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QUASI-LINEAR MOMENTUM & ENERGY TRANSPORT

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Abstract

The equations of quasilinear momentum and energy transport are studied to show that a suprathermal level of low frequency fluctuations can cause highly enhanced electron energy transport without effectively changing the plasma resistivity.

I. INTRODUCTION

Based on the toroidal quasilinear theory of Kaufman[1], a general formalism[2] was recently developed to compute the radial transport caused by supra-thermal, low-frequency electromagnetic fluctuations. In Ref. 2, the appropriate equation for particle transport was derived and discussed. In this paper, we study the equations governing energy, especially, momentum transport. In particular, we find expressions for a number of transport coefficients (including the viscosity η, the electrical conductivity σ, the heat conductivity K, etc.). A complete evaluation of these anomalous coefficients clearly requires a detailed knowledge of the fluctuation spectrum. However, even without such knowledge, significant information can be gained by examining ratios of various coefficients. For example, we show that the electron thermal conductivity K is anomalous[3] although the electrical conductivity σ is essentially classical[4]. Since the anomalous value of K is generally attributed to low-frequency microinstabilities[5-7], we calculate the level of the fluctuations which could cause the experimentally observed value of K. We then calculate the anomalous electrical conductivity σ_a resulting from this level of fluctuation and find that the ratio of $(\sigma_a)^{-1}$ to the classical resistivity $(\sigma_c)^{-1}$ is negligibly small.

A study of the momentum transfer equation is especially important to the problem of "current drive" in realistic tokamak geometry. The presence of inhomogeneities (gradients in density and temperature, and magnetic shear) can cause effective local radial transport of the parallel momentum which might ultimately result in steepening the current profile. This, in turn, could feed tearing instabilities with deleterious effects on plasma confinement. This will be discussed in a later publication.

After a brief review of the formalism, we derive the momentum and energy balance equation in Sec. II. Section III is devoted to a study of these equations, and Sec. IV contains a discussion of the results.

II. NOTATION AND DERIVATION.

We begin this section with a brief review of the notation and formalism of Ref. 2 which should be referred to for details.

In the toroidal quasilinear theory, the lowest order distribution function f evolves according to

$$\frac{\partial f(\underset{\sim}{J},t)}{\partial t} = \frac{\partial}{\partial \underset{\sim}{J}} \cdot \underset{\approx}{D} \cdot \frac{\partial}{\partial \underset{\sim}{J}} f \tag{1}$$

where $\underset{\approx}{D}$ is the quasilinear diffusion operator[2], and the actions $\underset{\sim}{J}$ are adiabatic invariants, which along with the associated angles[2] $\underset{\sim}{\theta} = (\theta_g, \phi, \theta)$, form the most convenient phase space variables. The three actions are:

$$M = \frac{m}{2}\frac{v_\perp^2}{\Omega} = \frac{m^2 c}{e}\,\mu$$

$$p = m\dot{\zeta}R^2 - \frac{e}{c}\psi(x) \tag{2}$$

$$J = \frac{e}{c}\oint \frac{d\beta}{2\pi}\,\alpha(\beta, H_0, p, M) \quad .$$

Thus M is proportional to conventional magnetic moment $\mu = (v_\perp^2/2B)$, p is the canonical angular momentum, and J measures the toroidal flux linked with the particle drift orbit. The actions are defined in terms of the flux co-ordinates (α, β, ζ) where α measures the toroidal magnetic flux, and β and ζ are respectively the poloidal and toroidal angles. Note that $\underset{\sim}{B} = \underset{\sim}{B}_t + \underset{\sim}{B}_p = \underset{\sim}{\nabla}\alpha \times \underset{\sim}{\nabla}\beta + \nabla\zeta \times \nabla\psi(\alpha)$, $\psi(\alpha)$ being the poloidal flux. From Eq. 2, it can be seen that for untrapped particles $J \simeq (e/c)\alpha$ and thus can serve as a radial co-ordinate. For trapped particles, however, J is a measure of the banana width. In Eq. (2) H_0 refers to the unperturbed Hamiltonian

$$H_0 = \frac{mv^2}{2} + e\Phi_0 \tag{3}$$

where Φ_0 is the equilibrium electrostatic potential. The Hamiltonian H_0 can be used to define the set of equilibrium gyro, toroidal, and poloidal frequencies

$$\underset{\sim}{\Omega}(\underset{\sim}{J}) \equiv \dot{\underset{\sim}{\theta}} = \frac{\partial H_0}{\partial \underset{\sim}{J}} . \tag{4}$$

Notice that $\underset{\sim}{\omega}(\underset{\sim}{J})$ like all other equilibrium quantities are independent of $\underset{\sim}{\theta}$ which are cyclic co-ordinates in linear theory.

To derive the equations of momentum and energy transfer, we start with Eq. 77 of Ref. 2:

$$\frac{\partial}{\partial t}\left\langle \overline{\chi} \right\rangle - \left\langle \overline{\frac{\partial \chi}{\partial t}} \right\rangle + \frac{1}{V'} \frac{\partial}{\partial \overline{\alpha}} V' \left\langle \overline{\chi \frac{\partial \chi}{\partial t}} \right\rangle = \frac{1}{m^3 V'} \int d^3J \; d^3\theta \; \delta\,(\alpha - \overline{\alpha})\, \chi \frac{\partial f}{\partial t} \quad F(\chi) \tag{5}$$

where χ denotes either parallel momentum $mv_{\parallel}$ or energy $mv^2/2$. The symbol $\langle\ \rangle$ stands for the flux surface average while the overhead bar implies an average with respect to the distribution function, i.e.,

$$\overline{\chi} = (V')^{-1} \oint d\beta d\zeta \; g^{\frac{1}{2}} \overline{\chi}$$

$$= (V')^{-1} \oint d\beta d\zeta \; g^{\frac{1}{2}} \int\!\!\int d^3v \; \chi f .$$

In Eqs. (5)-(6) $\overline{\alpha}$ labels an arbitrarily chosen flux surface (or equivalently a radial position), $V' = dV/d\overline{\alpha}$ with

$$V = \int_0^{\bar{\alpha}} d\alpha \int d\beta \, d\zeta \, g^{\frac{1}{2}},$$

where $g^{\frac{1}{2}}$, defined by $d^3x = g^{\frac{1}{2}} d\alpha d\beta d\zeta$, is the Jacobian of transformation from Cartesian co-ordinates to (α,β,ζ). To derive the right hand side of Eq.(5), we have used the fact that a canonical transformation connects $(\underset{\sim}{x}, m\underset{\sim}{v})$ to $(\underset{\sim}{J}, \underset{\sim}{\theta})$ implying $d^3J \, d^3\theta = m^3 \, d^3x \, d^3v$.

Substituting Eq. (1) into (5) and remembering that

$$\underset{\sim}{D} = \sum_{\underset{\sim}{\ell}} \underset{\sim}{\ell}\underset{\sim}{\ell} D_0(\underset{\sim}{J}) \tag{6}$$

where $\underset{\sim}{\ell} = (\ell_g, \ell_\phi, \ell_\theta)$ is a triplet of integers characterizing the Fourier decomposition in the ignorable and periodic co-ordinates, we can rewrite the right hand side of Eq. (5) in the convenient form

$$F = \sum_{\underset{\sim}{\ell}} F_{\underset{\sim}{\ell}} = \sum_{\underset{\sim}{\ell}} (\hat{F}_{\underset{\sim}{\ell}} + U_{\underset{\sim}{\ell}}) \quad , \tag{7}$$

with

$$\hat{F}_{\underset{\sim}{\ell}} = \left(\frac{2\pi}{m}\right)^3 (V')^{-1} \frac{\partial}{\partial \bar{\alpha}} \int d^3J \, D_0 \left(\underset{\sim}{\ell} \cdot \frac{\partial f}{\partial \underset{\sim}{J}}\right) A_{\underset{\sim}{\ell}} \quad , \tag{8}$$

$$U_{\underset{\sim}{\ell}} = \left(\frac{2\pi}{m}\right)^3 (V')^{-1} \int d^3J \, D_0 \left(\underset{\sim}{\ell} \cdot \frac{\partial f}{\partial \underset{\sim}{J}}\right) X_{\underset{\sim}{\ell}} \quad , \tag{9}$$

where

$$A_{\underset{\sim}{\ell}} = (2\pi)^{-3} \int d^3\theta \, a_{\underset{\sim}{\ell}} \, \delta(\alpha - \bar{\alpha}) \quad , \tag{10}$$

$$X_{\underset{\sim}{\ell}} = (2\pi)^{-3} \int d^3\theta \left(\underset{\sim}{\ell} \cdot \frac{\partial \chi}{\partial \underset{\sim}{J}}\right) \delta(\alpha - \bar{\alpha}) \quad , \tag{11}$$

and $a_{\underset{\sim}{\ell}}$ is a scale factor.[2] The diffusion coefficient D_0 is given by

$$D_0 = 2\pi e^2 \delta(\omega_a - \underset{\sim}{\ell}\cdot\underset{\sim}{\Omega}) |C^a_{\underset{\sim}{\ell}}|^2 \qquad (12)$$

where ω_a is the oscillation frequency of the eigenmode labeled by $a = a(\ell)$, ℓ being the toroidal mode number, and

$$C^a_{\underset{\sim}{\ell}} = (2\pi)^{-3} \int d^3\theta \; [\exp(i\underset{\sim}{\ell}\cdot\underset{\sim}{\theta})] (\underset{\sim}{E}\cdot\underset{\sim}{v}) \qquad (13)$$

is the coupling coefficient representing the work done by the wave on the particle. For applications to the low-frequency instabilities (which leave μ invariant, and hence have $\ell_g = 0$), it is conventional to put $\ell_\phi=\ell$, $\ell_\theta = -n$ implying $\underset{\sim}{\ell}=(0,\ell,-n)$.

We now have to specify χ and carry out the procedure delineated in Ref. 2 to obtain the appropriate transport equation. We shall derive and display the equation of parallel momentum transfer $(x = mv_\parallel \simeq mv_\zeta)$ in detail, and simply give the expressions for thermal conductivity from the energy transport equation. We also limit ourselves to a treatment of the far untrapped particles only: the trapped particles do not contribute substantially to parallel momentum transport.

Before deriving the equation proper, we remark that in the general analysis of Ref. 2, toroidicity can enter in two different ways: through the particle orbits (trapping, toroidal drifts, bananas, etc.), and through the structure of the eigenmode (ballooning representation). In this paper, we do not consider the toroidal effects on particle orbits though the analysis can be readily generalized. Even in this limit (far untrapped), the equation of momentum transfer has several features of interest.

In the far untrapped limit, the action $J \simeq e/c\alpha$, and therefore can be used as the radial co-ordinate. Equivalently, the distribution functions could be written as

$$f(J,p,M) = f(\alpha,p,M)$$

In the presence of a toroidal current, the simplest equilibrium distribution function which could qualitatively describe a tokamak plasma is

$$f = f_M(H_0,\alpha)\left(1 + \frac{V_\parallel(\alpha)}{T(\alpha)}\,\chi(\alpha,p)\right) \tag{14}$$

with

$$\chi = mv_\zeta = \frac{p + \frac{e}{c}\,\psi(\alpha)}{R} \tag{15}$$

where $f_M(H_0,\alpha)=[N(\alpha)/\pi^{3/2} v_t^3(\alpha)]\exp[-H_0/T(\alpha)]$ is a Maxwellian, $V_\parallel(\alpha)$ is the parallel drift speed, $v_t^2=[2T(\alpha)/m]$, $T(\alpha)$ is the temperature, and $N(\alpha)$ is the plasma density. Considerations of simplicity have prompted us to use the drifting Maxwellian instead of the more accurate Spitzer function; the difference however will not be qualitative.

Making use of Eqs. (2), (4), (14) and (15), we have

$$\underset{\sim}{\ell}\cdot\frac{\partial f}{\partial J} = f_M\left\{-\frac{\underset{\sim}{\ell}\cdot\underset{\sim}{\Omega}}{T}\left(1+\frac{V_\parallel \chi}{T}\right)+\frac{k_\parallel V_\parallel}{T}\right.$$
$$\left.-\frac{nc}{e}\left[A_1+\frac{w}{T}A_2+\frac{V_\parallel\chi}{T}\left(A_1+\frac{w}{T}A_2+A_3\right)\right]\right\} \qquad (16)$$

where $w=H_0-e\phi_0=mv^2/2$, $k_\parallel$ is the wave number in the direction of the ambient magnetic field, i.e.,

$$k_\parallel = \frac{\ell q-n}{qR} \quad , \qquad (17)$$

and

$$A_1 = \frac{1}{N}\frac{dN}{d\alpha}+\frac{e}{T}\frac{d\phi_0}{d\alpha}-\frac{3}{2}\frac{1}{T}\frac{dT}{d\alpha} \quad ,$$
$$A_2 = \frac{1}{T}\frac{dT}{d\alpha} \quad , \qquad (18)$$
$$A_3 = \frac{1}{V_\parallel}\frac{dV_\parallel}{d\alpha}-\frac{1}{T}\frac{dT}{d\alpha} \quad .$$

In deriving Eqs. (16)-(18), we have used

$$\underset{\sim}{\ell}\cdot\frac{\partial\chi}{\partial \underset{\sim}{J}} = \frac{\ell}{R}-\frac{nc}{e}\frac{e}{c}\frac{1}{Rq} = \frac{\ell}{R}-\frac{n}{qR}\equiv k_\parallel \quad .$$

Within the context of Eqs. (6)-(18), and noting that for the far untrapped particles

$$\int d^3J \simeq \frac{m^2R}{B}\int_{-\infty}^{+\infty}\frac{dv_\parallel}{mv_\parallel^2/2}\int^{\infty} dw\int d\alpha \quad , \qquad (19)$$

and

$$\underset{\sim}{\ell} \cdot \underset{\sim}{\Omega} = \ell\Omega_\phi - n\Omega_\theta \approx \ell \frac{v_\parallel}{R} - n\frac{v_\parallel}{qR} = k_\parallel v_\parallel \qquad (20)$$

which makes

$$\delta(\omega - \underset{\sim}{\ell} \cdot \underset{\sim}{\Omega}) = \frac{1}{|k_\parallel|}\,\delta\left(v_\parallel - \frac{\omega}{k_\parallel}\right) \equiv \frac{1}{|k_\parallel|}\,\delta(v_\parallel - v_p), \qquad (21)$$

we can evaluate every term in Eq. (5) to obtain

$$\frac{\partial}{\partial t}\left\langle mNV_\parallel\right\rangle - \left\langle eNE_T\right\rangle + \frac{1}{r}\frac{\partial}{\partial r}\, r\left(\sum_{\ell,n}\Gamma_{\ell n}\right) = \sum_{\ell,n} U_{\ell n} \qquad (22)$$

where $E_t = (cR)^{-1}(\partial\psi/\partial t)$ is the toroidal induction electric field, and $r = (2\alpha/B)^{1/2}$ is a convenient radial variable.[2] The quantities $\sum_{\ell,n}\Gamma_{\ell,n}$ and $\sum_{\ell,n} U_{\ell,n}$ denote respectively the radial <u>flux</u> of parallel momentum and the <u>source</u> of parallel momentum generated by the fluctuations. For a concise display of the flux and source terms, we define a quantity (with the dimensions of a frequency)

$$\nu_{jk} = \frac{4\sqrt{\pi}}{\omega_t}\sum_\ell \left|\frac{e\bar{E}_\ell}{mv_T}\right|^2 (\hat{\omega})^{k-j}\, S_{jk} \quad , \qquad (23)$$

with

$$S_{jk} = \sum_n \frac{n^j}{|\ell q - n|\,(\ell q - n)^k}\exp\left[-\frac{\hat{\omega}^2}{(\ell q-n)^2} - \frac{(\ell q-n)^2}{2\Delta^2}\right] \quad , \qquad (24)$$

where $\hat{\omega} = (\omega/\omega_t)$, $\omega_t = (v_t/qR)$ is the transit frequency, $\Delta = \Delta(\ell)$ is the measure of the submode width defined in

Ref. 2, and $\overline{E}_\ell$ is proportional to the fluctuation amplitude. Clearly ν_{jk} can be determined only if a detailed knowledge of the spectrum is available. Breaking up ν_{jk} into a sum over ℓ and n is done because a toroidal mode is labeled only by one quantum number ℓ; n is just a dummy index to be summed from $-\infty<n<\infty$. We can now conveniently and succinctly express

$$\sum_{\ell,n} \Gamma_{\ell n} = \Gamma_1 + \Gamma_2 + \Gamma_3 \quad , \tag{25}$$

$$\sum_{\ell,n} U_{\ell n} = -[U_1 + U_2 + U_3] \quad , \tag{26}$$

where

$$\Gamma_1 = -NmV_{\|}\rho_p^2[\nu_{24}(A_{1r}+A_{2r}+A_{3r}) + \nu_{26} A_{2r}] \quad , \tag{27a}$$

$$\Gamma_2 = -(\tfrac{1}{2})Nmv_t\rho_p^2[\nu_{23}(A_{1r}+A_{2r}) + \nu_{25}A_{2r}] \quad , \tag{27b}$$

$$\Gamma_3 = -Nmv_T\rho_p\left[\nu_{13} + \frac{V_{\|}}{v_T}(2\nu_{14} - \nu_{12})\right] \quad , \tag{27c}$$

$$U_1 = -NmV_{\|}\rho_p [\nu_{12}(A_{1r}+A_{2r}+A_{3r}) + \nu_{14}A_{2r}] \quad , \tag{28a}$$

$$U_2 = -\tfrac{1}{2} NmV_{\|}\rho_p [\nu_{\|}(A_{1r}+A_{2r}) + \nu_{13}A_{2r}] \quad , \tag{28b}$$

$$U_3 = -Nmv_t\left[\nu_{01} + \frac{V_{\|}}{v_T}(2\nu_{02} - \nu_{00})\right] \quad , \tag{28c}$$

where $\rho_p (v_t/\Omega)(qR/r)$ is the poloidal gyroradius, and $A_{ir} = (A_i/Br)$ signifies that α derivatives in Eq. (18) have been replaced by r derivatives. Notice that in Eqs. (27)-(29), the information about the fluctuation spectrum appears only through the ν_{jk}.

The equation of energy transport can be similarly derived, and has the form

$$\frac{\partial}{\partial t} < \frac{3}{2} NT > - < enV_{\parallel} E_{\parallel} > + < \nabla \cdot \Gamma_W(w) > - \frac{1}{r} \frac{\partial}{\partial r} r \sum_{\ell n} \Gamma_{\ell n}(w) = \sum_{\ell, n} \dot{U}_{\ell n}(w) \tag{29}$$

since

$$\frac{\partial}{\partial t} w = \frac{\partial}{\partial t} (H_0 - e\Phi_0) = ev_{\parallel} E_{\parallel} \tag{30}$$

Here Φ_0 is the equilibrium electrostatic potential (whose time dependence is neglected), and $E_{\parallel} \simeq E_t$ denotes the Ohmic heating, induced electric field. Γ_W represents the Ware-Galeev pinch effect for energy transport, as discussed (for the case of particle transport) in Ref. 2. The quantitites $\sum_{\ell,n} \Gamma_{\ell n}(w)$ and $\sum_{\ell,n} U_{\ell n}(w)$ occurring in Eq. (29) represent respectively the radial flux of energy, and the source of energy (wave heating) due to the presence of waves. These terms are readily expressed in terms of the ν_{jk}. In particular, the expression for electron thermal conductivity, the transport coefficient we use later, is

$$K_a = (\nu_{22} + \nu_{24} + \tfrac{1}{2}\nu_{26}) \rho_p^2 \tag{31}$$

Before closing this section, we remark that the terms Γ_1, Γ_2, V_1 and V_2 are proportional to the gradients in density, temperature, and $V_\parallel$. In particular, Γ_1 includes the fluctuation-induced viscosity (coefficient of $A_3 r$). The occurrence of momentum fluxes, in Γ_2, which are independent of $V_\parallel$ is easily understood from Eq. (16) and, in fact, quite similar to the neoclassical case.[8] One flux Γ_3 and source U_3 are exceptional in involving only a single power of the gyroradius. Because these terms are not only quantitatively dominant but also without neoclassical counterparts, we consider them in detail in the following sections.

Notice that the term proportional to $V_\parallel$ in U_3 has the form of $(J_\parallel / \sigma_{\parallel a})$, where $\sigma_{\parallel a}$ is the anomalous electrical conductivity. The remaining term in U_3 which is present even for $V_\parallel = 0$, is the dominant term used in most current drive studies.

III. Momentum Transfer Equation

From the expression for U_3 in Eq. (28c), we obtain the anomalous electrical conductivity

$$\sigma_{\parallel a} = \frac{e^2 N}{m(\nu_{00} - 2\nu_{02})} \quad . \tag{32}$$

It is clear that a complete evaluation of the transport coefficients requires a detailed knowledge of the fluctuation spectrum. This knowledge is, in general, not available. In spite of this, a significant amount of information can be extracted from our analysis by examining the ratios of the transport coefficients. An important example concerns the electron thermal conductivity, which is observed to much exceed the neoclassical value in Tokamak experiments. Let us assume that the enhancement is due to the presence of low frequency electromagnetic fluctuations. Further, we denote the enhancement by an anomaly factor A, which is then the ratio of the observed thermal conductivity K_0, to the neoclassical thermal conductivity $(K_{nc} \simeq \nu_c \rho_p^2)$, i.e.,

$$A = K_0/K_{nc} \tag{33}$$

From Eqs. (31) and (32) we obtain

$$(\nu_{22} + \nu_{24} + \frac{1}{2}\nu_{26}) = A\nu_c \quad , \tag{34}$$

and

$$\sigma_{\parallel a} = \frac{e^2 N}{m(\nu_{00} - 2\nu_{02})} = \frac{1}{A}\,\frac{\nu_{22} + \nu_{24} + \frac{1}{2}\nu_{26}}{\nu_{00} - 2\nu_{02}}\,\sigma_c \quad , \tag{35}$$

where $\sigma_c = (e^2N/m\nu_c)$ is the classical conductivity. To determine the ratio $(\sigma_{\parallel a}A/\sigma_c)$ we need to compare the effective collision frequencies ν_{jk}. We do this after a brief discussion on the nature of fluctuations.

It is generally believed that in the tokamaks the anomalous transport is caused by electromagnetic fluctuations (drift waves, drift-tearing modes, etc.) characterized by short perpendicular wave lengths such that $k_\perp \rho_s \simeq 1$, where ρ_s is the ion gyroradius with electron temperature. Since $k_\perp \simeq \ell q/r_n$, we note that typically $\ell q \simeq r_n/\rho_s \sim 100 \gg 1$. In addition, these microinstabilities have frequencies in the vicinity of $\omega_{e*} = (k_\perp \rho_s c_s/r_n)$, the electron diamagnetic drift frequency. For typical tokamak parameters, $\hat{\omega} = \omega/\omega_t \simeq (\omega_{e*}/\omega_t) \sim 0.2$. Making use of this information about the nature of the fluctuation spectrum, it is possible to make quite accurate analytical estimates for the S_{jk} and hence the ratios between various ν_{jk}. We have also computed the functions S_{jk} numerically with a choice of typical parameters,

$\ell q = 100$, $\hat{\omega} = 0.2$, and $\Delta = 1$. These are clearly a function of the distance from a mode rational surface and a few typical cases are displayed in Figs. 1 - 3. We find that for $\ell q \gg 1$, $S_{jk} = (\ell q)^j \bar{S}_{jk}$, where $\bar{S}_{jk}$ is essentially independent of ℓq. We shall list the maximum value of a few of the S_{jk} computed for the parameters given earlier. Let s_{jk} be the maximum value of S_{jk} (that is, maximized with respect to the fast variable, $\ell q-n$). Then

$$\begin{gathered} s_{00} = 3, \; s_{01} = 8, \; s_{02} = 50, \\ s_{13} = 340 \; \ell q, \\ s_{22} = 50\ell^2 q^2, \; s_{24} = 2.5 \times 10^3 \ell^2 q^2 \;, \\ s_{26} = 1.8 \times 10^5 \ell^2 q^2 \;. \end{gathered} \tag{36}$$

Next, we perform the ℓ-sum indicated in Eq. (23). Assuming that $\hat{\omega} \simeq 0.2$ and that $|E_\ell|^2$ varies slowly with ℓ, we obtain, approximately,

$$\frac{\nu_{00}}{\nu_{02}} \simeq 3, \quad \frac{\nu_{22}}{\nu_{24}} \simeq \frac{2}{3}, \quad \frac{\nu_{22}}{\nu_{26}} \simeq \frac{1}{2} \quad , \tag{37}$$

$$\frac{\nu_{00}}{\nu_{22}} \simeq \frac{1}{10} \; \frac{1}{\ell_0^2 q^2} \quad ,$$

where ℓ_0 is a representative ℓ, characterizing the spectrum. This leads us to the result

$$\frac{\sigma_{\parallel a}}{\sigma_c} \simeq \frac{100}{A} \ell_0^2 q^2 \simeq \frac{100 \times 10^4}{A} \quad , \tag{38}$$

since $\ell_0 q \simeq 10^2$ in typical experimental spectra. Hence, even for a large anomaly $A \simeq 100$, $\sigma_{\parallel a} \gg \sigma_c$. Thus, the anomalous resistivity $(\sigma_{\parallel a})^{-1}$ is much smaller than the classical resistivity $(\sigma_c)^{-1}$. Therefore, we have shown that in the presence of electromagnetic fluctuations, the plasma can show anomalously high thermal conductivity although its resistivity remains essentially classical. The fact that this conclusion is in agreement with experiment, strengthens our confidence in the assumption that microinstabilities are responsible for anomalous electron transport.

IV. DISCUSSION AND CONCLUSION.

In this paper, we have derived the equations of quasi-linear momentum transport, and discussed several of their implications. We have shown that useful information about plasma behavior in the presence of electromagnetic fluctuations can be extracted from the theory even when the details of the fluctuations spectrum are not known. In particular, we showed that a suprathermal level of fluctuations could cause highly enhanced electron thermal conductivity without appreciably changing the plasma resistivity from its classical value. This conclusion, which is in agreement with the tokamak experiments, lends support to the assumption that microinstabilities (drift or drift-tearing type) are responsible for anomalous thermal conductivity.

APPENDIX A

The following physical argument confirms the magnitude (and perhaps illuminates the origin) of the surprisingly large momentum flux, Γ_3 , due to electromagnetic perturbations.

The argument is simplified by consideration of an unrealistic special case: axisymmetric perturbations, which conserve the canonical angular momentum,

$$p = mRv_{\zeta} - \frac{e}{c} \psi .$$

Here ψ is the poloidal magnetic flux and v_{ζ} is the toroidal particle velocity. Since we consider low frequency perturbations which leave the magnetic moment invariant, the basic scattering process considered here is resonant energy exchange between particles and waves. The point is that quasilinear diffusion in action space does not conserve particle energy; even the total (particle plus wave) energy is conserved only globally, and not locally.

Let us suppose that the waves are externally generated, as in RF heating or current drive schemes. Then energy exchange may be presumed to increase particle energies. Because the magnetic moment is constant, $|v_{\zeta}|$ must increase; because p is constant, the poloidal flux seen by a particle on its orbit must change by the amount

$$\Delta\psi = \frac{c}{e}\, mR \,\frac{v_\zeta}{|v_\zeta|}\, \Delta(|v_\zeta|) \tag{A1}$$

where $\Delta(|v_\zeta|) > 0$ is the change in speed. Since the distribution function is roughly independent of $v_\zeta/|v_\zeta|$, as many charged particles will be displaced outward as are displaced inward. Thus, this process is not associated with any macroscopic particle (or energy) flux. However, since particles displaced in opposite radial directions have opposing toroidal velocities, a flux of toroidal momentum, Γ_3, results. Equation (A1) provides the estimate

$$\Gamma_{3r} \simeq Nm \left(\overline{v_\zeta \frac{\Delta(\psi/B_p R)}{\Delta t}}\right) \sim Nm \frac{mc}{eB_p} \left(\overline{|v_\zeta| \frac{\Delta|v_\zeta|}{\Delta t}}\right) \sim Nm\rho_p \nu v_t\ , \tag{A2}$$

where $\nu = (\Delta t)^{-1}$ measures the rate at which the toroidal speed changes. We note that Eq. (A2) agrees with the more detailed result of Eq. (27c).

A similar but more complicated argument can be constructed for the case of non-axisymmetric perturbations. One essential physical consideration in either case is the local, resonant change in particle energy.

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REFERENCES

1. A.N. Kaufman, Phys. Fluids 15, 1063 (1972).
2. R.D. Hazeltine, S.M. Mahajan, and D.A. Hitchcock, Phy. Fluids 24, 1164(1981).
3. H.P. Furth, Nucl. Fusion 15, 487 (1975), D.F. Duchs, D.E. Post, and P.H. Rutherford, Nucl. Fusion 17, 565 (1977), TFR Group, in Plasma Physics and Controlled Nuclear Fusion Research (International Atomic Energy Agency, Vienna, 1977) Vol. I, p. 35.
4. R. Goldston, (private communication)
5. W. Horton and R.D. Estes, Nucl. Fusion 19, 203 (1979) and references therein.
6. R.D. Hazeltine and H.R. Strauss, Phys. Rev. Lett. 37, 102 (1976).
7. J.D. Callen, Phys. Rev. Lett. 39, 1540 (1977).
8. R.D. Hazeltine, Phys. Fluids 17, 961 (1974).

FIGURE CAPTIONS

Fig. 1: Plot of $\bar{S}_{0,0} = S_{0,0}$ as a function of $\ell q - n_0$ for $\Delta = 1$, $\omega/\omega_T = 0.2$ and $n_0 = 100$.

Fig. 2: Plot of $\bar{S}_{13} = S(\ell q)^{-1} S_{13}$ as a function of $\ell q - n_0$ for $\Delta = 1$, $\omega/\omega_T = 0.2$, and $n_0 = 100$.

Fig. 3: Plot of $\bar{S}_{2,6} = (\ell q)^{-2} S_{2,6}$ as a function of $\ell q - n_0$ for $\Delta = 1$, $\omega/\omega_T = 0.2$, $n_0 = 100$.

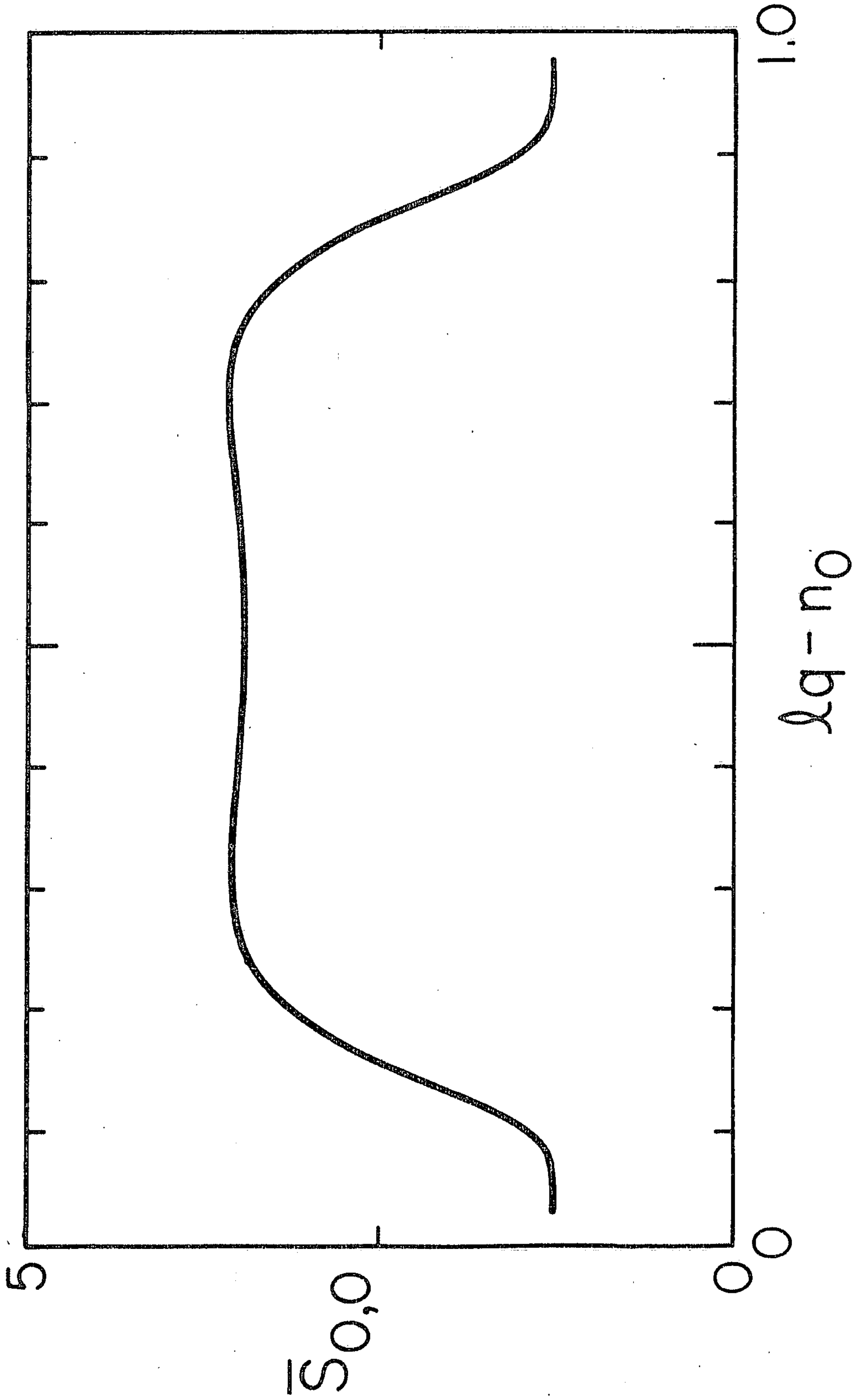

FIG. 1

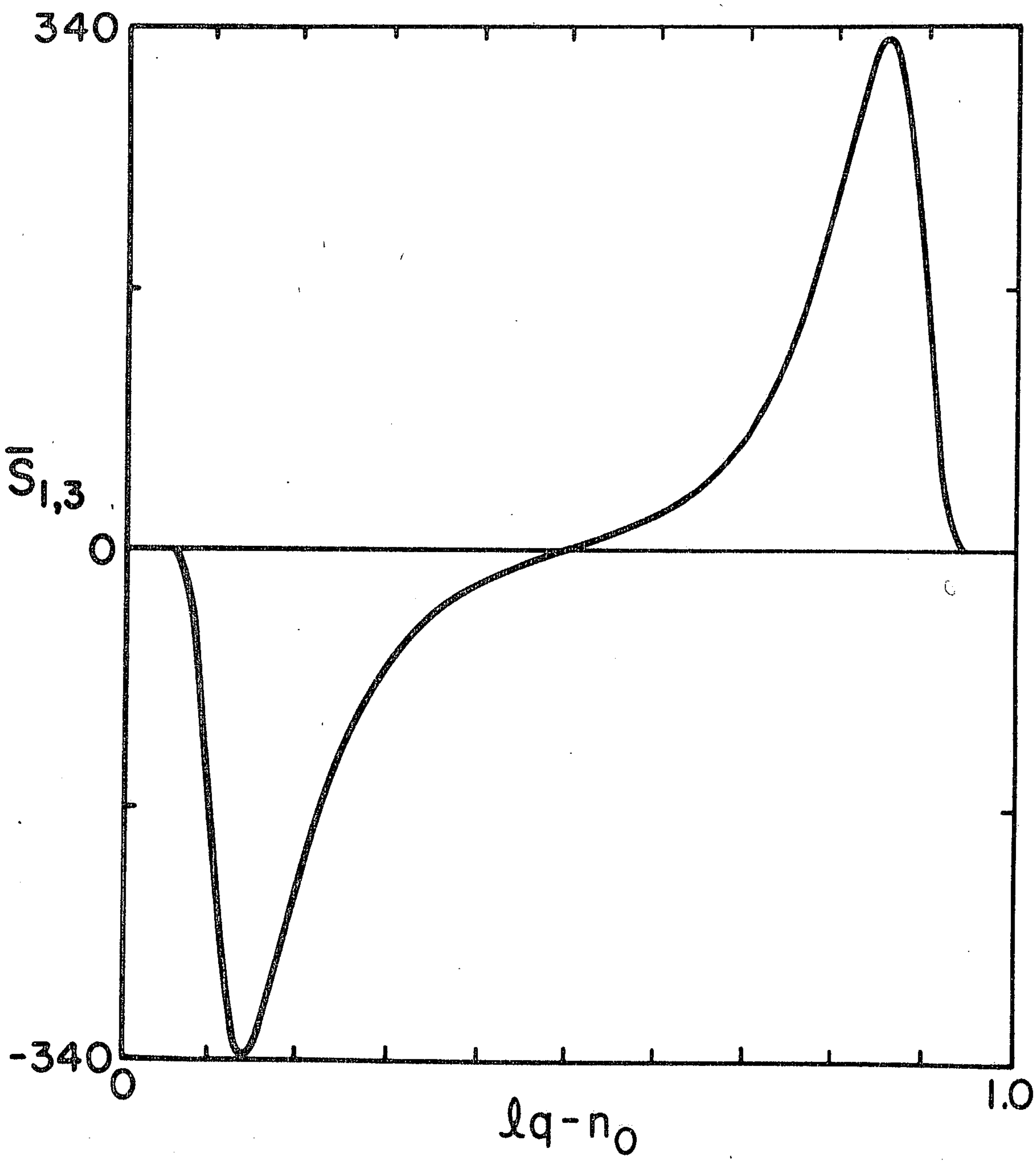

FIG. 2

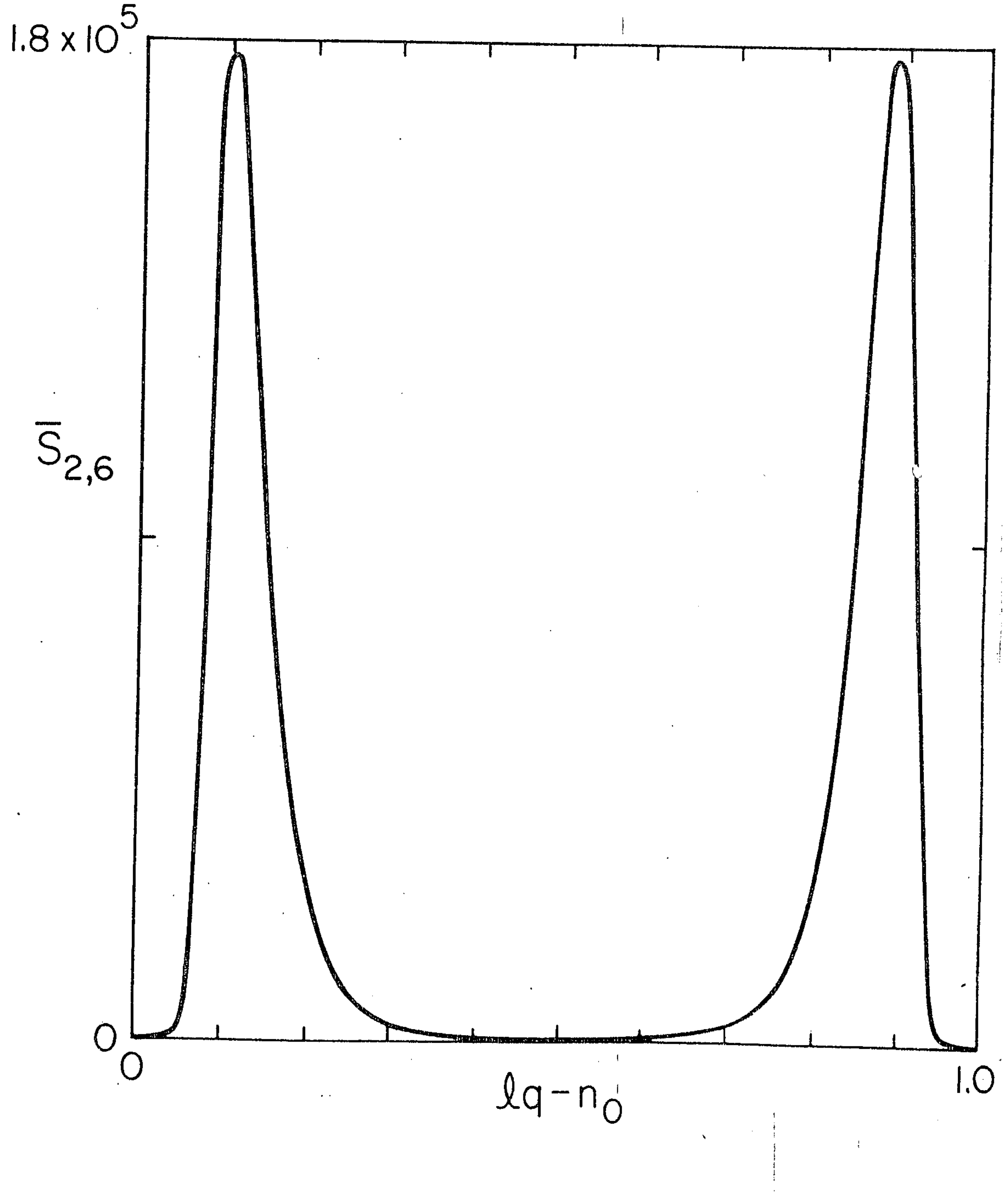

FIG. 3