DOE/ET/53088-169 IFSR #169

COLLISIONAL TRANSPORT FOR A SUPERTHERMAL ION SPECIES IN A PLASMA

F. Cozzani and W. Horton
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

April 1985

Collisional Transport for a Superthermal Ion Species in Magnetized Plasma

F. Cozzani and W. Horton

Institute for Fusion Studies

The University of Texas at Austin

Austin, Texas 78712-1060

Abstract

The transport theory of a high energy ion species injected isotropically in a magnetized plasma is considered for arbitrary ratios of the high energy ion cyclotron frequency to the collisional slowing down time. The assumptions of (1) low fractional density of the high energy species and (2) the average ion speed is faster than the thermal ions and slower than the electrons and are used to decouple the kinetic equation for the high energy species from the kinetic equations for background ions and electron. The kinetic equation is solved by a "Chapman-Enskog" expansion in the strength of the gradients; an equation for the first correction to the lowest order distribution function is obtained without scaling *a priori* the collision frequency with respect to the gyrofrequency. Various transport coefficients are explicitly calculated for the two cases of a weakly and a strongly magnetized plasma.

I. Introduction

Several situations of interest in plasma physics, with application to controlled thermonuclear research as well as to astrophysics and space physics, are characterized by the presence of a low density, high energy ion components in an otherwise locally Maxwellian plasma.

In the present work, we study the transport properties of such a superthermal ion species due to collisions with the ions and electrons of the local Maxwellian plasma. The background plasma consists of locally Maxwellian ion and electron distributions f_i, and f_e, with local densities $Z_i n_i(\mathbf{x}, t) = n_e(\mathbf{x}, t)$ and temperatures $T_i(\mathbf{x}, t) \simeq T_e(\mathbf{x}, t)$. Whenever the background ions are not of the same species, e.g., in a D-T plasma, they will be modeled by a fictitious ion of mean mass and charge.

The high energy ions are assumed to be injected isotropically into the background plasma, the word injection referring to any physical process by which the high energy ions appear into the plasma. The high energy component can actually be the same ion species as the background at a much higher energy, as well as a different species. In this latter case, the approximations to be presently discussed require the masses and charges of the two ion components not to differ by more than an order of magnitude.

The rate of injection is taken as a given function of $\mathbf{x}$; the appropriate source term in the kinetic equation for the high energy distribution is spatially localized and isotropic in velocity space. For simplicity, we shall neglect any explicit time dependence of the rate of injection, on the time scale of interest. The assumption of isotropic injection is realistic in situations like α-particle production in burning plasmas, but it is also valid for some magnetized plasmas of relevance in astrophysics and space physics. It is well known, for example, that coronal flares occur preferably in strongly active regions of the solar corona, in which the magnetic field lines exhibit a complicated structure. For a sufficiently large volume of space, the source term for the high energy ions can be modeled as an isotropic distribution.

The spectrum of injection energies is taken to be peaked around the value $\epsilon_I =$

$\frac{1}{2}m_h v_I^2$, the injection velocity v_I is assumed to satisfy

$$v_i \ll v_I \ll v_e, \tag{1}$$

$v_i(v_e)$ being the ion (electron) thermal velocity.

The high energy ions are designated by the subscript h and their number density is assumed to satisfy

$$n_h(\mathbf{x}, t) \ll n_i(\mathbf{x}, t), \tag{2}$$

but, because of Eq. (1), the number of high energy ions in the velocity space region of interest $v \gg v_i$ far exceeds the number of background ions.

No distinction is made between high energy and background ions whenever the former are slowed down to $v \simeq v_i$, as a result of collisions with electrons and background ions. This loss of thermalized high energy ions into the background ion component is represented by a cut-off-type sink term in the kinetic equation for the high energy species.

Also, it is assumed that no relative streaming velocity, to lowest order, exists among all the plasma components.

Figure 1 shows qualitatively the velocity space appropriate to the injection process. The critical velocity v_c where the frictional drag from the thermal ions and electrons are equal, defined in Eq. (8), is taken here to be less than the injection velocity v_I. The dashed line indicates the velocity $v_{\text{cutoff}} \gtrsim v_i$, below which the high energy ions are thermalized with the background ions.

II. Kinetic Equation for the High Energy Species

Under the above discussed approximations, the high energy ion distribution function satisfies the kinetic equation

$$\frac{\partial}{\partial t} f + \mathbf{v}\cdot\nabla f + \frac{eZ_h}{m_h}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\nabla_{\mathbf{v}} f$$
$$= C_{hi}(f, f_i) + C_{he}(f, f_e) + S - s. \tag{3}$$

The collision operators in Eq. (3) are of the Landau form (Braginskii 1965):

$$C_{ab}(f_a, f_b) \equiv -\frac{2\pi \Lambda Z_a^2 Z_b^2 e^4}{m_a}$$
$$\cdot \frac{\partial}{\partial \mathrm{v}_\beta}\int\left\{\frac{f_a(\mathbf{v})}{m_b}\cdot\frac{\partial f_b(\mathbf{v}')}{\partial v'_\gamma} - \frac{f_b(\mathbf{v}')}{m_a}\frac{\partial f_a(\mathbf{v})}{\partial v_\gamma}\right\} U_{\beta\gamma} d\mathbf{v}', \tag{4}$$

where

$$U_{\beta\gamma} \equiv \frac{1}{g^3}\left[g^2\delta_{\beta\gamma} - g_\beta g_\gamma\right] \quad , \quad g_\beta \equiv v_\beta - v'_\beta,$$

and Λ is the Coulomb logarithm.

Interparticle collisions, i.e., the C_{hh} term, are neglected in Eq. (3) because of the low density assumption, Eq. (2). Similarly, the collision operators C_{ih}, C_{eh}, representing the change in the background ion and electron distributions, due to collisions with high energy particles, are correspondingly neglected in the kinetic equations for background ions and electrons. In other terms, because of Eq. (2), the dynamics of the background ions and electrons is not significantly affected by the high energy particles and the background species are assumed to have relaxed to a locally Maxwellian state, on the time scale of interest.

As a final step in decoupling the kinetic equation for the high energy particles from the kinetic equations for background ions and electrons, the collision operators C_{he}, C_{hi} are simplified (Braginskii 1957, 1965) according to Eq. (1), by expanding to lowest relevant order in the small quantities v_I/v_e, v_i/v_I respectively, so to depend on the high energy ion distribution only:

$$C_{he} \simeq \tau_{s\ell}^{-1} \frac{\partial}{\partial v_k} (v_k f) \tag{5}$$

$$C_{hi} \simeq \frac{m_i}{m_h} \frac{v_c^3}{2} \tau_{s\ell}^{-1} \frac{\partial}{\partial v_j} \left(V_{jk} \frac{\partial}{\partial v_k} f + 2 \frac{m_h}{m_i} \frac{v_j}{v^3} f \right). \tag{6}$$

In Eqs. (5) and (6) the slowing down time $\tau_{s\ell}$ and the critical velocity v_c, the velocity at which the ion drag equals the electron drag, are respectively defined to be

$$\tau_{s\ell} \equiv \frac{m_h}{m_e} \frac{\tau_{ee}}{Z_h^2}, \tag{7}$$

$$v_c \equiv \left(\frac{3\pi^{1/2}}{4} \right)^{1/3} \left(\frac{m_e}{m_i} \right)^{1/3} Z_i^{2/3} v_e, \tag{8}$$

where

$$\tau_{ee} \equiv \frac{3 m_e^{1/2} T_e^{3/2}}{4 (2\pi)^{1/2} \Lambda e^4 n_e}.$$

Also, in Eq. (6) we defined

$$V_{jk} \equiv \frac{v^2 \delta_{jk} - v_j v_k}{v^3}.$$

In Eq. (3), the isotropic source with injection energy $\epsilon_I = 1/2 m_h v_I^2$ of width Δv_I is taken to be

$$S(\mathbf{x}, v^2) \equiv \frac{\dot{n}_h(\mathbf{x})}{4\pi v_I^3} \frac{2}{\pi^{1/2} \Delta \left[1 + \mathrm{erf}\left(\frac{1}{\Delta}\right)\right]} \cdot \frac{\exp\left[-\left(\frac{u-1}{\Delta} \right)^2 \right]}{u^2}, \tag{9}$$

where $u = v/v_I$ and $\Delta = \Delta v_I / v_I$. The spatially localized $\dot{n}_h(\mathbf{x})$ is the production rate of high energy particles. For the case of α particle production in a burning D-T plasma, $\dot{n}_h$ is given by

$$\dot{n}_h = n_T(\mathbf{x}) n_D(\mathbf{x}) \langle \sigma v \rangle_{DT}, \tag{10}$$

where σ is the D-T fusion cross section and the $\langle\,\rangle$ symbol denotes averaging over a Maxwellian distribution. In this particular case, the injection energy is $1/2 m_h v_I^2 =$ 3.52 Mev and the spread arises from D-T thermal velocities $\Delta = \Delta v_I / v_I \cong (T_i/\epsilon_I)^{1/2}$.

The source term S has been chosen so to satisfy

$$\int S d^3 v = \dot{n}_h. \tag{11}$$

The sink term $s(\mathbf{x}, v^2)$ in Eq. (3) represents the loss mechanism explained in the introduction; its role in the present analysis is merely to insure a steady state solution and it is sufficient to require

$$\int_V d^3x \int S d^3 v = \int_V d^3 x \int s d^3 v \tag{12}$$

and

$$s(\mathbf{x}, v^2) = 0$$

for

$$v > v_{\text{cutoff}}, \tag{13}$$

where

$$v_i \leq v_{\text{cutoff}} \ll v_I. \tag{14}$$

In Eq. (12), the space integration is extended over a volume $V \geq \left(\ell^h_{\text{mfp}}\right)^3$, where the "mean free path" $\ell^h_{\text{mfp}} \sim v_I \tau_{s\ell}$ is the distance traveled by the high energy ion, through the background plasma, before being slowed down to $v \simeq v_i$ by collisions with background ions and electrons. Note that, typically, $\ell^h_{\text{mfp}} \gg \ell^e_{\text{mfp}}$.

III. Moment Equations

Moment equations for the high energy species, expressing conservation of particles, momentum and energy respectively, are derived (Chapman and Cowling 1970; Boyd and Sanderson 1969; Balescu 1975; Hinton and Hazeltine 1976) taking velocity moments of Eq. (3):

$$\frac{\partial n}{\partial t} + \nabla \cdot \boldsymbol{\Gamma} = \dot{n}_h - \dot{\ell}, \tag{15}$$

$$\frac{\partial}{\partial t}(mn\mathbf{U}) + \nabla \cdot \overleftrightarrow{P} - eZn\left(\mathbf{E} + \frac{\mathbf{U}\times\mathbf{B}}{c}\right) = \mathbf{R}_e + \mathbf{R}_i, \tag{16}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}P\right) + \nabla \cdot \mathbf{Q} = Q_i + Q_e + \mathbf{U}\cdot(\mathbf{R}_i + \mathbf{R}_e + eZn\mathbf{E}) + H - h. \tag{17}$$

We defined (Hinton and Hazeltine 1976), omitting the h subscript for convenience, the particle density

$$n \equiv \int f d^3v, \tag{18}$$

the particle flux

$$\boldsymbol{\Gamma} \equiv n\mathbf{U} \equiv \int f\mathbf{v}d^3v, \tag{19}$$

the stress tensor

$$\overleftrightarrow{P} \equiv \int fm\mathbf{v}\mathbf{v}d^3v, \tag{20}$$

the energy flux

$$\mathbf{Q} \equiv \int f\frac{mv^2}{2}\mathbf{v}d^3v. \tag{21}$$

The stress tensor and energy flux are to be distinguished from the pressure tensor and heat flux respectively, which are expressed in terms of the peculiar velocity $\mathbf{w} \equiv \mathbf{v} - \mathbf{U}(\mathbf{x}, t)$. The pressure is defined to be

$$p \equiv \frac{1}{3}Tr\{\overleftrightarrow{P}\}, \tag{22}$$

where Tr denotes the trace.

Before discussing the other quantities appearing in Eqs. (15)-(17), it is necessary to point out a fundamental difference between the present theory and the ordinary theories of collisional transport. It is well known (Chapman and Cowling 1970; Braginskii 1965) that, whenever the lowest order* distribution function is a local Maxwellian, it is convenient to impose the following relations, expressing the existence of "privileged moments" in the theory:

$$\int f_M^{(0)} d^3v = \int f d^3v = n(\mathbf{x},t), \tag{23}$$

$$\frac{1}{n}\int f_M^{(0)} \mathbf{v} d^3v = \frac{1}{n}\int f\mathbf{v} d^3v = \mathbf{U}(\mathbf{x},t), \tag{24}$$

$$\frac{1}{n}\int f_M^{(0)} \frac{m}{2}(\mathbf{v}-\mathbf{U})^2 d^3v = \frac{1}{n}\int f\frac{m}{2}(\mathbf{v}-\mathbf{U})^2 d^3v = \frac{3}{2}T(\mathbf{x},t), \tag{25}$$

where

$$f_M^{(0)} \equiv \frac{n(\mathbf{x},t)}{[2\pi T(\mathbf{x},t)/m]^{3/2}} \exp\left[\frac{-m\left(\mathbf{v}-\mathbf{U}(\mathbf{x},t)\right)^2}{2T(\mathbf{x},t)}\right].$$

The parameters $n(\mathbf{x},t)$, $\mathbf{U}(\mathbf{x},t)$, $T(\mathbf{x},t)$ appearing in the local Maxwellian are therefore identified with the mean density, velocity, energy of the gas, as defined through the total (unknown) f. In the present theory, we anticipate that the lowest order distribution for the high energy particles will turn out to be non-Maxwellian and no privileged moments exist in the theory.

The friction forces $\mathbf{R}_e$, $\mathbf{R}_i$ and collisional energy exchange Q_e, Q_i are respectively defined as

$$\mathbf{R}_e \equiv \int m\mathbf{v}C_{he}(f)d^3v \tag{26}$$

$$\mathbf{R}_i \equiv \int m\mathbf{v}C_{hi}(f)d^3v \tag{27}$$

$$Q_e \equiv \int \frac{m}{2}(\mathbf{v}-\mathbf{U})^2 C_{he}(f)d^3v \tag{28}$$

$$Q_i \equiv \int \frac{m}{2}(\mathbf{v}-\mathbf{U})^2 C_{hi}(f)d^3v. \tag{29}$$

The particle injection rate $\dot{n}_h$ has already been defined through Eq. (11); the rate at which the high energy ions are lost into the background ion population by the thermalization

* Here, anticipating Sec. IV, we assume that the kinetic equation is solved by an appropriate series expansion for the distribution f.

process is defined to be

$$\dot{\ell}(\mathbf{x}) \equiv \int s d^3 v. \tag{30}$$

The rate of energy injection due to the source S, and the rate at which energy is directly channeled from the high energy species into the background ions are given by

$$H \equiv \int \frac{mv^2}{2} S d^3 v, \tag{31}$$

$$h \equiv \int \frac{mv^2}{2} s d^3 v, \tag{32}$$

where S, we recall, is given by Eq. (9), and h is usually negligible because of Eq. (13). The aim of the present work, as of any transport theory, is to provide a closed set of equations describing the time evolution of the density and mean energy for the high energy particles; we shall derive, therefore, explicit expressions for the transport fluxes appearing in Eq. (15) and Eq. (17) to the necessary order of approximation.

IV. The Method of Solution and the First Approximation $f^{(0)}$

Let us assume that the unknown distribution f can be expanded as

$$f(\mathbf{x}, \mathbf{v}, t) = f^{(0)}(\mathbf{x}, v^2, t) + f^{(1)}(\mathbf{x}, \mathbf{v}, t) + \cdots + f^{(j)}(\mathbf{x}, \mathbf{v}, t) + \cdots. \tag{33}$$

The first approximation to f, $f^{(0)}$, is taken to be isotropic in velocity space and, for $j = 1, 2, \ldots$, we assume

$$\left| \frac{f^{(j)}}{f^{(j-1)}} \right| \sim \frac{\ell^h_{\text{mfp}}}{L_n} \sim \frac{v_I \tau_{s\ell}}{L_n} < 1, \tag{34}$$

where L_n is a typical density (temperature) scale of the background plasma, e.g., $L_n^{-1} \equiv \frac{|\nabla n_e|}{n_e}$. Although the ordering given by Eq. (34) rules out some relevant applications, it makes possible to proceed without invoking *a priori* any scaling between the gyrofrequency Ω_h and $\tau_{s\ell}^{-1}$.

The kinetic equation is solved, in the region of velocity space characterized by $v > v_{\rm cutoff}$, order by order in the small parameter $v\tau_{s\ell}/L_n$, expanding the distribution f according to Eq. (33). The first approximation $f^{(0)}$ is therefore solution of

$$C_{he}\left(f^{(0)}\right) + C_{hi}\left(f^{(0)}\right) + S = 0, \tag{35}$$

which, in terms of the dimensionless velocity defined in Eq. (10), becomes

$$\left(\frac{\alpha}{u^2} + u\right)\frac{d}{du}f^{(0)} + 3f^{(0)} + \tau_{s\ell}S = 0. \tag{36}$$

In Eq. (36) the dimensionless critical velocity has been defined as

$$\alpha \equiv \left(\frac{v_c}{v_I}\right)^3. \tag{37}$$

The solution of Eq. (36) is

$$f^{(0)} = \frac{\dot{n}_h}{4\pi v_I^3}\tau_{s\ell}y(u;\alpha,\Delta), \tag{38}$$

$$y(u;\alpha,\Delta) \equiv \frac{1}{\alpha+u^3}\frac{1-\mathrm{erf}\left(\dfrac{u-1}{\Delta}\right)}{1+\mathrm{erf}\left(\dfrac{1}{\Delta}\right)}. \tag{39}$$

In Figs. (2a,b) and (3a,b) we plot, for selected values of α and Δ, the functions $y(u)$, $u^2y(u)$, $u^2S(u)$. Let us remark on the following:

(i) the expression for $f^{(0)}$ is valid only for $v > v_{\rm cutoff}$;

(ii) the parameter Δ cannot assume arbitrary small values for, if the slope of $f^{(0)}$ becomes arbitrary large, higher derivative terms in C_{he}, C_{hi} become comparable with the terms retained in Eqs. (5) and (6). For example, a correction to C_{he} of order $\sim \tau_{s\ell}^{-1}T_e/m_h\partial^2/\partial v_k^2 f^{(0)}$ has been neglected, provided that $f^{(0)}$, solution of Eq. (36), satisfies

$$\frac{2\epsilon_I}{T_e}f^{(0)} \gg \frac{1}{u}\frac{d}{du}f^{(0)},s$$

for $v > v_{\rm cutoff}$.

Actually, because of the large factor ϵ_I/T_e, the parameter Δ can become significantly small, as can be seen by Figs. 3a and 3b, without violating the above inequality and the simple analytical form given by Eqs. (38) and (39) does describe several physical situations of interest;

(iii) the lowest order distribution $f^{(0)}$ depends on space and time only through the quantities $\dot{n}_h(\mathbf{x})$, $T_e(\mathbf{x},t)$, $n_e(\mathbf{x},t)$. Let us note that, although $\dot{n}_h$ is strongly sensitive to the ion temperature profile in the case of α particle production (fusion production) the lowest order distribution $f^{(0)}$ does not depend explicitly on T_i. This is because, due to the scaling given by Eq. (1), the high energy particles "see" the background ions as practically stationary during collisions.

The first order approximation, $f^{(0)}$, gives the lowest order expressions for the density

$$n^{(0)} = \int f^{(0)} d^3v = \frac{\dot{n}_h \tau_{s\ell}}{4\pi v_I^3} \int y(u;\alpha,\Delta) d^3v \tag{40}$$

and mean energy per particle

$$\epsilon^{(0)} = \frac{1}{n^{(0)}} \int f^{(0)} \frac{m_h v^2}{2} d^3v = \frac{m_h v_I^2}{2} \frac{\int u^2 y(u;\alpha,\Delta) d^3v}{\int y(u;\alpha,\Delta) d^3v}. \tag{41}$$

In the above integrals, as well as in the forthcoming integrals, we extend the integration over all velocity space, even though we recall that the derived form of $y(u)$ is valid only for $v > v_{\text{cutoff}}$; by inspection of the integrals, it is straightforward to convince oneself that the error involved is negligible.

Also, to allow a simple tabulation of the integrals, we shall approximate, only under the integral sign,

$$\frac{1 - \text{erf}\left(\dfrac{u-1}{\Delta}\right)}{1 + \text{erf}\left(\dfrac{1}{\Delta}\right)} \approx \Theta(u-1),$$

$\Theta(x)$ being the Heaviside function which is valid in many situations of interest where $\Delta \ll 1$. We stress that the above simplification is by no means necessary for our analysis to be carried on and is invoked only for dispensing of the parameter Δ in the tabulation of the results, and to reduce double (triple) integrals to single (double) integrals. Equations (40) and (41) reduce therefore to

$$n^{(0)} \simeq \dot{n}_h \tau_{s\ell} \ell n \left(\frac{1+\alpha}{\alpha} \right), \tag{42}$$

$$\epsilon^{(0)} \simeq \frac{\epsilon_I I_{4,1}(\alpha)}{\ell n \left(\frac{1+\alpha}{\alpha} \right)}, \tag{43}$$

where

$$I_{n,1}(\alpha) = \alpha^{(n-2)/3} \int_0^{\alpha^{-1/3}} \frac{t^n}{t^3+1} dt \simeq \begin{cases} \dfrac{1}{n-2} & \text{small } \alpha \\ \dfrac{1}{\alpha(n+1)} & \text{large } \alpha \end{cases} \tag{44}$$

Similarly, the lowest order expression for the energy exchange with background ions and electrons are given by:

$$Q_{hi}^{(0)} \equiv \int \frac{m_h v^2}{2} C_{hi} \left(f^{(0)} \right) d^3v = -2\dot{n}_h \epsilon_I \alpha I_{1,1}(\alpha), \tag{45}$$

$$Q_{he}^{(0)} \equiv \int \frac{m_h v^2}{2} C_{he} \left(f^{(0)} \right) d^3v = -2\dot{n}_h \epsilon_I I_{4,1}(\alpha), \tag{46}$$

V. The Second-order Approximation $f^{(1)}$

Because of point (iii) of the discussion following Eq. (39), time and space derivatives acting on $f^{(0)}$ are expressed as time and space derivatives acting on $\dot{n}_h$, n_e, T_e. In particular, the time derivatives $\frac{\partial n_e}{\partial t}$, $\frac{\partial T_e}{\partial t}$ are eliminated (Chapman and Cowling 1970; Braginskii 1965) through lowest order moment equations for electrons, which, we recall, for a locally, non-drifting, Maxwellian electron distribution simply reduce to $\frac{\partial n_e}{\partial t} = 0$, $\frac{\partial T_e}{\partial t} = 0$. The second approximation $f^{(1)}$ therefore satisfies

$$C_{he}\left(f^{(1)}\right) + C_{hi}\left(f^{(1)}\right) - (\mathbf{v} \times \boldsymbol{\Omega}) \cdot \nabla_{\mathbf{v}} f^{(1)} =$$
$$\left[\mathbf{v} \cdot \nabla \ell n \left(\frac{\dot{n}_h}{n_e}\right) + \frac{3}{2}\frac{u^3}{\alpha + u^3}\mathbf{v} \cdot \nabla \ell n T_e\right] f^{(0)} + \frac{eZ_h}{m_h v_I^2}\mathbf{E} \cdot \mathbf{v}\frac{1}{u}\frac{d}{du} f^{(0)}, \tag{47}$$

where $\boldsymbol{\Omega} \equiv \Omega_h \hat{b}$; $\hat{b} \equiv \mathbf{B}/|\mathbf{B}|$.

To make the following notation more compact, let us indicate with ξ any of the driving forces appearing in Eq. (47) and with O_ξ the corresponding operator acting on $f^{(0)}$, e.g., for $\boldsymbol{\xi} = \nabla \ell n T_e$, $O^\xi = \frac{3}{2}\frac{u^3}{\alpha + u^3}$.

We can then write the general solution for $f^{(1)}$, when no $\mathbf{B}$ field is present, as (Chapman and Cowling 1970)

$$f^{(1)}(\mathbf{v}) = \sum_\xi \phi^\xi(u)\mathbf{v} \cdot \boldsymbol{\xi}. \tag{48}$$

In a magnetic field, this dependence is of the form (Braginskii 1965):

$$f^{(1)}(\mathbf{v}) = \sum_\xi \left[\phi^\xi(u)\mathbf{v} \cdot \boldsymbol{\xi}_{||} + \phi^\xi_\perp(u)\mathbf{v} \cdot \boldsymbol{\xi}_\perp + \phi^\xi_\Lambda(u)\mathbf{v} \cdot \boldsymbol{\xi}_\Lambda\right], \tag{49}$$

where

$$\boldsymbol{\xi}_{||} \equiv \hat{b}\hat{b} \cdot \boldsymbol{\xi} \quad ; \quad \boldsymbol{\xi}_\perp \equiv (\hat{b} \times \boldsymbol{\xi}) \times \hat{b} \quad ; \quad \boldsymbol{\xi}_\Lambda \equiv \hat{b} \times \boldsymbol{\xi}.$$

In Eqs. (48) and (49), the unknown scalar functions ϕ, $\phi_\perp$, ϕ_Λ are to be determined, and the summation is extended to all the driving forces appearing in Eq. (47). Evidently, it is sufficient to consider the case of a transverse gradient since $\phi(u)$ is obtained from $\phi_\perp(u)$

by writing $\Omega_h = 0$. In the case of $\xi = \frac{eZ_h}{m_h v_I^2}\mathbf{E}$, we shall consider only parallel electric fields.

Also, let us define

$$\Phi^\xi(u) = \phi_\perp^\xi(u) + i\phi_\Lambda^\xi(u). \tag{50}$$

The equation determining Φ^ξ, the part of $f^{(1)}$ driven by the single driving force ξ, is therefore

$$\left(\frac{u^3+\alpha}{u^2}\right)\frac{d}{du}\Phi^\xi + \left[\frac{4u^3 + (1 - m_i/m_h)\alpha}{u^3} - i\lambda\right]\Phi^\xi = \tau_{s\ell} O^\xi f^{(0)}, \tag{51}$$

where

$$\lambda \equiv \Omega_h \tau_{s\ell} = \frac{1}{Z_h}\Omega_e \tau_{ee}. \tag{52}$$

The general solution of Eq. (51) with Φ=0 for $u \to +\infty$ is

$$\Phi^\xi(u) = -\frac{\dot{n}_h \tau_{s\ell}^2}{4\pi v_I^3 (u^4 + \alpha u)}\left(\frac{u^3}{u^3+\alpha}\right)^{1/3\ \frac{m_i}{m_h}} \int_u^\infty \frac{x^3(x^3+\alpha)^{1/3\ m_i/m_h - i\lambda}}{(u^3+\alpha)^{-i\lambda}(x^3)^{1/3\ m_i/m_h}} O^\xi(x)dx. \tag{53}$$

In Eq. (53), x is just an integration variable and $y(x)$ is the velocity-dependent part of $f^{(0)}$, given by Eq. (39). Note that, except for the charge Z_h, the parameter λ depends only on background plasma quantities. Up to this point, a maximal scaling hypothesis $\Omega_h \tau_{s\ell} \sim 1$ has been employed; it is now convenient to consider the two opposite limits of Eq. (51):

$$\lambda \ll 1 \quad ; \quad \lambda \gg 1. \tag{54}$$

For the unmagnetized case, i.e., $\lambda \ll 1$, the solution of Eq. (51) is

$$\Phi^\xi(u) = \frac{-\dot{n}_h \tau_{s\ell}^2}{4\pi v_I^3}\frac{1}{u^4+\alpha u}\left(\frac{u^3}{u^3+\alpha}\right)^{1/3\ \frac{m_i}{m_h}} \int_u^\infty x^3 \left(\frac{x^3+\alpha}{x^3}\right)^{1/3\ \frac{m_i}{m_h}} O^\xi y(x)dx. \tag{55}$$

For the magnetized case, i.e., $\lambda \gg 1$, let us expand Φ^ξ as follows:

$$\Phi^\xi \equiv \Phi_\perp^\xi + i\Phi_\Lambda^\xi = \Phi_{(0)}^\xi + \frac{1}{\lambda}\Phi_{(1)}^\xi + \frac{1}{\lambda^2}\Phi_{(2)}^\xi + \cdots . \tag{56}$$

Substituting Eq. (56) in Eq. (51) we find:

$$\Phi^{\xi}_{(0)\Lambda} = \frac{\dot{n}_h \tau_{s\ell}}{4\pi v_I^3 \Omega_h} O^{\xi} y(u) \tag{57}$$

$$\Phi^{\xi}_{(0)\perp} = 0 \tag{58}$$

$$\Phi^{\xi}_{(0)\Lambda} = 0 \tag{59}$$

$$\Phi^{\xi}_{(1)\perp} = \frac{\dot{n}_h \tau_{s\ell}}{4\pi v_I^3 \Omega_h} \left\{ \frac{u^3+\alpha}{u^2} \frac{d}{du} O^{\xi} y(u) + \frac{4u^3 + \alpha(1 - m_i/m_h)}{u^3} O^{\xi} y(u) \right\}. \tag{60}$$

With the above derived expression for $f^{(1)}$, it is possible to close the moment equations for the lowest order density and energy by expressing the necessary fluxes to first, non-vanishing, order as follows:

The particle flux, driven by $\boldsymbol{\xi} = \nabla \ell n \left(\frac{\dot{n}_h}{n_e} \right)$, or $\nabla \ell n T_e$ is given by

$$\boldsymbol{\Gamma}^{\xi} = \boldsymbol{\xi} \cdot \int \Phi^{\xi}(u) \mathbf{v}\mathbf{v} d^3 v, \tag{61}$$

the energy flux is similarly

$$\mathbf{Q}^{\xi} = \frac{m_h}{2} \xi \cdot \int \Phi \xi(u) v^2 \mathbf{v}\mathbf{v} d^3 v. \tag{62}$$

The high energy ion current, driven by $\boldsymbol{\xi} = eZ_h \mathbf{E}/m_h v_I^2$, is given by

$$\mathbf{j} = eZ_h \boldsymbol{\xi} \cdot \int \Phi^{\xi}(u) \mathbf{v}\mathbf{v} d^3 v. \tag{63}$$

In Eqs. (61) and (62), $\Phi^{\xi}(u)$ is given by Eqs. (57)-(60) or Eq. (55), according to whether the plasma is magnetized or not; in Eq. (63), $\Phi^{\xi}(u)$ is given by Eq. (55).

Note that, in particular, the dissipative terms $\mathbf{U} \cdot \mathbf{R}_i$, $\mathbf{U} \cdot \mathbf{R}_e$ are of second order in the present ordering, and the fluxes given by Eqs. (61)-(63) are the only ones needed for the closure of the continuity and energy transfer equations.

In the next sections we give the explicit expression for the fluxes as definite integrals depending on the parameter α, the dimensionless critical velocity defined in Eqs. (8) and (37).

Tables of the integrals for significant values of α consistent with Eq. (1) are given in Sec. VIII.

VI. Fluxes for Magnetized Plasma ($\lambda \gg 1$)

VI.1. Fluxes in the magnetic surface

Particle Flux

$$\Gamma_\Lambda = \Gamma_\Lambda^{\nabla\left(\frac{\dot{n}_h}{n_e}\right)} + \Gamma_\Lambda^{\nabla T_e}$$
$$= \dot{n}_h \tau_{s\ell} \frac{\epsilon_I}{m_h \Omega_h} \left[\frac{2}{3} \hat{b} \times \nabla \ell n \left(\frac{\dot{n}_h}{n_e} \right) I_{4,1}(\alpha) + \hat{b} \times \nabla \ell n T_e \frac{I_{7,2(\alpha)}}{\alpha} \right]; \tag{64}$$

where

$$I_{n,2} \equiv \alpha^{\frac{n-2}{3}} \int_0^{\alpha^{-1/3}} \frac{t^n}{(t^3+1)^2} dt \simeq \begin{cases} \dfrac{\alpha}{n-5} & \text{small } \alpha \quad (n \geq 6) \\ \dfrac{1}{\alpha(n+1)} & \text{large } \alpha \end{cases} \tag{65}$$

Energy Flux

$$\mathbf{Q}_\Lambda = \mathbf{Q}_\Lambda^{\nabla\left(\frac{\dot{n}_h}{n_e}\right)} + \mathbf{Q}_\Lambda^{\nabla T_e}$$
$$= \dot{n}_h \tau_{s\ell} \frac{\epsilon_I^2}{m_h \Omega_h} \left[\frac{2}{3} \hat{b} \times \nabla \ell n \left(\frac{\dot{n}_h}{n_e} \right) I_{6,1}(\alpha) + \hat{b} \times \nabla \ell n T_e I \frac{9,2(\alpha)}{\alpha} \right]; \tag{66}$$

VI.2. Fluxes across the magnetic surface

Particle Flux

$$\Gamma_\perp = \Gamma_\perp^{\nabla\left(\frac{\dot{n}_h}{n_e}\right)} + \Gamma_\perp^{\nabla T_e} = -\frac{\dot{n}_h \tau_{s\ell}}{\lambda} \frac{\epsilon_I}{m_h \Omega_h}$$
$$\cdot \left\{ \left[\frac{1}{3} + \frac{2}{3} \frac{m_i}{m_h} \alpha I_{1,1}(\alpha) \right] \nabla_\perp \ell n \left(\frac{\dot{n}_h}{n_e} \right) + \left[I_{4,1}(\alpha) + \frac{1}{3} \frac{m_i}{m_h} I_{4,2}(\alpha) \right] \nabla_\perp \boldsymbol{\ell n T_e} \right\}; \tag{67}$$

Energy Flux

$$\mathbf{Q}_\perp = \mathbf{Q}_\perp^{\nabla\left(\frac{\dot{n}_h}{n_e}\right)} + \mathbf{Q}_\perp^{\nabla T_e} = -\frac{\dot{n}_h \tau_{s\ell}}{\lambda} \frac{\epsilon_I^2}{m_h \Omega_h}$$
$$\cdot \left\{ \left[\frac{1}{2} + \frac{2}{3} \frac{m_i}{m_h} \alpha I_{3,1}(\alpha) \right] \nabla_\perp \ell n \left(\frac{\dot{n}_h}{n_e} \right) + \left[3 I_{6,1}(\alpha) + \frac{m_i}{m_h} I_{6,2}(\alpha) \right] \nabla_\perp \boldsymbol{\ell n T_e} \right\}; \tag{68}$$

VII. Fluxes for Unmagnetized Plasma ($\lambda \ll 1$) or Parallel to the Magnetic Field

Particle Flux

$$\Gamma = \Gamma^{\nabla\left(\frac{\dot{n}_h}{n_e}\right)} + \Gamma^{\nabla T_e} =$$

$$= -\dot{n}_h \tau_{s\ell}^2 \frac{\epsilon_I}{m_h}\left[\frac{2}{3}\nabla \ell n\left(\frac{\dot{n}_h}{n_e}\right) J_1\left(\alpha, \frac{m_i}{m_h}\right) + \nabla \ell n T_e J_2\left(\alpha, \frac{m_i}{m_h}\right)\right]; \qquad (69)$$

$$J_1\left(\alpha, \frac{m_i}{m_h}\right) \equiv \alpha^{2/3}\int_0^{\alpha^{-1/3}}\left(\frac{t^3}{t^3+1}\right)^{1+\frac{1}{3}\frac{m_i}{m_h}} dt \int^{\alpha^{-1/3}}\left(\frac{z^3}{z^3+1}\right)^{1-\frac{1}{3}\frac{m_i}{m_h}} dz \qquad (70)$$

$$J_2\left(\alpha, \frac{m_i}{m_h}\right) \equiv \alpha^{2/3}\int_0^{\alpha^{-1/3}}\left(\frac{t^3}{t^3+1}\right)^{1+\frac{1}{3}\frac{m_i}{m_h}} dt \int_t^{\alpha^{-1/3}}\left(\frac{z^3}{z^3+1}\right)^{2-\frac{1}{3}\frac{m_i}{m_h}} dz. \qquad (71)$$

Energy Flux

$$\mathbf{Q} = \mathbf{Q}^{\nabla\left(\frac{\dot{n}_h}{n_e}\right)} + \mathbf{Q}^{\nabla T_e} =$$

$$= -\dot{n}_h \tau_{s\ell}^2 \frac{\epsilon_I^2}{m_h}\left[\frac{2}{3}\nabla \ell n\left(\frac{\dot{n}_h}{n_e}\right) J_3\left(\alpha, \frac{m_i}{m_h}\right) + \nabla \ell n T_e J_4\left(\alpha, \frac{m_i}{m_h}\right)\right]; \qquad (72)$$

$$J_3\left(\alpha, \frac{m_i}{m_h}\right) \equiv \alpha^{4/3}\int_0^{\alpha^{-1/3}} t^2\left(\frac{t^3}{t^3+1}\right)^{1+\frac{1}{3}\frac{m_i}{m_h}} dt \int_t^{\alpha^{-1/3}}\left(\frac{z^3}{z^3+1}\right)^{1-\frac{1}{3}\frac{m_i}{m_h}} dz \qquad (73)$$

$$J_4\left(\alpha, \frac{m_i}{m_h}\right) \equiv \alpha^{4/3}\int_0^{\alpha^{-1/3}} t^2\left(\frac{t^3}{t^3+1}\right)^{1+\frac{1}{3}\frac{m_i}{m_h}} dt \int_t^{\alpha^{-1/3}}\left(\frac{z^3}{z^3+1}\right)^{2-\frac{1}{3}\frac{m_i}{m_h}} dz. \qquad (74)$$

Electric Field Driven Current

$$\mathbf{j} = \frac{e^2 Z_h^2}{3}\frac{\dot{n}_h \tau_{s\ell}^2}{m_h}\mathbf{E}\left[J_5\left(\alpha, \frac{m_i}{m_h}\right) + \frac{I_{5,2}(\alpha)}{\alpha}\right], \qquad (75)$$

$$J_5\left(\alpha, \frac{m_i}{m_h}\right) = \int_0^{\alpha^{-1/3}}\left(\frac{t^3}{t^3+1}\right)^{1+\frac{1}{3}\frac{m_i}{m_h}} dt$$

$$\cdot \int_t^{\alpha^{-1/3}}\frac{1}{z^2}\left(\frac{z^3}{z^3+1}\right)^{1-\frac{1}{3}\frac{m_i}{m_h}}\left[\left(2 - \frac{m_i}{m_h}\right) + \frac{m_i}{m_h}\frac{z^3}{z^3+1}\right] dz. \qquad (76)$$

According to the approximations discussed in the introduction and after Eq. (41), we require $m_i/m_h \leq 2$ in Eqs. (70)-(76). In Tables I-VII, the values of the integrals defined in this section are computed, over a typical range of values for α, and m_i/m_h.

VIII. Tables of Integrals

Table I—Values of Integrals of type $I_{n,1}$ and $I_{n,2}$ Eqs. (64)-(66)

α	$I_{4,1}$	$\alpha I_{1,1}$	$I_{7,2}/\alpha$	$I_{6,1}$	$I_{9,2}/\alpha$
10^{-3}	0.489	0.011	0.482	0.249	0.248
10^{-2}	0.454	0.046	0.426	0.243	0.236
10^{-1}	0.337	0.163	0.259	0.201	0.167
1	0.126	0.374	0.044	0.086	0.033
10	0.019	0.481	$1.09 \cdot 10^{-3}$	0.013	$8.63 \cdot 10^{-4}$

Table II—Values of Integrals of Type $I_{n,1}$ and $I_{n,2}$ Eqs. (67)-(68)

α	$I_{4,2}$	$\alpha I_{3,1}$	$I_{6,2}$	$I_{5,2}/\alpha$
10^{-3}	$7.06 \cdot 10^{-3}$	$8.80 \cdot 10^{-4}$	$8.39 \cdot 10^{-4}$	1.970
10^{-2}	0.028	$7.44 \cdot 10^{-3}$	$6.63 \cdot 10^{-3}$	1.208
10^{-1}	0.078	0.049	0.035	0.497
1	0.082	0.164	0.053	0.064
10	0.018	0.237	0.013	$1.47 \cdot 10^{-3}$

Table III—Values of Integral J_1

α	$\frac{m_i}{m_h} = 0.5$	$\frac{m_i}{m_h} = 1$	$\frac{m_i}{m_h} = 2$
10^{-3}	0.380	0.375	0.366
10^{-2}	0.268	0.260	0.247
10^{-1}	0.111	0.105	0.096
1	0.012	0.011	0.010

Table IV—Values of Integral J_2

α	$\frac{m_i}{m_h} = 0.5$	$\frac{m_i}{m_h} = 1$	$\frac{m_i}{m_h} = 2$
10^{-3}	0.377	0.372	0.363
10^{-2}	0.257	0.250	0.237
10^{-1}	0.091	0.086	0.078
1	$4.81 \cdot 10^{-3}$	$4.41 \cdot 10^{-3}$	$3.78 \cdot 10^{-3}$
10	$\sim 10^{-5}$	$\sim 10^{-5}$	$\sim 10^{-5}$

Table V—Values of Integral J_3

α	$\frac{m_i}{m_h} = 0.5$	$\frac{m_i}{m_h} = 1$	$\frac{m_i}{m_h} = 2$
10^{-3}	0.081	0.081	0.081
10^{-2}	0.073	0.072	0.071
10^{-1}	0.042	0.041	0.039
1	$6.12 \cdot 10^{-3}$	$5.77 \cdot 10^{-3}$	$5.19 \cdot 10^{-3}$
10	$1.36 \cdot 10^{-4}$	$1.27 \cdot 10^{-4}$	$1.12 \cdot 10^{-4}$

Table VI—Values of Integral J_4

α	$\frac{m_i}{m_h} = 0.5$	$\frac{m_i}{m_h} = 1$	$\frac{m_i}{m_h} = 2$
10^{-3}	0.081	0.081	0.081

10^{-2}	0.071	0.070	0.069
10^{-1}	0.036	0.035	0.033
1	$2.53 \cdot 10^{-3}$	$2.39 \cdot 10^{-3}$	$2.15 \cdot 10^{-3}$
10	$\sim 10^{-5}$	$\sim 10^{-5}$	$\sim 10^{-5}$

Table VII—Values of Integral J_5

α	$\frac{m_i}{m_h} = 0.5$	$\frac{m_i}{m_h} = 1$	$\frac{m_i}{m_h} = 2$
10^{-3}	2.494	2.398	2.238
10^{-2}	1.259	1.181	1.051
10^{-1}	0.368	0.323	0.251
1	0.029	0.022	0.010
10	$5.14 \cdot 10^{-4}$	$3.22 \cdot 10^{-4}$	$3.11 \cdot 10^{-5}$

IX. Conclusions

In this work, we have studied the transport properties of the high energy ion component due to collisions with background thermal ions and electrons. The relevant approximations involved in the foregoing analysis have been discussed in the introduction. In Sec. IV, we used the lowest order solution for the distribution function to calculate the particle density and mean energy for the superthermal species, and the lowest order expressions for the collisional energy exchange with background ions and electrons; they are given by Eqs. (42)-(46). In Sections VI and VII, the first order fluxes necessary to close the even moment equations, Eqs. (15) and (17), were derived using the second approximation for the distribution function in the two limits of magnetized and unmagnetized plasma respectively; the expressions for the transport fluxes are given by Eqs. (64)-(76). Let us now consider two applications of the theory.

An example of interest in astrophysics is given by the solar corona, where flares accelerate ions at superthermal velocites (Chiuderi 1979). Typical values for the lower corona, the region above the steep temperature gradient which separates the chromosphere from the corona itself are as follows: the electron temperature reaches a rather flat value at $T_e \simeq 1 - 2 \cdot 10^6\ {}^\circ K$; the background plasma density is of the order of $10^9 - 10^8 m^{-3}$; the magnetic field varies from 1 Gauss up to a few thousand Gauss in active regions; values of 10-100 G can be considered significant. A typical scale for the system is taken to be the isothermal scale height (Cozzani 1981; Field 1965) $L_n \equiv RT_e/\mu g$, where R is the gas constant, μ the mean molecular mass ($\mu = 1/2$ for a fully ionized hydrogen plasma), g the solar acceleration.

We consider protons at $\epsilon_I = 1$ KeV as high energy particles; although acceleration of ions up to 100 KeV is a standard feature of solar flares (Chiuderi 1979), we are more interested in the low energy part of the flare acceleration spectrum since extremely high energy ions have a negligible Coulomb scattering cross section and do not significantly interact with the background plasma. Of course, the present theory does apply to other astrophysical situations where the superthermal ions reach energies far above the value considered here; in these instances, however, the typical scale of the system is usually much larger than in the solar case and the collisional localization condition, Eq. (34), is again satisfied.

Also, it should be remarked that a quantitative theory should consider the entire energetics of the physical process involved in the acceleration mechanism, not only the energy portion which is carried by the higher energy component which reaches the Earth with almost no interaction with the background plasma in the accelerating region.

For the given values, the characteristic parameters defined in the previous analysis are as follows: $v_c = 1.22 v_I$ (i.e., ion drag slightly exceeds electron drag at the injection energy), $\alpha = 1.80$; $\lambda = 10^7 - 10^8$; $RT/\mu g \equiv L_n \simeq 3 \cdot 10^5$ Km; $\lambda_{\rm mfp} \simeq 10^4$ Km. We see that $\lambda_{\rm mpf} \ll L_n$, as required by Eq. (34). In Fig. 2a we plot the functions $y(u)$, $u^2 y(u)$ for the corresponding value of the parameter α; the chosen value of Δ is arbitrary.

The transport coefficients integrals for $\alpha = 1.8$ are

$\mathrm{I}_{4,1} = 0.083$	$\mathrm{I}_{9,2}/\alpha = 0.015$
$\alpha I_{1,1} = 0.417$	$\mathrm{I}_{4,2} = 0.063$
$\mathrm{I}_{7,2}/\alpha = 0.020$	$\alpha I_{3,1} = 0.192$
$\mathrm{I}_{6,1} = 0.058$	$\mathrm{I}_{6,2} = 0.042$

for use in formulas (64), (66), (67) and (68).

As an example for the unmagnetized case, we consider the inertial fusion concept (Miyamoto 1980; Luciano et al. 1983): the background plasma consists in this case of the imploded core of the D-T pellet with a typical temperature of $T_e \simeq 10$ KeV. The high energy species is represented by the alpha particles resulting from the thermonuclear reactions, with $\epsilon_I = 3.52$ MeV. Their energy spectrum is taken to be very peaked around ϵ_I, as depicted in Fig. 3b.

In this case, $v_c = 0.30 v_I$ (i.e., electron drag dominates at the injection energy), $\alpha = 0.028$; treating the background D-T plasma as being composed of a fictitious ion of mean mass 5/2, in units of the proton mass, the ratio m_i/m_h appearing in the relevant integrals is taken to be equal to 5/8. Let us note that, in contrast to the previous example of a very diffuse plasma where the scaling given by Eq. (34) was satisfied by the large space scales involved, in this case the collisional localization condition is assured by the high density of the imploded pellet core, which has a density of the order $10^{25} - 10^{26}\mathrm{cm}^{-3}$.

Indeed we obtain $\ell_{\mathrm{mfp}} \simeq 10^{-3}\mathrm{cm} \lesssim$ radius of the imploded pellet core. In Fig. 3a we plot the functions $y(u)$, $u^2 y(u)$ for the corresponding values of the parameters α, Δ. The values of the transport coefficients integrals for $\alpha = 0.028$, $m_i/m_h = 5/8$ are

$$J_1 = 0.198 \qquad J_4 = 0.059$$

$$J_2 = 0.182 \qquad J_5 = 0.0785$$

$$J_3 = 0.062 \qquad I_{5/2}/\alpha = 0.877$$

for use in formulas (69), (72) and (75).

Acknowledgements

This work was supported in part by the United States Department of Energy Grant Number DE-FG05-80ET-53088 and the National Science Foundation grant NSF-ATM85-06646.

References

R. Balescu, 1975, *Equilibrium and non equilibrium statistical mechanics*, (Wiley, New York).

T.J.M. Boyd, J.J. Sanderson, 1969, *Plasma Dynamics*, (Nelson, London).

S.I. Braginkii, Soviet Phys. JETP, **5**, 965 (1957).

S.I. Braginskii, 1965, in *Reviews of Plasma Physics*, Vol. I, edited by M.A. Leontovich (Consultant Bureau, New York) p. 205.

S. Chapman, T.G. Cowling, 1970, *The Mathematical Theory of Non Uniform Gases*, 3rd ed., (Cambridge University Press, London).

C. Chiuderi, 1979, *High Energy Phenomena on the Solar Surface*, Stars and Star Systems, (Reidel, Durdrecht).

F. Cozzani, July 1981, *Problemi di stabilita magnetoidrodinamica in plasmi astrofisici*, Italian Ph.D. thesis (unpublished), University of Parma, Italy.

G.B. Field, Ap. J. **142**, 531 (1965).

F.L. Hinton, R.D. Hazeltine, 1976, *Theory of Plasma Transport in Toroidal Confinement Systems* (Fusion Research Center Report #99, The University of Texas at Austin).

J.F. Luciano, P. Mora and J. Virmont, Phys. Rev. Lett. **51**, 1664 (1983).

K. Miyamoto, 1980, *Plasma Physics for Nuclear Fusion* (M.I.T. Press, Cambridge).

Figure Captions

1. Velocity space appropriate to the injection process. The dashed line indicates the velocity $v_{\text{cutoff}} \simeq v_I$, below which the high energy ions are considered thermalized with the background ions.

2(a). Lowest order solution $y(u; \alpha, \Delta)$, given by Eq. (39), illustrating the distribution velocity for values of the parameters corresonding to the solar corona plasma discussed in Sec. IX.

(b). Function $u^2 S(u; \Delta)$, with $S(u; \Delta)$ given by Eq. (9), illustrating the distribution of injection speed around the mean value v_I, for a value of Δ corresponding to Fig. 2a.

3(a). Same as Fig. 2a for values of the parameters corresponding to the inertial fusion plasma discussed in Sec. IX.

(b). Same as Fig. 2b for a value of Δ corresponding to Fig. 3a.

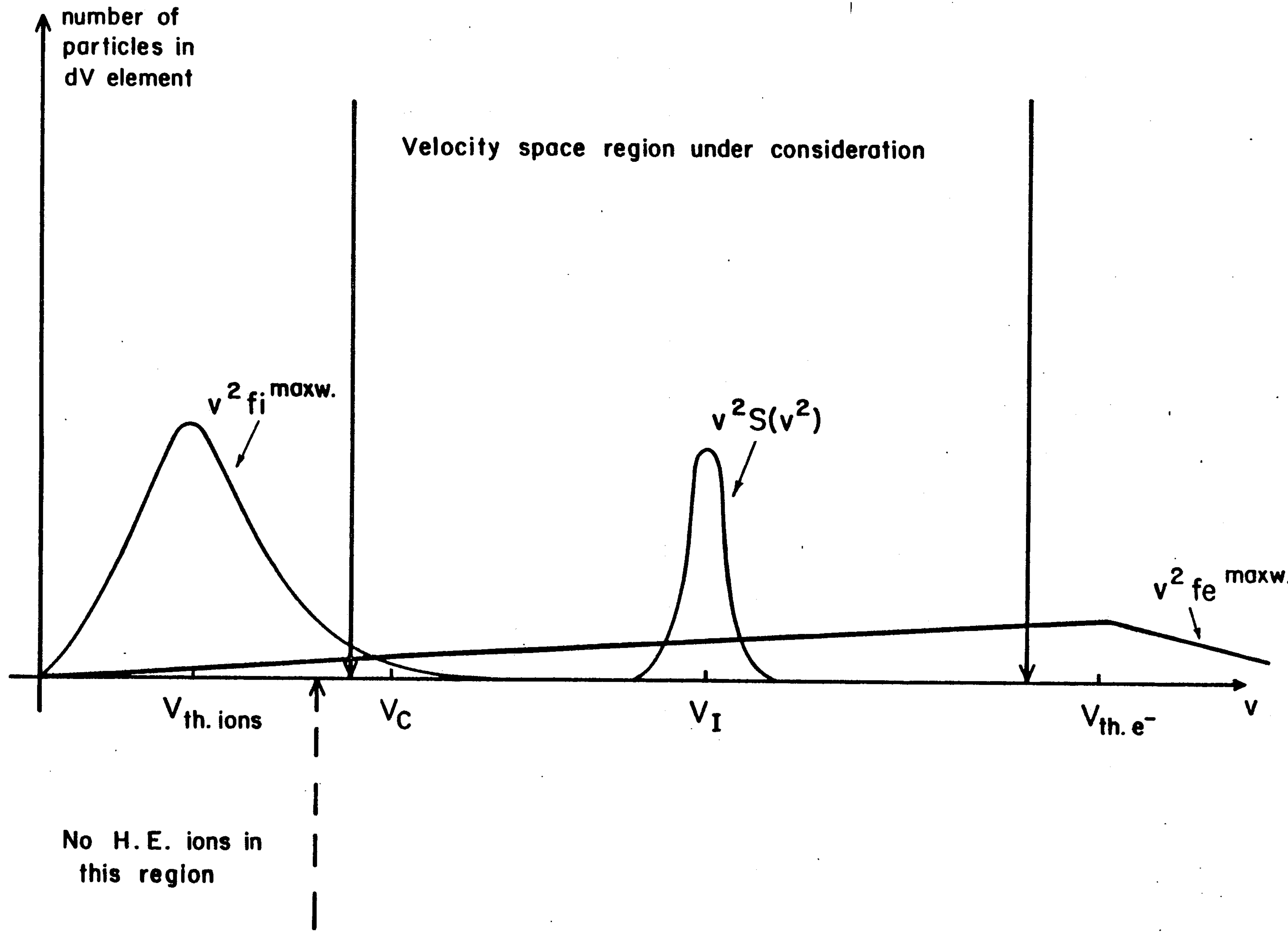
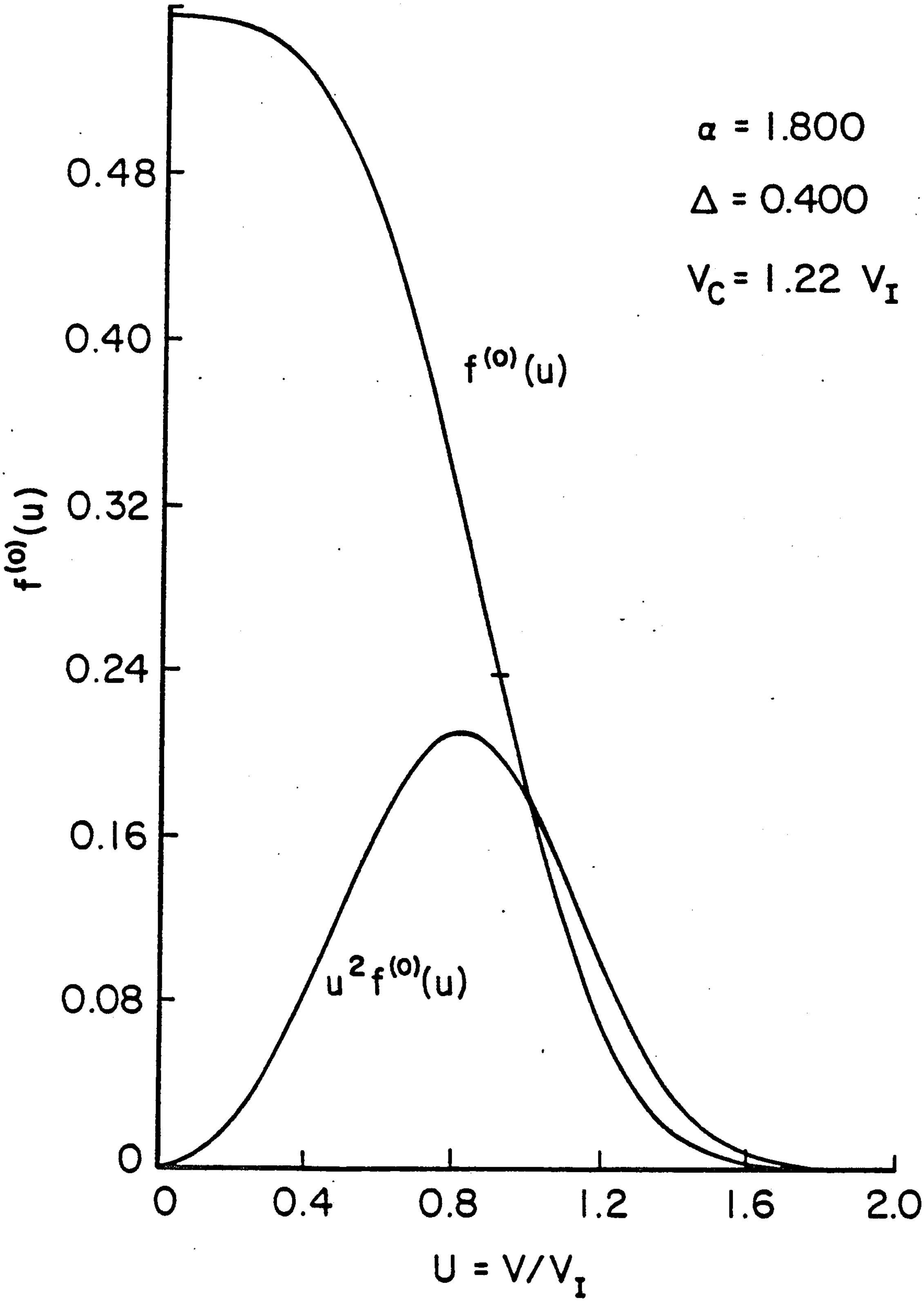

number of
particles in
dV element
Velocity space region under consideration
$v^2 fi^{maxw.}$
$v^2 S(v^2)$
$v^2 fe^{maxw.}$
$V_{th. ions}$
V_C
V_I
$V_{th. e^-}$
v
No H.E. ions in
this region

Fig. 1



α = 1.800
Δ = 0.400
V_C = 1.22 V_I
$f^{(0)}(u)$
$u^2 f^{(0)}(u)$
0.48
0.40
0.32
0.24
0.16
0.08
0
0.4
0.8
1.2
1.6
2.0
U = V/V_I

Fig. 2a

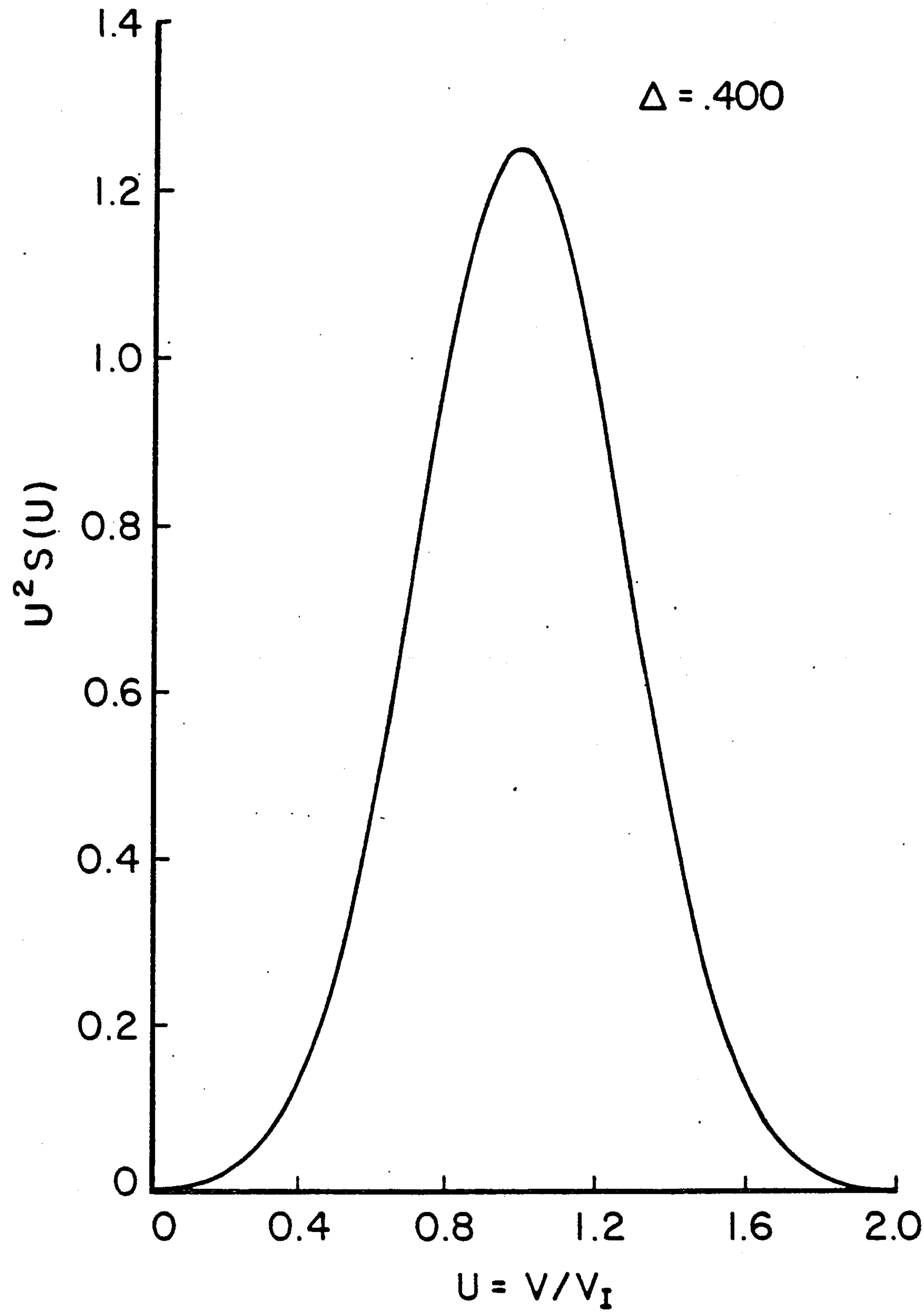
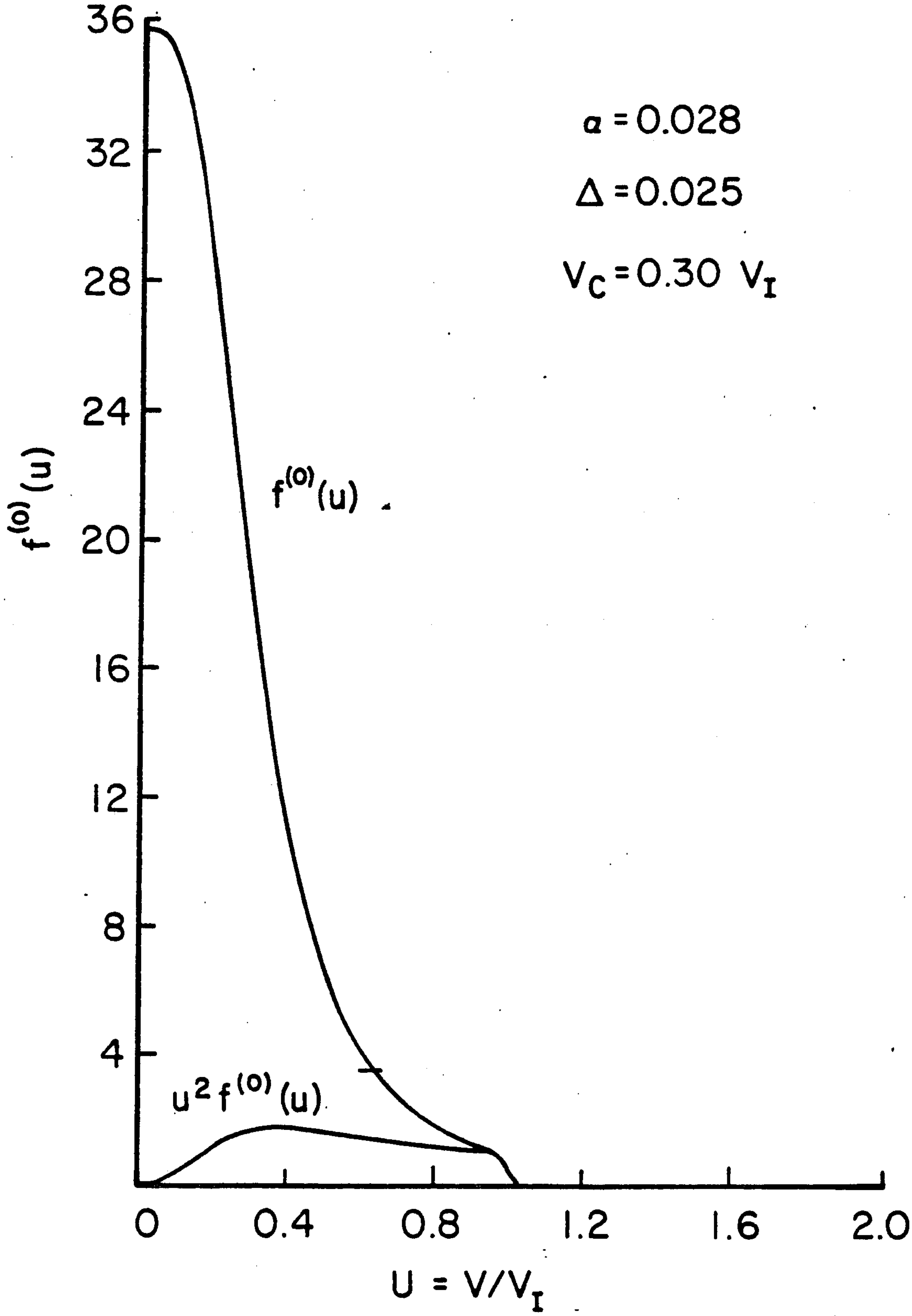


Fig. 2b



α = 0.028
Δ = 0.025
V_C = 0.30 V_I
$f^{(0)}(u)$
$u^2 f^{(0)}(u)$
36
32
28
24
20
16
12
8
4
0
0.4
0.8
1.2
1.6
2.0
U = V/V_I

Fig. 3a

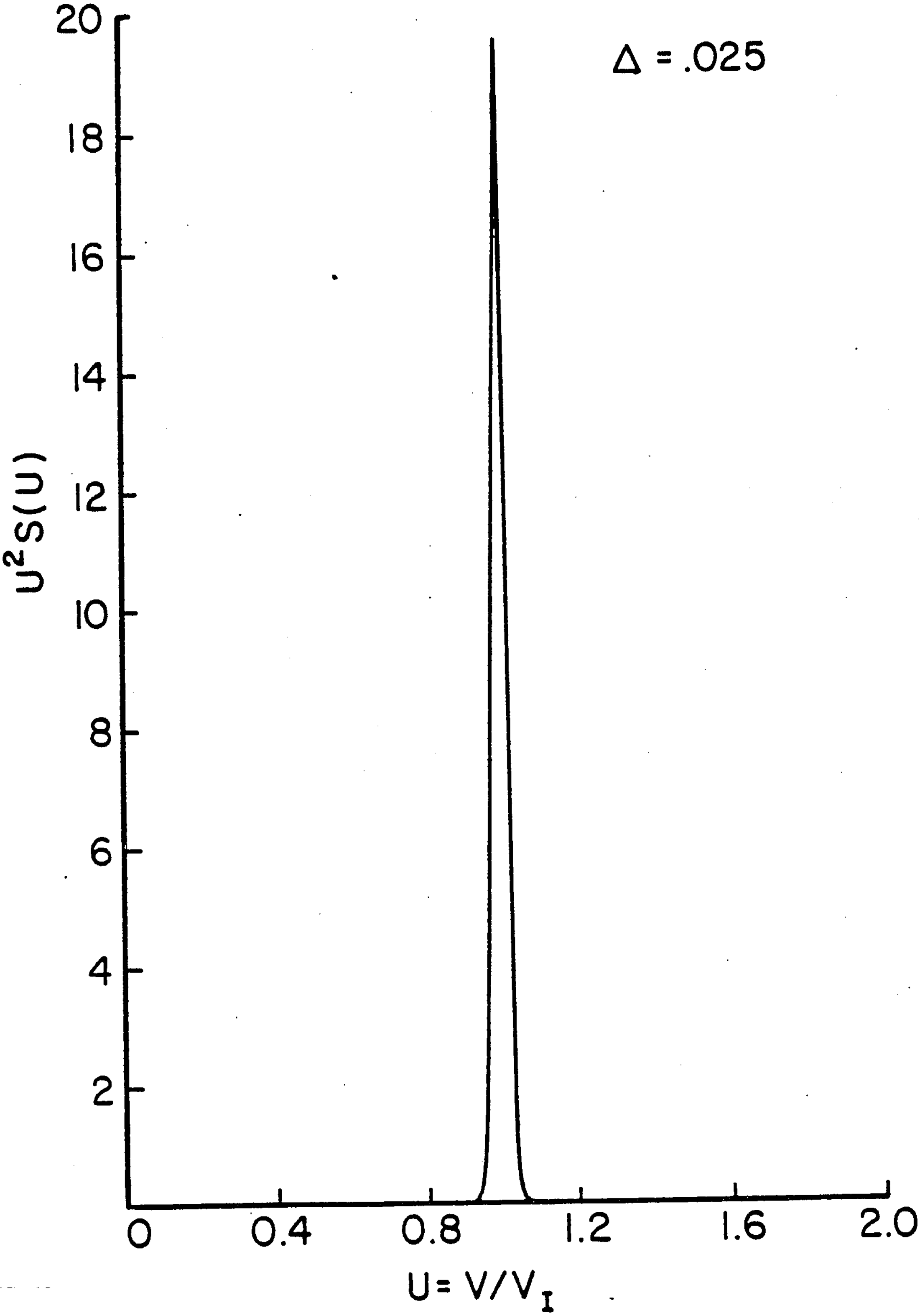


Fig. 3b