Wave localization and density bunching in pair ion plasmas

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By investigating the nonlinear propagation of high intensity electromagnetic (EM) waves in a pair ion plasma, whose symmetry is broken via contamination by a small fraction of high mass immobile ions, it is shown that this new and interesting state of (laboratory created) matter is capable of supporting structures that strongly localize and bunch the EM radiation with density excess in the region of localization. Testing of this prediction in controlled laboratory experiments can lend credence, *inter alia*, to conjectures on structure formation (via the same mechanism) in the MEV era of the early universe. © *2008 American Institute of Physics*. [DOI: 10.1063/1.3005382]

The physics of pair plasmas was turned into an even more exciting field of investigation when it descended from its astrophysical heights to the terrestrial laboratory. The story of laboratory pair plasmas had its defining moment with the successful creation of a "sufficiently" dense pair-ion (pi) plasma, consisting of equal-mass, positive and negative fullerene ions (C_{60}^{+} and C_{60}^{-}).[1] Unlike the electron-positron (e-p) plasma systems (both of the astrophysical[2–7] and laboratory[8] variety), the fullerene plasma has a long enough lifetime that the collective behavior peculiar to the plasma state can be experimentally investigated under controlled conditions.

The dynamics of pair plasmas is expected to be different from the standard electron-ion plasma where the different mass of the species, automatically breaks the symmetry between the constituents. On the other hand, if the pair plasmas are prepared in identical conditions, they must remain symmetric, for example, their thermal speeds and temperatures are likely to be similar.

This is indeed the case, as evident from the tremendous surge in theoretical activity[9–14] to interpret and understand the experimental results.[1] Most of these papers attempt to explain the experimental findings in terms of the linear and nonlinear properties that may be accessible to only pure pi plasmas. A somewhat different class of phenomena may occur when the PI plasma creation is accompanied by a significant fraction of free electrons; specific linear modes may develop in such plasmas.[11,12]

The aim of this paper, however, is totally different. We wish to propose, here, experiments that explore phenomenon that are associated with slightly contaminated plasmas, where the symmetry is broken, say by the presence of a small amount of heavier ions (heavier than the ions that constitute the main plasma). Our motivation is twofold: (1) to study, per say, a possible nonlinear bunching of electromagnetic (EM) waves through this new and exciting state of matter whose composition may be highly controllable (naturally at some future date), and (2) to create (from this restricted perspective) an approximate replica of the cosmic plasma of the MEV era which consists primarily of electrons and positrons with a small concentration of symmetry-breaking ions.[2,3,7] The hope is that the laboratory experiment will show that, in a minimally contaminated pair plasma, electromagnetism can provide mechanism for density bunching lending further credence to the idea of the electromagnetic origin of the large scale structure of the universe.

It is important to stress that, although cosmic considerations do provide a motivation, the principal intent of this paper is to investigate nonlinear phenomena in this potentially very interesting, versatile, and controllable new state of matter. Because of relatively low densities and heavier mass of the species, the corresponding frequencies and scale lengths will be in a novel range quite different from the e-p plasma counterpart. One of the challenges, therefore, will be to derive laboratory conditions suitable for the observance of bunching of the EM waves. Similar behavior could be expected in doped (or dust-contaminated) fullerene plasmas in laboratory.[7]

The fullerene plasmas of massive ions, however, are not quite suitable for this experiment since all the frequencies associated with the collective modes (plasma frequency, acoustic and Alfvén frequencies) tend to be rather low. Fortunately, the group of Hatekayama and Oohara have already made considerable progress in the production of the hydrogen, H^+–H^- plasmas.[15,16] Since the initial report, both the quality and quantity of this light pair ion plasma has been steadily improving. We will not dwell on how the H^+–H^- plasmas are or will be produced and diagnosed (see Refs. 15 and 16). We will simply assume that, in a not too distant future, it will become possible to "dial" in an H^+–H^- plasma with densities sufficiently "high" that collective modes can be experimentally excited and studied.[17]

The main thrust of this paper is to investigate the nonlinear interactions of EM waves with a primarily H^+–H^- plasma containing a small impurity of high mass ions (positive or negative). We will show that the symmetry breaking induced by the immobile heavy ions (the main components, H^+ and H^-, will have slightly different ambient densities to insure charge neutrality) leads to a finite electrostatic field

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which, in turn, makes the plasma capable of supporting stable localized EM wave structures of finite amplitude.

The pair ion plasma with a heavy ion contamination will be nonrelativistic both in velocity and temperature as opposed to the cosmic plasma it is supposed to mimic.[2,3] Although our eventual interest will be in the (H^+–H^-+heavy ion) plasma, we will analyze an arbitrary pair plasma whose principal constituents have a mass m, and charge $\pm q$. The equations of motion for the dynamic species are

$$\frac{d^{\pm}\mathbf{p}^{\pm}}{dt} + \nabla \hat{P}^{\pm} = \pm q\left[\mathbf{E} + \frac{1}{c}(\mathbf{u}^{\pm} \times \mathbf{B})\right], \tag{1}$$

where $q=Z|e|$; $\mathbf{p}^{\pm}=m\mathbf{u}^{\pm}$ ($\mathbf{u}^{\pm}$) are the hydrodynamic momenta (velocities) of the oppositely charged species, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields and $d^{\pm}/dt=\partial/\partial t+\mathbf{u}^{\pm}\cdot\nabla$ are the co-moving derivatives. In Eq. (1), we have invoked a generic equation of state $P^{\pm}=P^{\pm}(n^{\pm})$ to write the pressure term $(1/n^{\pm})\nabla P^{\pm}$ as $\nabla \hat{P}^{\pm}$.

It has been shown earlier[2,3] that (a) the perfectly symmetric system (1) cannot sustain a localized electrostatic field, and (b) the electrostatic field is essential for the localization of an electromagnetic wave passing through a pair plasma. Since the creation of localized nonlinear electromagnetic structures is the theme of this paper, a mechanism for symmetry breaking must be provided. We do this by doping the pi plasma with a heavy ion impurity (for the electrostatic mode excitations in e-p-i plasmas, see Ref. 7) so that the demands of charge neutrality

$$qn^{+} = qn^{-} + q'N_{\infty} \tag{2}$$

will cause a difference between the densities $n^{\pm}$ of the main constituents. Here q' is the charge of the heavy, nondynamic, uniformly distributed, contaminating specie, and ∞ is taken to be the fiducial point where the perturbations vanish. Equation (2) implies the relation $qn_{\infty}^{+}=qn_{\infty}^{-}+q'N_{\infty}$ for ambient densities; it also leads to

$$n^{+} - n^{-} = n_{\infty}^{+} - n_{\infty}^{-}. \tag{3}$$

By design, the density N_{∞} of the contaminant is much smaller than the density of the main pair $n_{\infty}^{\pm}$ ($N_{\infty} \ll n_{\infty}^{\pm}$), which evolve via the continuity equation,

$$\frac{\partial n^{\pm}}{\partial t} + \nabla \cdot (n^{\pm}\mathbf{u}^{\pm}) = 0. \tag{4}$$

The equations of motion, now, must be coupled with the Maxwell equations. In terms of the vector ($\mathbf{A}$) and the electrostatic (ϕ) potentials, the latter may be written as (Coulomb gauge $\nabla\cdot\mathbf{A}=0$)

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - c^2\Delta\mathbf{A} + c\frac{\partial}{\partial t}(\nabla\varphi) = 4\pi c\mathbf{J}, \tag{5}$$

$$\Delta\varphi = -4\pi\rho, \tag{6}$$

where the charge and current densities are defined by

$$\rho = q(n^{+} - n^{-}) - q'N_{\infty}; \quad \mathbf{J} = q(n^{+}\mathbf{u}^{+} - n^{-}\mathbf{u}^{-}). \tag{7}$$

In terms of the dimensionless variables

$$\mathbf{u}^{\pm} = \frac{\mathbf{u}^{\pm}}{c}, \quad n^{\pm} = \frac{n^{\pm}}{n_{\infty}^{\pm}}, \quad \hat{A} = \frac{q\hat{A}}{mc^2}, \quad \mathbf{r} = \frac{\omega_{-}}{c}\mathbf{r}, \quad t = \omega_{-}t, \tag{8}$$

where $\hat{A} \equiv [\mathbf{A};\varphi;(q^{-1}T^{\pm})]$ and $\omega_{-}=(4\pi q^2 n_{\infty}^{-}/m)^{1/2}$ is the Langmuir frequency of major negative species, the defining equations read [the continuity equations retain their form (4)]

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - \Delta\mathbf{A} + \frac{\partial}{\partial t}(\nabla\varphi) + [n^{-}\mathbf{u}^{-} - (1+\epsilon)n^{+}\mathbf{u}^{+}] = 0, \tag{9}$$

$$\Delta\phi = [n^{-} - (1+\epsilon)n^{+} + \epsilon], \tag{10}$$

$$\frac{\partial \mathbf{\Pi}^{\pm}}{\partial t} = \mathbf{u}^{\pm} \times \mathbf{\Omega}^{\pm} - \nabla\psi^{\pm}, \tag{11}$$

where the equations of motion (1), after standard manipulation, have been rewritten in a revealing form that contains the generalized flows,

$$\mathbf{\Pi}^{\pm} = \mathbf{u}^{\pm} \pm \mathbf{A}, \tag{12}$$

the generalized vorticities

$$\mathbf{\Omega}^{\pm} = \nabla \times \mathbf{\Pi}^{\pm}, \tag{13}$$

and the effective energies

$$\psi^{\pm} = \hat{P}^{\pm} + \frac{(\mathbf{u}^{\pm})^2}{2} \pm \varphi. \tag{14}$$

The symmetry-breaking small parameter $\epsilon=|q'|N_{\infty}/qn_{\infty}^{-} \ll 1$, appearing in Eqs. (9) and (10), will eventually be the source as well as the measure of the electrostatic field.

We are, now, all set to analyze the one-dimensional propagation ($\partial/\partial z \neq 0$, $\partial/\partial x=0$, $\partial/\partial y=0$) of a circularly polarized EM wave with a mean frequency ω_o and a mean wave number k_o,

$$\mathbf{A}_{\perp} = \tfrac{1}{2}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})A(z,t)\exp(ik_o z - i\omega_o t) + \text{c.c.} \tag{15}$$

Here $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the standard unit vectors, and the envelope $A(z,t)$ is a slowly varying function of z and t. Using the gauge condition ($A_z=0$ in the present context), we can calculate

$$\mathbf{u}^{\pm} \times \mathbf{\Omega}^{\pm} = \mathbf{u}^{\pm} \times \left(\hat{\mathbf{z}} \times \frac{\partial \mathbf{\Pi}_{\perp}^{\pm}}{\partial z}\right) = \hat{\mathbf{z}}\mathbf{u}^{\pm} \cdot \frac{\partial \mathbf{\Pi}_{\perp}^{\pm}}{\partial z} - u_z^{\pm}\frac{\partial \mathbf{\Pi}_{\perp}^{\pm}}{\partial z} \tag{16}$$

and substitute it in the transverse component of Eq. (11) to derive

$$\left(\frac{\partial}{\partial t} + u_z\frac{\partial}{\partial z}\right)\mathbf{\Pi}_{\perp}^{\pm} = 0. \tag{17}$$

The simplest solution $\mathbf{\Pi}_{\perp}^{\pm}=0$ is the most relevant here since, for the localized solutions we are seeking, all fields must go to zero at infinity. The consequential relation

$$\mathbf{u}_{\perp}^{\pm} = \mp \mathbf{A}_{\perp} \tag{18}$$

relates the transverse components of the hydrodynamic velocities and the vector potential. The longitudinal dynamics is obtained from the z component of Eq. (11) using Eq. (16).

Noting that all terms in this dynamics vary on a slow time scale, we can introduce the following variables for convenience: $\xi=z-v_g t$, $\tau=t$, where $v_g=k_0/\omega_0$ is the group velocity of the EM wave packet. Assuming $v_g\partial/\partial\xi \gg \partial/\partial\tau$, and integrating, we find

$$u_z^{\pm} = \tfrac{1}{2}u_z^{\pm 2} + \tfrac{1}{2}A_\perp^2 \pm \phi + (\hat{P}^{\pm} - \hat{P}_\infty^{\pm}). \tag{19}$$

The result holds in the transparent plasma limit ($\omega_0 \gg 1$) for which $v_g \simeq 1$. In the laboratory experiments, this condition, which in dimensional terms demands $\omega_0 \gg \sqrt{2}\omega_-$ ($=\omega_p$ the plasma frequency) can be easily arranged for pi plasmas.[1,15,16] In the nonrelativistic case under consideration (the velocities are normalized to c), the first term on the r.h.s. of Eq. (19) is readily neglected. However the second term $A_\perp^2 = u_\perp^{\pm 2}$ must be kept because $u_\perp^{\pm 2}$ is allowed to be $\gg u_z^{\pm 2}$. In Eq. (19), the constants of integration are determined from the boundary condition that, at infinity, EM fields and the plasma momenta vanish.

Similarly, the continuity equation yields

$$n^{\pm} = \frac{1}{1-u_z^{\pm}} \simeq (1+u_z^{\pm}). \tag{20}$$

From Eqs. (19) and (20), and the quasineutrality condition [obtained by neglecting $\triangle\phi$ in the Poisson equation (10)] one derives, after straightforward algebra, the following major relationships:

$$\phi \simeq \frac{\epsilon}{4}|A_\perp|^2 \tag{21}$$

and

$$u_z^- \simeq 2\epsilon^{-1}\phi, \quad n^+ + n^- \simeq 2(1-\epsilon\phi). \tag{22}$$

As expected, the electrostatic field [that will bring in the nonlinearity, necessary for localization, in the Maxwell equation (9)] owes its existence to symmetry breaking, and is proportional to the parameter $\epsilon \ll 1$. It is clear that the electrostatic potential will be absent in pure pair plasmas with no contamination. This phenomenon was first investigated for a relativistic plasmas in a cosmic setting.[2] In a pure pair plasma (equal density and equal temperature for the two species) the radiation pressure imparts equal longitudinal momenta to both the negative and positive ions (since their effective masses are equal) and, thus, causes no charge separation ($n_-=n_+$ and $\phi=0$).

To proceed further, let us determine the transverse current defined in Eq. (7) or Eq. (9) by resorting to Eqs. (18), (21), and (22). To the lowest order in the nonlinearity, we find

$$\mathbf{J}_\perp = -\left[(2+\epsilon) - \tfrac{1}{4}\epsilon^2|A_\perp|^2\right]\mathbf{A}_\perp. \tag{23}$$

Notice that the nonlinear term coincides with the nonrelativistic limit of the expression derived in Ref. 3. We see, that without an asymmetry ($\epsilon \equiv 0$), the nonlinear term disappears since the electrostatic potential (responsible for this term) ϕ is now zero.

When the transverse current, evaluated in Eq. (23), is substituted into Eq. (9), we obtain the nonlinear equation for the evolution of the slowly varying envelope function A. Following the standard methodology (see Ref. 3, for example), we can derive the final equation in the parabolic approximation,

$$2i\omega_0\frac{\partial A}{\partial\tau} + \frac{2}{\omega_0^2}\frac{\partial^2 A}{\partial z^2} + \frac{1}{4}\epsilon^2|A|^2 A = 0, \tag{24}$$

where the wave frequency ω_0 satisfies the dispersion relation $\omega_0^2=(2+\epsilon)+k_0^2$ implying $v_g \simeq 1$ for a transparent plasma for which $\omega_0 \gg 1$ has been assumed. With the additional obvious normalization, Eq. (24) is cast into the standard form of the nonlinear Schrödinger equation (NLSE),

$$i\frac{\partial A}{\partial\tau} + \frac{\partial^2 A}{\partial z^2} + \epsilon^2|A|^2 A = 0. \tag{25}$$

It is well known that the NLSE can be solved for solitons by the inverse scattering method. The usual stationary soliton solution is obtained by letting (Ω is a constant that corresponds to a nonlinear frequency shift)

$$A = A(\xi)e^{i\Omega^2\tau} \tag{26}$$

that leads to the nonlinear ordinary differential equation (ODE),

$$\frac{\partial^2 A}{\partial\xi^2} - \Omega^2\left[1 - \frac{\epsilon^2}{\Omega^2}A^2\right]A = 0. \tag{27}$$

With boundary conditions appropriate to a localized solution, $A=0=dA/d\xi$ as $\xi \to \pm\infty$, Eq. (27) yields

$$A = A_m \operatorname{sech}\left(\frac{\epsilon A_m}{\sqrt{2}}\xi\right), \tag{28}$$

a lump of the electromagnetic field potential with amplitude $A_m=\sqrt{2}/\epsilon$, and localization width $d=\sqrt{2}/\epsilon A_m=1$. In standard measures, therefore, the localization width is the collisionless skin depth of the main plasma ($d \sim \lambda_{\text{skin}}$).

We have thus demonstrated that the pair ion plasmas for which the symmetry is broken by a slight contamination (doping) of a heavier immobile ion can support stable localized EM wave structures even in the nonrelativistic limit appropriate to the current and near future laboratory experiments. To appreciate the importance of theoretical results in the context of a laboratory setting, let us invoke a hypothetical H^+–H^- plasma of density 10^{10} cm^{-3} that is expected to become available in the near future.[16,17] For these densities, the characteristic plasma frequency $\omega_p \simeq 1.86\times 10^8$ s^{-1} corresponding to a $\nu_p=\omega_p/2\pi \simeq 30$ MHz. The first condition for our derivation that the plasma be transparent ($\omega_0 \gg \omega_p$) can be easily satisfied by choosing the EM wave frequency to be in the few hundred MHz range.

Since the localization distance was shown to be about a skin depth, for $\omega_p=1.86*10^8$ s^{-1}, it comes out to be $d \sim \lambda_{\text{skin}}=c/\omega_p \sim 1.6$ m. Thus to experimentally observe density and electromagnetic field bunching in a doped H^+–H^- plasma, the plasma must extend over several meters (along the propagation direction). Naturally, for higher densities, the

localization distance will decrease and the length requirement on the experiment will be correspondingly relaxed.

By investigating the nonlinear propagation of high intensity EM waves in a pair ion plasma contaminated with a small fraction of a high mass immobile ions (for symmetry breaking), we have highlighted a very remarkable property of this new and interesting state of (laboratory created) matter—it can strongly localize the EM radiation with finite density excess in the region of localization. It must be stressed that this is just one example of a vast variety of physical phenomena (many with astrophysical consequences) associated with pair plasmas that can be explored in controlled experiments.

It is worth repeating that the equation (25) obeyed by the field envelope A comes out to be the NLSE which is known to have exact soliton solutions. Solitons carry a large density inhomogeneity, and form nondiffractive, nondispersive localized structures. Such objects have been called "heavy bullets of light," and their possible important role in cosmology as a source of structure formation has been already alluded to.[2,3] The solitons are also important for energy transfer investigations in plasmas.

There are several ways in which this simple idealized calculation can be extended and augmented. For example, we could, *inter alia* (1) investigate other modes of symmetry breaking, (2) introduce a guide magnetic field (see Ref. 7) for electrostatic mode envelope excitations in pair-ion plasmas doped by massive ions (or dust particles), and (3) increase the dimensionality by introducing weak transverse dependence.

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