady state: \( \partial / \partial t = 0 \)

\[ 0 = \overline{B} \cdot \nabla \overline{U} + \eta \frac{\partial}{\partial t} \overline{\sigma} \]

\[ 0 = \overline{B} \cdot \nabla \overline{\sigma} + \overline{B}_r \frac{\partial}{\partial t} \overline{\sigma} \]

\[ \alpha \overline{B}^2 = \frac{1}{r} \frac{2}{dr} \left[ \eta \overline{B}^2 \sum \left| \frac{B_{rk}}{k} \right|^2 \frac{d}{dr} \left( \frac{\overline{\sigma} \overline{B}}{B_z^2} \right) \right] \]

(Taylor: \( J \cdot B / B_z = \sigma = \text{const.} \))

b) Growing modes - at wall

\[ \alpha \overline{B}^2 = -\frac{i}{2} \sum \left( \frac{k \times B \cdot \sigma r}{k \cdot B} \right) \frac{2}{dr} \frac{2}{dr} \left| \frac{m - n q}{mq + nr^2/R^2} \right|^2 \]

\[ \frac{k \cdot B}{k \times B \cdot r} = \left( \frac{m - n q}{mq + nr^2/R^2} \right) \frac{r}{R} \, \text{RFP} \quad \alpha > 0 \]

\[ \text{Tok} \quad \alpha < 0 \]

c) Growing modes - at center

\[ U = \frac{\partial \phi}{\partial t} \sim r \]

\[ \frac{\partial^2}{\partial t^2} \phi = \frac{q \, dq}{dr} \frac{\partial}{\partial t} \left( \frac{3}{3r} \right)^2 \, \text{RFP} \quad \alpha > 0 \]

\[ \text{Tok} \quad \phi > 0 \]

("1st reconnection")
\[ \frac{\partial}{\partial t} \rho = \nabla \cdot \rho \nabla \cdot \nu + (\Gamma - 1) \gamma J^2 + \nabla \cdot \chi \cdot \nabla \rho \]

\[ \nu = (\nabla \nu - \frac{\partial A}{\partial t}) \times B / \bar{B}^2 + \nu_w \bar{B} / \bar{B} \]

Mean field

\[ \bar{J} \times \bar{B} \cdot \sigma r = \frac{\partial \bar{\rho}}{\partial t} \]

\[ \bar{\nu} = \alpha + (\xi z \bar{B}_z - \gamma \bar{J} \cdot \bar{B}) / \bar{B}^2 \]

\[ \frac{\partial \bar{\rho}}{\partial t} = -\nu \bar{B}^2 + (\Gamma - 1) \gamma \bar{J}^2 + \frac{1}{\Gamma} \frac{\partial r \chi_z}{\partial \nu} \frac{\partial \bar{\rho}}{\partial \nu} \]

\[ \mathcal{L}(\bar{\rho}) = \mathcal{L}(\bar{\nu}, \bar{\rho}) \]

\[ \nu = \nabla \cdot \rho \nabla \cdot \bar{\nu} + \nabla \cdot \rho \nabla \cdot \bar{\nu} + \nabla \cdot \rho \nabla \cdot \bar{\nu} - \frac{1}{\Gamma} \frac{\partial r \chi_z \bar{B} \cdot \nabla \rho \bar{B}_r}{\bar{B}_r} \]
Fluctuations

\[ \frac{\partial \hat{\psi}}{\partial t} - \frac{\mathbf{u} \cdot \nabla \hat{\psi}}{B^2} + \mathbf{v} \cdot \nabla \hat{\psi} = \mathbf{B} \cdot \nabla \left( \frac{\partial \hat{\phi}}{\partial t} - \frac{\mathbf{F} \cdot \nabla \hat{\phi}}{B^2} \right) + \mathbf{B} \cdot \nabla \cdot \nabla \hat{\psi} \]

\[ \frac{d}{dt} \mathbf{u} \cdot \nabla \hat{\psi} = \mathbf{B} \cdot \mathbf{v} \mathbf{\hat{u}} + \mathbf{\hat{u}} \cdot (\mathbf{\nabla} - \frac{2 \mathbf{\nabla} \cdot \mathbf{\hat{u}}}{B^2}) \]

\[ + \frac{2 B_0^2}{r B^4} \mathbf{v} \cdot \nabla \cdot \mathbf{\hat{u}} \]

\[ \frac{\partial \hat{\rho}}{\partial t} = - \mathbf{v} \cdot \nabla \hat{\rho} \cdot \mathbf{B} - \mathbf{F} \cdot \nabla \cdot \hat{\rho} - \frac{\mathbf{F} \cdot \mathbf{B} \cdot \nabla \hat{\rho}}{B^2} \]

\[ + \chi \nabla^2 \hat{\rho} + \mathbf{B} \cdot \nabla \left( \frac{\mathbf{v} \cdot \mathbf{B} \cdot \nabla \hat{\rho}}{B^2} \right) \]

\[ \hat{\rho} \frac{d}{dt} \mathbf{v} = - \mathbf{B} \cdot \nabla \hat{\rho} / B \]
\[ \beta = 0 \quad \text{Numerical example} \]

shows effect of \( \nu \)

\[ X_0 = 0 \quad X_3 = \text{const} \]

Initially \( \alpha = \nu = 0 , \ \beta = 0.8 , \ \beta \epsilon (0) = 1 \)

\[ t = 100 \quad \beta = 0.3 , \ \beta \epsilon (0) = 1.3 \]

\[ \text{dynamo} \]
INITIAL EQUILIBRIUM

\[ T = 0 \quad \beta_0 = 80\% \]

\[ B_2 \]

\[ E_\theta \]

\[ P \]

Time: 00.05, 02 and 01

Pressure: 00.05
$T = 100 \quad (\gamma = 10^{-3}) \quad \beta_0 = 30\%$
Energy vs Time

P.E. and K.E. vs. Time

D-E1 LogP.E. vs. Time

Magnetic energy by mode
$p(0)$ vs. time

Theta and F vs. time
NONLINEAR EVOLUTION OF TEARING MODES
IN REVERSED FIELD PINCHES

T. AMANO

NAGOYA UNIVERSITY
Nonlinear Evolution of Tearing Mode in Reversed Field Pinches

H. Ashida*, Y. Maejima*, A. Nagata**, T. Amano

*Electrotechnical Laboratory

**Fac. of Eng., Hiroshima Univ.

Institute of Plasma Physics, Nagoya Univ.
1). What is RFP?

Present status of experiment and theory

1). Code description, test, comparison with other simulations

2). Ergodic Field Lines, K-S entropy, Diffusion.

3). Dynamo Effect

4). Discussion,
Basic Equations

\[
\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + S^2 \mathbf{j} \times \mathbf{B}
\]

\[
\frac{\partial \omega}{\partial t} = \omega \cdot \nabla \mathbf{u} - \mathbf{u} \cdot \nabla \omega + S (\mathbf{B} \cdot \nabla \mathbf{j} - \mathbf{j} \cdot \nabla \mathbf{B})
\]

\[\omega = \text{rot} \mathbf{u}\]

\[\text{div} \mathbf{u} = 0\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \mathbf{B} \cdot \nabla \mathbf{u} - \mathbf{u} \cdot \nabla \mathbf{B} - \text{rot} \mathbf{n} \times \mathbf{j}
\]

\[\mathbf{j} = \text{rot} \mathbf{B}\]

\[S = \frac{\tau_r}{\tau_A} \quad \text{at Plasma Center}\]

\[\tau_r = \frac{4\pi a_w^2}{C^2 \eta} \quad : \text{Resistive Diffusion Time}\]

\[\tau_A = \frac{a_w \sqrt{4\pi \rho}}{B_0} \quad : \text{Alfvén Transit Time}\]

\[\nabla^2 p = -\nabla (\mathbf{u} \cdot \nabla \mathbf{u}) + \nabla \left[ S^2 (\mathbf{j} \times \mathbf{B}) \right]\]

cylindrical approx.
Mode Expansion

We use the cylindrical coordinates:

\((r, \theta, \gamma), \gamma = \phi/R\)

(i) \(f_{m,n}(r) \cos(m\theta + n\gamma)\)

\(V_r, \omega_\theta, \omega_\gamma, B_\theta, B_\gamma, j_\theta, j_\gamma\)

(ii) \(f_{m,n}(r) \sin(m\theta + n\gamma)\)

\(V_\theta, V_\gamma, \omega_r, B_r, j_r\)

(iii) \(m > 0\) \((-\infty < n < +\infty)\)

\(m = 0\) \(n < 0\)

\(\nabla \cdot \mathbf{V} = 0\) \((\text{div} \mathbf{V} = 0)\)
Boundary Conditions

\[ r = a \]

\[ v_r = 0 , \]
\[ n \times w = 0 \]
\[ B_r = 0 \]
\[ n \times B_r = 0 \]

\[ B_{\phi}(a) = \text{const} \]
\[ B_{\phi0}(a) = \text{const} \]

\[ \eta_{\alpha\beta} = \eta_\perp - \eta_\parallel \frac{B_\alpha B_\beta}{B^2} \]

\[ \eta_\perp = 2 \eta_\parallel \]

\[ E_j = \eta_\parallel(0) \dot{\varphi}(0) = \eta_\parallel \dot{\varphi} + \eta_{\phi0} \dot{\varphi} \]

(\[ \eta_\parallel, \eta_\perp \text{ are independent of time. } \])

or \[ \eta_\perp = 2 \eta_\parallel = \text{const (in } r) \]
Growth rate $\gamma$ versus $S$ for $m = 1$ and $\beta = 0$.


\[ q(r) = 0.1225(1 - 1.87r^2 + 0.8323r^4). \]
\[ \beta = 0. \]
\[ q(0) = 0.1225 > q(r) > q(1) = -0.005145. \]

field reversal surface \( r = 0.93 \).
Results. (incompressible case).

1) \( m/n = 1/10 \) single helicity
   complete reconnection of the magnetic island and expulsion of the magnetic axis
   followed by a slow, second reconnection and the re-emergence of the original magnetic axis.

2) \( m/n = 1/11 \) single helicity
   the magnetic islands saturates without expulsion of the magnetic axis.
Electron Heat Transport in a Tokamak with Destroyed Magnetic Surfaces;

Rechester, Rosenbluth, P.R.L 40 38 (1978).


\[ \vec{B} = B_x \hat{\vec{z}} + B_{\theta}(x) \hat{\vec{\theta}} + \delta \vec{B} \]
\[ \delta \vec{B} = \sum m b_{mn}(x) \exp \left[ i (m \theta - n \frac{2\pi}{R} ) \right] + \text{C.C.} \]

island width
\[ \Delta_{mn} = 4 \left[ \frac{2 R}{m} \left| b_{mn}(x) \right| \frac{d \ell}{dx} \right] \left| x = x_{mn} \right| \]

stochasticity parameter
\[ S = \frac{1}{2} \left( \Delta_{mn} + \Delta_{mn'} \right) \left/ \left| x_{mn} - x_{mn'} \right| \right. \]

\[ S > 1 \]
\[ \ell(x) = l_0 \exp \left( \frac{x}{\ell_c} \right) \]

diffusion of field lines
\[ \langle (\Delta r)^2 \rangle = 2 \ell D_{st} \]
\[ D_{st} (x) = \pi R \sum_{m,n} \frac{b_{mn}(x)}{B_x^2} \delta \left( \frac{m}{\delta (x)} - n \right) \]

\[ \tilde{h} = \frac{1}{L_c} = \lim_{Z \to \infty} \lim_{d_0 \to 0} \left[ \frac{1}{Z} \ln \left( \frac{d(Z)}{d_0} \right) \right] \]

\[ \tilde{h} = \frac{1}{4} \frac{(\frac{2}{3})!}{(\frac{1}{3})!} (3D_{st})^{\frac{1}{3}} = 0.3645 D_{st}^{\frac{1}{3}} \]

**direct measurement of D_{st}**

\[ D_{st} = \lim_{Z \to \infty} \frac{1}{LZ} \sum_{n=1}^{L} x_n^2 \]

\[ x_n = Y - Y_0 \]

\[ \chi_r = \frac{\langle (\Delta Y)^2 \rangle}{2\tau} = D_{st} \nu_{\text{the}} \]

for \( \lambda \gg L_c \) **Collision-less case**
K-S Entropy

$S = 10^9$

$10^1$

$10^2$

$10^3$

0

500

1000

1500
HBTX-1A:

\[ a = 26 \text{ cm} \]
\[ T_e = 100 \text{ -} 200 \text{ eV} \]
\[ n_e = 10^{13} \text{ - } 10^{14} \text{ cm}^{-3} \]
\[ B = 10^3 \]
\[ S = 10^5 \]

\[ \tau_E = \frac{a^2}{4\chi_r} = \frac{26^2}{4\times20\times(4\times10^{-2})^3} v_{\text{the}} \]
\[ = 1.3 \times 10^{-4} \text{ sec} \]

\[ B_z(\text{bias}) = 500 \text{ Gauss} \]

\[ B_z(\text{reverse}) = -120 \text{ Gauss} \]

\[ B_\theta(a) = 1.54 \text{ kG at } I_p = 200 \text{ kA} \]
Robinson's Model

Initial Equilibrium

Safety factor \( \zeta \)

Suydam parameter \( C_1 \)

(Suydam condition \( C_1 < \gamma \phi \))

\[
q_1(r) = \frac{S_0}{\kappa} \left( 1 + \frac{\gamma^2}{\alpha} + \frac{\gamma^4}{\beta} + \frac{\gamma^6}{\delta} + \frac{\gamma^8}{\phi} \right)
\]

\[
C_1(r) = -\frac{4\pi p'}{\rho B^2} \left( \frac{\phi}{\delta} \right)^2 = C_0 \left( 1 + C_1 \gamma^2 + \cdots \right)
\]

\[
B_T(r) = \frac{q_1(r)}{q_1(0)} \exp \left\{ \int_0^r \frac{R^2}{\kappa^2 \gamma^2 + r^2} \left( C_1 \gamma^2 - \frac{\phi \gamma}{\delta} - \frac{2r}{R^2} \right) \right\}
\]

\( S_0 = 2, \ \alpha = -8, \ \beta = -400, \ \gamma = 2450, \ \delta = -110000 \)

\( C_0 = 0.19, \ \ \phi = -0.85 \)

Plasma radius = 4.2
Aspect ratio = 4

\( S = 10^5, \ \beta = 8.2\% , \ \beta = -0.83, \ \theta = 2.87 \)

\( \eta_\perp = 2 \eta_\parallel, \ \eta_\parallel = 1 \)
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$Q = \frac{2}{a^2} \int_0^\infty r B_s \, dr$

$\text{PIC NAME: RTFL}(t) = \frac{\Phi(t)}{\Phi(0)}$

$\times 10^{-1}$

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X-AXIS: Time

$10^{-3} r L_C$
Discussion

1) K-S entropy  Diffusion
   D measurement,

2) more modes. ------ pseudo-spectral code,

3) dynamo effect.     lower $\Theta$ model.

4) $\text{div } v \neq 0$,
   energy equation,

   Joule heating, anomalous loss,
MAGNETIC RECONNECTION IN TOKAMAKS

W. PARK

PRINCETON PLASMA PHYSICS LAB
Magnetic Reconnection in Tokamaks

W. Park
D. Monticello
R. White
Outline

• Ways of Reconnection.
  Reconnection rate $\gamma_s (\xi, \mu)$.

• Comparison with numerical simulation results.
  • External coil-driven reconnection
    • $m=1$ nonlinear internal kink
    • $m=2$ nonlinear tearing mode

• Application and comparison with tokamak experiment.

• Simulation of enhanced (double) sawtooth oscillation.

(Park, Monticello, White, Phys. of Fluids (27) 137 (1984))
Ways of Reconnection

- Magnetic topology change

\[ J_s \sim \frac{\Delta B}{\delta}, \quad \dot{\Phi}_s = \xi J_s \]

A) \( J_s \to \infty \) as \( \xi \to 0 \).

\[ \dot{\Phi}_s = \xi J_s = \xi^{\alpha} \quad (0 \leq \alpha < 1) \]

- \( l \sim \delta \) : Petschek type layer
  \[ \dot{\Phi}_s \sim (1 + \mu/\xi)^{-1/2} \]

- \( l \sim 1 \) : Sweet-Parker type layer
  \[ \dot{\Phi}_s \sim \xi^{4/3} (1 + \mu/\xi)^{-1/4} \]

B) \( J_s \to \text{finite as } \xi \to 0 \) (i.e., no current singularity)

\[ \dot{\Phi}_s = \xi J_s \sim \xi \]
2-D Reduced tokamak eqs.

\[
\begin{align*}
\frac{d q}{d t} &= -\varepsilon J \\
\frac{d \nabla^2 u}{d t} &= -\vec{B} \cdot \nabla J + \mu \nabla^2 \nabla^2 u
\end{align*}
\]

\[\vec{B} = \nabla^2 \phi \times \vec{e} + B_0 \vec{e}\]

\[\vec{v} = \nabla u \times \vec{e}\]

\[\psi(\alpha, \phi) \quad \phi = \theta + \alpha \varepsilon \quad \varepsilon = \vec{r} \times \vec{\phi}\]

- \(\alpha\) - finite difference packed grid inside a layer.

- \(\phi\) - Fourier space

- Quantitative study.
Helical coil driven
Reconnection

\[ m = 4 \]

\[ \zeta = 10^{-7} \]
\[ S = 10^{-7} \]

\[ \Delta B \approx 0.02 \]

TIME = 400.00
\[
\begin{align*}
\frac{\zeta}{\mu} & \ll 1 \\
\Delta \psi_s &= \frac{\zeta}{\mu} J_s \\
\dot{J}_s &= K (\psi_s)^{\frac{1}{4}} (\frac{\zeta}{\mu})^{\frac{1}{4}} \\
J_s &= K (\frac{\zeta}{\mu})^{\frac{1}{4}} \\
K_{num} &\approx 4 \times 10^{-3} \\
K_{th} &\approx (\Delta B)^{\frac{3}{2}} \frac{1}{\eta^{\frac{1}{2}}} \approx 3 \times 10^{-3}
\end{align*}
\]

\[\eta = 10^{-8}, \mu = 10^{-5}\]

\[\eta = 10^{-7}, \mu = 10^{-5}\]

\[\eta = 10^{-7}, \mu = 10^{-3}\]
$M=1$ Internal Kink  
Kadomtsev-Mantell flip

Time

$0 \quad m=1$ flip
$0 \quad$ Sawtooth Oscill.

final Ohmic state
\[ \mu < \mu \] 

\[ S \sim 10 \] 

\[ J \] 

\[ U \] 

\[ \Delta B \] 

\[ \cos \gg \mu \rightarrow \text{higher on tearing} \]
\[ \psi_s = k \xi^{1/2} \]

\[ J_s = k \xi^{-1/2} \]

No steady state

\[ \eta = 2 \times 10^{-7} \]

\[ \eta = 10^{-6} \]

\[ \eta = 2 \times 10^{-6} \]
\( \gamma \ll \mu \)

\[
\begin{align*}
\frac{\dot{J}_s}{J_s} & \sim \varepsilon^{\frac{1}{2}} \left( \frac{\gamma}{\mu} \right)^{\frac{1}{4}} \\
J_s & \sim (\gamma/\mu)^{\frac{1}{4}}
\end{align*}
\]

\( \eta = 10^{-8}, \mu = 10^{-4} \)

\( \eta = 10^{-7}, \mu = 10^{-4} \)

\( \eta = 10^{-7}, \mu = 10^{-3} \)
PDX Tokamak

Electrode gap = 0.05 cm, Ion gap = 0.5 cm, Collisionless s.d. = 0.6 cm
Resistive layer thickness = 0.05 cm, \( \mu \leq \varepsilon = 0.5 \times 10^{-7} \)

\[ q = k \varepsilon^2 \left( \frac{A}{2} + 1 \right)^{1/4}, \quad k = \frac{\Delta B \varepsilon}{\ell^2} = 0.04 \]

\( \tau = 3000 \tau_A \approx 0.4 \text{msec} \)

![Graph showing magnetic field behavior over time]

\( m = \frac{1}{2} \text{ Internal impedance} \)

0 ST tokamak: 100 \( \mu \)s: 70 \( \mu \)s
m = 2 Tear'ing

stable if \( z > 0 \)

\( B_\phi \)

\( U \)

\( J \)

\( J_5 \)
Experiment (Soft X-Ray Emissivity)

(a)

Symmetry

(b)

(c) Theory

Down

40

20

0

-20

Up

TIME (msec)

400

401

Center

TIME

B6

B5

B4

B3

B2

B1

B0

B-1

B-2

B-3

B-4

B6

B5

B4

B3

B2

B1

B0
Nonlinear Evolution of highly conducting confined plasmas

- Topology change
  - \( \mathcal{J} \to \infty \) as \( \varepsilon \to 0 \) at the x-point:
    \[
    \mathcal{J}_s \sim \varepsilon^{1/2} (1 + \varepsilon \lambda^2)^{-1/4}
    \]
  - no singularity:
    \[
    \mathcal{J}_s \sim \varepsilon
    \]

- Doublet or Spheromak formation:
  \[
  \mathcal{J}_s \sim \varepsilon^{1/2} (1 + \varepsilon \lambda^2)^{-1/4}
  \]

- 3-dimensionally, current singularities can occur at any place \((x\text{-line} \to x\text{-point})\) determined globally.

- Magnetic helicity is conserved in the fast reconnection time scale

\[
\frac{d}{dt} \left( \int \mathbf{A} \cdot \mathbf{B} \, d\Omega \right) \sim \int \epsilon \mathbf{j} \cdot \mathbf{B} \, d\Omega \sim \int \epsilon \mathbf{j} \, d\Omega \sim \varepsilon
\]
**Time scales**

- Island width

\[ m = 1 : \frac{\gamma}{2} \]
\[ m > 1 : \frac{\gamma}{3} \]

Linear phase  Nonlinear phase  Saturation

\[ \left\{ \begin{array}{l}
\tau_A \sim 0.1 \mu\text{sec} \\
\tau_s \sim \frac{\gamma}{2} \sim 1 \text{ sec} \\
\gamma \sim 10^{-7}
\end{array} \right. \]

Time scales of magnetic island growth:

- \( M = 1 \) : 1 msec  fast
- \( M > 1 \) : 1 sec  slow
Enhanced (double) Sawtooth

Soft X-ray

Central Soft X-ray Emissivity

$I_p = 1.3 \text{ MA}$

$\bar{n}_e = 2.3 \times 10^{13} \text{ cm}^{-3}$

Center channel

Outside of $z = 1$

$r = 0 \text{ cm}$

$r = 40 \text{ cm}$
\[ \frac{d\psi}{dt} = \epsilon J = \epsilon a^2 \psi \]
HALF COALESCENCE OF NONLINEAR M=1 MODE

R. PELLAT

ECOLE POLYTECHNIQUE
Half coalescence of M-linear $m=1$ mode

RPELLET

dynamics of the reconnection process unclear

- if the wit kink is linearly stable, origin of the current sheet?
- fast time scale of the disruption

- partial reconnection

$\Rightarrow$ ideal MHD mode at the origin?

- chain of magnetic island is unstable to an ideal MHD mode, leading to coalescence of the islands.

The $m=1$ island is unstable to a half-coalescence motion, which destroys the up-down symmetry of the configuration.
let us start with standard SW:

$$L = \int \mathcal{D}a \left( \mathcal{L}^2 - \frac{i}{2} \mathcal{L} \times \mathcal{L} \right)$$

$$\mathcal{L} = \nabla \times (\mathcal{E} \times \mathcal{B})$$

helical deformations included

$$a = \theta - h\zeta$$  \text{h pitch.}  

$$h = c$$

$$\mathcal{B}_\perp = \mathcal{E}_3 \times \nabla \mathcal{F}$$

$$f_2 \text{ function of } G = F - B \approx h \frac{\zeta^2}{2}$$

$$\Delta F = f_2$$

by expansion in (ha), $h \ll a \gg 1$

$$\mathcal{E}_3 = 0$$

$$\Rightarrow 2L \approx \int \mathcal{D}a \left( \frac{1}{2} \mathcal{L}^2 + \frac{\partial \mathcal{L}^r}{\partial \mathcal{U}^i} \right) \mathcal{L} \times \mathcal{L} \mathcal{U}^i$$

$$\mathcal{L}_\perp = \mathcal{E}_3 \times \nabla \mathcal{U}$$

$$\mathcal{T} = \mathcal{B}_\perp - h \mathcal{B}_3 \times \mathcal{E}_3$$

$$\Rightarrow \text{Euler}$$
Euler equation:
\[ \mathbf{E} \cdot \nabla \psi - \frac{d}{dt} \mathbf{T} \cdot \nabla \psi = 0 \]

It is easy to prove that translations, rotations, and the solution of equation:
\[ \frac{\partial \psi}{\partial x, y} \]

are exact solutions of equation. By derivation of equilibrium equation, they stabilize for internal finite modes in cylindrical channels for different shapes:

\[ q = \begin{array}{cccc}
\bigcirc & \bigcirc & \triangle & \bigcirc \\
\end{array} \]

All corrections of order \( \varepsilon^2 \) if:
\[ F = F_0 + \varepsilon F_1 \]

\[ \square, \bigtriangleup \text{ stabilize} \]
\[ \bigcirc \text{ destabilizes} \]
let us recall curvature effects.

cylindrical destabilizing $O(\frac{5}{R})^2$

Shafarevich.

toroidal stabilizing

if $\beta_p > 1$ stable.

Bussac, Pellat, Edery, Foule'.


then non-linear cylindrical.

Rosenbluth, Rutherford, D Jasian.

Park, White, Monrbello, Jordon.

Eq: is in numerical code, but not true in toroidal.
Now with sensitivity:

Coppo, Galvao, Pellet, Rosenbluth, Rutherford

\[ \text{Int. J. Plasma Phys. 2 (1976) 553} \]

\[ \{ \text{internal resistive mode: } z \sim y^{-2/3}. \} \]

\[ \{ \text{due to non constant } \psi, \text{ } \psi w \sim 0.5(\psi) \}

them numerical simulations:

"Kadomtsev theory"

previous talk.

Let us now speak of our recent approach:

I first come back to \( \psi \).

the idea is the following:

\[ \Gamma \text{ has any shape.} \]

\[ C \text{ is for any but a circle.} \]
let us compute $SW$.

inside the core: we call $\psi_e$ rotation about magnetic axis. Outside in the island, rotation about the center of $C$: $\psi_R$.

$\psi_e = B \cdot D \psi_e$ \hspace{1cm} $\psi_R = B \cdot D \psi_R$

exact solutions apart from boundary conditions on $\Gamma$.

One set, inside the core:

$\psi_e + \psi_R + \psi_i$

outside the island:

$\psi_R + \psi_i$

on $\Gamma$: $\psi_R + \psi_i = \psi_e + \psi_R + \psi_2$

by integration by parts:

$SW = \int d\Omega \left\{ \nabla^2 \psi_i - \frac{2i}{\hbar} \nabla \psi_i \frac{\partial \psi_i}{\partial \rho} \right\} + \int d\sigma \left\{ \rho \psi_e \psi_{e \rho} - \psi_e \psi_{\rho e} \frac{\partial \psi_i}{\partial \rho} \right\} + \frac{2}{\hbar} \frac{\partial \psi_i}{\partial \rho} (\psi_e + \psi_i) \right\}$

if $\Gamma$ is the circle, we found the
result of:
Buseri, Pellat, Souli, and Tagger.

Physica letters, 105 A, 112 (1 October 84)

\[ \chi_0 = \int d\Omega \, \psi_k \overset{\approx}{=} \psi_0 \]

because on \( \Gamma \), \( \psi_e = 0 \), \( \psi_1 = \psi_2 = 0 \).

If non circular \( \Gamma \):
\[
\begin{align*}
& \text{corrections are all of order } \varepsilon^2, \\
& \text{when our result of order } \varepsilon (\sim \varepsilon). \\
\end{align*}
\]

Let us explain in more detail our result for circular core.
rotation:

\[ \dot{\xi} = e_3 \times \nabla \mathbf{U} \]

\[ \mathbf{U} \]

\[ (1) \]

\[ \dot{\xi} = e_3 \times \nabla \mathbf{U} \]

\[ (2) \]
Equilibrium

Separatrix \((\rho = b, r = a)\)

\[
\vec{B} = B_0 \vec{e}_z + kr B_0 \vec{e}_\theta + \vec{B}^*
\]

\[
\vec{B}^* = \vec{e}_z \wedge \nabla G (r, \theta - k\varpi)
\]

\[
G (r, \theta - k\varpi) = (r-a)(\rho-b) F(r, \rho)
\]
inside the island, transverse solid rotation:
\[ \tau = e z \times \nabla u_i \quad u_i = \frac{1}{2} \mathcal{P} \]
displacement inside the core:
\[ \tau = e z \times \nabla u_c \quad u_c = \frac{1}{2} \mathcal{P} - \frac{1}{2} \lambda (\mathcal{P} - \mathcal{S}) \]
then
\[ \mathcal{S} W = \frac{1}{2} \lambda \mathcal{P} \int_0^{2\pi} d\phi (\mathcal{S}_\phi)^2 \cos (\alpha - k z) \]

x point \( a - k z = 0 \) \( S = a - k \) island half width. \( \mathcal{S}_\phi \):
field at the inner nipenatrix.

to be a little bit more explicit:
\[ n \cdot \mathbf{D} \Psi_c = \frac{2}{\mathcal{P}} (e \mathcal{B} \mathcal{E}) = - \frac{2}{\mathcal{P}} (e \mathcal{B}) \]
\[ n \mathcal{B}_z = \Psi_k = \frac{1}{2} (z - a) \frac{2}{\mathcal{P}} F = \frac{1}{2} (z - a) F \frac{(a - c) \sin \theta}{\mathcal{P}} \]
with \( z \sin \theta = \rho \sin \theta \)
\[ \Psi_R = \rho \sin (a - c)(z - a) F = (a - c) \mathcal{B}_z \rho \sin \theta \]
then by integration by parts \( \rightarrow \mathcal{S} W \).
\[ \max \text{ of } \gamma \quad \lambda = \left( \frac{a}{b} \right)^2 \]

\[ \begin{cases} 
\lambda = 1 & \text{translating of the core} \\
\lambda = \frac{a}{b} & \text{core rolls along other refer matrix} 
\end{cases} \]

\[ \gamma \sim \left[ 1 - g(0) \right] \frac{V_{\text{ap}} \mu^{1/2}}{a} \]

\[ B_0^2 = [1 - g(0)] kaB_0 \quad V_{\text{ap}} = \frac{k^2 a^2 B_0^2 \rho}{\mu} \]

\[ \mu = \frac{2a^2}{(a+\epsilon)} \int_0^{\epsilon} \left[ -B_0^2/B^2 \right] \cos (\alpha - k \xi) \, d\alpha \]

\[ \gamma \sim V_{\text{ap}} \frac{a}{a^2} (1 - g(0)) \quad \text{small} \]

\[ \gamma \propto \text{island size, not } \mu^{1/2} \]

it is an ideal Fermi mode

with one big difference: the finite island.
Let us recall coherence. *attraction of current*

\[ A. \text{ Bondeson, Phys. Rev. Lett. 51, 1660 (1983)} \]

\[ R. \text{ Pellegr, Phys. Plasmas 9, 121 (1982).} \]

\[
\psi - \psi_0 \sin k y + \alpha \left( \psi_0 \sin k y - \frac{\psi_2 k^2 \cos 3k y}{3} \right)
\]

*not present for Bondeson.*

Equivalent of translating.

two extremal \( \alpha \to \infty \)

\( \alpha < a \) finite true extremum

\[ \text{half coherence} \]

*in fact, for one island instability*.

Let us comment about this. It has a growth \( \sim \left( \frac{g_0}{c} \right)^{3/4} (\Delta')^{1/2} \end{equation}\]

like Diamond's and all \( 3/2 \) mode
but it is not correct in true Rutherford growth because Rutherford current makes $\Delta' \approx 0$ for $M+0$ time scales. This instability can appear if one has faster modification of the outside $\Delta'$ than reactive Rutherford current can achieve.

In the case of the $m=1$ mode the FWH is due to the $B$ modulation along the inner separatrix $x$. If there is no such modulation then there is no source of energy, but this modulation is a consequence of the island formation. ($B^2 \approx 0$ at $x$ point)

For the self-evident case, there is repulsion of the two currents:

![Diagram of two currents with repulsion](image)
A SAWTOOTH OSCILLATION MODEL

T. JENSEN

GA TECHNOLOGIES
A SAWTOOTH OSCILLATION MODEL

BY TORKIL H. JENSEN + FRED W. ACCLAIM

GA TECHNOLOGIES INC. [BULL. AM. PHYS. SOC. 29 1370 (1984)]

FRAMEWORK: EXPERIMENT

THEORETICAL

COMPUTATIONAL

ASTROPHYSICS

TOKAMAKS

RAPIN

SPHEROMAKS

TRANSPORT WITHOUT RECONNECTION

MULTIDIMENSIONAL CONFIGURATION PARAMETER SPACE

POSSIBLE RESULT OF CONTRADICTING TENDENCIES

i) CONFIGURATION WITH \theta < 0

ii) SIMPLE FLUCTUATIONS (OSCILLATIONS)

iii) COMPLICATED FLUCTUATIONS (TURBULENCE)
Aims of this work:

A SIMPLE UNDERSTANDING

OF THE SAW TOOTH OSCILLATION

WORK IS IN PROGRESS

MANY GUESSES NEEDED AT THIS TIME
ANALOGY: (RELAXATION OSCILLATOR)

\[ F \left( \frac{dx}{dt} \right) = E \left( x_2 - x_1 \right) \]

\[ V \left( I_1 \right) = \frac{1}{2} \left[ I_1 dt - I_0 dt \right] \]

ARE SAWTOOTH OSCILLATE THAT SIMPLE?
**Simplifying Assumptions:**

1. Resistive mild
2. Neoclassical symmetry \([ (\vec{E} + \vec{E}_B) \cdot \vec{B} = 0 ]; \) boundary: Circular cross-section conductor.
3. Tokamak ordering \(( (R/A)^2 < 1 )\)
4. Description of equil. by two parameters only. (May include effect of some turbulence)

**Equilibrium:** \( \vec{E} \cdot \vec{E} + \theta \{ \psi \} - 1 = 0 ; \psi_R = 0 \)

Two parameters for description of \(M:\)

- \( \mu \) is maximum current density \((\mu > 0)\)
- \( x \) is peakedness \(0 < x < 1\)

\[
M_1 = \frac{\mu}{x(x_{\text{op}} - \psi_e)} \left\{ \frac{K+1}{4} (\psi-\psi_{\text{op}} + 1^2) + \psi_e \right\}
\]

\[
M = \frac{1}{2} [ M + \mu - 1 - M_1 ]
\]

\(\psi_e\) is largest \(\psi\)-value at magnetic axes.

\(\psi_{\text{op}}\) is second largest \(\psi\)-value at magnetic axes.
TRANSPORT WITHOUT RECONNECTION:

\( \frac{dx}{dt} > 0 \quad \frac{dy}{dt} > 0 \) (THERMAL INSTABILITY)

TRANSPORT SLOW EQUIL.

SOLUTIONS TO EQUILIBRIUM PROBLEM:

i) WHEN \( \mu < 1 \), \( \partial^2 y > 0 \), ONE MAGNETIC AXIS, ONE SOLUTION, AXISYMMETRIC, \( \theta > 1 \)

ii) WHEN \( \mu > 1 \) MANY SOLUTIONS

1 SOLUTION WITH 1 MAGNETIC AXIS (\( \theta < 1 \))

2 SOLUTIONS WITH 3 MAGNETIC AXES \( \uparrow \) UP-DOWN SYMMETRIC

MANY SOLUTIONS WITH MANY MAGNETIC AXES \( \uparrow \) ASYMMETRIC

UP-DOWN SYMMETRIC: \( y(x, \theta) = y(x, -\theta) \)

ASYMMETRIC: \( y(x, \theta) \neq y(x, -\theta) \)
STABILITY:

ASSUMPTION: CONSIDER ONLY PERTURBATIONS OF SAME HELICAL SYMMETRY AS EQUILIBRIUM [(ε + η_0 δ) · δ = 0]

FORMALISM FOR STABILITY:

STABLE IF LOWEST EIGENVALUE, λ_0, IS POSITIVE

\[ \frac{d^2 ϕ_ν + DH(ϕ_ν)}{dt^2} - <DH(ϕ_ν)> + λ_ν ϕ_ν = 0, \]

ϕ_νB = 0, DH(ϕ) = \( \frac{1}{ε} [M(ϕ + ε ϕ_ν) - M(ϕ)] \)

\[ ε \rightarrow 0 \]

\[ <S> = \frac{∫ S dϕ}{∫ dϕ} / ∫ dϕ \]

INCLUDES STABILITY AGAINST:

i) SLOW RESISTIVE MODE (5 α γ)

ii) TEARING MODE (5 α γ')

iii) KINIC (IDEAL) MODE (5 α γ°)

* JENKIN + THOMPSON, J. PLASMA PHYS. 19 227 (1978)
IN STABILITY OF UP-DOWN SYMMETRIC 3-AXIS EQUIL.

EQUIL: \( \psi(x,z) = \psi(x,-z) \)

DIFFERENTIATE EQUIL. E.Q. W.R.T. \( y \):

\[ \frac{\partial \psi}{\partial y} + \Delta \psi \frac{\partial \psi}{\partial y} = 0 \]

NOTE: \( \frac{\partial \psi}{\partial y} = 0 \) ON SURFACE INSIDE BOUNDARY.

COMPARE TO STABILITY EIGENVALUE PROBLEM WITH \( \varphi = \partial \psi/\partial y \).

BECAUSE OF SYMMETRY, \( \langle \Delta \psi \frac{\partial \psi}{\partial y} \rangle = 0 \)

SO \( \Rightarrow \) Marginally Stable (2:0) When \( \varphi \) Forced To Vanish Inside Boundary

\( \Rightarrow \) Unstable When \( \varphi = 0 \) On Boundary.

SEE ALSO: BUSLAC, PERLAT, SOUCE + TAGGER, INT. CONF. ON. PLASMA PHYS., LAUJANNE 1984

ARGUMENT ALSO HOLDS FOR 1-AXIS EQUIL. WHEN \( \Lambda > 1 \) (\( \varphi < 1 \))

\( \Rightarrow \) KRUSKAL LIMIT.
$x$ FIXED

Asymmetric

Stability Limit

Axisymmetric

Updown Sym.
WE ASSUME THERE EXISTS STABLE 3-AXIS EQUILIBRIA.

THERE MUST BE ASYMMETRIC.

\[ \mu \rightarrow 1 \]

STABILITY LIMIT

NO STABLE EQUIL.

LINES OF CONSTANT CURRENT

ONE 1-AXIS EQ.
STABLE (q > 1)

UNSTABLE

ONE 3-AXIS EQ.
UP-DOWN
ASYMM.
STABLE

ONE 3-AXIS EQ.
ASYMM.
UNSTABLE

MANY MANY AX.
EQUIL.
UNSTABLE?
NATURE OF INSTABILITY:

SIMILAR TO DROPLET-ELLIPSE NODE OBSERVED IN DOUGLET.

NONLINEAR EFFECT:

\[ i) \] ALWAYS NONLINEARLY DESTABILIZED \[ \rightarrow \text{EXPLOSIVE} \]

\[ ii) \] SPECIFIC DIRECTION

GUESSES: THIS INSTABILITY BRINGS EQUILIBRIUM BACK TO 1-AXIS EQUILIB.
(INNER ISLAND DISAPPEARS).

SINCE \( \psi(a) \) IS CONSERVED WE CAN ESTIMATE \( \tau, \mu \) OF RESULTING 1-AXIS EQUILIBRIUM.

\[*\]
This is very close to a relaxation oscillator. For given \( I \) we can find both \( \mu_{\text{max}} \) and \( \mu_{\text{min}} \).

\[ \mu \uparrow \]

\[ \begin{array}{c}
\mu_{\text{min}} \\
\mu_{\text{max}}
\end{array} \]

\( \mu \) has probably a monotonie relationship to soft x-ray intensity observed in tokamaks.
**Numerical Effort:**

1. Make an Equilibrium Code
   - Input: μ, ν, N0 of axes

2. Modify Stability Code

3. Make sure "direction" on nonlinear instability has right sign

   In Progress

   Exaried

   Easy
SOFT X-RAY IMAGING OF TRANSIENT AND DOUBLE SAWTEETH ON TEXT

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Soft X Ray Imaging of Transient and Double Sawteeth on TEXT

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US-JAPAN Workshop on Magnetic Reconnection

Austin, Texas

December 10 - 13, 1984
SERIES OF DOUBLE SAWTEETH ON SOFT X RAY SIGNAL

SHOT #: 39386

MSEC
DOUBLE SAWTEETH ON RADIOMETER

SHOT # 58199  \( r = 5 \text{ cm} \)

\[ \begin{align*}
1.35 \text{ Kev} \\
1.15 \\
0.6 \\
0.0
\end{align*} \]

\[ \begin{align*}
250 & \quad 260 & \quad 270 & \quad 280 & \quad 290 & \quad 300 \\
\end{align*} \]

SHOT # 58213  \( r = 19 \text{ cm} \)

\[ \begin{align*}
1.40 \text{ eV} \\
0.6 \\
0.0
\end{align*} \]

\[ \begin{align*}
250 & \quad 260 & \quad 270 & \quad 280 & \quad 290 & \quad 300 \\
\end{align*} \]
RELATIVE AMPLITUDE PROFILES OF DOUBLE SAWTEETH

1ST DISRUPTION

2ND DISRUPTION

RADIUS (cm)

dA

A

( % )

7.5

6.0

4.5

3.0

1.5

0.0

-15

-10

-5

0

5

10

15
M=1 MODE DURING DOUBLE SAWTEETH

SHOT NO. 39145  TIME(MSEC)  312.00 - 316.00
SECOND CURRENT RISE CHANGES IN SAWTEETH
SHOT # 56658

X MON

200 KA

I P

100

4 x 10^13 cm

N E

0 40 80 120 170 220 270 320 370
MSEC
\[ B_{p0}(r) = \frac{r}{R} B_T + \alpha r (r^2 - r_{s1}^2)(r^2 - r_{s2}^2) \]

\[ \Psi^*(r) = \frac{B_T}{R} \int \left( \frac{1}{g(r)} - 1 \right) dr \]
Summary

. Observed relative amplitude profiles of the transient sawteeth and the double sawteeth are consistent with partial reconnection model assuming hollow current profile.

. Because less heat is released to outside \( r_s \) during the 1st disruption of a double sawteeth, inside \( r_s \) is superheated and outside \( r_s \) is supercooled, which results in huge 2nd disruption ( \( dA/A > 20\% \)).

. Two \( m=1 \) mode were observed during double sawteeth.

. Radiation loss due to impurity near the center of the plasma and anomalous current penetration may be responsible for a hollow current profile.
HALF COALESCENCE OF THE M=1 ISLAND IN TOKAMAKS

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University of Texas at Austin
Institute for Fusion Studies
HALF-COALESCEENCE OF THE $m=1$ ISLAND IN TOKAMAKS

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INSTITUTE FOR FUSION STUDIES
UNIVERSITY OF TEXAS
\[ \delta = a - b \]

\[ \rho^2 = \delta^2 + r^2 - 2\delta r \cos \theta \]

Perturbation:

Core: \( r < a \), \( \rho < b \)

\[ \phi = \frac{r^2}{2} - \frac{a}{2} (\rho^2 - b^2) \]

Inland: \( r < a \), \( \rho > b \)

\[ \nu = \frac{\hat{z}}{2} \times \nabla \phi \]
\[ Y_{\perp} = r(1-x) \]  
\[ + \hat{\delta} \{ -2 \delta \sin \theta \} \]  
\[ + \hat{\delta} \{ -2 \delta \cos \theta \} \]  

\(m = n = 0\)  
\(m = 1, n = 1\)  
\(m = 1, n = 1\) 
\(\gamma \parallel v_{\perp}\):

\[ \text{Symmetric} \]
\[ \text{Anti-Symmetric} \]
Source of free-energy is the attracting potential between the two dipole-fields.
\[ \frac{\partial v'}{\partial t} = \nabla' \times B_0 + \nabla' \times B' \quad [m=1] \]

\[ + \nabla' \times B_{SS} + \nabla' \times B' \quad [m=0] \]

\( J_0, B_0 : m=0, n=0 \)

\( J_{SS}, B_{SS} : m=1, n=1 \) (EQUILIBRIUM ISLAND)

\( \nabla', B' : m=1, n=1 \) (PERTURBATION)

**TWO PROCESSES:**

1) \( m=1 \) TEARING

2) IDEAL ROTATIONAL INSTABILITY

If \( B_{SS} \sim \delta^2 \), then \( \Delta_{rot} \sim \delta^2 \) ?
"EQUILIBRIUM" for the linear calculations

17:45:50  11/30/84  T = 5.65E+02  ZETA = 0.00
VELOCITY MAX = 1.03E-05

17:45:50  11/30/84  T = 5.65E+02  ZETA = 0.00
B-FIELD MAX = 4.39E-02

17:45:50  11/30/84  T = 5.65E+02  ZETA = 0.00
MIN = -8.44E-03  MAX = 8.44E-03  W/N = 1/1
R(IN) = 6.54E-01  R(OUT) = 7.59E-01  W = 1.05E-01

17:45:50  11/30/84  T = 5.65E+02  ZETA = 0.00
MIN = -7.13E-04  MAX = 6.79E-02  W/N = 1/1
R(IN) = 0  R(OUT) = 0  W = 0

VENTURED HELICAL FLUX

TOTAL HELICAL FLUX
\[ z = 3.08 \]

11:58:01  11/30/84  \( t = 8.08 \times 10^2 \)  ZETA = 0.00
B-FIELD  MAX = 4.07E-01

11:58:01  11/30/84  \( t = 8.08 \times 10^2 \)  ZETA = 0.00
MIN = -1.03E-01  MAX = 6.60E-03  M/N = 1/1
R(IN) = 5.24E-03  R(OUT) = 2.93E-01  W = 2.88E-01

11:58:01  11/30/84  \( t = 8.08 \times 10^2 \)  ZETA = 0.00
VELOCITY  MAX = 1.79E-01
INITIAL CONDITIONS

FOR THE NONLINEAR CALCULATION

ASYMMETRIC ISLAND
\[ t = 142 \]
FOR COMPARISON, A SYMMETRIC ISLAND EVOLVES TO THE FOLLOWING STATE.
$S_0 = 10^4$
ARE VACUUM BUBBLES A CAUSE OF MAJOR DISRUPTIONS?

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ARE VACUUM BUBBLES A CAUSE OF MAJOR DISRUPTIONS?

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IDEAL KINK INSTABILITY

VACUUM BUBBLES

KADOMTSEV & POGUTSE

ROSENBLUTH, MONTICELLO, STRAUSS & WHITE

PLASMA
INCLUDING RESISTIVITY
\[ m = 2 \text{ tearing mode saturates benignly at an island width } w \lesssim 0.4a \]

MULTIPLE HELICITIES -

WADDELL, CARRERAS, HICKS & HOLMES

UNDER SOME CIRCUMSTANCES, THE ISLANDS OF THE 2/1, 3/2, 5/3, ... OVERLAP

→ FLUX SURFACES DESTROYED
Our conclusions

* When \( q(0) > 1.5 \), the \( m=2 \) tearing mode grows to a very large amplitude, its magnetic islands encompassing virtually the entire plasma cross section.

* These very large islands produced by the resistive tearing mode are related to the vacuum bubbles produced by the ideal kink instability.

* Is \( q(0) \) larger than 1.5 during major disruptions?
SCENARIO FOR MAJOR DISRUPTIONS

1. MAGNETIC ISLANDS ASSOCIATED WITH MODES OF DIFFERENT HELICITIES OVERLAP
   → FLUX SURFACES DESTROYED
   → TEMPERATURE & CURRENT PROFILES FLATTENED
   → $q(0) > 1.5$

   IF THE DISCHARGE RECOVERS AT THIS POINT
   → MINOR DISRUPTION

2. ONCE $q(0) > 1.5$, THE ISLANDS OF THE $m=2$ TEARING MODE RAPIDLY GROW TO FILL THE CROSS SECTION
   → HOT CENTRAL PLASMA EXPELLED TO EDGE
   → DISCHARGE TERMINATES

* THIS SCENARIO IS SUPPORTED BY EXPERIMENTAL OBSERVATIONS OF TWO-STAGE DISRUPTIONS
REDUCED RESISTIVE MHD

OHM'S LAW

\[- \frac{\partial \psi}{\partial t} - \mathbf{B} \cdot \nabla \psi = \eta \mathbf{J}\]

VORTICITY EQUATION

\[\frac{d}{dt} \mathbf{W} = \mathbf{B} \cdot \nabla \mathbf{J} + \mu \nabla^2 \mathbf{W}\]

AMPERE'S LAW

\[\nabla^2 \psi = -\mathbf{J}\]

WHERE

\[W = \nabla^2 \psi = \hat{\mathbf{z}} \cdot \nabla \times \mathbf{V}\]
\[\mathbf{B} = \hat{\mathbf{z}} \times \mathbf{V} \times \nabla \psi\]
\[\mathbf{V} = \hat{\mathbf{z}} \times \nabla \psi\]

UNITS

\[\frac{t}{\tau_A} \rightarrow t, \quad \tau_A \equiv R/c_A\]
\[a \nabla \rightarrow \nabla, \quad R \frac{\partial}{\partial z} \rightarrow \frac{\partial}{\partial z}\]
EQUILIBRIUM

\[ q(t) = q(0) \left[ 1 + \left( \frac{r}{r_0} \right)^2 \right] \]

\[ \frac{r_0 - 2\lambda}{r_0} = \left[ \frac{q(1)}{q(0)} \right] - 1 \]

\[ q(1) = 3.4 \]

\[ \lambda = 4 \]
RESISTIVITY

1) \( \eta (r) = \frac{\eta}{J(r)} \)

2) \( \frac{\partial \eta}{\partial t} + \mathbf{v} \cdot \nabla \eta = \kappa_0 \mathbf{B} \cdot \nabla \mathbf{B} \cdot \nabla \eta + \kappa_1 \nabla^2 \eta \)

NUMERICAL SIMULATION

\( \psi, \varphi, \eta \sim e^{i(m \theta - nz)} \)

\( \frac{m}{n} = 2 \)

UP TO 60 HARMONICS RETAINED

200 RADIAL GRID POINTS
\[ \eta = 5 \times 10^{-5} \]
* ISLAND ABRUPTLY PENETRATES TO THE CENTER WHEN $q(0) > 1.5$
Doubling Time $\tau$

\[ \eta \quad \tau \]

$5 \times 10^{-5}$

$5 \times 10^{-6}$

\[ \Rightarrow \quad \frac{\tau}{\eta} \sim \eta^{-0.2} \]
IDEAL KINK

\[ q(r) = q(0) \left[ 1 + \left( \frac{r}{r_0} \right)^{2\lambda} \right]^{1/\lambda} \]

\[ r_0^{-2\lambda} = \left[ \frac{q(r)}{q(0)} \right]^{\lambda} - 1 \]

\[ J(r) = \frac{2}{q(r)} \left[ \frac{q(0)}{q(r)} \right]^{\lambda} \]

\[ \lambda \to \infty, \quad J(r) \to \frac{2}{q(0)} \]

\[ \eta(r) \sim \frac{1}{J(r)} \]
- Resistive
+ Ideal

* Saturation of Resistive and Ideal Cases is the same when \( q(0) > 1.5 \)
SELF-CONSISTENT \( \eta \) EVOLUTION

\[
\frac{d \eta}{dt} + \nabla \cdot \nabla \eta = K_\parallel \nabla \cdot \nabla \eta + K_\perp \nabla^2 \eta
\]

\[
\eta(t, t = 0) = \frac{\eta}{I(H)} \quad , \quad \eta = 5 \times 10^{-5}
\]

\( K_\parallel = 1 \)

\( K_\perp = 10^{-5} \)
Central temperature has dropped by a factor of six.
\( m = 2 \) SAWTEETH

* REGION INSIDE SEPARRATRICE HAS \( q \approx 2 \)
AFTER RECONNECTION.

\[
\begin{array}{c}
q \\
3 \\
2 \\
1 \\
0 \\
\end{array}
\]

\[
\begin{array}{c}
r \\
1 \\
\end{array}
\]

* OHMIC HEATING LOWERS \( q \) BELOW 2;
LARGE ISLANDS GROW AGAIN

\( \rightarrow m = 2 \) SAWTEETH
Experimental Evidence

T-4 Mirnov & Semenov, IAEA (1976)

20 Major Disruptions in Nearly Identical Discharges

Two Stage Disruptions:

1. Pre-Disruption: Interaction of \( m = 2 \) and \( m = 1 \) Perturbations results in a radial expansion and flattening ofCurrent and Temperature Profiles. \( q(0) \) rises above 1.5.

2. Disruption Per Se: Development of \( m = 2 \) Perturbation in Flattened Current Profile; Expansion of Column To The Wall
- L. LAURENT
Température électronique au centre

Activité M.H.D.

Courant plasma
**JIPP T-II Tokamak**

**Tsujii, et al. (1983)**

* During major disruptions, the dominant mode is $m/n = 2/1$ modulated by a weaker $1/1$ mode when the $2/1$ amplitude becomes large.

* No $3/2$ mode is detected during major disruptions. The $3/2$ mode is only detected during partial (minor) disruptions in which the central region of the discharge is not affected.
**JIPP III**

![Graph with various traces labeled as B_in, B_out, V_loop, B_θ, Hard X, and Z=0cm, with Soft X-ray traces at -2, -4, -6, -8, -10, -12, and -14. The t-axis is labeled from 150 to 190 ms.]

- **Tsujii, et al.**
SUMMARY

* WHEN $q(0) > 1.5$, $m=2$ MAGNETIC ISLANDS GROW TO A VERY LARGE AMPLITUDE ENCOMPASSING VIRTUALLY THE ENTIRE PLASMA CROSS SECTION.

* THESE VERY LARGE ISLANDS ARE RELATED TO THE VACUUM BUBBLES FORMED DURING THE NONLINEAR DEVELOPMENT OF THE IDEAL KINK INSTABILITY.

* AS A RESULT OF THE FORMATION OF THESE LARGE ISLANDS, THE HOT CENTRAL PLASMA IS CONVECTED OUT TO THE EDGE.
Our scenario for major disruptions is supported by the observation of two stage disruptions on several tokamaks.

If this scenario is correct, then a possible method for avoiding major disruptions is to strongly heat at the center (using ECRH, for example) thereby driving $q(0)$ down.
ENERGY PRINCIPLE FOR TOROIDAL PLASMA WITH BOUNDARY

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< Energy Principle for Toroidal Plasma with Boundary >

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- Generalized fundamental global invariant.
- Energy principle for plasma with boundary.
- Typical numerical results.

Refs.
papers on Energy Principle, Variational Principle, Relaxation ....

\[ \mathcal{W} = \int \left( \frac{B^2}{2\mu_0} + \frac{P}{\gamma-1} \right) dV \]
\[ \mathcal{W}_m = \int \frac{B^2}{2\mu_0} dV. \]

  ideal MHD. \( \delta \mathcal{W} = 0 \rightarrow \nabla P = \vec{j} \times \vec{B} \).

  ideal MHD. \( \delta^2 \mathcal{W} > 0 \rightarrow \) ideal MHD stability.

  slightly resistive MHD \( \rightarrow \) global invariant \( K_0 = \int_0^L \vec{A} \cdot \vec{B} dV/\mu_0 \).
  mag. energy \( \mathcal{W}_m \rightarrow \) minimum \( \Rightarrow \mu_0 \vec{j} = \lambda_0 \vec{B} \).

  helical tearing mode \( \rightarrow \) global invariant \( K_2 = \int x d\phi/\mu_0 \).
  \( \mathcal{W}_m \rightarrow \) minimum \( \Rightarrow \mu_0 \vec{j} = \lambda_0 \vec{B} \).

  infinite sets of global invariants \( K[\omega], M[u], S[v] \).
  \( \mathcal{W} \rightarrow \) minimum.

  \( \delta K_0 = 0, \nabla P = \vec{j} \times \vec{B} \rightarrow \mathcal{W} \) minimum \( \rightarrow \)

- J.D. Jukes, H. Oshiyama, T. Tsukada et al. ---
Three unsophisticated questions:

(I) Why do we obtain the same result of 
\[ \mu \vec{j} = \lambda_0 \vec{B} \], starting from the two different global invariants \( K_0 \) and \( K_\chi \)?

(II) Why do we obtain the minimum-energy state of \( \mu \vec{j} = \lambda_0 \vec{B} \) which does not satisfy the experimental boundary condition of \( \vec{j} = 0 \) at the wall?

(III) Are all the infinite set of global invariants \( K[w], M[u], \) and \( S[v] \) new mutually independent, physically available, fundamental invariants?
logical structure of the Taylor's theory on the minimum-energy state.

< 1st step > ideal MHD timescale $\Delta t(\text{IMHD})$

    virtual displacement
    (initial state) $\rightarrow \vec{x} \rightarrow$ (final state)

local helicity $\Delta K_0^i = \Delta K_0^f$

$\downarrow$

< local invariant >

global $K_0^i = K_0^f$

< global invariant >

< 2nd step > quasi-ideal MHD timescale $\Delta t(\text{QIMHD})$

    slightly resistive, reconnection of field lines.

local $\Delta K_0^i \neq \Delta K_0^f$

global $K_0^i = K_0^f$ (conjecture)

< 3rd step > minimization of mag. energy $W_m$

under the global constraint $K_0^f = \text{const.}$

$\rightarrow \delta K_0^f = 0$

$\Rightarrow \delta W_m + \lambda_0 \delta K_0^f = 0 \Rightarrow \mu_0 \vec{j} = \lambda_0 \overrightarrow{B}$. 
Kadomtsev's model

2nd step global $K_x^i = K_x^f$

3rd step $\delta \overline{w_m} + \lambda_0 \delta K_x^f = 0 \rightarrow \mu \overline{j} = \lambda_0 \overline{B}$.

Question (I)? (Ko, Kx → μj = λ0B).

* Generalized fundamental global invariant

<1st step> $\Delta t$ (IMHD)

initial $\rightarrow \overline{j} \rightarrow$ final

local poloidal flux $\psi^i = \psi^f$
toroidal flux $\phi^i = \phi^f$ \} local

between two \{ $d\psi^i = d\psi^f$ \} invariants

neighbouring surface \{ $d\phi^i = d\phi^f$ \}

↓ combinations of the local invariants

mutually independent local invariants
$d\psi, d\phi, \psi d\psi, \phi d\psi, \psi d\phi, \phi d\phi, \psi\phi d\psi, \psi\phi d\phi, \psi d\phi$.

• higher order combinations → poorer constancy
  ex. $(\psi + \Delta \psi)/\psi = 1 + (\Delta \psi/\psi)$.
  $(\psi + \Delta \psi)^2/\psi^2 = 1 + 2(\Delta \psi/\psi) + (\Delta \psi/\psi)^2$

• ex. $\psi^\alpha d\phi$ — dependent $\rightarrow \psi d\phi$
global invariant up to the 2nd order combination

\[ K = \frac{1}{\mu_0} \int \left( a_1 \psi d\phi + a_2 \phi d\psi + a_3 \psi d\psi + a_4 \phi d\phi ight. \\
\left. + a_5 d\psi + a_6 d\phi \right) \quad (1) \]

( generalized fundamental global invariant )

\( a_1, \ldots, a_6 \): constants

\* \( K \supset K_0, K_x \)

cylindrical model by Kadomtsev

\[ K_0 = \frac{1}{\mu_0} \int_{V_0} \vec{A} \cdot \vec{B} dV = \frac{2\pi R}{\mu_0} \int (\phi d\psi - \psi d\phi) \]

\[ K_x = \frac{1}{\mu_0} \int (\psi - \alpha \phi) d\phi : \alpha = \frac{1}{R \gamma_5}, \quad \gamma_5 = \frac{m}{n}. \]

\[ \Rightarrow \left\{ \begin{array}{l}
K_0: \quad a_1 = -2\pi R, \quad a_2 = 2\pi R, \quad a_3 = a_5 = a_6 = 0 \\
K_x: \quad a_1 = 1, \quad a_4 = -\alpha, \quad a_2 = a_3 = a_5 = a_6 = 0
\end{array} \right. \]

< 2nd step > \( \Delta t \) (QIMHD)

local \( \psi^i = \psi^f, \phi^i = \phi^f, \quad d\psi^i = d\psi^f, \quad d\phi^i = d\phi^f \)

global \( K^i = K^f \) (conjecture)

< 3rd step > minimization of \( W_m \) subject to

\( K^f = \text{const} \rightarrow \delta K^f = 0 \rightarrow \delta W_m + \lambda \delta K^f = 0 \)

using the cylindrical model by Kadomtsev

\[ \Rightarrow \quad \mu_0 \vec{J} = \lambda_0 \vec{B}, \quad \lambda_0 \equiv \lambda (a_1 - a_2) \quad (2) \]

Euler-Lagrange eqs.
arbitrary set of constants \((a_1, \ldots, a_6)\)

\[ \Rightarrow \text{minimum mag. energy state } \mu_0 \vec{j} = \lambda_0 \vec{B}. \]

\(\circ\ K \supset \{ \text{global invariants on mag. fluxes for various types of relaxation process} \)

**Question (II)?** \((\vec{j} = 0 \text{ at the wall})\)

<2nd step>

\(\overset{\text{resistive}}{\text{implicit assumption of slightly plasma}}\)

\(\overset{\text{in whole area inside the wall in } \Delta t \text{(QIMHD)}}{\text{resistivity } \eta \text{ of plasma } \to \infty \text{ near the wall}}\)

\(\overset{\text{because of the plasma-wall interaction}}{\rightarrow \{ \text{ are no longer conserved.}}\)

\(\overset{\text{do not contribute to the global invariant.}}{\rightarrow \{ \text{because of resistive decay.}}\)

\[ \frac{dK_0}{dt} = \left( -2 \int_{V_0} \eta \vec{j} \cdot \vec{B} dV + \Phi \omega \psi - \psi \omega \Phi \right) / 2 \mu_0 \]

\(\eta \to \infty \text{ near the wall.}\)
The global $K^i_f \Rightarrow K^f_f$ with contribution functions $f_{\eta_i}(\psi, \phi)$, where

$$K^f_f = \frac{1}{\mu_0} \int \left( f_{\eta_1} a_1 \psi \phi d\phi + f_{\eta_2} a_2 \phi \psi d\psi + f_{\eta_3} a_3 \psi d\psi + f_{\eta_4} a_4 \phi d\phi + f_{\eta_5} a_5 \phi d\phi + f_{\eta_6} a_6 \phi d\phi \right) \quad (3)$$

boundary conditions on $f_{\eta_i}$ ($i=1, \ldots, 6$)

a thin but a finite layer of plasma-wall interaction just inside the wall

$$f_{\eta_i} = \frac{\partial f_{\eta_i}}{\partial \psi} = \frac{\partial f_{\eta_i}}{\partial \phi} = 0 \text{ at the wall} \quad (4)$$

$$f_{\eta_i} = 1 \text{ at the mag. axis} \quad (5)$$

**<3rd step>**

minimization of $\mathcal{W}_m$ subject to $K^f_f = \text{const.}$

$$\Rightarrow \delta \mathcal{W}_m + \lambda_0 \delta K^f_f = 0 \quad (\delta K^f_f = 0)$$

$$\Rightarrow \mu_0 \vec{j} = \lambda^R(\psi, \phi) \vec{B} \quad (6)$$

(Euler-Lagrange eqs.)

where

$$\lambda^R(\psi, \phi) = \lambda_0 \left\{ a_1 (f_{\eta_1} + \psi \frac{\partial f_{\eta_1}}{\partial \psi}) - a_2 (f_{\eta_2} + \phi \frac{\partial f_{\eta_2}}{\partial \phi}) - a_3 \frac{\partial f_{\eta_3}}{\partial \phi} + a_4 \phi \frac{\partial f_{\eta_4}}{\partial \psi} - a_5 \frac{\partial f_{\eta_5}}{\partial \phi} + a_6 \frac{\partial f_{\eta_6}}{\partial \psi} \right\} \quad (7)$$

(4)(5) $\Rightarrow$ (6)(7)

$$\Rightarrow \{ \vec{j} = 0 \text{ at the wall} \} \text{ is satisfied naturally!}$$
Question (III)? \( (K[w], M[u], S[v]) \)

<1st step> in \( \Delta t(\text{IMHD}) \)

Local integrated helicity \( dK = \oint A \cdot B \, d\mathcal{V}/\mu_0 \) between two mass \( dM = \oint \rho \, d\mathcal{V} \) surfaces

and entropy \( dS = \oint s \, d\mathcal{V} \) are all local invariants.

\[ \begin{align*}
\Rightarrow \quad & w(\psi, \phi) \, dK = \oint w(\psi, \phi) A \cdot B \, d\mathcal{V}/\mu_0 \\
& u(\psi, \phi) \, dM = \oint u(\psi, \phi) \rho \, d\mathcal{V} \\
& v(\psi, \phi) \, dS = \oint v(\psi, \phi) s \, d\mathcal{V} \\
\end{align*} \]

\( \therefore \psi, \phi \rightarrow \text{local invariants and const. on mag. surfaces} \)

\[ \begin{align*}
\Rightarrow \quad & K[w] = \frac{1}{\mu_0} \oint w(\psi, \phi) A \cdot B \, d\mathcal{V} \\
& M[u] = \oint u(\psi, \phi) \rho \, d\mathcal{V} \\
& S[v] = \oint v(\psi, \phi) s \, d\mathcal{V} \\
\end{align*} \]

\[ \Delta \text{< analogous arguments >} \]

if independent quantities \( X \) and \( Y \rightarrow \) invariants

then infinite set of \( X^\alpha Y^\beta \) and/or \( w(X, Y) \)

\[ \Rightarrow \text{invariants} \]

Are they all \underline{new mutually independent, fundamental, physically available invariants}?

\[ (X, Y, XY) \]
< 2nd step >

* higher order combinations → poorer constancy

\* K[w]  
\* M[u]  
\* S[v]  

\{ dependent invariants on \}

\{ K_0 \ [w = 1] \}
\{ M_0 \ [u = 1] \}
\{ S_0 \ [v = 1] \}

\* in Δt (RMHD)

\[\begin{align*}
K[w] & < K_0 \quad \rightarrow \quad K_f^f \\
M[u] & < M_0 \quad \rightarrow \quad M_f^f \\
S[v] & < S_0 \quad \rightarrow \quad S_f^f
\end{align*}\]

\{ with \}

\{ contribution \}

\{ functions f \}

\[\rightarrow \text{not independent fundamental global} \]
\[\text{invariants against } K_0, M_0, S_0.\]

< 3rd step >

Bhattacharjee et al.

\[\delta W - \sum_{\alpha} \lambda_{\alpha} \delta K[w_{\alpha}] - \sum_{\alpha} \mu_{\alpha} \delta M[u_{\alpha}] - \sum_{\alpha} T_{\alpha} \delta S[v_{\alpha}] = 0.\]

\[\alpha = 0, 1, 2, \ldots \ldots \text{ including } K_0, M_0, S_0.\]

\[\text{physically reasonable?}\]
\[\text{or available?}\]
Energy Principle for Different Timescales

* in $\Delta t (\text{IMHD})$

- local constraint on mass (L.C.M.)
- on entropy (L.C.E.)
- on mag. fluxes (L.C.M.G)

$\Rightarrow$ Kruskal & Kulsrud $\delta W = 0 \rightarrow \nabla P = \vec{j} \times \vec{B}$

Bernstein et al. $\delta^2 W > 0 \rightarrow$ ideal MHD stability

By relaxing the strongest constraints of the three local constraints step by step to global constraints

$\rightarrow \Delta t (\text{RMHD})$ for slightly resistive MHD Timescale

L.C.M.G $\rightarrow$ global constraint on mag. fluxes (G.C.M.G)

L.C.M $\rightarrow$ L.C.M.

L.C.E $\rightarrow$ L.C.E.

minimization of $W = \int (\frac{B^2}{2\mu_0} + \frac{p}{\gamma-1}) dV$ subject to (G.C.M.G), (L.C.M), (L.C.E.)

$\Rightarrow \vec{j} = \frac{\lambda_R(\Psi) \vec{B}}{\mu_0} + \frac{\nabla \phi \times \nabla P}{\vec{B} \cdot \nabla \phi}$ equilibrium state

Euler-Lagrange eqs.

Partially Relaxed State (P.R.S.)
\( \rightarrow 0 \) \( \Delta t(Tu) \) for turbulent plasma timescale

\[
G.C.M.G \rightarrow G.C.M.G
\]

\[
L.C.M. \rightarrow G.C.M. \quad \text{global.}
\]

\[
L.C.E. \rightarrow \overline{G.C.E.}
\]

\[
\delta W = 0 \quad \Rightarrow \quad \vec{j} = \frac{\lambda_{tu}(\psi)}{\mu_0} \vec{B} + \frac{\nabla \theta \times \nabla P}{\vec{B} \cdot \nabla \theta}
\]

( P. R. S. )

\( \rightarrow 0 \) \( \Delta t(Tr) \) for plasma transport timescale

\((G.C.M.G.), (G.C.M.), (G.C.E.)\)

and in MHD equilibrium state in the sense of time average (semilocal constraint)

\[
( \vec{j} = \frac{\lambda^x(\psi)}{\mu_0} \vec{B} + \frac{\nabla \theta \times \nabla P}{\vec{B} \cdot \nabla \theta}, \quad x = R \text{ or } tu )
\]

\[
\delta W = 0 \quad \Rightarrow \quad \vec{j}^r = \frac{\lambda^r(\psi)}{\mu_0} \vec{B} + \frac{\nabla \theta \times \nabla P}{\vec{B} \cdot \nabla \theta}
\]

\( p = nkT, \quad \rho = mn \)

\[
T = T ax f_s^{tr}(\psi), \quad f_s^{tr}(\psi) : \text{contribution function on entropy}
\]

\[
P = (kT ax f_s^{tr}/m)^{\frac{\gamma}{\gamma-1}} \exp \left( \frac{\gamma}{\gamma-1} - \frac{m \nabla f_m^{tr}}{kT ax f_s^{tr}} \right)
\]

\[
P = (kT ax f_s^{tr}/m)^{\gamma} \exp \left( \frac{\gamma}{\gamma-1} - \frac{m \nabla f_m^{tr}}{kT ax f_s^{tr}} \right)
\]

\( f_m^{tr}(\psi) : \text{contribution function on mass} \).
relaxation by the helical tearing mode with
single helicity ( \( \varphi_s = m/n \) ) [Kadomtsev].

\[ f_\eta(x) \sim x/x_0 \quad \{ \begin{array}{l}
  x = 0 \text{ at the wall} \\
  x = x_0 \text{ - mag. axis}
\end{array} \]

\[ \text{larger } x \Rightarrow \{ f_\eta(x) \rightarrow 1 \} \]

\[ \Rightarrow \lambda^R(x) = \lambda_0 \frac{x}{x_0} \]

\[ \text{PRSM. } \mathbf{j} = \frac{\lambda^R(x)}{\mu_0} \overrightarrow{B} + \frac{\nabla \theta \times \mathbf{\Omega}}{|\overrightarrow{B}| \cdot \mathbf{\theta}} \]

numerical procedure \( \rightarrow \) ref. 1
(cylindrical approximation).

Suydam criterion \([\text{in } \Delta t(\text{IMHD})]\).

\[ \rightarrow \text{finite } \beta \]
Fig. 1 A typical numerical result of Tokamak-like equilibria stable with respect to the Suydam criterion. \( \lambda_c r_w = 0.34 \). \( q_s = m/n = 1/1 \). \( \beta_{ax}( \text{at the magnetic axis}) = 0.008 \).

Fig. 2 A typical numerical result of RFP equilibria stable with respect to the Suydam criterion. \( \lambda_c r_w = 4.16 \). \( q_s = 1/10 \). \( \beta_{ax} = 0.067 \).
Minimum-energy states in $\Delta t (\text{RMHD})$.

$F - \theta$ diagram

$\theta_s = \frac{m}{n} = \frac{1}{l}$

Stability in $\Delta t (\text{IMHD})$

Pitch $P$

- Tokamak
- Screw pinch

Pitch minimum unstable in $\Delta t (\text{IMHD})$

- Spheromak
- RFP
marginal Suydam stable

PRSM(1): \( \bar{\lambda}(\psi) = \lambda_0 [1 - (\psi / \psi_0)^2] \)

PRSM(II): \( \bar{\lambda}(\psi) = \lambda_0 [1 - (\psi / \psi_0)^3] \)

PRSM(III): \( \bar{\lambda}(\psi) = \lambda_0 \)

O: Data on TPE-1R(M)

O: Data on TPE-1R(M)
comparison between OHTE and PRSM(II).

marginally stable with respect to the Suydam criterion.
\[ \theta_s = \frac{m}{n} = \frac{1}{20}, \frac{1}{10}, \frac{1}{5}, \frac{1}{1}, \]

PRSM (II):
\[ \lambda = \lambda_0 \left[ 1 - \left( \frac{V_c - V}{V_c} \right)^4 \right] \]

PRSM (T):
\[ \lambda = \lambda_0 \left( \frac{\chi}{\chi_0} \right) \]
- finite \( \beta \)
- \( \chi = \delta_s \phi - (\phi - \phi_w) \)
- marginally Suydam stable

F-Theta Diagram
Conclusions

(1) The generalized fundamental global invariant $K$ includes both the two global invariants, $K_0$ by Taylor and $K_x$ by Kadomtsev.

(2) The minimum-magnetic-energy state under the generalized global invariant $K$ is proved to become the same state of $\mu_0 j = \lambda_0 B$.

(3) When the plasma-wall interaction is taken into account in the global constraints on magnetic fluxes, the resultant minimum-energy state is proved to satisfy naturally the experimental boundary condition of $j = 0$ at the wall.

(4) Energy principles for three timescales of $\Delta t(\text{RMHD})$, $\Delta t(\text{Tu})$, and $\Delta t(\text{Tr})$ lead to the same vector form of equilibrium equations used in the partially relaxed state model (PRSM).

(5) Numerical results by the PRSM agree fairly well with experimental data on the two typical RFP experiments.

It should be noted that the present theory of energy principles is available not only to the RFP plasma but also to the plasma in the Tokamak and other toroidal devices.
SPHEROMAK AND MAGNETIC RECONNECTION

T. HAYASHI

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Spheromak and Magnetic Reconnection

T. Hayashi and T. Sato
(Hiroshima Univ.)

Reconnection in spheromak

formation
relaxation
instability

1. Tilting Instability
Magnetic Reconnection
(simulation
(experiment (Princeton S-1
proto S-1/c, Movie))

2. Relaxation
\( n = 2, 3, 4 \)
(simulation
(experiment

\[ \text{formation} \]
\[ \text{flux core} \]
\[ \text{coil} \]

\[ \text{Time} \]

\[ \text{O} \]

\[ \text{O} \times \text{O} \]

\[ \text{O} \times \text{O} \]

\[ \text{O} \times \text{O} \]

\[ \text{O} \times \text{O} \]

\[ \text{O} \times \text{O} \]
Experimental method

Proto S-1/c

Internal magnetic probe (3 components of B)

Flux core

Stabilizing conductor

Movie (slide)

Contours of magnitude of B_t

750-800 G
550-700 G

Formation stage
Tilting instability stage
\[ \psi, \phi = -0.3 + 0.5 \sin(\pi T/44) \text{ Wb} \] where \( T \) is normalized by the Alfvén transit time \( \tau_A \) which is approximately 2.3 \( \mu \text{sec} \) and \( r \) is the radial distance of a core surface point. We note, however, that \( \phi \), is so programmed that the toroidal-flux core current is crowbarred at \(-100\) kA once it reduces to \(-100\) kA (the crowbar time was roughly \( 35\tau_A \) in our case). The wall of the vacuum vessel is assumed to be a conductor so that the perpendicular magnetic field component is fixed at the wall. The simulation domain \((r, \theta, z)\) is divided into 37 \( \times 16 \times 37 \) meshes.

First, we show the formation stage of the spheromak. Figure 2 shows some field line trajectories at three different times. It is seen that a thin closed magnetic surface (torus) formed just inside the flux core develops into a fat torus (spheromak) with a gradual movement of the magnetic axis toward the central axis. The formation is completed at about \( 90\tau_A \) and the created spheromak keeps its shape thereafter. We have continued the run until \( 140\tau_A \) but it is found that its shape is maintained without any appreciable change. This indicates that the numerical diffusion time is much longer than \( 50\tau_A \).

With this success in creating a spheromak, we then go on to study the tilting instability. We have applied a small \( \alpha = 1 \) tilting perturbation at \( T = 105 \). With intent to see the dynamic behavior of the spheromak in response to the perturbation,
Simulation

Vacuum Vessel

Flux Core

Experiment

Stabilization Conductor
Magnetic Reconnection

forced reconnection  Tearing mode
system            open            closed
boundary          BU              (wall)
drive             forced          spontaneous

reconnection rate
boundary condition (indep. of $\eta$)  internal condition ($\eta$)

$B \nu$

In the tilting case, the external flow is caused by tilting motion, etc.

(S.P. $\dot{\psi} \sim \frac{B_0^3}{\rho^{3/4}} \frac{\eta^{3/2}}{\rho^{1/4}}$)
• Order estimation of the reconnection time scale

\[ T \sim \frac{4}{\dot{A}} \sim \frac{a}{\langle v \rangle} \sim \frac{1}{0.01} \]
\[ \sim 100 \, (T_A) \]

\[ \dot{A} \sim a \langle B_0 \rangle \]
\[ \dot{A} \sim \langle v \rangle \langle B_0 \rangle \iff \text{forced reconnection} \]

• \( S \sim 5000 \) (corresponding to Spitzer resistivity with \( \sim 20 \text{cy} \))
  
  \[ [ S_{\text{numerical}} \sim S \, ? \] 

• Estimation of \( \dot{j} \) in current layer

\[ \dot{j} \sim \frac{\langle v \rangle \langle B_0 \rangle}{\eta} \]  
\[ \sim \frac{1}{2} \]

\[ \text{cf. } \dot{j}_{\text{torus}} \sim 10 \]

• Simulation result

\[ S \sim 50000 \]

\[ S \sim 5000 \] \quad \text{similar time scale (However, } S_{\text{numerical}} \text{ might be concealing the physics. )}

\[ S \sim 500 \] \quad \text{decay + tilt (shrink)
Typical Spheromak with a flux hole
(created by numerical simulation)

The substantial amount of the toroidal field which was supplied from the flux core is contained in the spheromak.

The plasma current fluxes only inside of the spheromak torus, so a vacuum-like magnetic field is realised outside the torus.

The existence of a flux hole can be clearly seen. It turns out that the aspect ratio $R/a$ is about 2.0 in this example.
$l_w$: Wall Radius

$T_{\text{TOTAL KINETIC ENERGY}}$ (Normalized)

$T_{\text{TIME (Normalized)}}$

(a) $l_w = 1.5$
(b) $l_w = 0.75$
(c) $l_w = 0.67$

$n = 1$

$10^{-1}$

$10^{-2}$

$10^{-3}$

$10^{-4}$

$10^{-5}$

$10^{-6}$

$10^{-7}$

50

60

70

80
COIL SYSTEM & S-I FLUX-CORE
PF/TF/EF = 5.0/10.0/2.0  
PF/PC/TC = -275/180/600  
PM/TM/EM = 200/200/A  

12-SEP-84  SHOT # 9762  
T/F VOLTAGE SCAN  
2.9 mTorr HYDROGEN

AMPLITUDE OF N MODE
\langle BP \rangle

23-OCT-84  00:27:5 ZN01-09762  
\theta

\(t\) (msec)
Development of $a$ profile during formation stage

$(S-1 \text{ experiment})$

$T=0.5$, $T=0.4$, $T=0.31$, $T=0.25$

$T=0.25$

$T=0.22$
<table>
<thead>
<tr>
<th>( n = 3 )</th>
<th>Top</th>
<th>Front</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T = 104.8 )</td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>( T = 122.2 )</td>
<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td>( T = 126.6 )</td>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
<td><img src="image9" alt="Diagram" /></td>
</tr>
<tr>
<td>( T = 131.9 )</td>
<td><img src="image10" alt="Diagram" /></td>
<td><img src="image11" alt="Diagram" /></td>
<td><img src="image12" alt="Diagram" /></td>
</tr>
<tr>
<td>( T = 148.4 )</td>
<td><img src="image13" alt="Diagram" /></td>
<td><img src="image14" alt="Diagram" /></td>
<td><img src="image15" alt="Diagram" /></td>
</tr>
</tbody>
</table>
Numerical particle pushing adiabatic drifting particles. Drifting Maxwellian particles approach separatrix.
Model:

1) steady state
2) kinetic ions and electrons
3) collisionless
4) no resistivity imposed
5) drifting Maxwellian approaches separatrix
6) low $\beta$, may be extended easily to high $\beta$

Fields:

$\vec{E} = E_0 \hat{z}$, $E_0 \equiv \text{constant}$

$\vec{B} = B'(x\hat{y} + y\hat{x}) = -\nabla \phi$

$\phi = -B'xy$, $B' \equiv \text{constant} \ (< 0)$ (in sketch)

$\psi = -\frac{1}{2} B'(x^2 - y^2)$
Define "guiding center" as a point where $\mathbf{v}_\parallel = 0$.

$$\mathbf{A} = \mathbf{V}(x,y) \hat{z}$$

$$\mathbf{E} = -\nabla \phi = E_0 \hat{z} - \nabla f(x,y)$$

$$\mathbf{B} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

$$\mathbf{v}_d = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

$$\mathbf{F} = m \mathbf{v} + q \mathbf{A}$$

$$H = \frac{1}{2m} (\mathbf{p} - q \mathbf{A})^2 + q \phi$$

$$\dot{R}_z = -\frac{\partial H}{\partial R_z} = q E_0$$

$$m \dot{v}_z = qE_0(t - t_0) - q\psi(x,y)$$

The "guiding center" moves with the $v_\parallel = 0$ surfaces, i.e.

$$x^2 - y^2 = -\frac{2}{B}, E_0(t - t_0)$$

The guiding center may have $v \rightarrow \infty$ but the particle never does.
Power transferred to plasma near the X point

\[ P = \int \int f(w) (w_2 - w_1) \bar{\mathbf{v}} \cdot d\mathbf{s} \, d^3\mathbf{v} \]

where

- \( S \) is a position on the separatrix
- \( \bar{\mathbf{v}} \) is velocity of particle crossing \( S \)
- \( W_1 \) is kinetic energy of particle referred to Maxwellian entry point
- \( W_2 \) is kinetic energy of particle leaving vicinity of separatrix

When adiabatic invariants \( \mu = \frac{W_1}{B} \) and \( J = \int \mathbf{v}_{\parallel} \cdot d\mathbf{v}_{\parallel} \) break down near the X point, \( W_1 \neq W_2 \).
Analytic indication that $P$

is positive:

By symmetry, every orbit of a particle entering with $W_1$ and leaving with $W_2$ has a corresponding orbit with $W'_1 = W_2$ and $W'_2 = W_1$.

$x' = -y$ \hspace{1cm} $v'_{sx} = v_{sy}$
$y' = -x$ \hspace{1cm} $v'_{sy} = v_{sx}$
$z' = -z$ \hspace{1cm} $v'_{sz} = v_{sz}$
$t' = -t$

The power integral may be rewritten

$$P = \frac{1}{a} \iiint [f(w_1) - f(w_2)] (w_2 - w_1) \bar{u} \cdot d\bar{s} d^3\bar{u}$$

$$= -\frac{1}{2} \iiint f'(\bar{w}) (w_2 - w_1)^2 \bar{u} \cdot d\bar{s} d^3\bar{u}$$

$$= \frac{1}{2kT} \iiint f(\bar{w}) (w_2 - w_1)^2 \bar{u} \cdot d\bar{s} d^3\bar{u}$$

$P > 0$ \hspace{1cm} $\bar{w} = (w_1 + w_2) / 2$
Comments:

- The X point acts as a scattering center. The reconnection process is reversible and appears to cause no entropy increase. Particles leaving the separatrix are not precisely Maxwellian. They are slightly more "ordered" than those approaching the separatrix. To keep entropy fixed there must be a net energy gain.
Axial current in the nonadiabatic region near the X point is \( I \approx P/E_0 \).

\[ I \approx n \]

As \( I \) increases the X point becomes oblique.

The convenient symmetry is lost.

There are no shocks.
Conclusions:

- A model of driven reconnection in a collisionless nonresistive plasma has been developed.

- It has been shown analytically that energy is transferred from fields to the plasma.

- Plasma heating has been verified by a computer model, to be published.
The reconnection mechanism described here will influence the rate of island formation for the Field Reversed Theta Pinch.

A similar mechanism with $B_z \neq 0$ will affect reconnection in other devices.

There is no simple vacuum limit when $B_z \neq 0$. 
BUBBLE FORMATION DUE TO SURFACE TEARING MODES

G. KURITA

J. A. E. R. I.
Bubble Formation due to Surface Tearing Mode

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and T. Takeda

JAERI

* on leave from Fujitsu Limited
# Purpose

To re-investigate the bubble formation due to free boundary mode precisely, taking into account of the finite plasma resistivity.

# Method

The vacuum region is replaced by a highly resistive plasma, "pseudo-vacuum" model, to avoid the difficulty of tracing the plasma surface position precisely.
# Basic Equations

$$\frac{\partial U_{m/n}}{\partial t} = [U, \phi]_{m/n} + [J, \psi]_{m/n}$$

$$\frac{\partial \psi_{m/n}}{\partial t} = [\psi, \phi]_{m/n} + \sum_{m=m^{m''}, n=n^{n''}} \eta_{m''/n''} J_{m/n''} - E_{m/n}$$

$$\frac{\partial \eta_{m/n}}{\partial t} = [\eta, \phi]_{m/n}$$

$$[\psi, \phi]_{m/n} = \sum_{m=m^{m''}, n=n^{n''}} \frac{m'}{r} \left( 4_{m''/n''} \frac{\partial \phi_{m''/n''}}{\partial r} - \phi_{m''/n''} \frac{\partial 4_{m''/n''}}{\partial \gamma} \right)$$

$$J_{m/n} = \Delta 4_{m/n}$$

$$U_{m/n} = \nabla \cdot (P_0 \nabla \psi_{m/n})$$
# Numerical Method

to advance eq. without numerical instability.

For this two-dimensional diffusion eq.

$\frac{\partial \bar{q}(r,\theta)}{\partial t} = \bar{\gamma}(r,\theta) \Delta \bar{q}(r,\theta)$,

the numerical stability condition is

$\bar{\gamma}(r,\theta) > 0$ for all $(r, \theta)$.

To satisfy this condition, we introduce new variable $S_{mn}$, and solve

$\frac{\partial S_{mn}}{\partial t} = [S, \phi]_{mn}$

instead of $\gamma$ eq., and calculate $\bar{\gamma}(r,\theta)$ by assuming

$\bar{\gamma}(S) \equiv \bar{\gamma}(S_{t=0}) : S_{t=0} = r$

for all $\theta$. Then we can obtain

$\gamma_{mn} = \frac{1}{2\pi} \int_0^{2\pi} \bar{\gamma}(r,\theta) \cos m\theta d\theta$
Shaping Function

\[ J(r) = f_\gamma(r) S_\gamma(r) \]
\[ P(r) = f_\rho(r) S_\rho(r) \]
\[ \eta(r) = f_\eta(r) S_\eta(r) \]

\[ f_\gamma(r) = \begin{cases} (1 - C_\alpha)[1 - (\frac{r}{a})^{2C_\beta}] + C_\alpha & ; r \leq a \\ C_\alpha & ; r > a \end{cases} \]

\[ S_\gamma(r) = \begin{cases} S_\gamma(a - C_\gamma) & ; r < a - C_\gamma \\ (1 - C_\alpha)[\frac{1}{2} + \frac{1}{\pi}(\tan^{-1} \beta - \beta x^2)] + C_\alpha & ; a - C_\gamma \leq r \leq a + C_\gamma \\ S_\gamma(a + C_\gamma) & ; r > a + C_\gamma \end{cases} \]

\[ \gamma = \pi \frac{r - a}{C_\beta} \quad , \quad \beta = \frac{1}{1 + \gamma^2} \]

\[ x_\gamma = \pi \cdot C_\gamma / C_\beta \]

Diagram:
- \( S_\gamma(r) \)
- \( C_\delta \)
- \( C_\alpha \)
- \( a \)
- \( b \)
- \( r \)
Linearized Equations

\[ \chi \nabla \cdot (\rho \nabla \Phi) = F \Delta 4 - \frac{m dJ_0}{r} 4 \]

\[ \chi 4 = -F \Phi + \eta_0 \Delta 4 + \eta J_0 \]

\[ \delta \eta = -\frac{m}{r} \frac{dn_0}{dr} \Phi \]

\[ F = \frac{1}{\delta} (m - n \eta) \]

\[ \frac{2}{\eta \Phi} \rightarrow \gamma : \text{growth rate} \]

subscript \(0\) : equilibrium quantity

\( \delta \) : safety factor
Effect of External Density

\[ \frac{J_e}{J_i} \ll 1 \]
\[ \frac{\eta_i}{\eta_e} \ll 1 \]

- Internal Solutions
  \[
  \begin{align*}
  \Phi_i &= \frac{r^m}{V_i} \\
  4_i &= \frac{r^m}{V_i}
  \end{align*}
  \]

- External Solutions
  \[
  \begin{align*}
  \Phi_e &= \Phi_e (r^m - b^{2m} r^{-m}) \\
  4_e &= \Phi_e (r^m - b^{2m} r^{-m})
  \end{align*}
  \]

- Boundary Condition
  \[
  \Phi_i = \Phi_e \mid r = a \\
  4_i = 4_e \mid r = a
  \]

- Analysis
  \[
  \gamma^2 = \left[ 1 + \frac{P_e}{P_i} \frac{1 + (a/b)^{2m}}{1 - (a/b)^{2m}} \right]^{-1} \cdot \gamma^2_{\text{analytic}}
  \]

\[
\gamma_{\text{analytic}}^2 = \frac{2}{P_i a_b^2} (m-n_b^2) \left[ 1 - \frac{m-n_b^2}{1 - (a/b)^{2m}} \right]
\]
**Effect of External Resistivity**

\[
\begin{align*}
\frac{J_e}{J_i} & \ll 1 \\
\frac{P_e}{P_i} & \ll 1
\end{align*}
\]

\[
0 \quad \rightarrow \quad \left[ 1 - \left( \frac{r_s}{b} \right)^{2m} \right] \left[ 1 - \left( \frac{a}{r_s} \right)^{2m} \right] \frac{\eta_e^2}{2} \gamma^{1/5} + \beta \left[ 1 - \left( \frac{a}{b} \right)^{2m} \right] \frac{\eta_e^2}{2} \gamma^2 + (m - n q_b a) \left[ 1 - \left( \frac{r_s}{b} \right)^{2m} \right] \left[ m - n q_b a - 1 + \left( \frac{a}{r_s} \right)^{2m} \right] \gamma^{5/4} + \beta (m - n q_b a) \left[ m - n q_b a - 1 + \left( \frac{a}{b} \right)^{2m} \right] = 0
\]

\[
\beta = \frac{2m}{r_s} \left[ \frac{q_e^2 m^2}{32 q_b^4 P_e} \right]^{1/4} \eta_e^{3/4}
\]

\[
\beta \gg 1 \quad \rightarrow \quad \gamma = \gamma^{\text{analy}}
\]

\[
\beta \ll 1 \quad \rightarrow \quad \gamma = \frac{1}{2} \left[ \frac{q_e^2 m^2}{q_b^4 P_e} \right]^{1/5} \Delta'^{4/5} \eta_e^{3/5}
\]

\[
\Delta' = -\frac{2m}{r_s} \frac{1}{1 - (r_s/b)^{2m}} \frac{m - n q_b a - 1 + (a/b)^{2m}}{m - n q_b a - 1 + (a/r_s)^{2m}}
\]
\[ \gamma \]

--- : real vacuum

\[ \rho_e/\rho_i = 10^{-3} \]

\[ \rho = \text{const.} \]

\( q_a \)
Parameters of nonlinear calculation

- $P_i = P_e = 1$.
- $\eta(0) = 1 \times 10^{-6}$, $\eta(b) = 1$.
  \[ \eta(r) = E/J(r) \]
- $a/b = 0.66$
- 20.1 equal spacing radial mesh
- 24 Fourier components
- Single helicity
\[ J(r) = J_0(1 - r^{20}) \quad q_a = 1.95 \]
$J(r) = J_0(1-r^2)$

$q_a = 1.64$
\[ J(r) = J_0(1 - r^2) \quad q_a = 2.05 \]
\[ J(r) = J_0 (1 - r^1)^2 \quad \alpha = 1.90 \]
\[ J(r) = J_0 (1 - r^{10})^2 \quad q_a = 2.10 \]
\[ J(r) = J_0 (1 - r^{10})^2 \quad \alpha_a = 2.10 \]
\[ J(r) = J_0 (1 - r^{10})^2 \]
Mechanism of this bubble formation

1. The change of the profile of stream function from tearing eigenfunction to the surface tearing one as $\delta a$ decreases.

2. The plasma boundary moves inward due to this plasma flow, while the rational surface remains at the initial position. And the axis of magnetic island comes out to the vacuum region.

3. The mode grows like the kink mode but with restricted range of poloidal angle in the vacuum region and forms the bubble.
# Summary

The bubble is formed for $q_a$ nearly equal to 2. Therefore, it is a candidate of the major disruption in the discharge with $q_a \approx 2$. In order to understand the relation between the bubble formation and the major disruption, however, we need to take into account of the interaction between the surface current and the material limiter resulting from the bubble formation.

This problem remains for future study.
REVERSE FLOW VORTICES AND CURRENT SHEETS

IN NONLINEAR TEARING

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I. Introduction

A. Typical aspects of the solar flare

- Energy "release" of $10^{30-32}$ ergs on time scale of $10^{-3}$ sec.
- Observable consequences: Radiation (UV, X-ray, Y-ray), Energetic particles, Coronal disruptions, Mass ejections.

- Magnetic configurations:

  Magnetic loop or Arcade
  
  [Diagram of a magnetic loop with labels for susceptible to instabilities: $10^3$ km, fixed in photosphere, many modes stabilized.]

- Driven Reconnection:

  [Diagram showing loop merging due to convective motion and emerging flux.]
Physical parameters:

\[ \begin{align*} 
T &= 1.2 \times 10^6 \text{ K}, \quad \rho = 150 \text{ eV}, \quad \eta = 10^{9-10} \text{ cm}^{-3} \\
\theta &= 10^6, \quad \text{scale} = a = 10^7 \text{ cm} \\
S &= 10^{10-11}, \quad \beta = 0.01-0.1, \quad \Omega a = 0.1
\end{align*} \]

B. Energy conversion process

A reasonable scenario is as follows:

- Flare energy stored in stressed (nonpotential) coronal magnetic fields.
- The stochastic reconnection responsible for conversion process.
- Reconnection occurs in nonlinear phase of tearing instability.

Two questions regarding this approach:

1. Does the tearing mode saturate early in nonlinear regime; i.e., is the required energy converted?
2. Is the process fast enough?

Problem: \[ \text{growth time of tearing instability} \] (classical resistivity) \approx 1 \text{ day}. Need several growth times.
C. Present study

- Saturation properties (constant $\psi$ vs. long wavelength)
- Energy conversion
- Behavior of physical properties in nonlinear mode.

II. Model

MHD equations: incompressible, $\eta = \text{const.}$, $\nu = 0$, slab geometry

\[
\nabla \times \left( \rho_0 \frac{d \mathbf{v}}{dt} - \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} \right) = 0,
\]

\[
\frac{d \mathbf{B}}{dt} - \nabla \times (\nabla \times \mathbf{B} - \frac{\mathbf{v} \cdot \nabla}{4\pi} \nabla \times \mathbf{B}) = 0.
\]

\[\mathbf{v} = \nabla \phi \times \hat{e}_z, \quad \mathbf{B} = \nabla \psi \times \hat{e}_z.\]

Initial field: $\mathbf{B}_0 = B_0 \left[ \tanh \left( \frac{x}{a} \right) \hat{e}_x + \text{sech} \left( \frac{x}{\lambda} \right) \hat{e}_z \right]$

where $a =$ shear scale

Initial disturbance: Linear tearing mode with magnitude $3$ linear, nonlinear terms $= \text{equal}.$
Parameters (Non dimensionalize w.r.t. $b, \frac{a}{\Theta}, \beta \omega$)

$$s = \frac{\tau}{\tau_n}; \quad \tau_r = \frac{4 \pi a^2}{c^2 n}, \quad \tau_n = \frac{a \sqrt{4 \pi n} \rho}{\beta \omega}$$

$$\alpha = k a = \frac{\tau a}{\tau}$$

III. Integrated behavior
Long Wavelength

Maximum Linear Growth

Constant $\Psi$
\[ \frac{\Delta E_M}{E_{M_0}} = S = 10^6 \]
\[ \alpha = 0.042 \]

TIME (\( \tau_h \)) x 10^3
<table>
<thead>
<tr>
<th></th>
<th>Nonconstant $\Psi$</th>
<th>Constant $\Psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evolution time</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>116</td>
<td>119</td>
</tr>
<tr>
<td>Energy release</td>
<td>$4.2 \times 10^3$</td>
<td>$3.8 \times 10^5$</td>
</tr>
<tr>
<td>Energy source (1)</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>(2)</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>Energy budget</td>
<td>$M_Y$</td>
<td>$M_2$</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$K_2 + K_{r2} K_2$</td>
<td>&lt; 0.05</td>
<td>&lt; 0.09</td>
</tr>
<tr>
<td>thermal</td>
<td>92</td>
<td>48</td>
</tr>
<tr>
<td>$(J_n / J_0)_{max}$</td>
<td>2.5</td>
<td>2</td>
</tr>
<tr>
<td>$(E_n / E_0)_{max}$</td>
<td>$8 \times 10^{-4}$</td>
<td>$6 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
IV. **Spatial Structure**

Temporal sequence-of-events in nonlinear evolution
1. X-point \(\Rightarrow\) Current sheet (all parameter space)
2. Reverse flow vortex (all except \(S=100\), \(a=0.5\))
3. Second island or bifurcation (\(S\geq 10^3\), nonconstant \(\psi\))

- Question: What's going on in these simulations?
- Requirements on numerics in order to simulate this behavior (not present in all nonlinear tearing computations).
$S = 10^4$, $a = 0.05$

Linear Current

$J_{\text{max}} = 0.003$ (a)

Total Current ($t = 3000 \tau_n$)

$J_{\text{max}} = 25$ (b)

Reversed Current

0.01

30
Current sheet formation

Syrovatskii's result: Current sheets form at time of variation of a frozen-in magnetic field containing a neutral line or X-point (ideal MHD, $\beta_p = 0$)
x-component of momentum equation:

\[ \rho \frac{\partial v_x}{\partial t} = -\rho v_x \frac{\partial v_x}{\partial x} - \rho v_y \frac{\partial v_x}{\partial y} - \frac{1}{c} \frac{\partial s}{\partial x} - \frac{\partial p}{\partial x} \]

Formation of reverse vortex

x-component of induction equation:

\[ \frac{\partial B_x}{\partial t} = \frac{\partial}{\partial y} \left( \nu_x \frac{\partial B_x}{\partial y} - \nu_y B_x - B_x J_2 \right) \]

Formation of second island

Note: Nonlinear effect (terms not present in linear solution)

Relevant terms contain \( s \)-derivatives (\( \perp \) to current sheet)

Numerical resolution requirement: At \( s=10^4 \), current sheet thickness = 0.01. Use \( \Delta y=0.05 \) near \( y=0 \).

Result - No current sheets, reverse flow vortices, second islands

Conclusion - Fine numerical resolution essential.

Approximately 2 orders of magnitude smaller than linear tearing layer thickness.
Linear Tearing Mode

Normalized Eigenfunctions

$S = 10^{10}$, $10^6$, $10^2$

Dimensionless Distance, $\mu$

$a = 0.5$

$10^6$, $10^{10}$, $10^2$
Possible alternative explanation for reverse vortices: Build-up of back pressure due to symmetry conditions at center of island.

Result: Outflow in early stages. However, eventually obtain behavior as for symmetry conditions - with virtually no outflow.

Conclusion: Behavior governed by physics in tearing layer rather than by boundary conditions (for this case).
LIE TRANSFORM APPROACH TO DRIFT-RESONANT
INTERACTIONS IN FLUID AND PLASMA TURBULENCE

B. McNAMARA

LAWRENCE LIVERMORE NATIONAL LAB
LIE TRANSFORM APPROACH TO DRIFT-RESONANT
INTERACTIONS IN FLUID AND PLASMA TURBULENCE

BRENDAN McNAMARA, LLNL, US-JAPAN WORKSHOP ON MAGNETIC
RECONNECTION, IFS. PRC. 1984.

*A Dominant* Mechanism for generating high wave-number
modes at the onset of turbulence is the
drift-resonant interaction of a few large scale modes.

- **Resonant Coupling Between $k$ and Large $k'$ is like**
  \[ \epsilon V_k \cdot V_{k'} \left( \frac{a_k \sqrt{E} (k_i + 1)}{k_i} \right) \frac{1}{r_i \omega_D \sqrt{E}} \]
  
  *But Node-Node Coupling is like* \( \epsilon \)

- **BENARD CONVECTION DISPLAYS ISLANDS LIKE B IN TOKAMAK**

- **LIE TRANSFORM PRIMER**

- **Resonant Interactions of Coupled Oscillators**

- **'PUNBALL' MODEL OF DIFFUSION WITH MANY
  RESONANCES 'RFP?'

- **LIE TRANSFORM APPROACH TO REDUCED MHD TURBULENCE.**


Work begun at Culham Laboratory, 1983, with J.B. Taylor,
J.A. Connor, et al.
BENARD CONVECTION PROBLEM
(WAYNE ARTER, CULHAM '83)

\[ \frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \left( \nabla f^2 \right) \quad f = \cos x \cos y \sin z + \varepsilon(\omega^2 x + \omega y) \sin 2z \]

**Diagram:**
- HEATED FLUID LAYER
- \( \approx 2D \) Periodic Cells
FLOW LINE SURFACE OF SECTION

$\eta \equiv 0.1$
CHAIN OF 7 DRIFT RESONANCE ISLANDS
FLOW LOOKS
STOCHASTIC
REGULAR MOTION NEAR O-POINT ONLY.
BUT NOTE REMNANTS OF DRIFT SURFACES
HAMiLTONiAN FOR FLUId FLOW


COMPARE
\[ \nabla \cdot \mathbf{u} = 0 \]
\[ \frac{\partial f}{\partial t} + [H, f] = 0 \]

CONSERVES
\[ F = \int_A u_x \, dx \, dy \quad J = \int_C p \, dq \]

IDENTIFY CANONICAL VARIABLES AS

\[ p = \int_A u_x \, dy \quad q = x \quad t = z \]

\[ \therefore p = 2 \cos x \sin y \sin z + 4 \epsilon \left( y \cos 2x + \frac{1}{2} \sin 2y \right) \sin 2z \]

FROM EQUATIONS OF MOTION

\[ \frac{dp}{dt} = -\frac{\partial H}{\partial x} \]
\[ \frac{dq}{dt} = \frac{\partial H}{\partial \dot{q}} = u_x \]

\[ \frac{dx}{dt} = \frac{\partial H}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}} \left( \frac{\partial H}{\partial \dot{x}} \right) = \frac{u_z}{2} \]

THEN
\[ H = \int u_x \, dy = -\sin x \sin y \cos z - 2y \sin 2x \cos 2z \]

OR
\[ H = -\frac{p}{2} \tan q + \epsilon \int h_1(p, q, t) \]
THE BASIC LIE TRANSFORM

\[ L = e^{\epsilon [H_1, J]} \]

\[ L^{-1} = e^{-\epsilon [H_1, J]} \]

EASY INVERSE

DEFINITIONS

ORBIT OPERATOR: \([H_0, \ ]\)

AVERAGE OP.: \(\overline{H}_1 = \oint H_1 \, dq_1\)

PROPAGATOR: \(\hat{H}_1 = \int (H_1 - \overline{H}_1) \, dq_1 / \partial \overline{H}_0 \)

AND \([H_0, \hat{H}_1] = \bar{H}_1 - \hat{H}_1\)

RESULT OF APPLYING OPERATOR

\[ L \cdot H = \left(1 + \epsilon [\hat{H}_1, ] + \frac{\epsilon^2}{2} [H_1, [H_1, J]] + \cdots \right) \cdot (H_0 + \epsilon H_1) \]

= \(H_0 + \epsilon \overline{H}_1\)

\[ + \epsilon [\hat{H}_1, H_0] \]

\[ + \frac{\epsilon^2}{2} [\hat{H}_1, [\hat{H}_1, H_0]] + \epsilon^2 [\hat{H}_1, H_1] + \cdots \]

= \(H_0 + \epsilon \overline{H}_1 + \frac{\epsilon^2}{2} [\hat{H}_1, H_1 + \overline{H}_1] + \cdots \)

REPEAT PROCESS TO GET \(O(\epsilon^n)\) TRANSFORM

Resonant Perturbation of Coupled Oscillators in a Sheared System

\[ H = H_A(p_1) + p_2 - \varepsilon \sum_k V_{k'} \cos(k_1 q_1 - k_2 q_2) \]

\[ = H_A(p_1) + p_2 - \varepsilon V_k \cos(k \cdot Q) - \varepsilon \sum^{*} V_{k'} c_k \]

\( k^{th} \) Island Chain When Oscillates Near \( k^{th} \) Chain.

\( k_1 h'_A - k_2 = 0 \)

\( V_{mk} \equiv 0 \) for simplicity.

Lie Transform Generator to Average \( \Sigma^{*} \)

\[ \mathcal{L} = \varepsilon \epsilon [\Sigma^{*},] \]

\[ \Sigma^{*} = \Sigma^{*} \frac{V_{k'} s_{k'}}{k'_1 h'_A - k'_2} \]

All \( d_{k'} \neq 0 \).

Then

\[ \mathcal{L}.H = H_A + p_2 - \varepsilon V_k c_k \]

\[ - \frac{\varepsilon^2}{2} \sum^{+} \sum^{*} \left( \frac{c_{k'} c_{k''} V_{k'} [V_{k''}, k' \cdot Q] + s_{k'} s_{k''} V_{k''} [V_{k'}, k' \cdot Q]}{d_{k'}} \right) \]

\[ + O(\varepsilon^3) \]

\( \Sigma^{+} \) includes \( 2V_k \), \( \Sigma^{*} \) has no \( V_k \).

\( \varepsilon^2 \) Term is leading part of effect on \( k^{th} \) chain of \( (k,k') \) interactions and \( (k',k'') \) mode coupling.
SECONDARY ISLANDS.

Transform to frame of $K^{th}$ chain

$$Q_1 = k_1 q_1 - k_2 q_2 \quad Q_2 = q_2$$

When

$$c_{K'} c_{K''} \rightarrow \cos \left( \frac{(k_i + k_{i}')(k_i + k_{i}'')}{k_i} \right) Q_1 + Q_2 \left( \frac{k_i (k_i + k_{i}'') - (k_i' + k_{i}''')}{k_i} \right)$$

Then transform to drift coords. on the island.

$$Q_1 = a \sqrt{\varepsilon} \cos Q_D \quad \text{(Actually an elliptic fr. of } Q_D)$$

So

$$c_{K'} c_{K''} \rightarrow \sum_m J_m (a \sqrt{\varepsilon} (k_i' + k_{i}'')/k_i) \cos (m Q_D - L Q_2)$$

Resonance condition

$$m \omega_0 \sqrt{\varepsilon} = k_i' + k_{i}'' - \frac{(k_i' + k_{i}'')}{k_i} \neq 0$$

Require $m$ to be as small as possible,

$$m \sim \frac{1}{k_i \omega_0 \sqrt{\varepsilon}} \quad \text{independent of } K', K''$$

So

$$J_m \sim (a \sqrt{\varepsilon} (k_i' + k_{i}'')) \frac{1}{k_i \omega_0 \sqrt{\varepsilon}}$$

Resonant coupling to high mode $K'$ is

$$e^2 \left( \frac{V_{K'}}{d_{K'}^2} \right) V_K (a \sqrt{\varepsilon} (k_i' + 1)) \frac{1}{k_i \omega_0 \sqrt{\varepsilon}}$$
COUPLING DIAGRAM

Note: any mode, \( k' \), gives \( \infty \) number of drift resonances at \( k'' \) chain.

\[ k_2' = \frac{k_2}{k_1} \]

\[ k_2'' = \frac{k_2}{k_1} k'' + (k_1 k'' - k_1) \]

\( (\text{VE}) \)

(1) \( k \) interacts directly with modes \( k' \) near resonance line, \( v_K v_{k''} / d_{k''} \)

(2) \( k' \) and \( k'' \) beat at island, \( v_{k'} v_{k''} / d_{k''} \)

\( \epsilon \)-perturbation theory finds resonance only when \( k' = v_K + m k'' \), coupling \( \sim \epsilon^{n+m} \)
**Lie Transform of Liouville Eqn.**

\[ \partial f + [H, f] = 0 \]

\[ f = f(x, x', t) \]

Introduce conjugate pair \((\epsilon, \tau)\), energy & time \(K = \epsilon - H = 0\)

Then, in extended phase space, \([\epsilon, f] = 0\)

**Lie Transform**

\[ L \cdot [K, f] = [L \cdot K, L \cdot f] = 0 \]

or

\[ [K_0 + \epsilon \bar{K}_1 + \epsilon^2 [\bar{K}_1, K_1 + R_1] + \ldots, f(P, Q, T)] = 0 \]

**Pinball Machine' Diffusion Model**

- **Quasilinear Theory Gives**
  \[ [K_0 + \epsilon^2 K_2, f] = 0 \]
  \[ f = f_0 + \tilde{f} \]

Diffusion in \(J\)-space:

\[ [K_0, f_0] - \epsilon^4 \langle [K_2, [\tilde{f} K_2, f_0]] \rangle = 0 \]
TOKAMAK DISRUPTIONS

HISTORY OF TEARING MODE GROWTH

1961 (1) LINEAR PHASE, GROWS LIKE \( e^{\delta t} \)

\[ \delta \sim O(\eta^{3/5}) \]

1973 (2) RUTHERFORD PHASE, SATURATES AT AMPLITUDE \( O(\eta^{4/5}) \)

GROWS LIKE \( (\eta \delta)^2 \)

1979 (3) DISTORTION OF EQUILIBRIUM DESTABILISES OTHER MODES

(4) INTERACTION OF ALL MODES

\[ \rightarrow \text{EXPLOSIVE DISRUPTION!!} \]

\[ \Delta' = \left[ \frac{1}{2} \frac{a}{1 - b} \right] \]

[Diagram of layer and outer solution]
The reduced resistive magnetohydrodynamic equations for a current-carrying magnetofluid in a sheared slab are:

\[ \frac{\partial \psi_k}{\partial t} + \left[ \frac{\partial}{\partial r} \left( \sum_k ( -i k \gamma ) \phi_{-k} \psi_k \right) - i k_y \sum_k \frac{\partial \phi_{-k}}{\partial r} \psi_k \right] \]

\[ - \left[ \frac{\partial}{\partial r} \left( \sum_k ( -i k \gamma ) \psi_{-k} \phi_k \right) - i k_y \sum_k \frac{\partial \psi_{-k}}{\partial r} \phi_k \right] \]

\[ = -i k \parallel \phi_k + \eta \nabla_1^2 \psi_k, \quad (47) \]

and

\[ \frac{\partial}{\partial t} ( \nabla_1^2 \phi_k ) + \left[ \frac{\partial}{\partial r} \left( \sum_k ( -i k \gamma ) \phi_{-k} \nabla_1^2 \phi_k \right) - i k_y \sum_k \frac{\partial \phi_{-k}}{\partial r} \nabla_1^2 \phi_k \right] \]

\[ - \left[ \frac{\partial}{\partial r} \left( \sum_k ( -i k \gamma ) ( \nabla_1^2 \phi_{-k} ) \phi_k \right) - i k_y \sum_k \frac{\partial \phi_{-k}}{\partial r} \nabla_1^2 \phi_k \right] \]

\[ = -i k \parallel \nabla_1^2 \psi_k + i k_y \frac{d J_0 (r)}{dr} \psi_k + \left[ \frac{\partial}{\partial r} \left( \sum_k ( -i k \gamma ) \psi_{-k} \nabla_1^2 \psi_k \right) - i k_y \sum_k \frac{\partial \psi_{-k}}{\partial r} \nabla_1^2 \psi_k \right. \]

\[ - \left[ \frac{\partial}{\partial r} \left( \sum_k ( -i k \gamma ) \nabla_1^2 \psi_{-k} \psi_k \right) - i k_y \sum_k \frac{\partial \psi_{-k}}{\partial r} \nabla_1^2 \psi_k \right]. \]
Magnetic Surfaces for 2:1 Mode in Presence of 3:2 and Others.

The perpendicular electron thermal conductivity \( \bar{X}_\perp \) does not have much influence on the nonlinear process when the classical value for the perpendicular transport coefficient is used.\(^{21}\) Using an anomalous value for \( \bar{X}_\perp \), smoother temperature profiles can be obtained during the calculation. Figure 15 compares, for a fixed time, the current density and temperature profiles for two different values of \( \bar{X}_\perp \) (\( \bar{X}_\perp = 0.05 \) and \( \bar{X}_\perp = 5.0 \), respectively). The effect of \( \bar{X}_\perp \) on the current profile is not as noticeable as it is on the temperature profile, but there is a clear tendency for the smallest scale length fluctuations in the current density profile to disappear as \( \bar{X}_\perp \) is increased.

Finally, another point to consider is the effect of the initial conditions on the nonlinear evolution, i.e., the effect of changing the initial island widths \( W_0^{m,n} \) and \( W_0^{m,n} \) for the \( (m=2; n=1) \) and \( (m=3; n=2) \) modes, respectively. This is illustrated in Figs. 16(a) and 16(b), where the nonlinear growth rates of two modes are plotted as functions of time for different initial condi-
In deriving Eq. (97), contributions added in \( m' \) were dropped; thus, the renormalized reduced magnetohydrodynamic equations for a long-wavelength tearing mode instability in the presence of fully developed, densely packed turbulence are

\[
\gamma_k \psi_k + ik_\parallel \phi_k = (D^A_k + \eta) \frac{i \partial^2 \psi_k}{\partial x^2},
\]

(98)

\[
\gamma_k \frac{\partial^2 \phi_k}{\partial x^2} + ik_\parallel \frac{\partial \psi_k}{\partial x} = a_k \frac{\partial^2 \phi_k}{\partial x^2},
\]

(99)

\[
D^A_k = \sum_k \frac{k_y^2}{\gamma''} |\phi_{k'}|^2,
\]

(100)

and

\[
a_k = \sum_k \frac{\Delta \gamma' \delta(x'')}{\gamma''} \frac{m^2}{m''^2} k_y' |\psi_{k'}|^2.
\]

(101)

Because the case of interest is one characterized by high levels of fluid turbulence, \( D^A_k \gg \eta \), the collisional resistivity is neglected hereafter. Equations (98) and (99) admit two classes of solution. The first class corresponds to a tearing instability triggered by the anomalous ohmic diffusivity \( D^A_k \), with the dispersion relation

\[
\gamma_k^A = (\sqrt{2}/3)^4 (\Delta_{k'}^A)^4 (D^A_k)^3 (k_y/L_s)^2.
\]

(102)

Because \( D^A_k \sim (\gamma'')^{-1} \) and \( \gamma'' \sim \gamma_k \), the anomalous tearing mode growth rate is

\[
\gamma_k = \left( (\sqrt{2}/3) \Delta \gamma' \right)^{1/2} (\hat{D})^{3/8} (k_y/L_s)^{1/4},
\]

(103)

where

\[
\hat{D} = \sum_k k_y'' |\phi_{k'}|^2.
\]

(104)

The turbulently broadened tearing layer width is

\[
\lambda_k = (L_s/k_y)^{1/2} \hat{D}^{1/4},
\]

(105)

where \( \gamma_k > a_k \) is required for consistency with the neglect of \( a_k \) in the calculation of the growth rate. Note that while the anomalous ohmic diffusivity accelerates the tearing mode growth, \( \Delta \gamma' > 0 \) is still required for instability. The mode
LIE TRANSFORM OF REDUCED MHD EQNS.

AS WITH FLUID FLOW, \( \nabla \cdot \mathbf{B} = 0 \) SUGGESTS HAMILTONIAN FORMULATION.

\[
\mathbf{B} = B_0 \hat{z} + \nabla \psi_0(t) \times \hat{z} + \nabla \psi_1 \times \hat{z}
\]

FIELD LINES FOLLOW

\[
H = B_0 \mathcal{E} + \psi_0 + \psi_1 = H_0 + \psi_1
\]

with

\[
p = \frac{B_0}{2} \quad q = \Theta \\
(\mathcal{E}, \mathcal{z}) \text{ CONJUGATE}
\]

SO

\[
\mathbf{B} \cdot \nabla J = [H, J]
\]

\[
\frac{da}{dt} = \frac{dH}{dp} = \frac{1}{rB_0} \frac{dH}{dt} = \frac{1}{rB_0} \frac{d\psi}{dt} = \frac{1}{rB_0} \frac{d\Theta}{dz}
\]

\[
\frac{dz}{dt} = \frac{dH}{d\mathcal{E}} = \frac{\partial H}{\partial \mathcal{E}} = \frac{B_0}{\mathcal{E}}
\]

LIE TRANSFORM OPERATOR ON \( M^{th} \) RESONANCE IS

\[
\mathcal{L} = \exp \left[ \hat{\phi}_1^*, J \right]
\]

\[
\hat{\phi}_1^* = \sum_{n' \neq n} \phi_{n'} e^{i(n'\Theta + k'z)}
\]
TRANSFORM MOMENTUM EQN

\[ \mathbf{L} \cdot (\mathbf{B} \cdot \nabla J) = \frac{d\mathbf{u}}{dt} \]

\[ \Rightarrow [H_0 + \psi_1 + \frac{1}{2} [\hat{\psi}, \psi_1 + \psi_2], J] = \frac{d\mathbf{u}}{dt} \]

And

\[ [H_0 + \psi_1, J] = \Re H_0 \frac{\partial J}{\partial \eta} = \Omega_0 \frac{\partial J}{\partial \eta} \]

is the angle derivative of \( J \) around the island and

\[ \Omega_0 \sim O(\sqrt{\psi_1}) \]

TOKAMAK

with \( \psi_1 = \psi_2(\psi_1 \cos(2\theta - z)) + \psi_3 \cos(3\theta - 2z) \)

\[ \Omega_0 \frac{\partial J}{\partial \eta} + \left[ 6 \frac{\psi_2}{B_0^2} \left( \psi_2 \cos(\psi_1 \cos(2\theta - z)) + \cos(3\theta - 2z) \right) \right] J \frac{d\mathbf{u}}{dt} \]

FINAL RESONANT MODE COUPLING EQN IS

\[ \oint_{\partial D} \left( [H_2, J] \frac{d\mathbf{u}}{dt} \right) = 0 \]

Couples 2 modes to many larger \( M \) modes!
(a) 191 MODES
$t = 6.62 \times 10^{-3} \tau_R$
(d) $t = 7.01 \times 10^{-3}$ $n$

$m=2n$

$(m=3, n=2)$

$(m=2, n=1)$ Island filled with many modes!

$(m=3, n=2)$ initialised at lower amplitude.
rate of the \((m = 3; n = 2)\) mode is calculated, and its value can be compared with the nonlinear growth rate of this mode (Fig. 3). The agreement is good. It is also important to investigate the correlation of this anomalous growth with the buildup of the high-\(m\) fluid turbulence under various cir-

In Fig. 2, the growth rate and voltage trace for the \(m = 3, n = 2\) mode are being compared. It shows that the results are the same as the anomalous growth rate calculation and confirms the correlation.

FIG. 3. The growth rate of the \((m = 3; n = 2)\) mode as obtained from the model is compared with the calculation. Same case as in Fig. 2.

FIG. 5. Same case as Figs. 2 and 3: (a) mode kinetic energy; (b) voltage trace.
CONCLUSIONS

* DYNAMICS OF PARTICLES, FLUID FLOW LINES, MAGNETIC FIELD LINES ALL SHOW STOCHASTIC MOTION WHEN 2 OR MORE MODES OF LOW MODE # INTERACT

* LIE TRANSFORM DEVELOPS DRIFT EQNS. FOR ISLAND INTERACTION WHICH ARE LARGE, O(VE)

* LIE TRANSFORMS CAN BE APPLIED TO FLUID EQNS. TO GIVE NEW, LARGE COUPLINGS BETWEEN MODES.
RESISTIVE DYNAMICS OF MAGNETIC ISLANDS

WITH CURVATURE AND PRESSURE

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INTRODUCTION

Curvature and pressure are very important in the linear stability of resistive instabilities. Glasser, Greene and Johnson found it resulted in a LARGE CRITICAL $\Delta'_{c}$ for instability in tokamaks.

Rutherford considered the nonlinear theory of tearing modes driven by $\Delta'$.

Here we compute the nonlinear evolution including interchange effects.

We compute a Grad-Shafranov equation to describe MHD equilibria in the island vicinity. We then derive the resistive evolution.
We focus attention on the region near the rational surface, \( q_0 = \frac{m}{n} \).

Matching to exterior region introduces \( \Delta' \).

Islands are taken to be thin to make problem tractable.

Nonlinear islands grow more slowly than linear instabilities. So

1) Neglect inertia
2) Assume sound wave propagation along \( B \) causes \( B \cdot \nabla p = 0 \)

Make expansion in aspect ratio \( \epsilon \)

Local pressure profile is found by assuming large diffusion and given sources deep in plasma interior (e.g. ohmic heating, etc.)
Islands described by resistive MHD

\[ \nabla \mathbf{j}_n = -\mathbf{B} \cdot \nabla \rho \times \nabla \left( \frac{1}{\mathbf{B}^2} \right) \]

\[ \mathbf{j} \times \mathbf{B} = \nabla \rho \]

\[ \mathbf{E}_n = \eta \mathbf{j}_n \]

We employ flux coordinates

\[ \mathbf{B}_0 = \nabla \chi \times \nabla \left( q(x) \Theta - \Phi \right) \]

Define angle which follows the island:

\[ \alpha = \Theta - \Phi / q_0 \]

Define averaging operator to separate out resonant and non-resonant parts

\[ \tilde{f}(x, \alpha) = \frac{\int d\varphi f(x, \alpha, \varphi)}{\int d\varphi} \]

\[ \tilde{f} = f - \tilde{f} \]
The average vorticity equation is crucially important. Define

\[ [A, C] = \nabla \varphi \cdot \nabla A \times \nabla C \]

\[ h = -\frac{1}{2} B_0^2 + 2 p / B_0^4 \]

\[ B_0 + B_1 = \nabla \Phi \times \nabla \psi \]

\[ Q = J / B \]

The total helical flux, \( \Psi_h = \overline{\Psi} - \int dx q / q_0 \)

is an important quantity, since it acts as a flux function in the island vicinity (e.g. \( p = p(\Psi_h) \)). The averaged vorticity equation is

\[ [\Psi_h, Q] = [\overline{\rho}, \overline{h}] + [\overline{\rho}, \overline{h}] - [\overline{\psi}, \overline{Q}] \]

Eq (1)
Without pressure or curvature, 
Eq(1) implies

\[ Q = Q(\psi_n) \]

which is a central element of 
Rutherford's analysis. Interchange 
effect modify this.

The first term gives the effect of 
normal curvature, the others the 
geodesic curvature.

To compute the latter terms, we 
make an expansion for thin 
islands.

This is similar to the approx-
imation of localization near a 
rational surface used by 
Glasser, Greene, and Johnson.
Computation of the nonresonant terms proceeds similarly to linear theory, since the magnetic non-linearity is negligible for them. The result has the form

\[
\hat{\mathbf{r}}_{\text{h}} \cdot \mathbf{h} = \hat{\mathbf{r}}_{\text{h}} \cdot \mathbf{I} = \hat{\mathbf{r}}_{\text{h}} \cdot \mathbf{h}_g
\]

which makes the solution of Eq(1) very simple

\[
Q = Q_*(\psi_h) + G_1 \frac{\partial p(\psi_h)}{\partial \psi_h} \psi_h
\]

where \( G_1 = h + h_g \), and \( Q_* \) is an arbitrary function needed to specify the current on a flux surface. \( G_1 \) is proportional to the curvature expression \( E + F \) given by Glasser et al.
To obtain a Grad-shafranov equation for \( \overline{\Psi} \), we take the average of Ampere's Law. For thin islands, this is

\[
\frac{\partial^2}{\partial x^2} \overline{\Psi} = \overline{\nabla x^2 \bar{Q}} + \overline{\nabla \bar{x} x^2 \bar{Q}}
\]

We then obtain
\[
\frac{d^2}{dX^2} \psi_h = Q_{\text{eff}}(\psi_h) + (X - X_R)(G_1^* G_2) \frac{dp(\psi_h)}{d\psi_h}
\]

Where \( G_1 + G_2 \) is proportional to the expression in the Mercier stability criterion \( D_1 < 1/4, D_1 = E + F + H \)
\( q_h(\psi) \) is determined by Ohm's Law. As in Rutherford, we define the flux average \( \langle \psi_h \rangle \) over a surface of constant \( \psi_h \). Then

\[
\langle \partial \psi_h / \partial t \rangle = q_h(\psi_h)
\]

This determines \( \mathbf{J}_* \). The Grad-Shafranov equation can then be solved with two conventional approximations:

1) One harmonic dominates in the perturbed flux, so

2) The "constant \( \psi \)" approximation holds in the island interior. This requires that \( E+F \) be small.

Far from the island, Eq(2) gives

\[
\psi - c_1 (\chi - \chi_R) \sqrt{1 - \frac{D_1}{D_2}} + c_2 (\chi - \chi_R) \sqrt{1 - \frac{D_2}{D_1}}
\]

which are the Mercier solutions.

The finite \( \beta \) generalization of \( \Delta' \) is

\[
\Delta'_* = \frac{c_2}{c_{1+}} - \frac{c_2}{c_{1-}}
\]
When matched to the exterior, we obtain the evolution of $\psi$. This is most clearly written in terms of the island width $\Delta \chi$

\[
\frac{k_i}{\eta} \frac{\partial}{\partial t} \Delta \chi =
\]

\[
\Delta \star \Delta \chi^S + k_2 \frac{E + F}{\Delta \chi}
\]

where $S = -1 + \sqrt{1 - 4D_i}$, $k_2 \approx 6.3$
The finite $\beta$ generalization of $\Delta'$ is

$$
\Delta'_* = \frac{C_{2+}}{C_{1+}} - \frac{C_{2-}}{C_{1-}}
$$

When matched to the exterior, we obtain the evolution of $\psi$. This is most clearly written in terms of the island width $\Delta \chi$

$$
\frac{k_i}{\eta} \frac{\partial}{\partial t} \Delta \chi =
$$

$$
\Delta'_* \Delta \chi^{s_1} + k_2 \frac{(E+F)}{\Delta \chi}
$$

where $k_2 \approx 6.3$, $s_1 = -1 + \sqrt{1 - 4D_1}$
The driving term for magnetic free energy was given by Rutherford (for \( \beta \to 0 \)).

The interchange driving term is stabilizing for good curvature, but decreases in relative importance as \( \Delta \chi \) increases. The two are comparable when

\[
\Delta^* \Delta \chi^{\sqrt{1-4D_x}} = k_2 (E + F)
\]

Eq. (3)

This is the nonlinear generalization of the concept of \( \Delta_c' \).
There are several similarities between the linear results of Glasser et al. and those above.

1) For islands just barely into the nonlinear regime, with widths of order the linear theory resistive layer, the $\Delta'$ required for growth according to Eq(2) agrees with the critical $\Delta'$ of linear theory. Since the influence of pressure decreases as the island grows, a tearing mode in a region of good curvature which has $\Delta'>\Delta'_C$ is not stabilized in this nonlinear regime.

2) Only E+F contributes to the resistive evolution, despite the fact that the pressure drive for the ideal MHD case is E+F+H. This agrees with linear theory when E+F and H is small.
ALSO, note that Eq(3) gives the saturated island width for a resistive interchange instability (when $\Delta' < 0$). Of course, the dynamics if two islands overlap is not described by the present calculation.
RELAXATION OF TOROIDAL PLASMAS

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RELAXATION OF TOROIDAL PLASMAS

A. Bhattacherjee, Columbia University
A. H. Glasser, LANL

Other Co-workers (chronologically)

R.L. Dewar, PPPL, Australian National University
D.A. Monticello, PPPL
M.S. Chance, PPPL
J.C. Wiley, FRC, University of Texas
A. Khare, PPP, India
J.E. Sedlak, IFS
Review of basic ideas

1. Ideal plasmas characterised by denumerably infinite number of invariants

\[ \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{U} \times \mathbf{B}) = 0 \]

\[ \frac{\partial \mathbf{A}}{\partial t} - \dot{\mathbf{U}} \times \mathbf{B} + \nabla \chi = 0 \]

\[ \Leftrightarrow \quad \frac{dK}{dt} = 0, \quad K = \int_V d\tau \quad \frac{\mathbf{A} \cdot \mathbf{B}}{2} \] (over any volume V bounded by field lines (Hoffatt, 1969))

2. Taylor's conjecture (1974)

For small departures from ideal behaviour

\[ K_0 = \int_V d\tau \quad \frac{\mathbf{A} \cdot \mathbf{B}}{2} \] (V)

is the only remaining invariant. Plasma relaxes to minimum-energy state consistent with the invariance of \( K_0 \) and total toroidal flux.

Thus, minimising

\[ W = \int_V d\tau \left[ \frac{\mathbf{E}^2}{2} + \frac{\mathbf{B}^2}{2} \right] \]
subject to conservation of $K_0$, and toroidal flux

\[ 8W - \lambda S_\phi = 0 \]

\[ \Rightarrow \bar{J} = 2. \bar{B}, \quad \psi = 0 \]

Comments

1) Mechanism for conservation of $K_0$?

2) Model not generally good for tokamaks: predicts disrupted profiles

3) Taylor states: overkill for internal modes

\[ \sigma = \bar{J} \bar{B} / B^3 = \sigma_0 = \text{constant} \]

unstable with wall removed from plasma, always force-free.

Bear little resemblance to "optimised profiles" in fusion devices

Yet, Taylor's model is an undeniably attractive theoretical model for relaxation in RFP's

Might the basic idea be extended to understand relaxation in tokamaks or stellarators?
3. Ideal MHD (Kruskal and Kulsrud, 1958)
\[ \mathbf{b} = \partial x \times \partial \varphi + \partial \varphi \times \partial \theta \]
\[ \mathbf{a} = \varphi \partial \theta - \varphi \partial x \]

Under ideal variations
\[ s_{\varphi} = -s \partial \varphi, \quad s_{\theta} = q(\varphi) s_{\varphi} \]

\[ K \left[ \omega_{x} \right] = \int_{V} d\varphi \omega_{x}(\varphi, \varphi) \frac{\mathbf{a} \cdot \mathbf{b}}{2} \text{ is conserved} \]

for every volume \( V \) bounded by field lines equivalent to the class of invariants due to Moffatt

4. Quasi-ideal MHD (Kadomtsev-Monticello, 1975)

\[ m=1, n=1 \] resistive internal kink

\[ K_{\chi} = \int_{V} \omega_{x}(\chi) \frac{\mathbf{a} \cdot \mathbf{b}}{2} d\varphi \]

\[ \chi = q_{s} \varphi - \varphi; \quad q_{s} = \text{pitch of the mode} \]

Further arguments (Bhattacharjee, Dewar, Monticello, 1980, 1982) show \( \omega_{x}(\chi) = \chi^{\alpha} \), where \( \alpha \) is a positive integer
Ad hoc truncation

\[ x = 0, 1 \]

\[ K_0 = \int d\tau \frac{\mathbf{A} \cdot \mathbf{B}}{2} , \quad K_1 = \int d\tau x \frac{\mathbf{A} \cdot \mathbf{B}}{2} \]

Examine consequences of the simplest modification to Taylor's truncation procedure

\[ \tilde{F} = (2 \cdot + \frac{3\pi}{2} \chi) \mathbf{B} \]

Examine solutions for which \( \tilde{F} = 0 \) on wall

\[ \tilde{F} = 2 \cdot (1 + \frac{\chi}{\Xi}) \mathbf{B} \]

\[ \Xi = \Xi_p \text{ on boundary} \]

Equilibrium solutions obtained by scanning in \( \lambda_0 \).
Quasi-ideal model of reconnection (Kadomtsev, 1973)

Contours of auxiliary field \( B_0 - \frac{rB_z}{R_R} \)
Estimate of decay rates

\[ K_x = \int dx \, x^2 \frac{\bar{A} \cdot \bar{B}}{2} \]

\[ \frac{dK_x}{dt} = \frac{1}{2} \int dx \, \left[ x^2 \left\{ \frac{\partial \bar{A}}{\partial t} \cdot \bar{B} + \frac{\partial \bar{B}}{\partial t} \cdot \bar{A} \right\} + \bar{A} \cdot \bar{B} \frac{\partial x^2}{\partial t} \right] \]

\[ \frac{\partial \bar{B}}{\partial t} - \nabla \times (\nabla \times \bar{B}) = -\eta \nabla \times \bar{F} \]

\[ \frac{\partial \bar{A}}{\partial t} - \nabla \times \bar{B} = -\eta \bar{F} + \alpha \nabla \times \bar{A} \]

\[ \frac{dK_x}{dt} = \int dx \left[ -\eta \nabla \cdot \bar{B} \, x^2 + \bar{A} \cdot \bar{B} \frac{\partial x^2}{\partial t} - \frac{\eta \bar{F} \cdot (\nabla \times \bar{A})}{2} \right] \]

Also

\[ W = \int dx \, \frac{\bar{B}^2}{2} \]

\[ \frac{dW}{dt} = \int dx \left[ -\eta \nabla^2 - \frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 \right) \right] \]

Park, Monticello and White (1984)

\[ \tau^{-1} = \frac{1}{K_x} \frac{dK_x}{dt} \sim -\eta \left( x+1 \right) \]

\[ \tau^{-1} = \frac{1}{W} \frac{dW}{dt} \sim -\eta \frac{1}{5} \]

\[ \tau^{-1} / \tau^{-1} \sim \frac{5}{L} \left( x+1 \right) \sim \eta \frac{1}{2} \]

\( L \): macroscopic length

\( s \): singular layer width
$ZT-40$ DATA

$F = \frac{B_2(a)}{\langle B_2 \rangle}$

$\theta = \frac{B_\theta (a)}{\langle B_\theta \rangle}$

$\times \Delta$ Experimental Data

--- Taylor's Theory

--- Present Theory

Courtesy: D. Baker, LASL
Energy Principle with Global Invariants

Extremise \( W = \int d\tau \frac{B}{2} \) subject to the invariance of \( K_x = \int d\tau \frac{A \cdot B}{2} \)

\[ 3W = \frac{\Sigma}{2} 2_k \delta K_x = 0 \]

\[ \Rightarrow \bar{J} = \sum_{k} \frac{2_k (\kappa + 2)}{2} \chi^k \bar{B} \]

\[ \sigma = \frac{\bar{J} \cdot \bar{B}}{B^2} = \sum_{k} \frac{2_k (\kappa + 2)}{2} \chi^k \]

Consider equilibrium in periodic cylinder, with dependence only on \( r \)

\[ \frac{d\sigma}{dr} = \sum_{k} \frac{2_k (\kappa + 2)}{2} \frac{d\chi}{dr} (2s - 2) \]

\[ = 0 \quad \forall \ s \neq 2 \]

In the final state, model predicts that if the rational surface for the mode falls within the plasma, the parallel current will flatten itself at that surface. Or, the rational surface may be eliminated from within the plasma.
Fig. 1(a) Energy of equilibria in present theory compared with energy of Taylor states (marked by Δ). Arrows indicate direction of increasing λ. Labels P and T distinguish pinchlike and tokamaklike equilibria. Dashed lines indicate unstable equilibrium (energy stationary, but not minimum).
(b) Typical q-profile on the stable P-branch. (c) Typical q-profile on the stable T-branch.

\[ V = 2 \omega R^{-1} (2\pi \Phi_p)^{-2} \]
STABILITY OF RELAXED STATES

Ideal and resistive stability of relaxed states are examined by integrating Newcomb's equation, and computing \( d' \).

Have found completely stable (stable w.r.t. internal modes) tokamak-like solutions with \( q_{(\text{center})} \leq 1 \), \( q_{(\text{edge})} \leq 1.3 \).

RFP branch usually unstable to some \( m=1 \), large \( n \) instability.

Wisconsin RFP (Sprott, Prager, Kerst)

Proposed \( q \)-scaling studies from tokamak to RFP branch.

Have found transition region to be usually unstable in this model.
TRANSITION REGION BETWEEN TOKAMAK AND RFP BRANCH

ARATIO = 5.000  QS = 1.000  LAMDAO = -0.300  BETA0 = 1.000e-03
PS10 = -2.024e+00  PHIA = 1.258e-01

SIGMA VS. X

Q VS. X

BT VS. X

DP VS. X
SECOND VARIATION OF FREE ENERGY

\[ F = W - \sum_{\alpha} \lambda_{\alpha} K_{\alpha} \]

\[ \delta F = 0 \quad \text{equilibrium} \]

\[ \delta^2 F > 0 \quad \text{sufficient conditions for ideal and resistive stability} \]

In particular, have shown in a straight cylinder that if variations are restricted to the ideal class, then \( \delta^2 F \) reduces to Newcomb's form (Bhattacharjee and Dewar, 1982) Connection to resistive stability needs to be made more precise

Sufficient conditions for stability thus obtained without resolving the singular region of ideal theory.
FINITE PRESSURE EQUILIBRIA

Taylor states force-free even in the presence of finite pressure, \( \overline{f} = 2. \overline{B} \), \( \overline{b}_P = 0 \)

Consider resistive interchanges, with global invariants

\[
K_x = \int d\nu \, \nu^x \frac{\overline{A} \cdot \overline{B}}{2}
\]

\[
S_x = \int d\nu \, \nu^x \ln \left( \frac{b}{\rho} \right)
\]

\[
M_x = \int d\nu \, \nu^x \rho
\]

\[
\delta W - \frac{\Sigma}{\alpha} \left[ \lambda_x S_k + \nu_x S M_x + T_x S S_k \right] = 0
\]

\[
\overline{f} = \frac{\Sigma}{\alpha} \frac{2 \lambda_x (x+2)}{2} \nu^x \overline{B} - \rho'(\chi) (\nabla \times (\nabla \times (q_s \theta - x)))^{-1} \nabla \times \nabla (q_s \theta - x)
\]

\[
\rho = \rho \frac{\Sigma}{\alpha} T_x \nu^x
\]

\[
\rho \frac{\Sigma}{\alpha} T_x \nu^x = \frac{\rho}{y-1} \left[ \gamma - \ln \left( \frac{b}{\rho} \right) \right] \frac{\Sigma}{\alpha} T_x \nu^x
\]
Truncation \( \xi = 0 \)

\[ T = 2. \bar{B}, \quad \rho = \rho_0, \quad \rho = \text{constant} \]

Inclusion of total mass and total entropy as global constraints still gives Taylor states

\( \xi = 0, 1 \)

\[ T = (\lambda_0 + \frac{3}{2} \chi) \bar{B} - \left( \frac{4 \pi \rho_0 x^2}{\mu} \right)^{-1} \rho'(x) \]

\[ \nabla \times \nabla (q_0 - x) \]

\( \rho = \rho_0 + q(x), \quad \rho_0 \) and \( \rho \), are constants

\[ \frac{d\theta}{dr} = 0 \quad \text{at} \quad r = R_0, \quad \text{independent of the level of truncation} \]
APPLICATION TO PROFILE OPTIMISATION FOR 2-D AND 3-D EQUILIBRIA

1. Choose a dynamical model, e.g., resistive internal modes, resistive interchanges
2. Construct global invariants
3. Energy Principle with Global Invariants then specifies automatically so-called "free" functions such as current and pressure.

States expected to have robust stability

Implemented for axisymmetric states (Bhattacharjee and Wiley, 1982)

May be a reasonable way to go for stellarator equilibria, particularly in those cases for which perfect surfaces may not exist
SUMMARY

1. Have constructed a thermodynamic model, in the spirit of Taylor, to understand relaxation in tokamaks.

2. For a tearing mode of single helicity, imposition of additional constraints has given realistic profiles. Furthermore, a subset of these profiles have been shown to be ideally and resistively stable.

3. Model allows the construction of finite beta equilibria, with non-zero gradients.

4. Method may have useful applications for profile optimisation for 2-D and 3-D equilibria.
STATISTICAL PROPERTIES OF MHD TURBULENCE

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Statistical properties of MHD turbulence
Jiro Mizushima

① Mechanism of turbulent dynamo with helicity
M. Steenbeck, F. Krause & K. H. Rädler (1966)

② Phenomenological theory of MHD turbulence without helicity
R. H. Kraichnan (1965)
Y. Saito, J. Mizushima & N. Futami (1985)

③ Modified zero-fourth cumulant approximation for MHD turbulence without helicity
Y. Saito, J. Mizushima & N. Futami (1985)
Methods to study Ordinary (N.S.) Turbulence

Theoretical Methods

- Phenomenological argument
- EDQNM, Orszag
- Closure theory
- DIA Kraichnan
- M2FC approximation
- Tatsumi, Kida
- Takahashi
- Mizushima

Numerical Simulation

- Orszag
  - Less assumptions

Experimental Methods

Observational Methods
Mechanism of turbulent dynamo

Can large-scale magnetic fields be generated when $E^r$ and $H^\nu$ are injected?

M. Steenbeck, F. Krause & K. H. Rädler (1966)

<table>
<thead>
<tr>
<th>Large-scale</th>
<th>Middle-scale</th>
<th>Small-scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-energy</td>
<td>M-energy</td>
<td>M-energy</td>
</tr>
<tr>
<td>M-helicity</td>
<td>negative M-helicity</td>
<td>positive M-helicity</td>
</tr>
<tr>
<td>inverse cascade</td>
<td>$\alpha$-effect</td>
<td>Alfvén effect</td>
</tr>
</tbody>
</table>

- $\alpha$-effect: middle- or large-scale magnetic fields generation by $V$-helicity
  \[(\text{Residual helicity } H^r \cdot W - k^2 H^M)\]
- Alfvén effect: equipartition of $E^r(k)$ and $E^M(k)$ by $b_0$
- Cascade: Stretching of vortex lines
- Inverse cascade: inverse cascade of M-helicity
  cf. Two-dimensional N.S. turbulence
Phenomenological Theory of MHD Turbulence

1 NS Turbulence

Kolmogorov's Theory

\[ E(k) \sim \frac{l^3}{t^2} \]  

(2.1)

energy dissipation rate

\[ \varepsilon \sim \frac{l^3}{t^3} \]  

(2.2)

\[ k \sim \frac{1}{l} \]  

(2.3)

if \( E(k) \) is expressed only by \( \varepsilon \) and \( k \),

dimension analysis leads

\[ E(k) = K \varepsilon^{2/3} k^{-5/3} \]  

(2.4)

2 MHD Turbulence

R. H. Kraichnan (1965)

\[ \varepsilon \propto (b_o k)^{-1} \]  

(2.5)

\[ \varepsilon = A^2 b_o^{-1} \int [E(k)]^2 k^3 \]  

(2.6)

\[ \frac{3}{2} b_o^2 = \int_0^\infty E^M(k) \, dk \]  

(2.7)

\[ E^V(k) = E^M(k) = K (\varepsilon b_o)^{1/2} k^{-3/2} \]  

(2.8)
Extension of Kolmogorov's theory to decaying NS Turbulence.

T. Tatsumi, S. Kida & J. Mizushima (1978)

three assumptions

1. \[ E(k, t)/E_0 = t^{-\beta} F(k/t^\delta) \] (2.9)
   at \( k \sim k_E \) and inertial subrange and \( k \gg 1 \)

2. \[ E(k, t)/E_0 = Aa k^\alpha \] (2.10)
   at \( k \ll 1 \)

3. \[ E(k, t) = K \epsilon(t)^{2/3} k^{-5/3} \] (2.11)

Intersection

\[ E_s = Aa k_5^\alpha = K \epsilon(t)^{2/3} k_5^{-5/3} \] (2.12)

\[ k_5 = \epsilon(t)^{2/(3a+5)} \]

\[ E_s = \epsilon(t)^{2a/(3a+5)} \]

\[ \epsilon(t) \propto t^{-b} \]

\[ \epsilon(t) = -\frac{d\epsilon(t)}{dt} \propto t^{-(b+\nu)} \] (2.15)
\[ k_5 \propto t^{-\frac{2(b+1)}{3a+5}} \]
\[ E_5 \propto t^{-\frac{2a(b+1)}{3a+5}} \]
\[ \beta = \frac{2a(b+1)}{3a+5}, \quad \delta = -\frac{2(b+1)}{3a+5} \]  \( (2.17) \)

\[ \mathcal{E}(t) = \int_0^\infty E(k,t)dk = E_0 k_0 t^{-\beta+\delta} \int_0^\infty F(5)d\delta \]
\[ \propto t^{-2(a+1)(b+1)/(3a+5)} \]  \( (2.18) \)

Comparing \( (2.18) \) and \( (2.14) \)

\[ b = \frac{2(a+1)}{(a+3)} \]  \( (2.19) \)

\[ \beta = \frac{2a}{a+3}, \quad \delta = -\frac{2}{a+3} \]  \( (2.20) \)

when \( a = 2 \)

\[ \mathcal{E}(t) \propto t^{-6/5} : \text{Birkhoff's invariant exists} \]  \( (2.21) \)

when \( a = 4 \)

\[ \mathcal{E}(t) \propto t^{-10/7} : \text{Loitsiansky's invariant exists} \]  \( (2.22) \)
1. \( E'(k,t)/E_0 = E''(k,t)/E_0 = E(k,t)/E_0 = t^{-\beta} F(k/t^\delta) \) 
   \( \text{at} \ k \sim k_E \text{ and inertial subrange and } k \gg 1 \)  
2. \( E(k,t)/E_0 = Aa_k^a \) 
   \( \text{at} \ k \ll 1 \)  
3. \( E(k,t) = K [\xi(t) b_0(t)]^{1/2} k^{-3/2} \)  
\[ E(t) \propto t^{-b}, \ b_0(t) \propto t^{-b/2} \]  
\[ \xi(t) = - \frac{dE(t)}{dt} \propto t^{-(b+1)} \]  
\[ k_s(t) \propto t^{-(3b+2)/2(2a+3)} \]  
\[ E_s(t) \propto t^{-(3b+2)/2(2a+3)} \]
\[ \beta = a(3b+2)/2(2a+3), \quad \alpha = -(3b+2)/2(2a+3) \]  
\[ (2.28) \]

\[ \tilde{C}(t) = \int_0^\infty E(k, t)dk = E_0 k_0 t^{-\beta+\delta} \int_0^\infty F(\xi) d\xi \]
\[ \approx t^{-(a+1)(3b+2)/2(2a+3)} \]  
\[ (2.29) \]

Comparing (2.29) with (2.26)

\[ b = 2(a+1)/(a+3) \]

\[ \beta = \frac{2a}{a+3}, \quad \delta = -\frac{3}{a+3} \]  
\[ (2.30) \]

(2.30) is the same with (2.26) for N S turbulence.
\[ \frac{\partial \hat{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} + (\mathbf{b} \cdot \nabla) \mathbf{b} , \]

\[ \frac{\partial \hat{b}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{b} = (\mathbf{b} \cdot \nabla) \mathbf{u} + \lambda \Delta \mathbf{b} , \quad (3.2) \]

\[ \nabla \cdot \mathbf{u} = 0 , \quad (3.3) \]

\[ \nabla \cdot \mathbf{b} = 0 , \quad (3.4) \]

\[ \left( \frac{\partial}{\partial t} + \nu k^2 \right) \hat{\mathbf{v}}_i (k, t) = -ik_1 \delta_{ij} (k) \int \frac{dp dq}{p+q=k} \hat{\mathbf{v}}_i (p, t) \hat{\mathbf{v}}_j (q, t) \]

\[ + ik_1 \delta_{ij} (k) \int \frac{dp dq}{p+q=k} \hat{\mathbf{b}}_1 (p, t) \hat{\mathbf{b}}_j (q, t) , \quad (3.5) \]

\[ \left( \frac{\partial}{\partial t} + \lambda k^2 \right) \hat{\mathbf{b}}_i (k, t) = -ik_1 \delta_{ij} \int \frac{dp dq}{p+q=k} \hat{\mathbf{v}}_i (p, t) \hat{\mathbf{b}}_j (q, t) \]

\[ + ik_1 \delta_{ij} \int \frac{dp dq}{p+q=k} \hat{\mathbf{b}}_1 (p, t) \hat{\mathbf{v}}_j (q, t) , \quad (3.5) \]

\[ \text{where} \]

\[ k^2 = \delta_{ij} - \frac{k_i k_j}{k^2} \quad \text{and} \quad k = |k| . \]

\[ \mathbf{E}^M (k, t) = 2\pi k^2 \langle \hat{\mathbf{b}} (k, t) \cdot \hat{\mathbf{b}}^* (k, t) \rangle . \quad (3.8) \]
\[ \frac{3E^V(k, t)}{\partial t} + 2k^2 \frac{\partial}{R} E^V(k, t) \]

\[ = \frac{R}{2} \int dp dq \{ \frac{1 - \exp[-(k^2 + p^2 + q^2)t/R]}{k^2 + p^2 + q^2} \} \left[ k^2 E^V(p, t) - p^2 E^V(k, t) \right] \]

\[ \times E^V(q, t) \frac{1}{q} (xy + z^3) \]

\[ + \frac{1 - \exp[-(k^2 + \gamma p^2 + q^2)t/R]}{k^2 + \gamma p^2 + q^2} \left[ k^2 E^M(p, t) - p^2 E^V(k, t) \right] \]

\[ \times E^M(q, t) \frac{1}{q} z(1-y^2) \} , \] (3.9)

\[ \frac{3E^M(k, t)}{\partial t} + 2\gamma k^2 \frac{\partial}{R} E^M(k, t) \]

\[ = \frac{R}{2} \int dp dq \{ \frac{1 - \exp[-(\gamma k^2 + \gamma p^2 + q^2)t/R]}{\gamma k^2 + \gamma p^2 + q^2} \} \left[ k^2 E^M(p, t) - p^2 E^M(k, t) \right] \]

\[ \times E^V(q, t) \frac{k}{pq} (1-y^2) \]

\[ + \frac{1 - \exp[-(\gamma k^2 + p^2 + \gamma q^2)t/R]}{\gamma k^2 + p^2 + \gamma q^2} \left[ k^2 E^V(p, t) - p^2 E^M(k, t) \right] \]

\[ \times E^M(q, t) \frac{k^2}{p^2 q} z(1-y^2) \} , \] (3.10)
The kinetic and magnetic energies

\[ \mathcal{E}^V(t) = \frac{1}{2} \langle |u(x,t)|^2 \rangle = \int_0^\infty \mathcal{E}^V(k,t) dk , \]

\[ \mathcal{E}^M(t) = \frac{1}{2} \langle |b(x,t)|^2 \rangle = \int_0^\infty \mathcal{E}^M(k,t) dk . \]

The kinetic and magnetic enstrophies

\[ \Omega^V(t) = \langle (\partial u_1/\partial x_1)^2 \rangle = \int_0^\infty k^2 \mathcal{E}^V(k,t) dk , \]

\[ \Omega^M(t) = \langle (\partial b_1/\partial x_1)^2 \rangle = \int_0^\infty k^2 \mathcal{E}^M(k,t) dk . \]

Taylor's micro-scales of the kinetic and magnetic fields

\[ \lambda^V(t)^2 = \langle u_1^2 \rangle / \langle (\partial u_1/\partial x_1)^2 \rangle = 5\mathcal{E}^V(t)/\Omega^V(t) , \]

\[ \lambda^M(t)^2 = \langle b_1^2 \rangle / \langle (\partial b_1/\partial x_1)^2 \rangle = 5\mathcal{E}^M(t)/\Omega^M(t) . \]

The micro-scale Reynolds numbers

\[ R^V_\lambda(t) = \langle u_1^2 \rangle \lambda^V(t) R = \left[ \frac{2}{3} \mathcal{E}^V(t) \right]^{1/2} \lambda^V(t) R , \]

\[ R^M_\lambda(t) = \langle b_1^2 \rangle \lambda^M(t) R = \left[ \frac{2}{3} \mathcal{E}^M(t) \right]^{1/2} \lambda^M(t) R . \]

Initial conditions

\[ \mathcal{E}^V(k,0) = C^V \mathcal{F}(k) , \]

\[ \mathcal{E}^M(k,0) = C^M \mathcal{F}(k) , \]

where

\[ \mathcal{F}(k) = \beta_a k^a \exp(-\alpha_a k^2) , \]

\[ \alpha_a = \frac{a+1}{10} , \quad \beta_a = 3 \left( \frac{a+1}{10} \right)^2 / \Gamma \left( \frac{a+1}{2} \right) . \]
Table I. The values of the parameters for the present numerical integrations.

<table>
<thead>
<tr>
<th>CASE</th>
<th>R</th>
<th>$Pr$</th>
<th>$a$</th>
<th>$C^V$</th>
<th>$C^M$</th>
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<tbody>
<tr>
<td>I</td>
<td>800</td>
<td>1</td>
<td>2</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>II</td>
<td>800</td>
<td>1</td>
<td>2</td>
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<td>1.0</td>
</tr>
<tr>
<td>III</td>
<td>800</td>
<td>1</td>
<td>4</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>IV</td>
<td>800</td>
<td>1</td>
<td>4</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Fig. 2 The time evolutions of the kinetic and magnetic energy spectra for cases II and IV.

(a) $E^V(k,t)$ for case II.
(b) $E^M(k,t)$ for case II.
(c) $E^V(k,t)$ for case IV.
(d) $E^M(k,t)$ for case IV.
Fig. 3. The total energy spectra $E(k,t)$ for cases II and IV.

(a) $E(k,t)$ for case II.
(b) \( E(k,t) \) for case IV.
Fig. 4. The energy spectra $E^V(k, t)$ and $E^M(k, t)$ at $t=5$ for cases II and II.

(a) $E^V(k, 5)$ and $E^M(k, 5)$ for case I.
(b) $E^V(k, 5) \text{ and } E^M(k, 5) \text{ for case II.}
The decays of the kinetic, magnetic and total energies, i.e., $\mathcal{E}^{V}(t)$, $\mathcal{E}^{M}(t)$ and $\mathcal{E}(t)$ for cases II and III.

(a) $\mathcal{E}^{V}(t)$, $\mathcal{E}^{M}(t)$ and $\mathcal{E}(t)$ for case III.
(b) $\mathcal{E}^V(t)$, $\mathcal{E}^M(t)$ and $\mathcal{E}(t)$ for case IV.
Fig. 4.6 The time dependences of the enstrophy $\Omega^V(t)$ and $\Omega^M(t)$ for cases II and IV.

(a) $\Omega^V(t)$ and $\Omega^M(t)$ for case II.
(5) $\Omega_v(t)$ and $\Omega^M(t)$ for case IV.
Fig. 3.7. The time developments of the micro scale Reynolds number $R^V_\lambda(t)$ and $R^M_\lambda(t)$ for cases II and IV.

(a) $R^V_\lambda(t)$ and $R^M_\lambda(t)$ for case II.
(b) $R^V_\lambda(t)$ and $R^M_\lambda(t)$ for case IV.
Table II. The power indexes of the energy decay law, $\mathcal{E}^v(t) \propto t^{-b^v}$, $\mathcal{E}^m(t) \propto t^{-b^m}$ and $\mathcal{E}(t) \propto t^{-b}$.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
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<th>$b^m$</th>
<th>$b$</th>
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<td>2</td>
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<td>1.15</td>
<td>1.19</td>
</tr>
<tr>
<td>CASE II</td>
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<td>1.17</td>
<td>1.17</td>
</tr>
<tr>
<td>Eq. (2.16)</td>
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<td>1.43</td>
<td>1.43</td>
</tr>
<tr>
<td>CASE III</td>
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<td>1.38</td>
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<tr>
<td>CASE IV</td>
<td>4</td>
<td>1.36</td>
<td>1.34</td>
<td>1.36</td>
</tr>
</tbody>
</table>

$\langle V \times B \rangle \equiv \alpha B$
RELAXATION, TURBULENCE, AND DYNAMO ACTION

IN DRIVEN SYSTEMS

D. SCHNACK

SCIENCE APPLICATIONS, INC.
RELAXATION, TURBULENCE, AND DYNAMO ACTION IN DRIVEN SYSTEMS.

D. Schnack
SAI La Jolla
**Driven System**

A magnetoplasma system with external reservoirs of energy and flux.

**Examples**

Plasma confinement experiment with applied voltage.

Convective heating from within (stars, earth).

Coronal loop driven by differential rotation at feet.
Many Driven Systems Exhibit anomalously long lifetimes:

- Terrestrial and Solar Magnetic Fields
- Positive (non-reversed) toroidal flux in RFP
- Spheromak (d.c.)

How do such systems evolve?

- Laminar State
  - transport (destruction) → Unstable state
  - dynamo?
  - Relaxed, turbulent state → Relaxation onset of turb. dynamos?

- Is there such a separation between transport, instability, relaxation, and turbulence?
- Does the loop close?
- Properties of each step (Relaxation, instab., turb.)
- What is responsible for dynamo?
CODES FOR RELAXATION, DYNAMO, AND TURBULENCE STUDIES

SPECTR3 - fully compressible
INCOMPR - incompressible
NOT SO INC - (0,0) compressible; (m,n) incompressible

ALL HAVE:
• Resistive MHD
• "Primitive" equations
• 3-D cylindrical geometry
• Pseudo-spectral θ, z; dealiased
• Finite difference Cr
• Explicit, leap-frog advection (except NOT SO INC, where (0,0) is implicit)
• Implicit resistive diffusion.

NUMERICAL EXPERIMENTS.
• GUIDE TO ANALYTIC UNDERSTANDING.
**RELAXATION**

The tendency of non-ideal magnetoplasma systems to spontaneously minimize energy subject to constraints.

- Taylor hypothesis (Resistive plasma $\Rightarrow$ reconnection $\Rightarrow$ topology breaking)
  
  $W = \int \mathbf{B}^2 \, dv$, $K = \int \mathbf{A} \cdot \mathbf{B} \, dv$, $\frac{W}{K} \rightarrow \min$.

  - Is $K$ an invariant?
  - Is $K$ the proper invariant?
  - Is $K$ the only invariant?

- Transport processes increase $W/K$ (Caramana, et al. 1983)
  
  What mechanisms cause $W/K$ to decrease?
  - Instabilities?
  - Turbulence?

- Interplay between transport and relaxation.

- Effect of relaxation on confinement.
RELAXATION DUE TO MHD

INSTABILITIES (RFP)
(onset of turbulence)

- Plasma driven away from (W/k) min by transport.

- Resistive modes $\Rightarrow$ reconnection $\Rightarrow$ relaxation

- Many modes unstable
  - $m=1$, $|n|>>1$
  - resonant
  - non-resonant

- Single helicity evolution
  - 2 reconnection phases
  - $g(0)$ lowered, then raised

- 3-D evolution
  - $m=1$ saturation affected by other modes
  - mode coupling to $m=0,2$
  - reconnection $\Rightarrow$ stochasticity

- W/k minimized during growth and saturation of modes.
m=1, n=-11 single helicity

(1, -11) shows similar behavior.
$m=1, n=-11$ 3-D SIMULATION

*First reconnection*
*Growth retarded*
*Second reconnection begins*
$m = 1, n = -10$ 3-D Simulation

(1, -10)  $t = 0$

(1, -10)  $t = 140$

(1, -10)  $t = 60$

(1, -10)  $t = 180$

(1, -10)  $t = 100$

(1, -10)  $t = 200$

"first reconnection"

No second reconnection

"Bounce" at $t = 120$
(l, -10) resonant modes
(l, -11) non-resonant modes

Show rapid rise of $g(0)$:

First reconnection

Second reconnection

Slower rate of rise
Comparison of single-helicity and three-dimensional results

- Satellites at lower level in 3D than single helicity
- Energy drain to m=0, m=2

$N_r=65$, $N_\theta=8$, $N_z=64$
3-D SIMULATION - LOW C

Field line plots - surfaces of section

At t = 10, 20, 30, and 40, the flux surfaces remain consistent with the simulation parameters.
3-D SIMULATION - HIGH ☀

Field line plots

Confinement lost

"leak" toroidally localized

Rehealing of surfaces
3-D SIMULATION - LOW

$m=0$ magnetic islands.

Small $m=0$ cannot interact with $m=1$
3-D SIMULATION - HIGH large $m=0$ can interact with $m=1$ $m=0$ magnetic islands
Fig. 9. Energy in the radial component of the magnetic field as a function of time for the high-$B$ case.
(Magnetic Energy)/Helicity vs Time

\[ W_m = W_{m_1} + W_{m_0} + W_{m_2} \]
\[ \frac{\text{TOTAL ENERGY}}{\text{HELICITY}} \text{ VS. TIME} \]

\[ \frac{(WM+WK)}{K} \text{ VS TIME} \]

\[ WK = \int \rho v^2 d^3 r \]

\[ \frac{WM+WK}{K} \]

\[ t/\tau A \]
COMMENTS

1. W/K minimized during nonlinear growth of resistive MHD modes. \[\Rightarrow\] relaxation due to instabilities.

2. Process of relaxation from an unstable ("bad") state to a stable ("good") state causes destruction of flux surfaces, i.e., you don't get something for nothing.
**Dynamo Action**

The observed continuous conversion of externally supplied poloidal flux into toroidal flux so as to maintain a non-trivial "steady-state" in the presence of resistive diffusion.

Example: RFP

- Requires reconnection to "lock in" toroidal flux.
- Cannot be modeled with transport (Caramana's Bala)
- Relationship to relaxation, turbulence, and instability?
- Compressible or Incompressible?
- Role of boundaries, and boundary conditions.
- Continuous, or "quantized"?

For \( t > t_s \), only negative \( \Phi \) can be taken from, or given to, external circuit.
FLUX GENERATION DURING 3-D SIMULATION

• 3-D evolution of modes,
  \( H_0 = 1.77, \quad \epsilon = 0.2 \)
  \( N_x = 65, \quad N_y = 8, \quad N_z = 65 \)

• Flux generation during growth phase; diffusion after saturation.

• \( \langle E_0 \rangle \) vs. \( r \) during growth phase, and after saturation.

• Relaxation: \( \frac{\text{WL}}{K} \) decreases during growth phase, slow increase after saturation.
STRUCTURE OF "DYNAMO" ELECTRIC FIELD

- Flux change inside field reversal surface:

\[ \dot{\Phi}(r_v) \approx -r_v \int_0^{2\pi} E_\theta(r_v) d\theta \]

\[ = -r_v <E_\theta(r_v)> \]

where:

\[ <E_\theta(r_v)> = \eta <S_\theta> - \langle \nabla \times B \rangle_\theta \]

For

\[ \dot{\Phi}(r_v) \leq 0 \Rightarrow <E_\theta(r_v)> \leq 0 \]

Le., \( \delta E_\theta = \langle \nabla \times B \rangle_\theta \) cancels resistive diff.

Examine time evolution and radial structure of \( <E_\theta(r)> \) during nonlinear evolution of unstable RFP.
diffusion alone

\[ \phi_z(r, t) \]

\[ \langle E_\theta(r, t) \rangle \]

\[ \langle E_\theta(r, t) \rangle E_m \]

guasilinear contributions to \( \langle E_\theta(r, t) \rangle E_m \) from all modes with poloidal mode \( m \).

\( \langle E_\theta \rangle < 0 \implies \text{"dynamo"} \)

\( \langle E_\theta \rangle > 0 \implies \text{"anti-dynamo"} \)

Contribution from \( m = 0 \)

Contribution from \( m > 1 \)

Contribution from \( m = 1 \)

\( t \in [10, 20, 30, 40, 50, 60, 70, 80, 90] \)
$t = 14$

$\delta E_\theta = - \langle \mathbf{\nabla} \times \mathbf{B} \rangle_\theta$

$E_\theta = \eta \langle J_\theta \rangle + \delta E_\theta$
$E_{\theta} = \eta \langle \mathbf{j} \rangle + \delta E_{\theta}$

$\langle E_{\theta} \rangle$
$E_\theta = \langle \Phi | \vec{L} \times \vec{E} \rangle$

$E = \nabla \phi + \nabla \times \vec{E}$

$\dot{E}_\theta = \langle \Phi | \dot{\vec{L}} \times \vec{E} \rangle$

$\dot{E}_\theta = \langle \Phi | \vec{L} \times \dot{\vec{E}} \rangle$

$t = 2.0$

$E_\theta$

$E_\theta$

$E_\theta$

$\langle \Phi \rangle$

$\langle \Phi \rangle$

$\langle \Phi \rangle$
$E_\theta = \eta \langle J_\theta \rangle + SE_\theta$

$SE_\theta = -\langle v \times B \rangle_\theta$

$t = 25$

$E_\theta$

$\langle E_\theta \rangle$

$r/a$
$t = 32$

$S E_\theta = -\langle \mathbf{v} \times \mathbf{B} \rangle_\theta$

$E_\theta = \eta \langle J_\theta \rangle + S E_\theta$
$t = 37$

$SE_0 = -\langle \nu \times B \rangle_0$

$E_0 = \eta \langle J_0 \rangle + SE_0$
Role of compressibility in RFP Dynamo

3 cases examined:

Case 1: Incompressible: no sustainment (incomp) (diffusion)

Case 2: Not-so-incompressible, [(0,0) compressible, (m,n) incompressible]: sustainment, no reversal (notsoinc) (non-reversed Ohmic state)

Case 3: Fully compressible: sustainment with reversal (reversed Ohmic state) (spectra)

\( H_0 = 1.71 \quad \varepsilon = 1 \quad N_r = 65, N_o = 8, N_z = 16, \quad \text{no Ohmic heating} \)

All cases dominated by (1,-2) mode. \( S = 10^3 \)

Examine differences in \( \langle E \Theta(r) \rangle \) at \( t = 100 \) (after mode saturation) for each case.
(0,0) Compressible \( j \) (m, n) incompressible.

Compressional heating,
no loss.

\[ \beta_p = 0.04 \]
\[ \beta_p = 0.13 \]
\[ \beta_p = 0.28 \]
\[ \beta_p = 0.42 \]

Energy loss, \( \beta_p \times 0.01 \)

CASE 2
CASE 1 INCOMPRESSIBLE

E-THETA VS RADIUS M=0 N=0 TIME=1.00046E+02

\[ SE_0 = -\langle \mathbf{y} \times \mathbf{D} \rangle_0 \]

\[ E_0 = \eta \langle \mathbf{J}_0 \rangle + SE_0 \]
CASE 2 \{ (0,0) compressible \\ (m,n) incompressible \} not-so-incomp.

$E_0 = -\langle y \times B \rangle_0$

$E_0 = \eta \langle J_0 \rangle + SE_0$
CASE 3 FULLY COMPRESSIBLE

E-THETA VS. RADIUS M = 0 N = 0 TIME = 1.00004E+02

\( \langle SE_0 \rangle = -\langle v \times B \rangle_0 \\
\langle E_0 \rangle = \eta \langle J_0 \rangle + \langle SE_0 \rangle \)
**COMMENT:**

1. Compressible \( \langle E_0(r) \rangle \) differs only slightly from incompressible \( \langle E_0(r) \rangle \) (with \( (0,0) \) compressible). Perhaps it is just this particular case that gives this difference between compressible and incompressible models. (See, for example, the Ohmic state of Aydemir and Barnes.)

2. Flux is generated (for \( \omega_2 \)) during short "relaxation" process. If transport, relaxation and "turbulent" phases well separated, flux generation should appear "quantized" (seen experimentally at high \( D \), not seen at low \( D \)). When time-scales not well-separated, perhaps quasi-steady-state between transport relaxation results (turbulence?)
Turbulence

A statistical steady state resulting from the non-linear saturation of MHD instabilities.

- Present computers probably insufficient to study full turbulence.
  Onset of turbulence can be modelled (previous result).

- Spectra (Fourier decomposition)

- Cascades and inverse cascades (motion thru k-space)

- Power laws, inertial range

- Effects in sustainment, relaxation, dynamo, and transport.
**Spectral Evolution**

- Peak of $f(k)$:
  \[
  \langle k^2 \rangle = \frac{\int f(k) k^2 dk}{\int f(k) dk}
  \]

- "Spread" of $f(k)$
  \[
  \langle k - \langle k \rangle \rangle^2 = \frac{\int f(k) (k - \langle k \rangle)^2 dk}{\int f(k) dk}
  \]

Look at $\langle k^2 \rangle^{\frac{1}{2}}$ and $\langle k - \langle k \rangle \rangle^2$ vs time for $f(k) = \left\{ \begin{array}{l} \text{Helicity Spectrum} \\ \text{Energy Spectrum} \end{array} \right.$
NW PEAK K VS. TIME

\[
\langle \rho_{\alpha} \rangle_{\frac{1}{2}}
\]
HeliCity Spectrum

(0,0) removed
Conclusions

- Onset of turbulence can be computed.

- $W/K$ is minimized during non-linear growth and saturation of long-wavelength resistive MHD modes (relaxation).

- Relaxation process may cause degradation of confinement.

- Steady-state dynamo computed for $\varepsilon=1$; transient flux generation for $\varepsilon=2$. Long wavelength modes sufficient.

- Compressibility of $(0,0)$ essential for RFP studies; compressibility of $(m,n)$ may be important, but needs to be clarified.

- No evidence of inverse cascade for $K_j$; however, turbulent state not adequately computed.
THE DYNAMICS OF TEARING MODES

S. COWLEY

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The Dynamics of Tearing Modes with R.H. Kulsrud and T.S. Hahm.
We have addressed the regime applicable to T.F.T.R. and P.I.T. and shown stability when

\[ D'a < \frac{4 (Re)(\frac{a}{\rho})}{\beta p} \ln\left(\frac{\rho}{\delta}\right) \]

We conclude from experimental evidence that:

TEARING MODES ARE PRESENT WHEN THEY ARE LINEARLY STABLE

we propose the reason for this is:

NONLINEAR INSTABILITY
In a real discharge we expect the low \( m,n \) rational surfaces to be broken up into islands or possibly stochastic regions, the break-up can arise in many ways here are two:

- Imperfect coil alignment.
- Toroidal coupling to other modes.

In examining tearing modes one should consider an equilibrium with islands (or a stochastic field?) at the rational surface.
There are 3 types of tearing mode.

1. **Collisional**: Resistive mode of F.K.R.

   Width of the current channel is governed by $E_{\parallel} \to 0$ in the outer regions.

2. **Semi-Collisional**: Introduced by Drake & Lee.

   Width $\Delta$ of the current channel is where parallel diffusion of electrons limits conductivity.

   \[
   \Delta \ll \rho_i
   \]

   \[
   E_{\parallel} \to 0 \quad \text{for} \quad x \geq \rho_i.
   \]

   \[
   \mathcal{J}_{\parallel}(x) = \frac{\delta'(x)}{\rho_e} \frac{E_{\parallel}(x)}{\Delta}
   \]

   \[
   \Delta \sim \left(\frac{v_{ei}}{\omega_i} \right)^{1/2} \rho_e \frac{l_s}{l_n}
   \]
(3) **Collisionless**: Current limited by electron inertia, width $\delta$ where electrons 'see' D.C. $E_{\parallel}$ i.e. for large $k_{\parallel}$ electrons 'see' A.C. $E_{\parallel}$ and $Z_{\parallel}(x)$ is reduced.

$$\omega^{*} = k_{\parallel} v_{te}$$

$$\delta \sim \frac{\rho_{e} \rho_{s}}{\epsilon_{n}} \ll \rho_{i}$$

We consider regimes (2) & (3) since these have been incorrectly treated in the literature. Regime (2) covers most tearing modes in present day large tokamaks.
We make the following general assumptions:

(i) \( \omega^* \gg \nu_i \)

(ii) \( \omega^* \gg \kappa_{ii} V_{\text{thi}} \)

(iii) \( \frac{l_n^2}{\ell_s^2} \ll 1 \)

(iv) \( \frac{\rho_e \partial \rho_e}{\partial x} \ll 1 \)

(v) \( A_1 \ll A_{ii} \)

(vi) \( \delta, \Delta \ll \rho_i \)
\[ 0 = \frac{\pi}{2} \]

\[ B = B_0 \left( \frac{\rho}{\rho_0} \right) \]

\[ \frac{\pi}{2} x + \frac{\pi}{2} y \]

\[ \mathbf{E} \rightarrow 0 \]

\[ \mathbf{H} \rightarrow 0 \]
GENERAL EQUATIONS

AMPERES LAW

$$(\omega / \beta_p) \left( \frac{d^2 A_n}{dt^2} \right) = \frac{1}{t} \int dq \, \Phi(q) F(q) e^{i\omega t}$$

$$(\omega / \beta_p) \Delta^2 A_n = - (A_n + \phi(t)) \delta(t) - j_{\parallel}$$

QUASI-NEUTRALITY

QUASI-NEUTRALITY

$$(A_n - t \phi(t)) \delta(t) = \int dq \, \Phi(q) F(q) e^{i\omega t}$$

$$-ik_n J_n - \Phi \cdot \mathbf{J}_\perp$$

$\phi(t)$ and $\phi(q)$ are a Fouier transform pair - they are generalised functions.

$t$, $2\pi$ are different for the two modes.

$t$ is a scaled $x$. 
(3) \[ F(q) = (\frac{\omega^*}{\omega} + \frac{1}{T_i}) (\Pi_n - 1) - \frac{\omega^*}{\omega} \frac{q^2}{2} \frac{\Pi_n}{\epsilon^2} \] Ion, Boltzmann response

\[ \Pi_n = \exp - \left( \frac{q^2}{2\epsilon^2} \right) I_n \left( \frac{q^2}{2\epsilon^2} \right) \]

\[ \tau = \frac{\delta}{\rho_i} \text{ for collisionless mode.} \]

\[ \tau = \frac{\Delta}{\rho_i} \text{ for semi-collisional mode.} \]

Collisionless:

Electron conductivity comes from the Drift-Kinetic equation; \( t \) is defined as.

(4) \[ t = \frac{X}{\delta} \]

and \( \beta(4) \) is

(5) \[ \beta(4) = -\frac{1}{2} \left\{ (1 - \frac{\omega^*}{\omega}) \frac{1}{t^2} \frac{Z'}{(1+1)} + \frac{\omega^*}{\omega} \frac{\rho_p}{2} \frac{1}{(1+1)^3} \right\} \]
Solving Braginskii's Equations for the electrons. In the ordering:

\[(6) \quad \frac{k_{ii} v_e^2}{v_{ei}} \sim \omega^*\]

Note \(\omega^* \ll v_{ei}\) and \(k_{ii} v_e \ll v_{ei}\)

so that \(\omega^* \ll \overline{k_{ii}} v_{true} \ll v_{ei}\)

\[(7) \quad t = \sqrt{i} \frac{\Delta}{\lambda} = \sqrt{i} \frac{\mu_n}{\rho e} \frac{l_n}{l_s} \left(\frac{{\omega^*}}{v_{ei}}\right)^2\]

\[(8) \quad \alpha(t) = \frac{1 - \frac{{\omega^*}}{\omega} (1 + 1.71 \eta e) + 2.13 (1 - \frac{{\omega^*}}{\omega}) +^2}{1 + 5.08 t^2 + 2.13 t^4}\]
The boundary condition is as usual.

\[ \Delta' = \frac{1}{A_{xx}} \frac{dA_{xx}}{dx} \bigg|_{x=+\infty} - \frac{1}{A_{yy}} \frac{dA_{yy}}{dx} \bigg|_{y=-\infty} \]

and \( E_{xx} \to 0 \) as \( |x| \to \infty \)

Integrate (9) w.r.t. \( t \).

\[ \left( \frac{\omega}{\beta} \right)^2 \frac{1}{\beta} (\Delta') \int_{-\infty}^{\infty} A_{xx}(a) = \int_{-\infty}^{\infty} dt \left( A_{xx} + \Delta(t) \right) \Delta(t) \]

\( \beta \approx 0 \) very small

small since \( \beta \approx 1 \)

\( A_{xx} \) is nearly constant

Hence dominant current must cancel upon integration.

This makes \( w \approx 0 \)
MAGNETISED: versus UNMAGNETISED

Two limits to Eqn. (2)

\[-t \left( \frac{\Delta_n}{\delta} + \phi(t) \right) \phi(t) = \int dq \phi(q) F(q) e^{i\xi q} \]

Magnetised: \( \frac{\rho_i \xi}{\delta} \ll 1 \) in our case for \( t > \rho_i / \delta \)

\[ t \left( \frac{\Delta_n}{\delta} + \phi(t) \right) \phi(t) = \left[ \frac{T_e}{e} + \frac{\omega^*}{\omega_1} (1 + \xi^2) \right] \frac{\rho_i^2}{\delta^2} \frac{d^2 \phi}{dt^2} \]

Adiabatic or Unmagnetised
or Boltzmann ions

\[-t \left( \frac{\Delta_n}{\delta} + \phi(t) \right) = F(\infty) \phi(t) = -\left( \frac{\omega^*}{\omega_1} + \frac{T_e}{\omega_1} \right) \phi(t) \]

Boltzmann ions

Valid where \( \phi \gg \int dq e^{i\xi q} \phi(q) \Gamma_o (q/e) \)

or \( \phi \gg \frac{1}{2\rho_i} \frac{1}{\sqrt{2\pi}} \int dt' \phi(t') \exp \left( \frac{(t-t')^2}{\rho_i/\delta} \right) K_0 \left( \frac{(t-t')^2}{\rho_i/\delta} \right) \)
Non-local response

\[ \phi \gg \frac{\Sigma}{2\rho_0 \sqrt{2\pi}} \int_{-\infty}^{\infty} dt' \phi(t') \exp \left( \frac{\rho_i}{\rho_0} t \right) K_0 \left( \frac{(t-t')^2}{\rho_i \delta} \right) \]

Shows (12) is valid for \( t \approx 1 \ll \rho_i / \delta \)

to order \( \delta / \rho_0 \)
Expansion Technique

Treat $A_{\parallel}$ as known solve

Quasi-Neutrality Eqn. (2) for $\phi(q)$ then

substitute into (1) however (2) is an

integral Eqn. we solve in powers of $\delta / \rho$:

Call $\phi_u(t)$ the unmagnetised solution

the solution of Eqn.(13) i.e.

\begin{equation}
(13) \quad -t(A_{\parallel} + \phi_u(t)) \phi(t) = F(\infty) \phi_u(t)
\end{equation}

valid for $t \ll \rho_1 \delta$

Write (2) as:

\begin{equation}
(14) \quad (F(\infty) - \phi_u(\infty)) \phi_u(t) + \left( t^2 \phi(t) - \phi_u(\infty) \right) (\phi(t) - \phi_u(t))
\end{equation}

\[= \int dq \left( F(q) - \phi_u(\infty) \right) \phi(q) e^{iqt} \]
\[ \lim_{t \to \infty} t^2 \phi(t) = \phi(\infty) = (1 - \frac{\omega^*}{\omega})(\text{Boltzmann response}) \]

Define a function \( \chi(t) \) which equals \( \phi(t) \) for \( t \ll \frac{\rho}{15} \).

Rewrite 2. again.

\[ (F(\omega) - t^2 \phi(t)) \chi(t) = -t \phi(t) \Omega(\omega) \]

\[ + \int dq \left[ F(q) - F(\omega) \right] \phi(q) e^{iqt} \]

\[ = \int dq \left[ F(q) - F(\omega) \right] \phi(q) e^{iqt}. \]

\( A \) produces a small effect for \( t \ll (\frac{\rho}{15}) \); we have shown this is order \( \frac{1}{\rho} \).

\( B \) is finite for \( t \ll \frac{\rho}{15} \) only.
\( \phi(t) \) in \( \mathbb{B} \) can be approximated by \( X(t) \)

Dropping \( \mathbb{A} \) as small we obtain the \( \text{Orth Order:} \)

\[
(16) \quad X_0(t) = \phi u(t) \quad \text{as before.}
\]

\[
(17) \quad \phi_0(z) = \frac{F(z) - \delta(z)}{F(z) - \delta(\infty)} \phi u(z)
\]

We can iterate substituting \( \phi_0(z) \) into
\( \mathbb{A} \) evaluating \( X_1(t) \) substituting in \( (16) \) etc.
\( (F(\omega) - z^2 d(t)) \chi_n(t) = \int dq \left( F(\omega) - F(q) \right) \phi_{n-1}^{(i)} e^{i2^t} \)

\( \phi_n(q) = \frac{1}{F(q) - \delta(\omega)} \int \frac{d\tau}{2i} e^{-i2^t} \left\{ \left( z^2 d(t) - \omega \right) \left[ \chi_n^{(t)} + \phi_{n-1}^{(i)} - \chi_{n-1} \right] \right\} \)

Schematically we have

\( \phi_{n-1}^{(i)} \chi_{n-1} \) KNOWN

Non-local response propagates

Eqn(18) \( \phi_{n-1} \) from outside to inside to change \( \chi \) by \( \chi_n \)

\( \chi_n^{(t)}, \phi_{n-1}, \chi_{n-1} \) KNOWN

Taking \( \chi_n \)

and \( \phi_{n-1} - \chi_{n-1} \) as sources in the inside change \( \phi \) by \( \phi_n \) by non-local propagator.

\( \chi_n, \phi_n \) KNOWN
Treating $\beta_p < 1$ we expand $A_{11}$ in the first term as $A_{11} = \text{constant}$.

Integrating Eqn (1) Amperes law over $\Omega$ keeping $O(\delta_p)$ and $O(\beta_p)$ only we obtain:

\[
\left( \frac{\omega^*}{\omega} \right)^2 \frac{1}{\beta_p} \left( \Delta a \sqrt{\frac{\gamma}{a}} \right) \leq F(\infty) \int_0^t \frac{\Theta(t)}{t} + \left( \frac{\omega^*}{\omega} \right)^2 \beta_p \left[ \int_0^t \Theta(t) \, dt \right]^2
\]

\[
+ \int_{-\infty}^{\infty} dq \, \Theta(q) \left[ \frac{F(\infty) - F(q)}{F(q) - \Theta(\infty)} \right] \left[ \Theta(q) F(\infty) - \Theta(q) + \frac{T(q) F(\infty)}{F(q) - \Theta(\infty)} \right]
\]

\[
\Theta(t) = -\frac{-t \delta(t)}{F(\infty) - t^2 \delta(t)}
\]

\[
T(t) = -(t^2 \delta(t) - \Theta(\infty))^{-1}
\]
Results

It can be shown that the \( O(1) \) and \( O(\beta_p) \) terms give

\[
\text{Im} \, \omega = 0 \quad \text{for both modes}
\]

If \( (\delta') < \begin{array}{c}\frac{4}{(\beta_p)} \left( \frac{\alpha}{\rho_i} \right) \beta_p \ln(\rho_i) \end{array} \)

mode is stable!

Collisionless

\[
\gamma_{\text{damping}} = \frac{\omega^* \rho_e \delta}{\sqrt{c_s}} \ln(\rho_i / \delta)
\]

Semi-collisional

\[
\gamma_{\text{damping}} \leq \frac{3.71 \omega^* (1 + 0.4 \beta_e) \rho_e}{4 \pi V_z} \ln(\rho_i / \delta)
\]
Physics!

Parallel current driven by:

\[ F_e = \text{Force on electrons} = \nabla_{\parallel} P_e + n_e E_{\parallel} \]

Drop \( \nabla_{\perp} T_e \) for this discussion.

\( \nabla_{\parallel} P_e \) arises from parallel flows and tilting the field line. From parallel flows,

\[ n_e' = n_e \frac{K_n v_{\parallel}}{w} \]

Tilting field line

Field line displaced \( \delta_x \)

Electrons displaced \( \delta_x' \)
The electron displacement comes from the \( E \times B \) velocity

\[
\xi_x' = \frac{k e \phi}{\omega B_0}
\]

The displacement of the field line is

\[
\xi_x = \frac{\delta B_x}{i k_{11} B_0} = \frac{k A_{11}}{k_{11} B_0}
\]

The parallel pressure gradient is then

\[
\frac{\partial P_{11}}{\partial t} = +ik_{11} (\xi_x - \xi_x') \cdot \nabla n_0 + n_0 k_{11}^2 v_{\perp e} \frac{\omega}{\omega^*} v_{\parallel}
\]

\[
= -\frac{\omega^*}{\omega} e E_{11} n_0 + n_0 k_{11}^2 v_{\perp e}^2 \frac{\omega}{\omega^*} v_{\parallel}
\]

\[
\mathbf{F}_{11} = e n_0 \left(1 - \frac{\omega^*}{\omega} \right) E_{11} + n_0 k_{11}^2 v_{\perp e}^2 \frac{\omega}{\omega^*} v_{\parallel}
\]

Tilt term

Driving force

\( \mathbf{F}_{11} \) balanced by: Friction in semi-collisional case

inertia in collisionless case.
Simple Physical Picture of Stabilization

\[ I \rightarrow P/\delta \]

\[ t \]

\[ 1 + 2 = 0 \text{ dominant current canceling.} \]

Correction to current \( \Delta J_{\|} \) is approx

\( \Delta J_{\|} \approx \text{"Magnetised ion current response"} - \text{incorrect} \]

\( \text{"Unmagnetised ion current response"} \)

for \( t > \rho/\delta \) only
magnetised current from

\[ J_{\|} = \frac{D \cdot S_1}{ik_{\|}} \]

\( J_1 \) = polarisation current of ions.

\[
\int dt \Delta J_{\|} = \int_{\phi_1}^{\phi_2} \left\{ J_{\| \text{mag}} - S_{\| \text{unmag}} \right\}
\]

using \( J_{\|} \leq A_{\|} Z_{\|} \) for unmag.

\[
\frac{\delta}{a} \left( \Delta' a - \frac{\beta_0 a}{P_i} \right) = -\frac{4\pi}{c} \int_{-\infty}^{\infty} dt S_r(t) \text{ unmag.}
\]

stabilizing from \( S_{\|} \)

Magnetisation couples ions and field lines

field 'shakes' ions at \( \omega^* \) causing currents to flow that stabilize.
Nonlinear Aspects.

Equilibrium islands. $D\Gamma e$ and $D\phi = 0$ around island. All $w^*$ effects go away in fact when $D\Gamma e = 0$, $\phi = 0$ and ion stabilization goes in linear mode.

The evolution of the island from equilibrium is described well for widths of the island $w^* \gg \psi_0$ by resistive or k.d., Equilibrium island is given by $\psi_0$.

\[
\Delta'\psi \frac{(\psi + \psi_0)^{y_2}}{(\psi^2 + \psi_0^2)^{y_2}} = \frac{16\pi A}{\eta} \left( \frac{2B_0}{\varepsilon_5} \right)^{y_2} \frac{d\psi}{df}
\]

Exponential growth for $\psi \ll \psi_0$

for $\psi \gg \psi_0$

$\psi^{y_2} = \eta \left( \frac{2B_0}{\varepsilon_5} \right)^{y_2} \int_0^t \Delta'(t') dt'$
CONCLUSION

We have shown that tearing modes are stable when

\[ \Delta' a < 4 \left( \frac{\beta}{\pi} \right) \left( \frac{a}{\rho_i} \right) \beta_r \ln \left( \frac{\rho_i}{\delta} \right) \]

The question of stability however must be addressed in the non-linear regime since ripple \( \sim 1\% \) will make equilibrium islands of size greater than \( \rho_i \). Non-linear growth is expected.
COMPUTER MODELING OF

FAST COLLISIONLESS RECONNECTION

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Computer Modeling

of

Fast Collisionless Reconnection

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- F. Brunel
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Acknowledgment:
P.T. Bell
TALK OUTLINE

1 Brief Review of Other Simulation Work on Collisionless Tearing and/or Reconnection and/or Coalescence

2 Our Simulation Work
   - Model and Configuration
   - Reconnection Events
   - Energetics
   - Application to Solar Flares

3 Summary and Conclusions
OTHER WORK: BRIEF REVIEW

All use magnetostatic (Darwin) particle or kinetic collisionless codes up to $2 - \frac{1}{2} D$.

- Dickman, Morse, Nielson, Phys Fluids 13, 1708 (1969).
- Terrasawa, JGR 86, 9001 (1981).
of that anisotropy which was initially responsible for the instability. This large increase in axial thermal motion should have a large effect on end loss from $\delta$-pinches in times short compared with ion-ion collision times, and in fact is consistent with experimental findings on this point. Some plots like those of Fig. 3 together with a brief discussion of the code appear in a previous paper."

V. ASTRONLIKE CONFIGURATIONS

Figure 5 shows the initial profile for the case $\epsilon = -0.5$, $r = 3.0$, and $T = 1.0$ in which the diamagnetic current is sufficient to reverse $B_r$ and the plasma is well localized midway between the center axis and the flux conserver at $r_{\text{max}} = 6.0$. The same reflective conditions as above apply at

---

Fig. 5. Initial profiles of $n(r)$ and $B_r(r)$ for the reversed field equilibrium from which the tearing mode simulation of Fig. 6 was begun.
$\tau = 12 \omega_p^{-1}$
(e) $\Omega t = 44$

(f) $\Omega t = 52$

(g) $\Omega t = 60$

(h) $\Omega t = 68$

Fig. 3
\[ \frac{\tilde{\Gamma}}{\Omega} \]

\[ V^*^{3/2} \]

\[ V_{\infty}/V_{th} \]

\[ \gamma \propto b \]

\[ b(t) = \frac{b(t_0)}{1 - b(t_0) \Gamma (t - t_0)} \]

\[ \Gamma_{GCA} \propto \left( \frac{V_{\infty}}{V_{th}} \right)^{\frac{3\alpha}{2}} \]

**Fig. 6**
OUR WORK

- Particle Simulations of Collisionless Forced Tearing, Fast Reconnection and Coalescence


- Various MHD (Resistive, Collisional) Simulations of Some C-Mentioned for the sake of comparison.


P.F. 26, 832 (1983)
Simulation Model

- 2.5D Electromagnetic Finite Size Particle Code in Two Space (x, y)
- Three Velocities and Fields (x, y, and z) Dimensions

- Periodic Boundary Conditions

- Typical Parameters

\[ L_x \times L_y = 1200 \times 320 \quad (a = 2l_e) \]
\[ 2560 \times 160 \quad (a = 2l_e) \]
\[ M_i/m_e = 10 \]
\[ T_e/T_i = 2 \]

Speed of Light \[ C / V_{Te} = 5 \quad (a = 2l_e) \]
\[ C / V_{Te} = 6 \quad (a = 2l_e) \]
**Simulation Configuration**

\[ J_2^{ext} = J_0 \sin \left( \frac{t}{t_a} \right) \quad t < t_a = \frac{L_2}{V_e} \]
\[ = J_0 \quad t \geq t_a \]

128 x 32 case (Positions Fixed Only)  
256 x 16 case

\[ \frac{W_{ec}}{W_{pe}} = 0.77 \quad \frac{W_{cd}}{W_{pe}} = 0.077 \]
\[ S_e = 1.5 \lambda_c \quad S_c = 2.9 \lambda_c \]

\[ V_a = 1.2 V_{be} \quad \beta = 0.2 \]

We observed very low donor density changes

Ratio and Collisionless Skin Depth
**General Features of the Simulations**

"Thick" plasma sheet is formed

\[ \lambda (\lambda) \]

\[ \lambda \leq \frac{\lambda}{\lambda} = \frac{\lambda}{\lambda} \]

\[ k \lambda \geq 1 \quad \text{stable configuration} \]

\[ k \lambda \leq 1 \quad \text{threshold} \]

\[ k \lambda \leq 1 \quad \text{unstable w.r.t. tearing} \]

Most unstable at small k \lambda

k \lambda maximum growth mode

Periodic sheet structure or dimension

- k \lambda, k \lambda

\[ \text{i.e. number of islands in our case } \leq \left( \frac{k \lambda}{\lambda} \right) \]

Once the islands have formed, they are unable to coalesce

\[ \gamma \leq k \lambda \left( \frac{k \lambda}{\lambda} \right)^2 \]

Dislname & Glinski, 1978
PLASMA CURRENT DENSITY
$\beta = 0.2$

$\omega_{pe} t = 50$

$\omega_{pe} t = 125$

$\omega_{pe} t = 250$

$\omega_{pe} t = 375$

$x/\lambda_e$
PLASMA CURRENT DENSITY
\( \beta = 0.06 \)

\( \omega_{pe} t = 25 \)

\( \omega_{pe} t = 50 \)

\( \omega_{pe} t = 100 \)

\( \omega_{pe} t = 125 \)

\( \omega_{pe} t = 225 \)
\[ \psi_p(t) = \frac{1}{2} \left( \frac{u}{u_c} \right)^{1/2} \psi_0 \left( \frac{x}{L} \right) \left( \frac{u}{u_c} \right)^{1/2} + \frac{1}{2} \left( \frac{u}{u_c} \right)^{1/2} \psi_0 \left( \frac{x}{L} \right) \left( \frac{u}{u_c} \right)^{1/2} \]

- Convective process
- Disconnection induced by stretching \( \psi < t^{4.5} \)
- Convection \( \psi \propto t^2 \)
- Inconsequential process

\[ \psi(t) \propto \text{Squaring} \text{ function} \]

All the way
$B_t/B_p = 0.2$

$B_t/B_p = 4$

Pairera Flux versus Time
EVENLY THAT CAN BE
RELENTED THOUGH
ATTACHMENT OF THE
CURRENT ELEMENTS

\[ W_e = \frac{2 \pi^2}{c^2} \ln \left( \frac{L}{a} \right) \]

\[ E_{\text{magn}} \approx \frac{1}{8} W_e \]
in one to two Alfvén times. Significantly, there appear to be amplitude oscillations in the temperature, whose frequency matches \( \omega \approx k_1 v_{\perp} \), where \( k_1 = 2\pi/a \). This temperature oscillation behavior can be attributed to the overshooting of coalescing and colliding two-current blobs. Once two-current blobs coalesce, they are bound by the common magnetic flux, and the coalesced larger island vibrates, "freezing" more oblate and less in turn. Within the coalesced island, the two colliding plasma blobs cause turbulent flows, and the originally directed flow energy quickly dissipates into heat, thereby reducing the amplitude of the temperature oscillations (Fig. 2a).

As a result of this process, the momentum distribution of plasma ions in the poloidal direction shown in Figure 2b exhibits intense bulk heating (including adiabatic heating). The temperature in this direction was increased in our simulation by a factor of 60. The momentum distribution of ions in the toroidal direction (Fig. 2c) shows three regimes, the first being the bulk, the second being the exponential section \( f_2(p_z) = \exp(-p_z/p_0) \), and the third being flat distribution up to the relativistic factor \( \gamma \approx 2 \) in the relativistic region, where \( p_0^2/2M \approx 10^8 \times \) (bulk temperature). The heating in the poloidal direction is due to the intense process of the adiabatic heating and turbulent dissipation as a result of colliding plasma blobs. The heating in the toroidal direction is due to heating and acceleration by the inductive toroidal electric field induced by the annihilated poloidal magnetic flux as a result of coalescence.

**III. EXPLANATION OF OBSERVATION**

The flare loop slowly expands after it emerges from the photosphere as the toroidal field curvature of the loop makes the centrifugal motion. In time, the toroidal current \( J_t \) builds up, increasing the poloidal magnetic field \( B_p \). As the poloidal field \( B_p \) reaches the critical value that is of the order of magnitude \( B_t \), the adjacent flare current loop can now coalesce rapidly facilitated by the fast reconnection process governed by equation (1), the faster second phase. Such a fast coalescence of flare current loops proceeds explosively once in its nonlinear regime in a matter of one to two Alfvén times, releasing more than one-tenth of the poloidal magnetic energy into (ion) kinetic energy. Since the flare loop magnetic field (100 gauss) with current rod size \( 10^8 \) cm is \( W \approx 0.5 \times 10^{30} \) in \( (L/a) \sim 1.5 \times 10^{53} \) ergs \( \cdot \) \( \text{cm}^{-1} \) and the energy available in length \( d \sim L(\sim 10^8 \) cm) is \( E = 1.5 \times 10^{50} \) ergs for \( a = 10^8 \), \( d = L = 10^5 \) and \( E = 1.5 \times 10^{53} \) ergs for \( a = 10^8 \), \( d = L = 10^10 \). Released ion energy, therefore, is \( E_{\text{ion}} \approx -\frac{1}{2} E \) and is in between \( 2 \times 10^{38} \) ergs and \( 2 \times 10^{39} \) ergs due to the coalescence. This amount of energy is in the neighborhood of the solar flare energy (Sturrock 1980).
1980 June 7 Event

The impulsive burst at 0312 UT on 1980 June 7 is one of the most outstanding events which occurred during the SMM period. The burst is composed of seven successive pulses of a quasiperiodicity of ~8 s each of which was observed almost simultaneously in hard X-rays, γ-ray lines, and microwaves. From the comparison of the time profiles between hard X-ray and γ-ray emissions recorded with the SMM/GRS, Chupp have reported that the time of peak intensity of the prompt γ-ray line emission (4.1-6.4 MeV; due to de-excitation of excited ions like $^{12}$C and $^{16}$O) lags the corresponding time of the hard X-ray emission (40-140 keV and 305-355 keV) by ~2 s.

The burst was observed also with the 17-GHz polarimeter and interferometer at Nobeyama. It also shows seven successive pulses. However, each of the microwave pulses is composed of at least two.

Fig. 1. Time profiles of hard X-ray (80-140 keV), prompt γ-ray line (4.1-6.4 MeV) and microwave (17 GHz) for the 1980 June 7 event. Both the hard X-ray and prompt γ-ray line emissions were observed with the SMM Gamma-Ray Spectrometer with 1 and 2 sec resolution, respectively. The microwave emission was observed with the 17-GHz polarimeter at Nobeyama at the sampling rate of 0.2 sec (effective time constant of 0.3 sec).

Characteristics of condensation process

\[ B_f > B_p \text{ no condensation} \]
\[ B_f < B_p \text{ condensation} \]

Condensation happens within \(1-2\) Kepler times

Expect available

\[ W_c \approx \frac{2.5}{\epsilon_3} \ln \left( \frac{5}{\epsilon} \right) \]

\[ F_{\text{ion}} \approx (\frac{1}{10}) W_c \]

Sharp increase in temperature with condensation

Temperature oscillations with \( W \sim R^2 V^2 \)

Intensive nature of process:
Switch from \( B_f > B_p \) to
\( B_f < B_p \) as current builds up in the loops

For close loop \( B \)-field
\( a \approx 10^6 \) cm, \( b = 1.5 \times 10^{24} \) erg
\( a = 10^9 \), \( b = 10^{10} \) cm, \( W = 1.5 \times 10^{21} \) erg

Energy of process:
\( 2 \times 10^{38} - 2 \times 10^{39} \) erg

Temperature oscillations

Solar \( \gamma \)-ray

Amplitude oscillations
with period \( \approx 1 \) Alfvén

Time
EXPLOSIVE COALESCENCE

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Explosive Coalescence

T. Tajima
I.F.S.

Magnetic Collapse

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Acknowledgments

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J. Todoroki, M. Rosenthal, J. Dawson,
F. Coroniti
1. Introduction

2. Simulation Results and Indexes of Explosion

3. Dependence of Reconnection Rate on Current Peakedness

4. Self-similarity of Magnetic Collapse and Indexes of Explosion

5. Heuristic Theory of Explosive Coalescence

6. Theory Model and Solution by Stretch Factors

7. Explosive solutions

8. Oscillatory Structure (single, double, triple peaked amplitude osc.) and Sagdeev Potential

9. Comments on Sawtooth/Precursor and Relation to
phenomena

- suddenness of impulsive phase of flare
- amount of energy (up to $10^{31}$ erg) in microsec ($\sim 1$ Alfvén time), number of particles ($\sim 10^{34-36}$)
- 40-300 keV hard X-ray energy
- realignment of quadrupole structure
- post-flare phase (quiet soft X-rays)
- amplitude oscillations and double hump.
(or)

Coalescence

tokamak internal disruptions, etc.

Anti-Coalescence

Expulsion

Half-Coalescence

$F \times B_z$ (ideal MHD process)

Reconnection process
$\Omega_{et} = 0.2$ (weak rotation)
Hot tail distribution during explosive coalescence
### Indices of Explosion [exponents to the $1/(t_o-t)$] During Coalescence

<table>
<thead>
<tr>
<th></th>
<th>$\eta_{et}=0$</th>
<th>$\eta_{et}=0.2\omega_{pe}$</th>
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<th>$\eta_{et}=2.0$</th>
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<td>$L_x \times L_y=128 \times 32$</td>
</tr>
<tr>
<td></td>
<td>(NB: $L_x \times L_y=256 \times 32$ many islands)</td>
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<td></td>
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<tr>
<td>Simulations</td>
<td>Leboeuf et al.</td>
<td>Tajima &amp; Sakai</td>
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<td>$B^2$</td>
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<td>$8/3$</td>
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<td>$4$</td>
<td>$4$</td>
<td>$4$</td>
</tr>
<tr>
<td>Ion Temperature</td>
<td>in $x$-direction</td>
<td>$6/3$</td>
<td>$8/3$</td>
<td>$8/3$</td>
</tr>
<tr>
<td></td>
<td>$T_{ix}$</td>
<td>$6/3$</td>
<td>$3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Exponential</td>
<td>$24.30^{-1}$</td>
<td>$270^{-1}$</td>
<td>$190^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Compressional Alfven</td>
<td>$6.30^{-1}$</td>
<td>$6.00^{-1}$</td>
<td>$8.80^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Oscillation Period $\tau_{os}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Incompressibility is assumed. Derivation from observation might be due to plasma rotation in $\eta_{et}=1$ case.

S: simulations  T: theory
$E = 0.85$
$\epsilon = 0.7$
study of magnetic reconnection driven by instabilities. The third stage appears when the instability begins to saturate, and the rate of destruction of flux levels off. The demarcation between these three stages is not always sharp, and is sensitively dependent on the Reynolds number.

In Fig. 1, we show a typical sequence of frames following the time evolution of the poloidal flux function $\Psi$, and in Fig. 2, a typical flow field in the $x$-$y$ plane.

We now discuss our numerical results on the dynamics of the coalescence instability evolving from equilibria for which $\epsilon = 0.3$ (also studied in Refs. 3 and 8 for incompressible plasmas) and $\epsilon = 0.7$. In what follows, when we refer to the scaling with time of the rate of reconnection, it should be interpreted as the rate of destruction of magnetic flux in the "second stage" of the evolution of the instability.

A. $\epsilon = 0.3$

In Fig. 3 we compare the rates of reconnection as we vary the magnetic Reynolds number for a given number of the subclass of equilibria given by Eq. (9) with $B_{in} = 3.5$, $B_{ib} = 0$, and $\epsilon = 0.3$. For all values of the Reynolds number ($s = 5 \times 10^2 - 2 \times 10^3$), the destruction of flux proceeds linearly with time. The reconnection process does exhibit self-similar characteristics as the Reynolds number varies, in qualitative agreement with the results of Biskamp and Welter. As mentioned earlier, the separation in time of the three stages of evolution of the coalescence process is less sharp for larger than for smaller values of the resistivity. The explanation for this observation is that for larger values of resistivity, the destruction of flux is effective even in the first stage of the instability and blurs the separation between the first and the second stages.

We have computed the destruction of flux for a member of the second subclass of equilibria, with zero toroidal field, given by Eqs. (9) with $B_{in} = 3.5$ and $B_{ib} = 0$. Again, as shown in Fig. 4, the destruction of flux proceeds approximately linearly in time. It is seen thus, that for $\epsilon = 0.3$, the rate of reconnection is insensitive to the magnitude of the toroidal field, and the effect of plasma compressibility is negligible. We remark parenthetically that our numerical results for this case with zero toroidal fields are qualitatively different.

FIG. 1. Typical sequence of frames following the time evolution of the poloidal flux function $\Psi$. Here (a) displays the initial, and (d) the final configuration.

FIG. 2. Plasma flow field in the $x$-$y$ plane at some instant of time during the evolution of the coalescence instability.

FIG. 3. Poloidal-flux-destroyed ($\Delta \Psi$) as a function of time ($t$) for different values of the Reynolds number. The initial configuration is force-free, with $\epsilon = 0.3$, $B_{in} = 3.5$, and $B_{ib} = 0$. 

$\epsilon = 0.3$
Reconnection Experiment.

ite plasma density goes up to $\rho_0$, the characteristic density of the internal region. At some distance along the $z$ axis greater than $L$ this flux tube is located outside the diffuse internal region. There, the magnetic field is $B$ and the density is $\rho_e$. The angle $\alpha$ is $\alpha = B_\perp/B$, where $B_\perp$ is the average radial magnetic field in the diffusion layer. This field is given by $B_\perp = \psi/2\pi RL$, where $\psi$ is the total reconnected flux and $L$ is the length along $z$ traveled by the $\psi$ flux lines. Setting $L = L_0 + vt$ we obtain (Brunel et al. 1982)

$$L = vt\psi_c/(\psi - \psi_c),$$

where $\psi_c = 2\pi RL$. Brunel et al. assume that the fast phase of reconnection sets in when the diffusive length is shortened by the angularity of the lines such that axial convection on perpendicular lines matches radial convection into the diffusive layer. Equating (4) and (3) gives for the reconnected flux:

$$\psi = \psi_0 (t/t_0)^{\rho_1/\rho_e} + \psi_c,$$

where $\psi(t_0) = \psi_0$.

Qualitative agreement with this model can be seen in Fig. 6 which shows an abrupt change in slope of reversed trapped flux for each of the two cases. The reconnected flux $\psi_1 - \psi_{1r}$ is plotted versus $t - t_0$ for 7 mTorr in Fig. 7. Here $22.5 \pm 1.5$ kHz corresponds to the average rate of the reconnected flux during the plateau phase $\psi_{1r}$ is the trapped flux (from the axis to the circle). Density measurements were not possible in the region between SRC cells because of influence by the mirror coils. Thus the ratio $\rho_e/\rho_0$ in Eq. (5) could not be determined experimentally. However, the value of the exponent $t = 4 \pm 1$, indicating $\rho_e$. This would be expected since $\rho_e$ represents plasma on open field lines and $\rho_1$ that originally confined region. The most fast feature of the model is that it predicts a second, fast phase of reconnection region of the $x$-circle, a result which is obtained experimentally.

Merging velocity $w$ of the field lines was experimentally from data similar to those Fig. 2 and 3 which give the motion of the $x$-circle radius $r_x$ during the fast phase. The data directly obtained from the motion of the $x$-circle at $t = 6$ usec is $r_x = 2.7 \times 10^5$ cm/sec. $w$ can be compared for consistency with that for $w$ using Eq. (1). At $t = 6$ usec $w = 4.5$ kHz/usec is obtained from $w = 2.2 \times 10^5$ cm/sec in agreement with $r_x$. (The Alven velocity calculated using $v = 0.7 \times 10^5$ cm/sec is $v = 7 \times 10^5$ cm/sec). We note that the magnetic field (Bav$_A$/c$^2$) for the $p_0 = 7$ mTorr data are 150 for the slow phase and 50 for the fast phase, using $a = 1$ cm and the $n$ values from Section 5.

4.2 Change of Resistivity by Radial Convection of Current Carriers

Both in Fig. 2 and Fig. 3 the breaks in $\psi$ (at 5.5 usec and 3.2 usec) occur approximately when the separatrix approaches the $x$-circle where the onset of the sudden increase in $\psi$ is measured. As a qualitative second interpretation it is reasonable to assume that a corresponding lower density of current carriers will be convected to the $x$-circle, leading to increased, anomalous resistivity in the diffusive layer and therefore an increased $\psi$.

5. Plasma Resistivity

The values for the resistivity $\eta$ in the region of the $x$-circle before and during the second phase of the reconnection were calculated using the equation

$$\psi = 2\pi RL.$$
**Coalescence and Current Peakedness ($E'$)**

<table>
<thead>
<tr>
<th>$E$</th>
<th>0</th>
<th>0.3</th>
<th>0.7</th>
<th>0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>Sheet-pinch</td>
<td>Coalescence</td>
<td>Fast Coalescence</td>
<td>Explosive Coalescence</td>
</tr>
<tr>
<td></td>
<td>tearing inst</td>
<td>Sweat-Parker Process</td>
<td>Brunel-Tajima Process</td>
<td>Tajima-Sakai process</td>
</tr>
<tr>
<td>recon. flux</td>
<td>$e^{\nu t}$</td>
<td>$\eta^\frac{1}{2} t$</td>
<td>$\eta^\frac{1}{2} t^\alpha$</td>
<td>$\eta^0$</td>
</tr>
<tr>
<td>$\Delta \psi$</td>
<td>$\delta &lt; \eta^* $</td>
<td>$\eta^\frac{1}{2} t$</td>
<td>$(\alpha \approx 1)$</td>
<td>$(t_0 - t)^{\frac{2}{3}}$</td>
</tr>
</tbody>
</table>

$E$: Faddeev Equilibrium

\[ \psi_0 = \frac{B_y}{k} \ln (\cosh k_x + E \cos k_y) \]

$(0 = E < 1)$
### Temporal Rates of Driven Reconnection in Nonlinear Stages

<table>
<thead>
<tr>
<th>Process</th>
<th>Reconnected Flux vs. Time</th>
<th>NB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweet-Parker Process</td>
<td>$\Delta \psi = \eta^{1/2} B_y (v_A/L)^{1/2} t$</td>
<td></td>
</tr>
<tr>
<td>*Brunel-Tajima Process</td>
<td>$\Delta \psi = \psi_0 (\frac{t}{t_A})^{2/3}$</td>
<td>$\psi_0 = \eta^{1/2} B_y (v_A/L) t_A$</td>
</tr>
<tr>
<td>*Explosive Process</td>
<td>$\Delta \psi = \frac{\eta^0}{(t_0-t)^{4/3}}$</td>
<td>$t_\theta = \frac{\sqrt{2}}{3} a_0^{3/2} t_A$</td>
</tr>
<tr>
<td>Petschek Process</td>
<td>$\Delta \psi = \eta^0 t$</td>
<td>$\eta^0$ independent of $\eta$</td>
</tr>
</tbody>
</table>

NB: The Rutherford process is essentially a nonlinear stage of non-driven spontaneous tearing instability.
Self-similarity of Magnetic Collapse

\[ \Delta \sim (t_0 - t)^\alpha \quad \text{etc.} \]

\underline{temporal universality}

cf. Kadanoff's spin-block model for phase transition

\[ g(r) \sim (r - r_0)^\alpha \]

\underline{Kolmogorov's turbulence spectrum}

\[ E(k) \sim k^{-\alpha} \]

\underline{spatial universality}

cf. Lifshitz' explosion of universe

\[ a(t) \sim t^\alpha \]

\( \alpha = 1 \) or \( \frac{2}{3} \).

\underline{etc. etc.}
### Indexes of Explosion and Universality

(Phase Transition etc.)

<table>
<thead>
<tr>
<th></th>
<th>Magnetic Collapse of Coalescence Inst.</th>
<th>Phase Transition</th>
<th>Turbulence</th>
</tr>
</thead>
<tbody>
<tr>
<td>toward critical point</td>
<td>$E^{-1} - E_c^{-1}$</td>
<td>$T - T_c$</td>
<td>$R_0^{-1}$</td>
</tr>
<tr>
<td>scale</td>
<td>$t_0 - t \to 0$</td>
<td>$r^{-1} \to 0$</td>
<td>$k^{-1} \to 0$</td>
</tr>
<tr>
<td>order parameter</td>
<td>?</td>
<td>density fluctuation $\delta n(r)$</td>
<td>vorticity $\Omega(k)$</td>
</tr>
<tr>
<td>asymptotic</td>
<td>$\Delta \phi \sim (t_0 - t)^{-\frac{4}{9}}$</td>
<td>correlation fnc. $g(r) \sim r^{1+\delta} f(\frac{r}{\xi})$</td>
<td>energy spectrum $E(k) \sim k^{-\frac{5}{3}} f(k_{ed})$</td>
</tr>
<tr>
<td>Universal self-similarity</td>
<td>$E_x^2 \sim (t_0 - t)^{-\frac{8}{3}}$</td>
<td>$\int_0^\infty r^2 g(r) dr \sim \Delta T^{-\delta}$</td>
<td>$\int_0^\infty \Omega(k) dk &lt; R_0^{-1}$</td>
</tr>
<tr>
<td>Indexes of explosion (phase transition etc.)</td>
<td>$E_x \sim (t_0 - t)^{-\frac{8}{3}}$</td>
<td>$\nu_{vis} = \nu_0 \left(\frac{\Delta T}{T_c}\right)^{-\nu}$</td>
<td>$k d = k_0 R_0^{\nu}$</td>
</tr>
<tr>
<td></td>
<td>$a \sim (t_0 - t)$</td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td></td>
<td>$L \sim (t_0 - t)^2$</td>
<td></td>
<td>etc.</td>
</tr>
</tbody>
</table>
Heuristic Equations

Brunel-Tajima (extended) Eqs.

\[ \frac{\partial^4}{\partial t^4} = v B_y \]

continuity

\[ \rho_i u a = \rho_e v L \]

dynamic eq. (non explosive cases)

\[ P_i + \frac{B_i v^2}{\rho_i} = P_e + \frac{B_e v^2}{\rho_e} + \frac{B_e v^2}{\rho_e} \]

(inside)

\[ \frac{\partial^4}{\partial t^4} = \eta D^4 \]

(explosive case)

\[ \frac{\partial v_x}{\partial t} = -\frac{1}{\rho_0 \alpha x} v_x^2 + \frac{1}{\delta \pi \rho} \frac{\partial^2}{\partial x^2} B_y^2 \]
- non-explosive case (Drumel-Tajima reconnection rate)

\[ \Delta \gamma = \psi_5 / s_e \left( \frac{t}{t_0} \right) \]

reconnection angle

\[ \alpha = \frac{a}{L^*} = \frac{4}{uB_y e^t} \propto t^{d-1} \]

(Sweet-Parker \rightarrow Petschek)

- explosive case

Let \( v_k = \frac{V(x)}{t_0 - t} \)

\( B_y = \frac{B(x)}{t_0 - t} \)

\( 4 = \frac{\psi(x)}{t_0 - t} \)

\( q = \frac{\Phi(x)}{t_0 - t}. \)

\( \therefore \) Quadratic nonlinearity.
\[
\begin{align*}
\psi &= \psi' \sqrt{V} \\
2V &= -\left[ (\frac{\psi'}{\psi})^2 - [V^2] \right]
\end{align*}
\]

\[
\rightarrow \quad \left[ \psi^2 - \psi^2_0 \right]^{1/2} = \frac{d\psi}{dx}
\]

\[
\psi(x) = \frac{1}{4} (x-x_0)^2
\]

\[
\rightarrow \quad B^2 \propto (t_0-t)^2 \\
E^2_x \propto (t_0-t)^4 \\
E_2 \propto (t_0-t)^2 \\
J_2 \propto (t_0-t)^{-1}
\]

\[
L = (V_{Ai}/V_x) a \\
V_{Ai} \sim \text{const.}
\]

\[
a \sim t_0 - t \\
\therefore L \sim (t_0 - t)^2
\]
Reconnection angle \( \alpha = \frac{q}{L} \left( \frac{t_0 - t}{t_0 - t} \right) \) (small pert.)

(Sweet-Parker \( \rightarrow \) Petscheck)
Theoretical Model

\[ \frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0 \]

\[ m n_j \frac{\partial \mathbf{v}_j}{\partial t} = n_j \mathbf{v}_j (\mathbf{E} + \frac{\mathbf{V}_j \times \mathbf{B}}{c}) - \nabla p \]

\[ \nabla \times \mathbf{B} = \mu_0 \sum n_j \mathbf{v}_j \]

\[ \nabla \cdot \mathbf{E} = 4\pi \sum n_j \mathbf{v}_j \]

\[ \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \]

\[ \frac{\partial P_j}{\partial t} + \mathbf{v}_j \cdot \nabla P_j + \Gamma P_j \nabla \cdot \mathbf{v}_j = 0 \]

---

Assumptions

\[ \frac{\partial}{\partial x} \gg \frac{\partial}{\partial y}, \frac{\partial}{\partial z}. \]

Self-similarity in \( x \) solution expanded near \( x = 0 \).
Solution by Stretch factors (scale)

Introduce scale factors $a$ and $b$

\[ \nu_x = \frac{a}{a} x \]
\[ \nu_i x = \frac{b}{b} x \]

Consistency:
\[ n_e = \frac{n_0}{a} \]
\[ n_i = \frac{n_0}{b} \]

\[ \{ \text{continuity eq.} \} \]

Substitute these into dynamic eqs of e- and ions and Maxwell's eqs.

We get:

\[ \ddot{a} = -\omega_p^2 \left( \frac{a}{b} - 1 \right) - \frac{B_0^2 b}{4\pi n_0 \lambda^2} a^3 \left( \frac{b + m}{a} \right) + \frac{2P_{oe}}{mn_0 \lambda^2} \]

\[ \ddot{b} = \omega_i^2 \left( 1 - \frac{b}{a} \right) - \frac{m}{M} \frac{B_0^2 b^2}{4\pi n_0 \lambda^2} a^4 \left( \frac{b + m}{a} \right) + \frac{2P_{oi}}{mn_0 \lambda^2} \]

($\lambda$: $B$-field scale)
where the Sagdeev potential

\[ V(a) = -\frac{V_A^2}{\lambda^2 a} + \frac{C_s^2}{2\lambda^2 a^2} \]
First Integral

\[ a^2 = \frac{2VA^2}{\lambda^2 a} - \frac{c_s^2}{\lambda^2 a^2} + \varepsilon, \]

where

\[ \varepsilon : \text{"energy" in Sagdeev potential.} \]

\[ \varepsilon \sim 0 \implies \text{Explosive sol.} \]

\[ \varepsilon < 0 \implies \text{oscillatory sol.} \]

\[ T_{os} = \varepsilon \int_{a_t}^{a_i} \frac{a^4}{\sqrt{\varepsilon \left( a + \frac{VA^2}{\lambda^2 \varepsilon} \right)^2 - \frac{VA^2}{\varepsilon \lambda^2} - \frac{c_s^2}{\lambda^2}}} \, \mathrm{d}a \]

\[ = \frac{2\pi}{\lambda^2} \beta \frac{1}{\varepsilon^{3/2}} \]

amplitude oscillations before reconnection
"Sagdeev" potential $V(a)$

Diagram:
- $V(a)$ axis
- $a_0$, $a_{11}$, $a_{12}$, and $2a_1$ markers
- $V_{\text{min}}$ point
- $\epsilon/2$ marker
- "oscillations" and "explosive" labels
Results for explosive phase:

\[ v_x = -\frac{2}{3} \frac{x}{t_0 - t} \]

\[ n = \left( \frac{2}{3} \frac{\lambda}{\eta} \right)^{\frac{2}{3}} \frac{\lambda^{\frac{1}{3}} n_0}{V_A^{\frac{2}{3}} (t_0 - t)^{\frac{1}{3}}} \]

\[ E_x = -\frac{2}{9} \frac{M}{e} \frac{x}{(t_0 - t)^2} \]

\[ B_y = \left( \frac{2}{3} \frac{\lambda}{\eta} \right)^{\frac{2}{3}} \frac{B_0 \lambda^{\frac{1}{3}} x}{V_A^{\frac{2}{3}} (t_0 - t)^{\frac{1}{3}}} \]

\[ E_z = E_z^{(1)} \frac{1}{(t_0 - t)^{\frac{2}{3}}} + E_z^{(2)} \frac{1}{(t_0 - t)^{\frac{1}{3}}} \]

\[ (Self\text{-}similar\ phase) \]
"Sagdeev" potential, $V(a)$

- pressure effect
- charge separation

$V(a)$

$0 \quad a_{\text{min}}$

$J \times B$
The diagram illustrates the variation of $B_y^2$ with respect to time $t$. It shows two cycles with peaks at $a = a_{t1}$ and $a_{t2}$, and a trough at $a_{t1}$. The interval between two cycles is marked as $T$. The graph also includes a time axis from 0 to $a_{t1}$.