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Zonal flow generation by internal gravity waves in the atmosphere

W. Horton,[1,2] T. D. Kaladze,[3] J. W. Van Dam,[1] and T. W. Garner[2]

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[1] A novel mechanism for the generation of low-frequency large-scale zonal flows by higher-frequency, small-scale, finite-amplitude internal gravity (IG) waves is analyzed in the atmosphere from the troposphere to the ionosphere *E* layer. The nonlinear generation mechanism is based on the parametric excitation of convective cells by finite-amplitude IG waves. A set of coupled equations describing the nonlinear interaction of IG waves and zonal flows is derived. The generation of zonal flows is due to the Reynolds stress and mean stratification forces produced by finite-amplitude IG waves. The onset mechanism for the instability is governed by a generalized Lighthill instability criterion. Explicit expressions for the maximum growth rate as well as for the optimal spatial dimensions of the zonal flows are derived. The growth rates of zonal flow instabilities and the conditions for driving them are determined. A comparison with existing results is carried out. The present theory can be used for the interpretation of IG wave observations in the Earth's atmosphere and laboratory experiments. Some earthquake-related phenomena are briefly discussed.

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1. Introduction

[2] It is widely thought that nonlinear energy transfers from small- to large-scale length fluctuations (inverse cascade) is an important mechanism for the spontaneous generation of large, coherent structures both is atmospheres and magnetized plasmas. In this work we present a detailed calculation for the generation of sheared flows in the atmosphere and lower ionosphere from the parametric decay of finite amplitude internal gravity waves. The analysis finds the conditions on the wave number vector and amplitude for small-scale gravity waves to spontaneously excite vertically sheared zonal flows.

[3] Both ground-based and satellite observations clearly show that at different layers of the atmosphere, there exist large-scale quasi-two-dimensional band-like flows with nonuniform velocities along the meridians. Such flows (winds) parallel or nearly parallel to the lines of latitude are termed zonal (sheared) flows. Important examples include the zonal flows in Jupiter and the stratospheric polar jet in Antarctica. These zonal flows create transport barriers that have a crucial influence on mixing and confinement. For example, strengthening of the Antarctic ice polar jet in spring leads to dramatically enhanced seasonal destruction of lower stratospheric ozone zonal flows.

[4] Zonal flows may give rise to long-lasting vortices through secondary shear flow instabilities; for example, the Jupiter red spot may have been formed this way. Recently, it has been realized that zonal flows play a crucial role in the regulation of the anomalous transport in a tokamak. For these reasons, the problem of existence and stability of large-scale structures, such as convective cells, zonal flows, and jets, and their role on transport has been intensively investigated both in laboratory plasmas [see, e.g., *Hasegawa et al.*, 1979; *Terry*, 2000; *Diamond et al.*, 2005] and in geophysical fluid dynamics [see, e.g., *Busse*, 1994; *Rhines*, 1994].

[5] The generation of zonal flows is still not fully clarified. That zonal flows may be produced by a nonuniform heating of the atmospheric layers by solar radiation has been well known [*Gill*, 1982]. Recently, however, there have been active investigations of the generation and evolution of large-scale structures on the basis of nonlinear interactions propagating in nonuniform magnetoplasmas waves with the inhomogeneous zonal wind (shear flow) [see, e.g., *Smolyakov et al.*, 2000; *Manfredi et al.*, 2001; *Shukla and Stenflo*, 2002; *Kaladze et al.*, 2005]. It has been found that drift waves strongly couple with zonal flows whose dynamics is governed by the drift wave form of the Reynolds stress tensor. The driven flow is nonlinearly coupled with the Hasegawa-Mima equation in the drift wave-zonal flow theory. The generation mechanism is based on the parametric excitation of zonal flow due to the Reynolds stresses produced by finite-amplitude drift waves. Generally, zonal flows vary on slow timescales compared with finite-frequency waves. In the absence of zonal flows

[1]Institute for Fusion Studies, University of Texas at Austin, Austin, Texas, USA.
[2]Center for Ionospheric Research, Space and Geophysics Laboratory, Applied Research Laboratories, University of Texas at Austin, Austin, Texas, USA.
[3]I. Vekua Institute of Applied Mathematics, Tbilisi State University, Tbilisi, Georgia.

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the weak turbulence theory of the internal gravity waves has been worked out in detail and shown to have an anisotropic Kolomogorov-like spectra with indices of $-3/2$ to -2 by *Caillol and Zeitlin* [2000]. The weak turbulence calculations neglects the effects of vortex-wave interactions and the suppression of the turbulence level by the zonal flows. Thus, the actual wave number spectrum may be steeper and consistent with the observations that show a -3 spectral index for smaller-scale wavelengths. The study of the evolution of the combined wave/vortex turbulence with zonal flows remains an open problem.

[6] Similarly, the atmospheric medium builds up conditions which are favorable to the formation of electromagnetic nonlinear stationary solitary wave structures also [*Pokhotelov et al.*, 1996, 2001; *Chmyrev et al.*, 1991; *Kaladze et al.*, 2003; *Lawrence and Jarvis*, 2003]. In reality, several planetary atmospheres can support both propagating waves and zonal flows, and they thus constitute dynamic systems which exhibit complex nonlinear interactions. As there is a well-known analogy between drift waves and Rossby waves [see, e.g., *Nezlin and Chernikov*, 1995; *Horton and Ichikawa*, 1996] the idea of generation of zonal flows by tropospheric Rossby waves on the basis of kinetic equation for wave packets was put forward by *Smolyakov et al.* [2000]. Using the formulation of parametric instabilities on the basis of three waves resonant nonlinear interaction, the theory of zonal flows generation by monochromatic Rossby waves was further developed by *Shukla and Stenflo* [2003], *Onishchenko et al.* [2004], and *Kaladze et al.* [2007a]. These works show that zonal flows in a nonuniform rotating neutral atmosphere can be excited by finite-amplitude Rossby waves. The method is to calculate the interaction of the pump wave (Rossby wave) with a sheared flow and the two pump wave satellites. This approach is an alternative to the weak-turbulence approach used by *Smolyakov et al.* [2000]. The driving mechanism of this instability is due to the Reynolds stresses which are inevitably inherent for finite amplitude small-scale Rossby waves. Owing to this essential nonlinear mechanism spectral energy transfers from small-scale Rossby waves to large-scale enhanced zonal flows (inverse cascade) in the Earth's neutral atmosphere including the lower ionosphere.

[7] Zonal flow generation has been considered within a simple model of Rossby-wave turbulence, using the classical nonlinear two-dimensional Charney equation [*Terry*, 2000]. The necessary condition for zonal flow generation is similar to the Lighthill criterion for modulation instability in nonlinear optics [*Lighthill*, 1965], as shown here for gravity waves.

[8] In the present paper we develop a modified parametric excitation mechanism for the problem of zonal flow generation by the low-frequency branch of acoustic gravity waves or internal gravity (IG) waves. In contrast to Rossby waves IG waves are described by two potential fields (ψ, χ), which produces two driving stresses, the Reynolds stress and the stratification stress. In addition, the zonal flow has only one component of velocity along parallels which depends on the vertical z-coordinate.

[9] The article is organized as follows: in section 2 the spectral analysis of the problem is carried out and zonal flow evolution equations in terms of velocity potential function and density perturbations are derived considering the three waves resonant parametric interaction between the zonal flows, the primary (pump) IG modes generating the flows, and the side-band amplitudes, which depend on both the primary modes and zonal flows. Then, we calculate the driving forces of zonal flows, which are the mean Reynolds stress and the stratification force. Owing to the assumption of a distinct timescale and space-scale separation between the turbulent oscillations and the zonal flow, the zonal flow dispersion equation is derived in section 3, which is a biquadratic equation with respect to the zonal flow mode frequency. Zonal flow instabilities with the corresponding growth rates in different regimes are discussed in section 4. Our discussion and conclusions are presented in section 5.

2. Zonal Flow Evolution Equations

[10] Low-frequency acoustic gravity waves or internal gravity (IG) waves can be described by the following system of equations [see, e.g., *Stenflo*, 1996]

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\Delta\psi - \dfrac{1}{4H^2}\psi\right) - J(\psi, \Delta\psi) = \dfrac{\partial \chi}{\partial x}, \\ \dfrac{\partial \chi}{\partial t} - J(\psi, \chi) = -\omega_g^2 \dfrac{\partial \psi}{\partial x}, \end{cases} \tag{1}$$

where $\psi(x, z)$ is the velocity stream function, $\chi(x, z)$ is the normalized density perturbation, g is the free fall acceleration, H is the density scale height, and $\omega_g = (g/H)^{1/2}$ is the Brunt-V*äisälä* or buoyancy frequency. The local Cartesian coordinate system (x, y, z) is chosen with the east directed x axis along parallels, the northward directed y axis along meridians, and the z axis is directed vertically up. Thus the velocity $\mathbf{v} = \partial_z\psi\hat{\mathbf{x}} - \partial_x\psi\hat{\mathbf{z}}$ whose direction (Lie) derivative $\mathbf{v}\cdot\nabla$ gives rise to the Jacobian in equation (1). The vorticity gives rise to $\Delta = \partial^2/\partial x^2 + \partial^2/\partial z^2$ the two-dimensional Laplacian, and $J(a, b) = (\partial a/\partial x)(\partial b/\partial z) - (\partial a/\partial z)(\partial b/\partial x)$ is the Jacobian. It follows from the system (1) that small-amplitude waves with frequency $\omega_{\mathbf{k}}$ and wave vector $\mathbf{k} = (k_x, k_z)$ satisfy the dispersion relation

$$\omega_{\mathbf{k}}^2 = \frac{k_x^2\omega_g^2}{k_x^2 + k_z^2 + \dfrac{1}{4H^2}}. \tag{2}$$

[11] Equations (1) and (2) describe IG waves propagating along the Earth's parallels neglecting the Coriolis acceleration from $f = 2\Omega\sin\lambda$ where λ is the latitude and Ω is the angular rotation velocity of the planet. The neglect of f and $\beta = df/dy = 2\Omega/R_{\rm p}\cos\lambda$ in the internal wave dynamics is valid for $\omega_{\mathbf{k}}^2 \gg f^2$ where $f \cong 7\times10^{-5}$ rad/s. The β effect adds the vorticity term $\beta\partial_x\psi$ to equation (1) which is small for $(k^2 + 1/4H^2)\omega_{\mathbf{k}} > (k_x/R_{\rm p})2\Omega\cos\lambda$. The minimum IG frequency $\min(\omega_{\mathbf{k}}/f) = 2k_xH\omega_g/|f| \simeq k_xH(N/\Omega) \gg 1$.

[12] The nonlinear Jacobian terms in the system (1) allow us to consider coupling between different modes. We consider three-wave interaction, in which the coupling between the pump IG and side-band modes generates low-frequency large-scale modes, so called zonal flows. Since the zonal flow varies on a much longer timescale than the comparatively small-scale IG waves, one can use a multiple scale expansion, assuming that there is a sufficient spectral gap separating the large- and small-scale motions. Following the standard procedure to describe the evolution

of the coupled system (IG waves plus zonal flows), we split the perturbations $X = (\psi, \chi)$ in the system (1) into three components [see, e.g., *Onishchenko et al.*, 2004; *Kaladze et al.*, 2007a]:

$$X = \tilde{X} + \hat{X} + \bar{X}, \tag{3}$$

where

$$\tilde{X} = \tilde{X}_{+}(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega_{\mathbf{k}}t) + \tilde{X}_{-}(\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{r} + i\omega_{\mathbf{k}}t) \tag{4}$$

describes the spectrum of pump IG modes ($\tilde{X}_{-}(\mathbf{k}) = \tilde{X}^{*}_{+}(\mathbf{k})$, the sign * means the complex conjugative),

$$\hat{X} = \hat{X}_{+}(\mathbf{k})\exp(i\mathbf{k}_{+}\cdot\mathbf{r} - i\omega_{+}t) + \hat{X}_{-}(\mathbf{k})\exp(i\mathbf{k}_{-}\cdot\mathbf{r} - i\omega_{-}t) + c.c. \tag{5}$$

describes the spectrum of side-band modes, and

$$\bar{X} = \bar{X}_0 \exp(-i\Omega t + iq_z z) + c.c. \tag{6}$$

describes the large-scale zonal flow modes varying only along vertical direction z. The sign c.c. stands for the complex conjugate. The following energy and momentum conservation conditions are fulfilled that $\omega_{\pm} = \Omega \pm \omega_{\mathbf{k}}$ and $\mathbf{k}_{\pm} = q_z\hat{\mathbf{z}} \pm \mathbf{k}$, and the pairs ($\omega_{\mathbf{k}}$, $\mathbf{k}$) and (Ω, $q_z\hat{\mathbf{z}}$) represent the frequency and wave vector of the IG pump and zonal flow modes, respectively. Considering the local approximation the amplitude $\bar{X}_0$ of the zonal flow mode is assumed to be constant.

[13] To obtain the relations between the spectral components of pump IG modes, we substitute equation (4) into the system (1) and follow the standard quasi-linear procedure. We neglect the contribution of the small nonlinear terms for the high-frequency relations, but for the low-frequency zonal flow modes we will keep such nonlinear terms. Then, we obtain for the IG pump modes

$$\begin{cases} \omega_{\mathbf{k}}(k^2 + \frac{1}{4H^2})\tilde{\psi}_{\pm}(\mathbf{k}) = k_x\tilde{\chi}_{\pm}(\mathbf{k}), \\ \omega_{\mathbf{k}}\tilde{\chi}_{\pm}(\mathbf{k}) = \omega_g^2 k_x\tilde{\psi}_{\pm}(\mathbf{k}), \end{cases} \tag{7}$$

where $k^2 = k_x^2 + k_z^2$. Solution of this homogeneous system gives the dispersion relation in equation (2) for IG modes and the eigenmodes ($\tilde{\psi}_{\pm}$, $\tilde{\chi}_{\pm}$) = [1, $\pm\omega_g(k^2+1/4H^2)^{(1/2)}$]$\tilde{\psi}_{\pm}$.

[14] The contribution of small nonlinear terms is essential in case of the low-frequency zonal flow modes. To obtain the equations describing the turbulence and amplitude evolution of the zonal flow modes, we substitute equations (3)–(6) into the system of equations (1), we get

$$\begin{cases} \frac{\partial}{\partial t}\left(\Delta - \frac{1}{4H^2}\right)(\tilde{\psi} + \hat{\psi} + \bar{\psi}) \\ -J(\tilde{\psi} + \hat{\psi} + \bar{\psi}, \Delta(\tilde{\psi} + \hat{\psi} + \bar{\psi})) = \frac{\partial}{\partial x}(\tilde{\chi} + \hat{\chi}), \\ \frac{\partial}{\partial t}(\tilde{\chi} + \hat{\chi} + \bar{\chi}) \\ -J(\tilde{\psi} + \hat{\psi} + \bar{\psi}, \tilde{\chi} + \hat{\chi} + \bar{\chi}) = -\omega_g^2\frac{\partial}{\partial x}(\tilde{\psi} + \hat{\psi}). \end{cases} \tag{8}$$

Averaging over the fast small-scale fluctuations gives for the zonal flow evolution the following system of equations

$$\begin{cases} \frac{\partial}{\partial t}\left(\Delta\bar{\psi} - \frac{1}{4H^2}\bar{\psi}\right) = < J(\tilde{\psi} + \hat{\psi}, \Delta(\tilde{\psi} + \hat{\psi})) >, \\ \frac{\partial}{\partial t}\bar{\chi} = < J(\tilde{\psi} + \hat{\psi}, \tilde{\chi} + \hat{\chi}) >, \end{cases} \tag{9}$$

where the angular brackets denote the averaging over fast oscillations.

[15] As to the turbulent part contributions, the nonlinear terms are given only by the combinations $\tilde{\psi}$ and $\hat{\psi}$ with $\bar{\psi}$ and $\bar{\chi}$, i.e., from equation (8) we have

$$\begin{cases} \frac{\partial}{\partial t}\left(\Delta - \frac{1}{4H^2}\right)(\tilde{\psi} + \hat{\psi}) - J(\tilde{\psi} + \hat{\psi}, \Delta\bar{\psi}) \\ -J(\bar{\psi}, \Delta(\tilde{\psi} + \hat{\psi})) = \frac{\partial}{\partial x}(\tilde{\chi} + \hat{\chi}), \\ \frac{\partial}{\partial t}(\tilde{\chi} + \hat{\chi}) - J(\tilde{\psi} + \hat{\psi}, \bar{\chi}) \\ -J(\bar{\psi}, \tilde{\chi} + \hat{\chi}) = -\omega_g^2\frac{\partial}{\partial x}(\tilde{\psi} + \hat{\psi}). \end{cases} \tag{10}$$

Owing to the structure (6) of zonal flow the system of equations (9) may be reduced to the following one:

$$\begin{cases} \frac{\partial}{\partial t}\left(\Delta\bar{\psi} - \frac{1}{4H^2}\bar{\psi}\right) = \frac{\partial^2}{\partial z^2} < \frac{\partial\tilde{\psi}}{\partial x}\frac{\partial\hat{\psi}}{\partial z} + \frac{\partial\hat{\psi}}{\partial x}\frac{\partial\tilde{\psi}}{\partial z} >, \\ \frac{\partial}{\partial t}\bar{\chi} = -\frac{\partial}{\partial z} < \tilde{\psi}\frac{\partial\hat{\chi}}{\partial x} + \hat{\psi}\frac{\partial\tilde{\chi}}{\partial x} > . \end{cases} \tag{11}$$

Finally, from equation (11) we obtain the following dynamic equations for zonal flow

$$\begin{cases} -i\Omega\bar{\psi}_0 = R_{\perp}, \\ -i\Omega\bar{\chi}_0 = R_{\parallel}, \end{cases} \tag{12}$$

where

$$\begin{cases} R_{\perp} = \frac{q_z^2}{q_z^2 + \frac{1}{4H^2}} < \frac{\partial\tilde{\psi}}{\partial x}\frac{\partial\hat{\psi}}{\partial z} + \frac{\partial\hat{\psi}}{\partial x}\frac{\partial\tilde{\psi}}{\partial z} >, \\ R_{\parallel} = -iq_z < \tilde{\psi}\frac{\partial\hat{\chi}}{\partial x} + \hat{\psi}\frac{\partial\tilde{\chi}}{\partial x} > . \end{cases} \tag{13}$$

The right-hand sides of equation (12) represent driving forces of zonal flows, which are mean Reynolds stress ($R_{\perp}$) and mean stratification force ($R_{\parallel}$), respectively. Both terms are induced by the short-scale IG waves.

[16] Using the Fourier decompositions given by equations (3)–(6), we have

$$\begin{cases} R_{\perp} = \frac{q_z^2 k_x}{q_z^2 + \frac{1}{4H^2}}[2k_z(\tilde{\psi}_{-}\hat{\psi}_{+} + \tilde{\psi}_{+}\hat{\psi}_{-}) \\ \quad + q_z(\tilde{\psi}_{-}\hat{\psi}_{+} - \tilde{\psi}_{+}\hat{\psi}_{-})], \\ R_{\parallel} = q_z k_x(\tilde{\psi}_{-}\hat{\lambda}_{+} - \tilde{\psi}_{+}\hat{\lambda}_{-}), \end{cases} \tag{14}$$

where

$$\hat{\lambda}_{\pm} = \hat{\chi}_{\pm} - k_x\frac{\omega_g^2}{\omega_{\mathbf{k}}}\hat{\psi}_{\pm}. \tag{15}$$

In deriving equation (14) we used the expressions for $\tilde{\chi}_\pm$ from the second equation of the system (7).

[17] In order to calculate the functions $R_\perp$ and $R_\parallel$ (see equation (14)) the side-band amplitudes $\hat{\chi}_\pm$ and $\hat{\psi}_\pm$ should be found. From the system of equations (10) we find

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\Delta - \dfrac{1}{4H^2}\right)\hat{\psi} - J(\tilde{\psi}, \Delta\bar{\psi}) - J(\bar{\psi}, \Delta\tilde{\psi}) = \dfrac{\partial \hat{\chi}}{\partial x}, \\ \dfrac{\partial \hat{\chi}}{\partial t} - J(\tilde{\psi}, \bar{\chi}) - J(\bar{\psi}, \tilde{\chi}) = -\omega_g^2 \dfrac{\partial v}{\partial x}. \end{cases} \quad (16)$$

Let us find equations for $\hat{\psi}_\pm$ and $\hat{\chi}_\pm$ according to equations (5) and (6). Then we get from equation (16)

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\Delta - \dfrac{1}{4H^2}\right)\hat{\psi}_\pm - \dfrac{\partial \hat{\chi}_\pm}{\partial x} - J(\tilde{\psi}_\pm, \Delta\bar{\psi}_0) - J(\bar{\psi}_0, \Delta\tilde{\psi}_\pm) \\ = 0, \\ \dfrac{\partial \hat{\chi}_\pm}{\partial t} + \omega_g^2 \dfrac{\partial \hat{\psi}_\pm}{\partial x} - J(\tilde{\psi}_\pm, \bar{\chi}_0) - J(\bar{\psi}_0, \tilde{\chi}_\pm) = 0. \end{cases} \quad (17)$$

From these equations we find corresponding amplitudes satisfy the relations

$$\begin{cases} i\omega_\pm\left(k_\pm^2 + \dfrac{1}{4H^2}\right)\hat{\psi}_\pm \mp ik_x\hat{\chi}_\pm \pm k_x q_z(k^2 - q_z^2)\tilde{\psi}_\pm\bar{\psi}_0 \\ = 0, \\ i\omega_\pm\hat{\chi}_\pm \mp ik_x\omega_g^2\hat{\psi}_\pm \pm q_z k_x\tilde{\psi}_\pm\left(\bar{\psi}_0\dfrac{k_x\omega_g^2}{\omega_{\mathbf{k}}} - \bar{\chi}_0\right) = 0, \end{cases} \quad (18)$$

where $k_\pm^2 = k^2 + q_z^2 \pm 2k_z q_z$, $\omega_\pm = \Omega \pm \omega_{\mathbf{k}}$.

[18] The solution of equation (18) may be written as

$$\begin{cases} \hat{\psi}_\pm = i\dfrac{k_x q_z}{D_\pm}\tilde{\psi}_\pm\left[\pm\omega_\pm(k^2 - q_z^2)\bar{\psi}_0 + k_x\left(\bar{\psi}_0\dfrac{k_x\omega_g^2}{\omega_{\mathbf{k}}} - \bar{\chi}_0\right)\right], \\ \hat{\chi}_\pm = i\dfrac{k_x q_z}{D_\pm}\tilde{\psi}_\pm\left[k_x\omega_g^2(k^2 - q_z^2)\bar{\psi}_0 \right. \\ \qquad \left. \pm\,\omega_\pm\left(k_\pm^2 + \dfrac{1}{4H^2}\right)\left(\bar{\psi}_0\dfrac{k_x\omega_g^2}{\omega_{\mathbf{k}}} - \bar{\chi}_0\right)\right], \end{cases} \quad (19)$$

where

$$D_\pm = \omega_\pm^2\left(k_\pm^2 + \frac{1}{4H^2}\right) - k_x^2\omega_g^2. \quad (20)$$

Using expressions (19) for the auxiliary expression (15), we obtain

$$\hat{\lambda}_\pm = \mp\frac{\alpha_\pm}{D_\pm}\left\{\left[1 - \frac{\bar{\psi}_0}{\bar{\chi}_0}\frac{\omega_{\mathbf{k}}}{k_x}\left(k^2 + \frac{1}{4H^2}\right)\right]\left(\frac{1}{4H^2}\frac{\Omega}{\omega_{\mathbf{k}}} + 2q_z^2\frac{\Omega}{\omega_{\mathbf{k}}} + 2k_z q_z \pm 2k_z q_z\frac{\Omega}{\omega_{\mathbf{k}}} \pm q_z^2\right) + (k^2 - q_z^2)\frac{\Omega}{\omega_{\mathbf{k}}}\right\}, \quad (21)$$

where

$$\alpha_\pm = ik_x q_z\omega_{\mathbf{k}}\bar{\chi}_0\tilde{\psi}_\pm. \quad (22)$$

[19] Considering Ω/ω_k and q_z/k to be small parameters (meaning that the typical scales of the zonal flows are much larger then the scales of the IG waves) we can write the following ordering:

$$D_\pm = \pm D^{(1)} + D^{(2)} \pm D^{(3)} + D^{(4)}, \quad (23)$$

where

$$D^{(1)} = 2q_z k_z\omega_{\mathbf{k}}^2 + 2\Omega\omega_{\mathbf{k}}\left(k^2 + \frac{1}{4H^2}\right), \quad (24)$$

$$D^{(2)} = \Omega^2\left(k^2 + \frac{1}{4H^2}\right) + q_z^2\omega_{\mathbf{k}}^2 + 4q_z\Omega k_z\omega_{\mathbf{k}}, \quad (25)$$

$$D^{(3)} = 2q_z k_z\Omega^2 + 2\Omega q_z^2\omega_{\mathbf{k}}, \quad D^{(4)} = q_z^2\Omega^2. \quad (26)$$

[20] Similarly, expression for the value $\hat{\chi}_\pm$ in the system (19) may be represented as

$$\hat{\chi}_\pm = \mp\frac{\alpha_\pm}{D_\pm}\left\{\left[1 - \frac{\bar{\psi}_0}{\bar{\chi}_0}\frac{\omega_{\mathbf{k}}}{k_x}\left(k^2 + \frac{1}{4H^2}\right)\right]\left[\frac{\Omega}{\omega_{\mathbf{k}}}\left(k^2 + q_z^2 + \frac{1}{4H^2} \pm 2k_z q_z\right) \pm \left(2k^2 + \frac{1}{4H^2}\right) + 2k_z q_z\right] \mp (k^2 - q_z^2)\right\}. \quad (27)$$

[21] Now we begin the perturbative expansion of the expression in equations (19)–(22) with respect to the small parameters q_z and Ω. Then the expression (27) may be represented approximately as

$$\hat{\chi}_\pm \approx \hat{\chi}_\pm^{(0)} + \hat{\chi}_\pm^{(1)}, \quad (28)$$

where

$$\hat{\chi}_\pm^{(0)} = \mp\frac{\alpha_\pm}{D^{(1)}}\left[k^2 + \frac{1}{4H^2} - \frac{\bar{\psi}_0}{\bar{\chi}_0}\frac{\omega_{\mathbf{k}}}{k_x}\left(k^2 + \frac{1}{4H^2}\right)\left(2k^2 + \frac{1}{4H^2}\right)\right], \quad (29)$$

$$\hat{\chi}_\pm^{(1)} = -\frac{\alpha_\pm}{D^{(1)}}\left\{\left[1 - \frac{\bar{\psi}_0}{\bar{\chi}_0}\frac{\omega_{\mathbf{k}}}{k_x}\left(k^2 + \frac{1}{4H^2}\right)\right] \times \left[\frac{\Omega}{\omega_{\mathbf{k}}}\left(k^2 + \frac{1}{4H^2}\right) + 2k_z q_z\right]\right\} \mp \frac{D^{(2)}}{D^{(1)}}\hat{\chi}_\pm^{(0)}. \quad (30)$$

[22] Analogous perturbation we carry out for the expression (21). We find

$$\hat{\lambda}_\pm^{(0)} = 0. \quad (31)$$

Thus

$$\hat{\lambda}_\pm \simeq \hat{\lambda}_\pm^{(1)} + \hat{\lambda}_\pm^{(2)}, \quad (32)$$

where

$$\hat{\lambda}_{\pm}^{(1)} = -\frac{\alpha_\pm}{D^{(1)}}\left\{\left[1-\frac{\bar{\psi}_0}{\bar{\chi}_0}\frac{\omega_{\mathbf{k}}}{k_x}\left(k^2+\frac{1}{4H^2}\right)\right] \times\left(\frac{1}{4H^2}\frac{\Omega}{\omega_{\mathbf{k}}}+2k_zq_z\right)+k^2\frac{\Omega}{\omega_{\mathbf{k}}}\right\}, \quad (33)$$

$$\hat{\lambda}_{\pm}^{(2)} = \pm\frac{\alpha_\pm\Omega}{D^{(1)2}}\left\{\left[1-\frac{\bar{\psi}_0}{\bar{\chi}_0}\frac{\omega_{\mathbf{k}}}{k_x}\left(k^2+\frac{1}{4H^2}\right)\right] \times\left[\frac{\Omega^2}{4H^2\omega_{\mathbf{k}}}\left(k^2+\frac{1}{4H^2}\right)+4\omega_{\mathbf{k}}k_z^2q_z^2-2k^2k_zq_z\Omega - 2\omega_{\mathbf{k}}k^2q_z^2-q_z^2\frac{\omega_{\mathbf{k}}}{4H^2}+2\Omega\frac{k_zq_z}{4H^2}\right]+D^{(2)}\frac{k^2}{\omega_{\mathbf{k}}}\right\}. \quad (34)$$

3. Zonal Flow Dispersion Equation

[23] If we examine the contribution of $\hat{\lambda}_{\pm}^{(1)}$ to the second expression of system (14) we become convinced that it is zero. Thus in terms of $\hat{\lambda}_{\pm}^{(2)}$ in the expression $R_\parallel$ in equation (14) reduces to

$$R_\parallel = q_zk_x(\tilde{\psi}_-\hat{\lambda}_+^{(2)} - \tilde{\psi}_+\hat{\lambda}_-^{(2)}), \quad (35)$$

with $\hat{\lambda}_{\pm}^{(2)}$ of (34). This expression (35) for the stratification stress can be represented in the following form:

$$R_\parallel = i\frac{k_x^2q_z^2\Omega\omega_{\mathbf{k}}}{D^{(1)2}}I_{\mathbf{k}}(f_\parallel^\chi\bar{\chi}_0+f_\parallel^\psi\bar{\psi}_0), \quad (36)$$

where

$$f_\parallel^\chi = \left(k^2+\frac{1}{4H^2}\right)\left[\frac{\Omega^2}{\omega_{\mathbf{k}}}\left(k^2+\frac{1}{4H^2}\right)+2k_zq_z\Omega-\omega_{\mathbf{k}}q_z^2\right] + 4\omega_{\mathbf{k}}k_z^2q_z^2, \quad (37)$$

$$f_\parallel^\psi = -\frac{\omega_{\mathbf{k}}}{k_x}\left(k^2+\frac{1}{4H^2}\right)\left[\frac{\Omega^2}{4H^2\omega_{\mathbf{k}}}\left(k^2+\frac{1}{4H^2}\right)+4\omega_{\mathbf{k}}k_z^2q_z^2 - 2k^2k_zq_z\Omega-2\omega_{\mathbf{k}}k^2q_z^2-q_z^2\frac{\omega_{\mathbf{k}}}{4H^2}+2\frac{k_zq_z\Omega}{4H^2}\right], \quad (38)$$

where $I_{\mathbf{k}} = 2\tilde{\psi}_+\tilde{\psi}_- = 2|\tilde{\psi}_+|^2$.

[24] Using the expression for $\hat{\psi}_\pm$ from the system (19), analogous calculations carried out for the value $R_\perp$ in the system (14) give

$$R_\perp = 2i\frac{k_x^2q_z^3\Omega}{D^{(1)2}\left(q_z^2+\frac{1}{4H^2}\right)}I_{\mathbf{k}}(f_\perp^\chi\bar{\chi}_0+f_\perp^\psi\bar{\psi}_0), \quad (39)$$

where

$$f_\perp^\chi = k_x\left[\left(k^2+\frac{1}{4H^2}\right)(k_z\Omega-q_z\omega_{\mathbf{k}})+4k_z^2q_z\omega_{\mathbf{k}}\right], \quad (40)$$

$$f_\perp^\psi = \omega_{\mathbf{k}}\left[q_z\omega_{\mathbf{k}}\left(2k^2+\frac{1}{4H^2}\right)\left(k^2+\frac{1}{4H^2}-4k_z^2\right) + 2k_z^2k^2\omega_{\mathbf{k}}q_z - -\Omega\left(k^2+\frac{1}{4H^2}\right)\frac{k_z}{4H^2}\right]. \quad (41)$$

Using equations (36)–(41) the main equations (12) may be reduced to

$$\begin{cases}\bar{\psi}_0 = I_\perp^\psi\bar{\psi}_0+I_\perp^\chi\bar{\chi}_0,\\ \bar{\chi}_0 = I_\parallel^\psi\bar{\psi}_0+I_\parallel^\chi\bar{\chi}_0,\end{cases} \quad (42)$$

where

$$\begin{cases}I_\perp^\psi = -2\frac{k_x^2q_z^3}{D^{(1)2}\left(q_z^2+\frac{1}{4H^2}\right)}I_{\mathbf{k}}f_\perp^\psi,\\ I_\perp^\chi = -2\frac{k_x^2q_z^3}{D^{(1)2}\left(q_z^2+\frac{1}{4H^2}\right)}I_{\mathbf{k}}f_\perp^\chi,\\ I_\parallel^\psi = -\frac{k_x^2q_z^2\omega_{\mathbf{k}}}{D^{(1)2}}I_{\mathbf{k}}f_\parallel^\psi, \quad I_\parallel^\chi = -\frac{k_x^2q_z^2\omega_{\mathbf{k}}}{D^{(1)2}}I_{\mathbf{k}}f_\parallel^\chi.\end{cases} \quad (43)$$

[25] Further, we can represent expression (24) as

$$D^{(1)} = 2\omega_{\mathbf{k}}\left(k^2+\frac{1}{4H^2}\right)\left(\frac{k_z\omega_{\mathbf{k}}q_z}{k^2+\frac{1}{4H^2}}+\Omega\right), \quad (44)$$

or

$$D^{(1)} = 2\omega_{\mathbf{k}}\left(k^2+\frac{1}{4H^2}\right)(\Omega-q_zV_g), \quad (45)$$

where

$$V_g = \frac{\partial\omega_{\mathbf{k}}}{\partial k_z} = -\frac{k_z\omega_{\mathbf{k}}}{k^2+\frac{1}{4H^2}} \quad (46)$$

is the z-component of the IG waves group velocity. Thus for the system (42), (43) we have

$$(I_\perp^\psi, I_\perp^\chi, I_\parallel^\psi, I_\parallel^\chi) = \frac{(a_\perp^\psi, a_\perp^\chi, a_\parallel^\psi, a_\parallel^\chi)}{(\Omega-q_zV_g)^2}. \quad (47)$$

[26] These formulas can be called the transport coefficients. The functions ($a_\perp^\psi$, $a_\perp^\chi$, $a_\parallel^\psi$, $a_\parallel^\chi$) mean

$$a_\perp^\psi = -\frac{k_x^2q_z^3I_{\mathbf{k}}f_\perp^\psi}{2\omega_{\mathbf{k}}^2\left(k^2+\frac{1}{4H^2}\right)^2\left(q_z^2+\frac{1}{4H^2}\right)}, \quad (48)$$

$$a_\perp^\chi = -\frac{k_x^2q_z^3I_{\mathbf{k}}f_\perp^\chi}{2\omega_{\mathbf{k}}^2\left(k^2+\frac{1}{4H^2}\right)^2\left(q_z^2+\frac{1}{4H^2}\right)}, \quad (49)$$

$$a_{\parallel}^{\psi} = -\frac{k_x^2 q_z^2 I_{\mathbf{k}} f_{\parallel}^{\psi}}{4\omega_{\mathbf{k}}\left(k^2 + \frac{1}{4H^2}\right)^2}, \tag{50}$$

$$a_{\parallel}^{\chi} = -\frac{k_x^2 q_z^2 I_{\mathbf{k}} f_{\parallel}^{\chi}}{4\omega_{\mathbf{k}}\left(k^2 + \frac{1}{4H^2}\right)^2}. \tag{51}$$

From equation (42) we arrive at the following zonal flow dispersion relation

$$1 - (I_{\perp}^{\psi} + I_{\parallel}^{\chi}) + I_{\perp}^{\psi} I_{\parallel}^{\chi} - I_{\parallel}^{\psi} I_{\perp}^{\chi} = 0. \tag{52}$$

Thus, in general, we deal with a biquadratic zonal flow dispersion relation with respect to $(\Omega - q_z V_g)$, with V_g given in equation (46).

4. Zonal Flow Instabilities: Generalized Lighthill Criterion

[27] We analyze equation (52) in the case of monochromatic wave packet of the primary modes. As all values of equation (52) are proportional to $q_z^2 \ll k_z^2$ all right-hand sides of equation (52) are relevant only if the value $(\Omega - q_z V_g)$ is also a small parameter. Then the functions $f_{\perp}^{\psi}, f_{\perp}^{\chi}, f_{\parallel}^{\psi}, f_{\parallel}^{\chi}$ defined by equations (36)–(41) can be calculated for $\Omega \approx q_z V_g$. We find

$$\begin{cases} f_{\perp}^{\psi} = \omega_{\mathbf{k}}^2 q_z\left(2k^2 + \frac{1}{4H^2}\right)\left(k_x^2 - 2k_z^2 + \frac{1}{4H^2}\right), \\ f_{\perp}^{\chi} = -k_x q_z \omega_{\mathbf{k}}\left(k_x^2 - 2k_z^2 + \frac{1}{4H^2}\right), \\ f_{\parallel}^{\psi} = q_z^2 \frac{\omega_{\mathbf{k}}^2}{k_x}\left(k^2 + \frac{1}{4H^2}\right)\left[2k_x^2 + \frac{1}{4H^2} - k_z^2 \frac{4k^2 + \frac{1}{4H^2}}{k^2 + \frac{1}{4H^2}}\right], \\ f_{\parallel}^{\chi} = q_z^2 \omega_{\mathbf{k}}\left(2k_z^2 - k_x^2 - \frac{1}{4H^2}\right). \end{cases} \tag{53}$$

Now we can find the necessary combinations occurring in the dispersion equation (52), namely

$$\begin{aligned} I_{\perp}^{\psi} + I_{\parallel}^{\chi} &= \frac{a_{\perp}^{\psi} + a_{\parallel}^{\chi}}{(\Omega - q_z V_g)^2} \\ &= \frac{k_x^2 q_z^4 I_{\mathbf{k}}}{4\left(k^2 + \frac{1}{4H^2}\right)^2}\left(k_x^2 - 2k_z^2 + \frac{1}{4H^2}\right) \\ &\times \left[1 - \frac{2\left(2k^2 + \frac{1}{4H^2}\right)}{q_z^2 + \frac{1}{4H^2}}\right]\frac{1}{(\Omega - q_z V_g)^2}, \end{aligned} \tag{54}$$

and the second term

$$I_{\perp}^{\psi} I_{\parallel}^{\chi} - I_{\parallel}^{\psi} I_{\perp}^{\chi} = 0. \tag{55}$$

[28] Thus the dispersion equation (52) reduces to the form

$$(\Omega - q_z V_g)^2 = -\Gamma^2, \tag{56}$$

where Γ^2 means the squared zonal flow growth rate defined by

$$\Gamma^2 = \frac{k_x^2 q_z^4 I_{\mathbf{k}}}{4}\frac{V_g'}{\omega_{\mathbf{k}}}\left[1 - \frac{2\left(2k^2 + \frac{1}{4H^2}\right)}{q_z^2 + \frac{1}{4H^2}}\right] \tag{57}$$

where

$$V_g' = \frac{\partial V_g}{\partial k_z} = \frac{\partial^2 \omega_{\mathbf{k}}}{\partial k_z^2} = \omega_{\mathbf{k}}\frac{2k_z^2 - k_x^2 - \frac{1}{4H^2}}{\left(k^2 + \frac{1}{4H^2}\right)^2}. \tag{58}$$

in equation (57). For the modulational instability Γ^2 should be positive, i.e.,

$$\frac{V_g'}{\omega_{\mathbf{k}}}\left[1 - \frac{2\left(2k^2 + \frac{1}{4H^2}\right)}{q_z^2 + \frac{1}{4H^2}}\right] > 0. \tag{59}$$

We note that V_g' changes sign when $k_x = \pm(2k_z^2 - \frac{1}{4H^2})^{1/2}$. This occurs on the IG waves caustics, where $V_g' = 0$.

[29] For the IG waves we may assume $k^2 \gg \frac{1}{4H^2}$. As for the term $q_z^2 \ll k^2$, we may consider two special cases: (1) $q_z^2 \gg \frac{1}{H^2}$ and (2) $q_z^2 \ll \frac{1}{H^2}$.

[30] 1. In this case $(q_z^2 H^2 \gg 1)$ the necessary condition for instability is

$$\frac{V_g'}{\omega_{\mathbf{k}}} < 0. \tag{60}$$

This condition is similar to the Lighthill criterion for modulation instability in nonlinear optics [*Lighthill*, 1965]. Instability occurs when $k_x^2 > 2k_z^2$ and this condition thus applies to IG pump waves with wave vectors in the cone

$$-\frac{k_x}{\sqrt{2}} < k_z < \frac{k_x}{\sqrt{2}}. \tag{61}$$

The maximum growth rate is attained at the axis of the cone when $k_z = 0$. In this case zonal flow growth rate is of the order

$$\Gamma \approx q_z k k_x \left(\frac{I_{\mathbf{k}}|V_g'|}{\omega_{\mathbf{k}}}\right)^{1/2} \approx k_x q_z I_{\mathbf{k}}^{1/2} \cong |k_x q_z \tilde{\psi}_+|. \tag{62}$$

Expression (62) describes the initial (linear) stage of zonal flow growth due to the parametric instability of small-scale IG waves.

[31] 2. In this case $(q_z^2 H^2 \ll 1)$ the instability condition is expressed by the same condition (60) and the zonal flow

growth rate is decreasing q_zH times in comparison with the expression (62)

$$\Gamma \approx q_z^2 k k_x H \left(\frac{I_{\mathbf{k}}|V_g'|}{\omega_{\mathbf{k}}}\right)^{1/2} \approx k_x q_z^2 H I_{\mathbf{k}}^{1/2}. \qquad (63)$$

[32] Both nonlinear growth rates (62) and (63) increase as k. Physically, this instability is the manifestation of an inverse cascade. This analysis shows that the spectral energy of the small-scale IG waves turbulence is transferred into the large scales of zonal flows, i.e., the IG wave energy is converted into the energy of slow zonal motions. The analysis does not show if the spectrum fills in with many scales bridging the region between q_z and k_x, k_z. Nonlinear solutions to be presented in the future suggest that in most cases the spectrum does fill in intermediate scales when the small-scale intensity I_k is maintained.

5. Conclusion and Discussion

[33] In the present study we have investigated the nonlinear generation of low-frequency and large-scale zonal flows driven by relatively small-scale internal gravity (IG) waves in the Earth's neutral atmosphere. The investigated zonal flows propagate along the parallels and the corresponding flow velocity depends only on the vertical z-coordinate. The modified parametric approach [see, e.g., *Kaladze et al.*, 2007b] is developed considering the monochromatic primary modes. Accordingly, we study the interaction of a pump IG waves, two satellites of the pump waves (side-band waves) and a sheared zonal flow. The driving mechanism of this instability is due to the Reynolds stress $R_\perp$ and mean stratification force $R_\parallel$ in the equation governing the evolution of zonal flows.

[34] We have obtained zonal flow dispersion relation given by equation (56). The appropriate instability condition is found. According to our investigation (see equations (57), (58)) the possibility of zonal flow generation by IG modes in the upper atmosphere is rigidly connected with the sign of $\partial^2\omega/\partial k_z^2$ for these modes (the dispersion of the group velocity of the primary modes). Accordingly, a particular feature of this instability is that it appears solely for IG waves that are localized in a cone bounded by the caustics for which $V_g' = 0$. This feature can lead to the formation of a so-called caustic shadow in the spectrum of the IG waves.

[35] The most complete survey of observations of IG waves in the troposphere are found in the book by *Gossard and Hooke* [1975]. Investigated IG waves have the typical period of oscillations $\tau \geq 10$ min (with the corresponding frequency $\omega \sim 10^{-2}\ s^{-1}$). Photometric and visual observations of noctilucent clouds in mesosphere regions ($z \approx 80$ to 85 km) disclose the presence of IG waves with a vertical displacement amplitude $\zeta_0 = 0.5$ to 4 *km*, wavelengths $\lambda \approx 5$ to 50 km, and phase velocities $v_p \approx 10$ to 20 m/s.

[36] As is seen from our results the maximum growth rate of the zonal flow generation is of the order of (see equation (62))

$$\Gamma \simeq \frac{q_z}{k_x}\omega\tilde{\psi}_+. \qquad (64)$$

Here the stream function $\tilde{\psi}_+$ of pump modes is normalized by the value $v_p\lambda$. For the tropospheric IG modes we may choose $q_z/k_x \approx 10^{-1}$, $\omega \approx 10^{-2}$ s^{-1} and $\tilde{\psi} \approx 10^{-1}$. Then we obtain the following numerical value for the zonal flow growth rate $\Gamma \approx 10^{-4}$ s^{-1}. This estimate is consistent with existing observations and our investigation provides the essential nonlinear mechanism for the transfer of spectral energy from small- scale IG waves to large-scale enhanced zonal flows in the Earth's neutral atmosphere.

[37] *Rasmussen et al.* [2006] illustrate the generation of zonal flow in a simple fluid experiment performed in a rotating container with a radially symmetric bottom topography. It seems reasonable to simulate generation of zonal flows to confirm the theory of IG waves excitation provided in the present paper. An important new area of research both experimentally and computationally would be to determine how the turbulence may evolve beyond the parametric instability given here for the decays of high k modes in to zonal flows into a fully developed turbulence spectrum with the Kolmogorov spectral index of the -3 as seen in the atmosphere. Both laboratory experiments and computer simulations may be required to shed light on the evolution of the turbulence spectrum with vortices and zonal flows both of which can modify the turbulent energy spectrum.

[38] Finally, we would like to discuss some applications of IG waves to the earthquake forecast problem. Tectonic processes occurring in the Earth's crust generate Rayleigh waves propagating over the Earth's surface from the epicenter at supersonic velocities ($\approx$3 km/s). Rayleigh waves give rise to atmospheric-pressure disturbances by vertical pulsed action on the air. Excited IG waves propagate vertically almost without damping to a height of 100 km (their amplitude increases exponentially as the height increases) creating favorable conditions for nonlinear interactions. Appearance of large-scale zonal flow fluctuations may be taken as confirmation of this phenomenon. Zonal flows themselves give rise to long-lasting vortices by shear instability. The additional mixing of neutrals related to IG vortices and the transport of atmospheric components in the vertical direction, increases the neutral density in the atmosphere at this height. The increase in the density of oxygen atoms increases the efficiency of their recombination and, consequently, increases the intensity of green ($\lambda = 557.7$ nm) night-sky radiation observed before strong earthquakes. Hence, the increase of the green night-sky radiation can be used as an immediate (24–48 h) sign of earthquakes.

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T. W. Garner, Center for Ionospheric Research, Space and Geophysics Laboratory, Applied Research Laboratories, University of Texas at Austin, Austin, TX 78712, USA.
W. Horton and J. W. Van Dam, Institute for Fusion Studies, University of Texas at Austin, Austin, TX 78712, USA. (horton@physics.utexas.edu)
T. D. Kaladze, I. Vekua Institute of Applied Mathematics, Tbilisi State University, 2 University Street, 0143 Tbilisi, Georgia.