FINITE ORBIT ANALYSIS FOR LONG WAVELENGTH MODES
IN A PLASMA WITH A HOT COMPONENT

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Abstract

The z-pinchof model is used to calculate finite Larmor radius effects of a plasma with a hot component plasma annulus. The equations are analyzed for layer modes and the finite Larmor radius stabilization condition is calculated. Stability requires $k^2 \rho_h^2 R_\delta / \Delta \geq 1$, where $k$ is the wavenumber in the z-direction, $\rho_h$ the hot species Larmor radius, $\beta_h$ the hot particle beta and $\Delta$ the thickness of the pressure profile. In addition a new instability is found, caused by the interaction of the precessional modes associated with inner and outer edges of the hot particle pressure profile.
I. Introduction

We investigate finite orbit effects on long wavelength, curvature driven modes for a plasma with a hot component.\textsuperscript{1-6} Our method is to construct a variational form for an axisymmetric, linear pinch using the Vlasov and Maxwell equations. Specializing to flute instabilities of a z-pincho, we introduce the orbit expansion, and recover results of zero-Larmor radius\textsuperscript{5} theory and finite-Larmor radius\textsuperscript{7,8} theory in the eikonal approximation. New analysis is presented to describe the finite-Larmor radius effects on long wavelength layer modes. One result of the analysis is that the finite Larmor radius terms become small for long axial wavelengths if the perturbed magnetic field, $\delta B$, is proportional to the radial gradient of the unperturbed field, $\delta B/\delta r$, just as is in the case for long wavelength modes in the conventional magnetohydrodynamic (MHD) ordering.\textsuperscript{9} For these modes stability will arise if $k^2 \rho_h^2 R/\Delta_B \geq 1$, where $k$ is the wave number in the $z$-direction, $R$ the curvature radius, and $\Delta_B$ the magnetic field gradient in the hot pressure region.

In addition this paper improves on the analysis of layer modes,\textsuperscript{6,10,11} which is here calculated for arbitrary beta rather than for $\beta_h < 1$, where $\beta_h$ is the beta of the hot component. The improved dispersion contains a new mode of instability. For a thin annulus of hot particles the new mode is caused by the precessional oscillation of the plasma currents from the negative pressure gradient region interacting with the precessional oscillation from the positive pressure gradient region.

In Sec. II the derivation of the finite Larmor radius equations is presented. In Sec. III, these equations are analyzed for radially long
wavelength modes assuming $k\Delta < 1$, where $\Delta$ is the thickness of the hot plasma annulus. In Sec. IV an interpretation of these results for realistic geometry is given.

II. Derivation of Equations

We begin by constructing a quadratic form that embodies the equation of motion.

$$\delta W = \int dr \frac{E^+}{i\omega} \cdot (\nabla \times B_1 - J_1(E)),$$

where $E^+$ is the adjoint and $J_1(E)$, the perturbed current, is found from the Vlasov equation. We use the following representation for $E$

$$E = -v\phi+i\omega A + i\omega b\times v a,$$

where $b = B/|B|$ and $\phi$, $A$, $a$ are all proportional to $\exp[-i\omega t + ikz + i\psi]$ for a linear, axisymmetric pinch. The perturbed current $J_1$ is simply

$$J_1 = \int dv \, ev f_1,$$

where a sum over species is implied and $f_1$ is the solution to

$$\left. \frac{d}{dt} \right|_0 f_1 = -\left. \frac{d}{dt} \right|_0 f_0, \quad \frac{d}{dt} = \frac{\partial}{\partial t} + \nabla \cdot v + \frac{e(E+\gamma B) \cdot \partial / \partial v}{m}.$$

$f_0$ is a function of $E$, $P_z$, $P_\psi$ where $E = \gamma v^2/2$, and $P_z$, $P_\psi$ are the canonical moments in the $z$ and $\psi$ directions respectively. The zeroth
order electrostatic potential is assumed to be zero. After considerable algebra we find

\[ \frac{\delta W}{\pi L_z} = \int B_1^+ B_1^- \, rdr \]

\[ -e^2 \int r dr d\varphi \left( -\frac{1}{r} + \frac{1}{r^2} \right) \sum \frac{\xi_n \lambda^+ \lambda}{\omega-k_z \bar{v}_z - k_\delta - n\Omega} \]

(5)

where

\[ \lambda = rf_{P_a} \hat{z} + f_{P_z} \hat{z} \]

\[ s = \hat{r} \times \hat{b} \]

\[ \xi_n = \frac{1}{T} \int_0^T \exp \left[ -i \Phi_0 + ik_\delta \xi \right] \left( \theta - r \cdot A - r \cdot b \right) |_{r=r(\tau)} \]

\[ T = \frac{2\pi}{\Omega} = \int d\tau = \int \frac{dr}{|v_r|} \]

\[ z(\tau) = z_o + \bar{v}_z \tau + \delta z(\tau) \]

\[ \psi(\tau) = \psi_o + \delta \tau + \delta \psi(\tau) \]

\[ L_z \equiv \text{axial system length} \]

Use of the time-reflection symmetry of the orbits,
\[ r(\tau) = r(-\tau), \quad v_r(\tau) = -v_r(-\tau), \quad \dot{\phi}(\tau) = \dot{\phi}(-\tau), \quad v_z(\tau) = v_z(-\tau) \]

yields a simple relation between solutions of the adjoint operator

\[ \Phi = \Phi(r) \exp[i(kz + \ell \phi - \omega t)] \]

\[ \Phi^* = \Phi(r) \exp[-i(kz + \ell \phi - \omega t)] . \] (6)

Identical relations hold for \( a \) and \( A \). In terms of the \( r \) dependent amplitudes \( \Phi(r), a(r), A(r) \), the quadratic form is then self-adjoint.

For flute modes in a \( z \)-pinch \( (\varepsilon = \frac{1}{2}, \lambda = 0) \), Eq. (5) simplifies to

\[
\frac{\delta W}{\pi L_z} = \int rdr B_1^2 e^{2} dv \left[ (\Phi^2 + 2v_z a_r \Phi) f_c - v_z f_p a_r^2 \right. \\
- \left. \sum_{n=-\infty}^{\infty} \frac{(\omega f_c + kf_p)_{z}}{\omega - k\nu - n\Omega} (\Phi + v_z a_r - ikv_r a_r^2)_{n}^2 \right], \] (7)

where \( B_1 = a_r - k^2 a \) in this limit. A more useful form of Eq. (7), suitable for applying a finite Larmor radius expansion, can be obtained through the use of several identities:

\[ v_z = -\frac{d}{dt} \left( \frac{\Phi}{\Omega} \right) + v_D \] (8a)

\[ v_r = \frac{\dot{v}_z}{\Omega} \] (8b)

\[ \int v_r^2 g_p dv_z = \int v_z g dv_z \] (8c)

\[ \int v_z g_r dv_r = \int v_D g_r dv_r = -\int g_p dv_z \] (8d)
where \( \Omega_c = eB(r)/mc \) and \( v_D = v_B + v_{r\|}, \quad v_B = -\frac{v_{r\|} B_r}{\Omega_c}, \quad v_{r\|} = \frac{v_{\phi}^2}{\Omega_c r} \) and \( g \) is any function of \( \epsilon, P_z \) and \( P_\phi \). Equations (8a,b) follow exactly from the unperturbed orbit equations. We also use the identity:

\[
\sum_n \frac{i_n^2}{\omega - kv - n\Omega} = -\frac{1}{iT} \oint \frac{d\tau}{0} \xi(\tau) \xi'(\tau - \tau) \exp[i(\omega - kv)\tau'], \quad (9)
\]

where \( \xi = (4 + v_z a_r + ikv_r a) \exp(-ik\delta z) \). For manipulations of Eq. (9) a useful relation is

\[
\oint d\tau \oint d\tau' \alpha(\tau) \beta(\tau - \tau) \exp[i(\omega - kv)\tau'] = \oint d\tau \oint d\tau' \beta(\tau) \alpha(\tau - \tau) \exp[i(\omega - kv)\tau'], \quad (10)
\]

for any periodic functions \( \alpha, \beta \) which is easily shown through a change of variables in the integration. Noting that \( \int r dr dv \) and \( m^{-3} \int d\epsilon d\phi d\theta \) can be used interchangeably in Eq. (7), we obtain the following after use of the identities (8)–(10) and integrations by parts:

\[
\frac{\delta W}{\pi L_z} = \int r dr [B_1^2 + e^2 \int d\epsilon \epsilon^2 \frac{\omega^2}{k^2} \xi^2 B_2^2] + \\
- \frac{(\omega \xi + k f_p_z)}{ik^2} \int d\tau' \Lambda(\tau) \Lambda(\tau - \tau) \exp[i(\omega - kv)\tau']) \right], \quad (11)
\]

where \( \Lambda = (\omega \xi B + iv_r B_1) \exp(-ik\delta z) \) and the radial displacement \( \xi = -\frac{1}{B}(k\phi + a_r) \) has been introduced.

To proceed further we introduce the orbit expansion \( \delta = \rho/\Delta << 1 \), where \( \Delta = \text{scale length} \). The frequency \( \omega \) is assumed to be small
[\omega_0 = \vartheta(\delta^2)]. In the low frequency limit, the integral in Eq. (11) can be evaluated to \( \vartheta(\delta^3) \) (the necessary order to pick up the FLR corrections) by expanding the exponential \( \exp[i(\omega-k\vec{v})\tau'] \approx 1 + i(\omega-k\vec{v})\tau' \) when it multiplies an oscillatory term. We find

\[
\int_0^\infty d\tau' \Lambda(\tau) \Lambda(\tau'-\tau) \exp[i(\omega-k\vec{v})\tau'] = \int_0^\infty d\tau \left( \frac{\Lambda_2}{-1(\omega-k\vec{v})} - 2\Lambda_{\text{even}} \int_0^\tau d\tau' \Lambda_{\text{odd}} + i(\omega-k\vec{v})[\left(\int_0^\tau d\tau' \Lambda_{\text{odd}}\right)^2 - \left(\int_0^\tau d\tau' \Lambda_{\text{even}}\right)^2] \right), \tag{12}
\]

where \( \Lambda = \frac{1}{T} \int d\tau \Lambda(\tau), \Lambda = \Lambda - \Lambda, \Lambda_{\text{even}}(-\tau) = \Lambda_{\text{even}}(\tau) \) and \( \Lambda_{\text{odd}}(-\tau) = -\Lambda_{\text{odd}}(\tau) \). Expanding \( \Lambda \) to \( \vartheta(\delta^3) \) and inserting it into Eq. (12) yields the desired quadratic form containing the FLR corrections. To simplify the analysis we evaluate the FLR contributions for the hot species in the limit \( \omega, kv_{\kappa_1} \ll k\vec{v}_B \left( \nu_{\kappa_1} = \frac{\nu_{\lambda}}{2M}, \nu_B = \frac{-\nu_{\lambda}^2}{2M} \right) \) since this is the limit where the FLR terms are most significant. We also assume that the background ion species dominates the inertia \( (m_1n_1 >> m_nn_h) \) and satisfies the condition \( \omega >> kv_{B_1} \). The result then is

\[
\frac{\delta W}{nL_z} = \int d\tau \left\{ (1+G_1)(B_1 + \xi B')^2 + \frac{2(1-G_2)}{r} \xi B(B_1 + \xi B') + (P'' - BB' + G_3 B^2/r) \right\} \xi^2/r

- n_i m_i \omega(\omega-\omega^*) \left( \xi^2 + \frac{1}{k^2 B^2} \left[ (\xi B)' + B_1 \right]^2 \right)

+ \rho_h^2 \left( B' \left( \frac{B_1}{B'} \right)^2 + k^2 B_1^2 \right), \tag{13}
\]
where

\[
\omega^* = k f \frac{mv_z^2}{2} f_{Pz} \ \text{d}v
\]

\[
G_1 = -\sum_i e^2 \int d\mathbf{v} \left( \frac{\omega f_{\mathbf{v} + k f_{\mathbf{p}_z}}}{\omega - k\mathbf{v}} \right) v_B^2 \mathbf{v}_B^2 \frac{B^2}{B'}^2
\]

\[
G_2 = -\sum_i \frac{m^2}{B^2} \int d\mathbf{v} \left( \frac{\omega f_{\mathbf{v} + k f_{\mathbf{p}_z}}}{\omega - k\mathbf{v}} \right) v^2 v_z \frac{v^2 v_z}{2}
\]

\[
G_3 = -\sum_i \frac{m^2}{B^2} \int d\mathbf{v} \left( \frac{\omega f_{\mathbf{v} + k f_{\mathbf{p}_z}}}{\omega - k\mathbf{v}} \right) v^4 + kf_{\mathbf{p}_z} v^3
\]

\[
\rho_h = \frac{-m^2 h}{B^2} \int d\mathbf{v} \frac{f_{P_z}}{v_B} \frac{v^6}{16\mathbf{v}_0^2}
\]

The quantities \(G_{1,2,3}\) summed over species, are the same as defined in Ref. 6. Note that \(G_1\) is defined so that in the low frequency, weak curvature limit, the \(v_B\) terms cancel in the numerator and denominator. The leading terms in \(G_1\) are then the same as found in the zero-orbit theory:

\[
G_1 = e^2 \int d^3v \ f_{P_z} \ v_B \ B^2 \ B' \ B = -ef d^3v \ \frac{mv^2 f_{P_z}}{B'}
\]

\[
= -\frac{J_{z hot}}{B'} = -1 - \frac{B_0}{rB'} + \frac{J_{z cold}}{B'}
\]

\[14\]

Since the FLR corrections are neglected on terms proportional to \(\omega\) or
the curvature $\kappa = -1/r$, the only FLR modification of the zero-orbit result is the term proportional to $\rho_h^2$. Details of the derivation of the FLR correction are included in the appendix. The eikonal results for the FLR correction can be recovered by letting

$$\rho_h^2 \left( \frac{B_1}{B'} \right)^2 + k_1^2 \rho_h^2 B_1^2$$

in Eq. (13). An interesting property of the FLR term is that it vanishes in the limit $k^2 \to 0$ if $B_1 \propto B'$. Here $B_1 \propto B'$ corresponds to a rigid motion of the entire plasma annulus, so it is physically plausible that the FLR effect vanishes for this case.

The Euler equations resulting from Eq. (13), found by varying $\delta W$ with respect to $\xi$ and $B_1$, are:

$$B'(1+G_1)(B_1+\xi B') + \frac{(1-G_2)}{r} B(B_1+\xi B')$$

$$+ \left( P_{||} - G_2 \rho B^2 + G_3 B^2 / r \right) \xi / r - n_1 m_1 \omega(\omega-\omega^*) \xi$$

$$+ \frac{1}{r} \left( \frac{n_1 m_1 \omega(\omega-\omega^*)}{k^2 B^2} \right) ((\xi B)' + B_1) = 0$$

(15)

$$B'(1+G_1)(B_1+\xi B') + \frac{(1-G_2)}{r} B - \frac{n_1 m_1 \omega(\omega-\omega^*)}{k^2 B^2} ((\xi B)' + B_1)$$

$$- \frac{1}{r} \left( r \rho_h^2 B_1^2 (B_1/B') \right)' + k_1^2 \rho_h^2 B_1 = 0$$

(16)

If the FLR terms can be neglected, Eq. (16) becomes an algebraic equation for $B_1$. Inserting the zero-FLR solution for $B_1$ into Eq. (15)
yields Eq. (39) of Ref. 6 in the limit $\omega/\Omega_{ci} \ll 1$. (Reference 6 neglects $\omega_1^*$. In the finite FLR case, Eqs. (15) and (16) must be solved as two coupled second order differential equations.

Equation (16) shows the increased significance of the FLR correction in the weak curvature-low frequency limit. Since the term $(1+G_1) = \theta(\kappa)$, the FLR term is the same order as the other terms in Eq. (16) when $\rho_h^2/\Delta^2 \sim \kappa \Delta_B$ ($\Delta$ = pressure gradient scale length, $\Delta_B$ = magnetic field scale length $\equiv \Delta/\beta$). A limit of interest is $1 \gg \rho_h^2/\Delta^2 \gg \kappa \Delta_B$, the "strong FLR" limit, in which case the FLR terms dominate Eq. (16). Both the strong and weak FLR limits will be analyzed in the next section.

III. Long-wavelength modes

A tractable limit in which Eq. (15) and (16) can be analyzed is the case $k \Delta \ll 1$, i.e., long axial wavelengths. This case is of special interest in the strong FLR limit when FLR can stabilize all of the short wavelength curvature driven modes. We will consider both the strong and weak FLR limits since the latter has not been carried out for arbitrary $\beta$.

We will examine stability for equilibria of the type shown in Fig. 1, i.e., a hot plasma annulus centered about radius $R$ with a background of cooler plasma having a finite pressure gradient only in the outer half of the annulus. This is the model used in Ref. 6 to simulate an Elmo Bumpy Torus (EBT) type configuration. Similar results can be obtained for other profiles, such as disc-like hot plasma distributions to model tandem mirrors or other devices. For simplicity, we assume that $f(\varepsilon,P_\phi,P_z) = g(\varepsilon,P_z) \delta(P_\phi)$ so $G_2 \equiv G_3 = P_\parallel = 0$
and set $\omega^*=0$ for the background plasma. Equations 15 and 16 then become

$$B'(1+G_1)(B_1 + \xi B') + \frac{B}{r} (B_1 + \xi B') - n_1 m_1 \omega^2 \xi + B \phi' = 0 \quad (17a)$$

$$\left(1 + G_1\right)(B_1 + \xi B') + \frac{\xi B}{r} - \phi - \frac{1}{B'} \left[\rho_h^2 \xi^2 \left(\frac{B_1}{B}\right)\right]' + k^2 \rho_h^2 B_1 = 0 \quad (17b)$$

where the quantity $\phi \equiv \frac{n_1 m_1 \omega^2}{k^2 B^2} \left[(\xi B)' + B_1\right]$. In the long wavelength limit, two types of modes are generally found: $\omega = \gamma_{\text{MHD}}$ or $\omega = \omega_{k\perp}$ where $\gamma_{\text{MHD}} \equiv \left(\frac{k \rho_h}{n_1 m_1 r}\right)^{1/2}$ and $\omega_{k\perp} \equiv -\langle \frac{k v_{\text{th}}}{r \rho_h} \rangle$. We are also usually interested in the limit $\omega_{k\perp}/\gamma_{\text{MHD}} \gg 1$, one of the decoupling conditions necessary for stability in systems with unfavorable curvature. If we use these estimates for $\omega$ in Eq. (17a) then we find $\phi \sim \varphi(1/k\Delta)$. To lowest order in $k\Delta$, we have

$$\phi \sim (\xi B)' + B_1 = 0 \quad (18)$$

To proceed further analytically, some ordering of the finite Larmor radius terms is necessary.

(a) Weak FLR Limit

Consider first the weak FLR limit where the terms $\propto \rho_h^2$ are neglected. Iterating on the solution for $\phi$ using Eq. (18) in Eq. (17b) gives

$$\phi = -(1+G_1) \xi \frac{B'}{r} + \frac{\xi B}{r} \quad (19)$$
Using Eqs. (18) and (19) in Eq. (17a) gives an equation for $\xi$:

$$
[ (1+G_1) \xi B^2 ]' - \frac{\xi B^2}{r} = 0 .
$$

We are considering the limit $A_B/r \ll 1$, so $r$ is treated as a constant in Eq. (20). A boundary condition on $\phi$ is found from the continuity of $\phi$ at the two edges of the hot particle annulus:

$$
\phi(r^+) = \frac{n_1 m_1 \omega^2}{k B^2} \xi'(r^+) = \frac{n_1 m_1 \omega^2 k \phi(r^+)}{k B^2}
$$

$$
= \phi(r^-) = -(1+G_1) \xi'(r^-) + \frac{\xi(r^-)}{r} ,
$$

where with $\epsilon > 0$,

$$
r^+ = \lim_{\epsilon \to 0} R \pm (A+\epsilon) ,
$$

$$
r^- = \lim_{\epsilon \to 0} R \pm (A-\epsilon) .
$$

The quantity $K$ is precisely determined from the solution outside of the hot particle annulus. If $\omega < kv_A$, $K \equiv |k|$ for the inner edge of the annulus and we assume that $n_1$ vanishes on the outer edge of the annulus.

The quantity $1 + G_1$ is given by

$$
1 + G_1 = - \frac{B}{r B'} \left[ 1 - \frac{\omega}{\omega_{k \perp}} - \Phi \right] ,
$$
where \( \omega_{\kappa l} = \frac{kP'}{m\Omega_{h} \gamma} \) and \( \beta = \frac{-rP'_{c}}{B'^{2}} \)

is the Nelson–Lee–Van Dam parameter.\(^3\) For simplicity we take profiles such that \( \omega_{\kappa l} \) is a constant and \( \beta = 0 \) (i.e., \( P_{c}/B'^{2} \) constant) on the inner half of the annulus and \( \beta \) is a constant in the outer half of the annulus (a ramp–like background pressure gradient). Inserting Eq. (22) in Eq. (20) gives

\[
\left( \frac{B'^{3}}{B'} \left( 1 - \frac{\omega}{\omega_{\kappa l}} - \beta \right) \xi' \right)' + BB' \xi' = 0 .
\]  

(23)

Equation (23) has solutions of the form \( \xi = B^{\lambda} \) with

\[
\lambda = -1 \pm \left( \frac{-\omega}{\omega_{\kappa l} - \beta} \right)^{1/2}
\]

for the outer half of the annulus and

\[
\lambda = -1 \pm \left( \frac{-\omega}{\omega_{\kappa l} + \omega} \right)^{1/2}
\]

for the inner half of the annulus. A dispersion relation is found by requiring continuity of \( \xi \) and \( \left( 1 - \frac{\omega}{\omega_{\kappa l}} - \beta \right) \xi' / B' \) at the midpoint of the annulus and applying continuity of \( \xi \) at the annulus boundaries as well as the boundary conditions from Eq. (21). The result is a transcendental equation for \( \omega \):
\[
\omega^2 = \frac{k^2B^2}{K_n m_1} \frac{(\tilde{\mu} - \mu)}{1 + \tilde{\mu} - \mu + (1 - \omega/\omega_{k\perp})\mu\tilde{\mu}} \tag{24}
\]

where

\[
\mu = \left(\frac{-\omega/\omega_{k\perp}}{1 - \omega/\omega_{k\perp}}\right)^{1/2} \left[\frac{1 - (1 + \beta_h)\left[(-\omega/\omega_{k\perp})/(1 - \omega/\omega_{k\perp})\right]^{1/2}}{1 + (1 + \beta_h)\left[(-\omega/\omega_{k\perp})/(1 - \omega/\omega_{k\perp})\right]^{1/2}}\right]
\]

\[
\tilde{\mu} = \left(\frac{-\omega/\omega_{k\perp} - \beta}{1 - \omega/\omega_{k\perp} - \beta}\right)^{1/2} \left[\frac{1 - (1 + \beta_h)\left[(-\omega/\omega_{k\perp} - \beta)/(1 - \omega/\omega_{k\perp} - \beta)\right]^{1/2}}{1 + (1 + \beta_h)\left[(-\omega/\omega_{k\perp} - \beta)/(1 - \omega/\omega_{k\perp} - \beta)\right]^{1/2}}\right]
\]

and \(\beta_h \equiv 2\mu_{\perp}(R)/B^2(R)\), \(B_\perp = B(R - \Delta)\).

The dispersion relation Eq. (24) simplifies considerably for \(\Delta/R \ll \beta_h \ll 1\). Expanding in \(\beta_h\) yields

\[
\omega^2 = \frac{k^2p_{\perp}(R)}{K_n m_1 R} \frac{\beta(1 + \beta_h)}{(1 - \omega/\omega_{k\perp})(1 - \omega/\omega_{k\perp} - \beta) + \beta_\perp^2/2} \tag{25}
\]

where the finite \(\beta_h\) correction has been kept. Setting \(\beta_h = 0\) recovers the dispersion relation found in Refs. 6 and 8. Further simplifications occur in the strongly decoupled limit \(\omega_{k\perp}^2/\gamma_{\text{MHD}}^2 \gg 1\). In this limit two types of modes are found. The first is the curvature driven interchange mode with
\[ \omega^2 = \frac{k^2 P_{ih}(R)}{Kn_i m_i R} \frac{\beta(1+\beta_h)}{1-\beta(1-\beta_h/2)} \]  

(26)

Finite \( \beta_h \) causes a slight shift of the Nelson-Lee-Van Dam stability condition with \( \beta < 1/(1-\beta_h/2) \) for stability.

The second type of mode in the strongly decoupled limit are the precessional modes associated with the inner and outer halves of the annulus. The result, found to first order in \( \beta_{hot} \) is

\[ \frac{\omega}{\omega_{k\perp}} = 1 - \frac{\beta}{2} \pm \frac{\beta}{2} \left(1 - \frac{2\beta_h}{\beta}\right)^{1/2}, \]  

(27)

showing instability when \( \beta < 2\beta_h \). This new instability is due to finite-\( \beta_h \) induced coupling of the negative and positive energy precessional modes which are nearly degenerate at small \( \beta \). A stabilization mechanism is "detuning" or removal of the near degeneracy through the radial variation of the curvature, so that the precessional modes on the inner and outer parts of the pressure gradient are at different frequencies even when \( \beta=0 \). This detuning mechanism is discussed in Sec. IV.
(b) **Strong FLR Limit**

The other tractable limit for analyzing Eqs. (17a,b) is when the radial FLR term dominates Eq. (17b). When $\rho_h^2/\Delta^2 \gg \kappa\Delta_B$ we have

$$\rho_h^2 B'^2 \left( \frac{B_1}{B'} \right) = 0 \ . \quad (28)$$

The non-singular solutions of Eq. (28) as $B'\to0$, are of the form:

$$B_1 = C\pm B' \ . \quad (29)$$

The ± subscripts on $C$ in Eq. (29) denote the outer (+) and inner (−) halves of the annulus. To see that $B_1/B'$ is allowed to be discontinuous at the point $B'=0$ we consider the variational form Eq. (13). If we approximate the discontinuity by a continuous function between $r = R+\delta$ and $R-\delta$ and then let $\delta\to0$, we find a vanishing contribution to the energy. Combining Eq. (18) with Eq. (29) we find

$$\xi = (\xi\pm C\pm) \frac{B\pm}{B} - C\pm \ , \quad (30)$$

where $\xi\pm$, $B\pm$ are $\xi$, $B$ evaluated at the outer and inner edges of the annulus. These constants are partially determined by the continuity of $\xi$ at the midpoint of the annulus and the inner and outer boundaries. Two more constraints come from annihilating the large radial FLR term in Eq. (17b) by multiplying by $B'$ and integrating from the edge to the center:
\[ \int_R^{R+\Delta} \ln\left(-\frac{BB_1}{r} - B'B'(1+G_1 + \frac{B}{B'})(B_1+B') + k^2\rho_0^2B_1B' - \right) = 0 \quad (31) \]

Integrating the \( \phi \) term by parts and using Eq. (17a) gives the conditions

\[ C_\pm \left[ \int_R^{R+\Delta} dr \left( \frac{BB'}{r} + k^2\rho_0^2B^2 \right) - B\phi \right]_R^{R+\Delta} = 0 \quad (32) \]

From Eq. (17a) we can relate \( \phi(R+\Delta) \) to \( \phi(R) \)

\[ \phi(R+\Delta) = \phi(R) + \left[ \int_R^{R+\Delta} dr \frac{B^2}{B'^2} \left( 1 + \frac{B}{rB'} + G_1 \right) \right]_R^{R+\Delta} \quad (33) \]

Continuity of \( \phi \) at the edges of the annulus gives

\[ \phi(R+\Delta) = 0 \quad (\text{assuming } n_1(R+\Delta) = 0) \quad (34) \]

\[ \phi(R-\Delta) = \frac{n_1m_i\omega^2k\xi_-}{k^2B_-} \]

Combining Eqs. (32) through (34) along with continuity of the midplane from Eq. (30),

\[ (\xi_+ + C_+) \frac{B_+}{B_-} - C_+ = (\xi_- + C_-) \frac{B_-}{B_+} - C_- \quad (35) \]

\((B_+ = B_- \text{ to lowest order in the curvature)} \) gives a dispersion relation:
\[ \omega^2 = \frac{k^2 P_{1h}(R)}{K N_1 m_i} \frac{B_+}{B_0} \frac{\hat{\beta}(1-\lambda)^2}{(1-\lambda-Y)(1-\lambda-Y-\hat{\beta})+(1-\lambda)\hat{\beta}(\frac{B_+}{B_0}-1)} \]  

(36)

where

\[ Y = \frac{B_r B_0}{P_{1h}(R)} \int_R^{R+\Delta} dr \frac{B'}{B} \frac{\omega}{\omega_{k1}} = \frac{\omega}{\omega_{k1}} \text{ if } \beta_h << 1 \]

\[ \hat{\beta} = \frac{B_r B_0}{P_{1h}(R)} \int_R^{R+\Delta} dr \frac{B'}{B} \beta = \beta \text{, if } \beta_h << 1 \]

\[ \lambda = \frac{k^2 r}{P_{1h}(R)} \int_R^{R+\Delta} \rho_{h}^{2} B_r^2 = 3(k^2 \rho_{h}^{2} \frac{r}{\Delta_B}) \]

Note that no special profiles have been assumed. Equation (36) simplifies in the strongly decoupled limit \( \omega_{k1/\gamma_{\text{MHD}}}^2 >> 1 \). As found for the weak FLR case, two types of modes result. The interchange mode dispersion relation is

\[ \omega^2 = \frac{k^2 P_{1h}}{K N_1 m_i} \frac{B_+}{B_0} \frac{\hat{\beta}(1-\lambda)}{1-\lambda-\hat{\beta}(2-\frac{B_+}{B_0})} \]  

(37)

In the low \( \beta_h \) limit and for \( \lambda \to 0 \) Eq. (37) reduces to the low \( \beta_h \) zero FLR case found in Eq. (26). Note that although the radial FLR operator was the dominant term in Eq. (17b), the stability criterion is unchanged by this effect in the limit \( k^2 \to 0 \) (\( \delta \to 0 \)). This is analogous to the stability problem for cylinders with \( B = B_0 \) where the
FLR effect is small if \( m\Delta/r << 1 \) (\( m = \) azimuthal mode number) and nearly rigid displacements of the plasma. One should also note that in a cylinder the FLR effects exactly cancel if \( m=1 \). The case \( \lambda \neq 0 \) is similar to the eikonal result. Stability is assured for all \( \beta \) when \( \lambda > 1 \) \( (\Delta \ll \rho_H^2 \geq \lambda B/\rho) \).

The dispersion relation for the precessional mode in the strongly decoupled limit is

\[
Y = 1 - \lambda \left( 1 - \frac{\hat{R}}{2 \beta} \right) \left[ 1 - 4 \left( \frac{B_{e+}}{B_0} - 1 \right) \left( \frac{1 - \lambda}{\beta} \right) \right]^{1/2}. \tag{38}
\]

At low \( \beta_H \), \( \lambda = 0 \) and for the profile \( \beta = \) piecewise constant, Eq. (38) reduces to Eq. (25), again predicting instability for \( \beta < 2\beta_H \). The axial FLR term again stabilizes this mode when \( \lambda > 1 \).

IV. Discussion

The description of the response of a hot component plasma including finite Larmor radius effects has been developed for the z-pinch model and long wavelength layer modes \((k\Delta < 1)\) have been analyzed. This work improves on past analysis in that we can solve the equations for arbitrary large values of \( \beta_H \) (the beta of the hot component), whereas previous analysis assumed \( \beta_H < 1 \). The resulting dispersion relation gives two types of modes. One is a mode near the precessional frequency and the other is at lower frequency and more closely related to MHD modes.

We find that the finite Larmor radius (FLR) effects change the character of the equations somewhat if \( \beta_H^2 \rho_H^2 R/\Delta^3 > 1 \), but does not
necessarily alter the zero FLR stability picture. The perturbed magnetic field is then constrained to have a piecewise rigid response on each side of the pressure gradient. As a result, previously derived decoupling conditions for the hot particle energy, or the Lee–Van Dam–Nelson critical beta criterion, are only modestly shifted from the results of zero FLR theory if \( k^2 \rho_h^2 R / \Delta < 1 \). However, if \( k^2 \rho_h^2 R / \Delta > 1 \), our theory predicts stability for all modes. In our model we can have \( k = 0 \), and we cannot achieve a robust stability. This is similar to the result for a cylindrically shaped plasma with an imposed curvature, \(-1/R\). In such a case robust stability requires \((m^2 - 1) \rho_h^2 R / r_p^2 \Delta > 1\), where \( m \) is the mode number and \( r_p \) the plasma radius. (This result will be derived in a subsequent paper.) The cylindrical result shows that a robust stability cannot be achieved from the FLR terms for \( m = 1 \). However, it was recently shown that wall effects\(^{11,12}\) can stabilize the \( m = 1 \) of a cylindrical disc-shaped plasma with finite axial extent. However, a thin annulus shaped plasma will not be stabilized in a robust manner.

The annular shaped plasma is subject to the Lee–Van Dam–Nelson critical core beta limit,\(^3,4\) and even when the core beta is below the critical limit the precessional mode can destabilize. Equation (25) describes a new mechanism for the precessional mode to destabilize. If, \( \bar{\beta} = P_c R / B^2 \Delta \ll 1 \), Eq. (25) predicts that the precessional modes associated with the inner and outer parts of the hot particle pressure gradient interact and destabilize. Equation (25) is unstable for sufficiently small \( \bar{\beta} \). In order to observe a threshold, we should take into account that \( \omega_{\perp} \) is slightly different on the inner and outer parts of the layer as \( \xi \) varies with radial position. The precessional
mode in the strongly decoupled limit is found by setting the
denominator of Eq. (25) to zero. Taking into account the shifts in
\( \omega_{\perp} \), the dispersion relation then becomes

\[
[\omega_{\perp}(r_0) - \omega] [\omega_{\perp}(r_1)(1 - \bar{\beta}) - \omega] = -\omega_{\perp}(r_1)\omega_{\perp}(r_0)\beta_h \bar{\beta}/2 ,
\]

(39)

where \( r_0 \) is the center of the inner half of the layer and \( r_1 \) the center
of the outer half of the layer. The stability condition, assuming \( \beta_h \),
\( \Delta r < 1 \), is then,

\[ \frac{\Delta \omega_{\perp}}{\omega_{\perp}} > \bar{\beta} + (2\beta_h \bar{\beta})^{1/2} , \text{ if } \frac{\Delta \omega_{\perp}}{\omega_{\perp}} > 0 \]

\[ \bar{\beta}_h + \left| \frac{\Delta \omega_{\perp}}{\omega_{\perp}} \right| > (2\beta_h \bar{\beta})^{1/2} , \text{ if } \frac{\Delta \omega_{\perp}}{\omega_{\perp}} < 0 , \]

(40)

where \( \Delta \omega_{\perp} = \omega_{\perp}(r_1) - \omega_{\perp}(r_0) \). In a realistic plasma configuration \( k=\pi r \),
where \( r \) is the plasma radius, and therefore \( \Delta \omega_{\perp}/\omega_{\perp} > 0 \). Thus, with a
finite width annulus, the new precessional instability can be averted
at small values of \( \bar{\beta} \), but should eventually appear when \( \bar{\beta} \geq \Delta \omega_{\perp}/\omega_{\perp} \). In
addition, a recent calculation\(^{13} \) that accounts for line--bending in an
axially dependent model shows that stability of the coupled
precessional mode arises if, roughly, \( \beta_h |m| < 1 \), where \( m \) is the mode
number.

It should be noted that our detailed calculation assumed that the
hot pressure gradients were of equal magnitude on either side of the
annulus. One can readily show that if one has a skewed hot pressure
profile, such that \( k\Delta^{-1}_{\max} < 1 \), where \( \Delta^{-1}_{\max} \) is the smaller pressure
gradient, that our results are essentially unchanged. The main
assumption is that there is a hot plasma annulus such that the hot
pressure goes to zero on either side of the annulus. The FLR term, in Eq. (36) needs to be modified to differentiate between the appropriate pressure gradients on either side on the layer, but otherwise there is no essential difference from the results of the text as a result of skewness in the pressure profile. However, somewhat different conclusions would arise if $\kappa \Delta b \geq 1$, a case that violates the fundamental ordering of this paper. For present day experiments $\kappa \Delta b \approx 1$ may be appropriate. This possibility will be treated in forthcoming investigations.
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Appendix: Derivation of the FLR Corrections

In the section we include some of the detailed calculations leading to the FLR corrections, starting with Eq. (12) and employing the orbit expansion. As mentioned in the text, the corrections are evaluated in the low frequency, weak curvature limit.

The FLR corrections appear only for terms involving the perturbed magnetic field $B_1$, so here we will consider only the $B_1$ terms in Eq. (12). The other terms, after relatively straightforward algebra, lead to forms derived previously, and will not be discussed in detail.

Defining $\lambda = i v_r B_1 e^{-i k \delta z}$, we need to calculate

$$\int_0^\infty d\tau' \lambda(\tau) \lambda(\tau'-\tau) \exp[i(\omega-k\nu)\tau']$$

$$\approx \int_0^\infty d\tau \left\{ \frac{\lambda^2}{-i(\omega-k\nu)} - 2\kappa_{even} \int_0^\infty d\tau' \kappa_{odd} \right\}$$

$$\quad + i(\omega-k\nu) \left[ \left( \int_0^\infty d\tau' \kappa_{odd} \right)^2 - \left( \int_0^\infty d\tau' \kappa_{even} \right)^2 \right] + \frac{(\omega-k\nu)^2}{\nu^2}$$

where $\lambda = \int \frac{d\tau' \lambda}{T} \kappa = \lambda - \bar{\lambda}$. Expanding the exponent in the expression for $\lambda$ through third order, we have

$$\lambda = i v_r B_1 + k v_r \delta z B_1 - \frac{ik^2}{2} \delta z v_r B_1 - \frac{v_r \delta z^3}{6} k^3 B_1.$$  \hspace{1cm} (A.2)

From Eq. (A.2) we have
\[ \lambda^2 = k v_r \delta z B_1^2 - \frac{1}{3} k^4 B_1^2 \frac{v_r \delta z}{v_r \delta z^3}. \] (A.3)

In the second term we can use \( v_r = v_1 \sin \Omega_c \), \( \delta z = -\frac{v_r}{\Omega_c} \) to sufficient accuracy giving:

\[ \lambda^2 = k v_r \delta z B_1^2 - \frac{k^4 v_1^6}{16 \Omega_c^4} B_1^2. \] (A.4)

\( \delta z \) is needed to higher accuracy in the first term. From Eq. (8a) we have

\[ \delta z = \int d\tau (v_z - \bar{v}) = -\frac{v_r}{\Omega_c} + \int d\tau (v_D - \bar{v}). \] (A.5)

Using Eq. (A.5) we find

\[ k v_r \delta z B_1 = -\frac{k v_r^2}{\Omega_c^2} B_1 + k v_r \int d\tau (v_D - \bar{v}) B_1. \] (A.6)

To evaluate the second term we use \( v_r = \frac{\dot{v}_z}{\Omega_c} \)

\[ k v_r \int d\tau (v_D - \bar{v}) B_1 = k t_z \int d\tau (v_D - \bar{v}) B_1 = + k v_z v_r \int d\tau (v_D - \bar{v}) \left( \frac{B_1 B'}{\Omega_c B} - \frac{B_1'}{\Omega_c} \right). \]

\[ -\frac{k v_z}{\Omega_c} (v_D - \bar{v}) B_1. \] (A.7)

Using \( v_z = \frac{-\dot{v}_r}{\Omega_c} \) to lowest order in the curvature
\[ \frac{k v_r}{d \tau (v_D - \bar{v})} = \frac{k v_z v_r}{n_c} \int d \tau (v_D - \bar{v}) \left( B_1 B'/B - B_1 \right) + \frac{k v_r}{n_c^2} \left( \frac{v_r^2}{n_c} - B' - \bar{v} \right) B_1 \]

\[ = -\frac{k v_r}{16 n_c^3} \left[ B_1 \left( \frac{B'}{B^2} - B_1 \right) - 4B' \left( B_1 \right)' B^2 \right] - \frac{k B'B_1}{n_c B} \left( \frac{v_r^3}{3} \right) \]

\[ = -\frac{k v_r}{16 n_c^3} \left( B_1 \left( \frac{B'}{B^2} - \frac{2B''}{B} \right) + B_1 \frac{B'}{B} \right) . \quad (A.8) \]

Combining results gives

\[ \lambda^2 = \left( \frac{k v_r^2 B_1}{n_c^2} \right)^2 + \frac{k^2 + 5B_1}{16 n_c^4} \left[ B_1 \left( \frac{B'}{B^2} - \frac{2B''}{B} \right) + B_1 \frac{B'}{B} \right] - \frac{k^4 v_r^2 B^2}{16 n_c^4} . \quad (A.9) \]

The first term in Eq. (A.9) gives the term \( \propto G_1 \) in \( \delta W \) to lowest order in the orbit expansion. To extract the \( G_1 \) term we define \( v_B = -v_r^2 B'/n_c B \) and we have,

\[ \left( \frac{k v_r^2 B_1}{n_c^2} \right)^2 \equiv \left( k v_B B_1 \right)^2 \]

\[ = v_B k^2 v_B B_1 B_1 + k^2 v_B B_1 \left( v_B B_1 - v_B B_1 \right) . \quad (A.10) \]

The first term gives the \( G_1 \) term defined in the text. Inserting the orbit expansion into the second term gives
\[ \frac{\nu_B^2 B_{11}^2}{\eta_c^2} = \nu_B k^2 v_B \frac{B_{11}^2}{B^2} - \frac{k^2 v_1^6}{16 \eta_c^4 B^2} \left( \frac{B_{11}}{B} \right)'^2 \]  
\( A.11 \)

Thus,

\[ \lambda^2 = \nu_B k^2 v_B \frac{B_{11}^2}{B^2} - \frac{k^2 v_1^6}{16 \eta_c^4} \left[ B' \left( \frac{B_{11}}{B} \right)'^2 + B_{11} B' + \frac{k^2 B_{11}^2}{B} \right] \]  
\( A.12 \)

Next we need to evaluate the term:

\[ \frac{d \tau}{\nu_B} \int d\tau' \left[ \frac{d \tau}{\nu_B} \right] = \frac{d \tau}{\nu_B} \int d\tau' \left[ \frac{d \tau}{\nu_B} \right] \]

\[ \approx \frac{d \tau}{\nu_B} \int d\tau' \nu_B \delta z B_{11} \]  
\( A.13 \)

to sufficient accuracy. Using the orbit expansion in the right-hand side of \( A.13 \) we find

\[ \frac{d \tau}{\nu_B} \int d\tau' \nu_B \delta z B_{11} = 2ikB_{11}'(\bar{r}) \frac{d \tau}{\nu_B} \int d\tau' \nu_B \delta z \nu_B \]

\[ + \nu_B \int \nu_B \delta z \delta r d\tau' \]

\[ + 2ikB_{11}^2(\bar{r}) \frac{d \tau}{\nu_B} \int \nu_B \delta z \delta r d\tau' \]  
\( A.14 \)

where \( r = \bar{r} + \delta r \), \( \bar{r} \) = mean radius. The term \( \approx B_{11}' \) in \( A.14 \) can be evaluated using zero-order orbits:

\[ 2ikB_{11}' \frac{d \tau}{\nu_B} \int \nu_B \delta z \delta z + \nu_B \int \nu_B \delta z \delta r d\tau' \approx 2ikB_{11}' \frac{\nu^4}{16 \eta_c^3} \]  
\( A.15 \)
The term $\propto B_1^2$ requires greater care:

$$2ikB_1^2 \int d\tau \left( \nu_r \int d\tau' \nu_r \delta \tau \right) = 2ikB_1^2 \int d\tau (\nu_r \delta \tau)$$

$$= 2ikB_1^2 \int d\tau \frac{\delta r^2}{2} \frac{\delta \nu_r}{\delta \tau} = 2ikB_1^2 \int d\tau \left[ -\frac{d}{d\tau} \left( \frac{\nu_r}{\Omega_c} \right) + \nu_D \right] \frac{\delta r^2}{2}$$

$$= 2ikB_1^2 \int d\tau \left[ \frac{\nu_r}{\Omega_c} + \frac{1}{2} (\nu_D \nu_r) \delta \tau \right] \quad (A.16)$$

where several integrations by parts have been used. We need $\delta \tau$ to sufficient accuracy to evaluate (A.16)

$$\nu_r = \frac{\dot{v}_z}{\Omega} = \frac{d}{d\tau} \left( \frac{v_z}{\Omega} \right) - v_z \left( \frac{1}{\Omega_c} \right) = \frac{d}{d\tau} \left( \frac{v_z}{\Omega} \right) + \frac{v_z v_r B'}{\Omega_c B}.$$ 

Then,

$$\delta \tau = \frac{v_z}{\Omega_c} + \int d\tau' v_z \frac{v_r B'}{\Omega_c B} = -\frac{\dot{v}_r}{\Omega_c} + \int d\tau' v_z v_r \frac{B'}{\Omega_c B}. \quad (A.17)$$

Hence, we have

$$2ikB_1^2 \int d\tau \left( \nu_r \int d\tau' \nu_r \delta \tau \right) = 2ikB_1^2 \int d\tau \left[ -\frac{d^3}{d\tau^3} \frac{v_r^3}{3} \frac{1}{\rho^3} + \frac{v_r^2}{\Omega} \int d\tau' v_z v_r \frac{B'}{\Omega B} + \frac{1}{2} (\nu_D \nu_r) \delta \tau \right]. \quad (A.18)$$

Integrating once more by parts:
\[ 2i \kappa B_1^2 \delta \tau (v_r \int \tau \, dr' \nu_r \delta z) = 2i \kappa B_1^2 \delta \tau \left[ -\frac{v_r^4}{\Omega_c^2} \frac{B'}{B} + \right. \]

\[ \frac{v_r^2}{\Omega_c} \int \tau \, dr' \frac{v_z v_r B'}{\Omega_c B} + \frac{1}{2} (v_D - \nu) \delta r^2 \right] . \] \tag{A.19}

The right-hand side of (A.19) can be evaluated using zero-order orbits:

\[ 2i \kappa B_1^2 \delta \tau (v_r \int \tau \, dr' \nu_r \delta z) = 2i \kappa B_1^2 \left( -\frac{v_r^4}{8 \Omega_c^3} \frac{B'}{B} \right) . \tag{A.20} \]

Combining with (A.15) we have

\[ -2 \delta \tau \chi_{\text{even}}^r \frac{\delta \tau \chi_{\text{odd}}}{\delta \tau} = \frac{i \kappa v_r^4}{8 \Omega_c^3} \left( B_1 B_1' + 2 B_1^2 \frac{B'}{B} \right) \]

\[ \frac{k^2 v_r^6}{16 \Omega_c^4} \left( \frac{1}{i \kappa v_B} \right) \]

\[ = \frac{k^2 v_r^6}{16 \Omega_c^4} \left( \frac{2 B_1^2 B_1'}{B^2} + B_1 B_1' \frac{B'}{B} \right) \] \tag{A.21}

where we have written the result divided by \( i \kappa v_B \) in order to combine with the FLR contributions from the \( \chi^2 \) term (\( \omega - i \kappa \nu \) is replaced by \( -i \kappa v_B \) in the FLR terms in the low-frequency, weak curvature limit).

Finally, we need the terms

\[ i (\omega - k \nu) \left[ \left( \int \tau \, dr' \chi_{\text{odd}} \right)^2 - \left( \int \tau \, dr' \chi_{\text{even}} \right)^2 \right] \approx -i \kappa v_B \left[ \int \tau \, dr' iv_r B_1 \right]^2 . \] \tag{A.22}
The right-hand side of A.22 can be evaluated using zeroth order orbits

\[- \int d\tau ikv_B \left[ \int d\tau' iv_B \right]_2 = i kv_B \delta r^2_B = - \frac{k^2 v_B^6}{16 \Omega_c^4} \left( 2 B_1^2 \frac{B'^2}{B^2} \right). \tag{A.23} \]

Combining results from (A.12), (A.21) and (A.23) we find

\[ \int_{0}^{\infty} d\tau' \lambda(\tau) \lambda(\tau' - \tau) \exp(i(\omega - k\nu)\tau') \]

\[ = \int d\tau \left( \frac{v_B k^2 v_B B_1^2}{B^2} \right) \frac{B'^2}{B^2} \]

\[ = \int d\tau \left( \frac{v_B}{-i(\omega - k\nu)} \right) \frac{k^2 v_B^6}{16 \Omega_c^4} \left[ B'^2(B_1)^2 + k^2 B_1^2 \right]. \tag{A.24} \]

The first term on the right-hand side gives the \( G_1 B_1^2 \) term appearing in \( \delta W \) defined in Eq. (13) of the text. The remaining terms are the FLR correction.
References


Figure Caption

Figure 1 - Radial pressure and magnetic field profiles for a hot plasma annulus containing core pressure.