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QUASILINEAR EVOLUTION OF TEARING MODES
DURING MAGNETIC RECONNECTION

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Abstract

Particle simulations of magnetic reconnection show that the presence of a magnetic field parallel to the current sheet changes the character of reconnection from a laminar process to a weakly turbulent one. An analysis of the quasilinear spectrum of tearing modes is presented which may account for some features of the 2-1/2D simulations. The turbulent, incompressible plasma flow carrying reversed magnetic flux to the tearing layer is shown to produce a positive definite anomalous resistivity. The relationship to the negative anomalous resistivity of Biskamp is discussed.

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I. Neutral Current Sheet and Tearing Instability

Extensive 2-1/2D particle simulations¹ of collisionless magnetic reconnection show that there are two regimes depending on the ratio of the reconnecting poloidal magnetic field to the toroidal magnetic field. In the absence of the toroidal magnetic field parallel to the current sheet, the reconnection involves a compressible, singular laminar flow leading to the Sweet-Parker process² followed by a faster second phase.³ In the presence of a strong enough toroidal or parallel guide field, however, the flow becomes essentially incompressible and the simulations¹ show that the magnetic reconnection is a turbulent process. Such a case may become relevant to the reconnection process associated with the coalescence instability⁴ or the Kadomtsev disruption model,⁵ for example. In general, any reconnection process with a strong toroidal field driven by other instabilities, other island activities, or flow activities may involve the present process. We, however, exclude consideration of any three dimensional effects such as multiple helicity.

As noted in Ref. 1, the transition of the laminar to turbulent reconnection was observed only in the collisionless simulation but not in the magnetohydrodynamic (MHD) particle simulation. This suggests that the turbulence involves electron dynamics in an essential way, since the MHD smears out the electron dynamics. This onset of turbulence may be related to the onset of magnetized electron dynamics. In the present work, we concentrate on analysis of the resultant turbulence; we do not discuss the onset of or transition to the turbulent plasma, however.

The flow and magnetic activity are characterized by a spectrum of k_x, k_y fluctuations with time scales comparable to the maximum growth rate for collisionless tearing modes $\Delta\omega \sim \gamma_{\max} \leq k_y |\Delta B_y| v_e / B$. Figure 1 shows typical examples of the contours of constant poloidal flux in the turbulent phase.¹ In the following, we analyze some aspects of the quasilinear evolution of a spectrum of tearing mode turbulence.

The geometry we consider is a neutral current sheet $j_z(x)$ of thickness $\Delta x = x_0(t)$. The reversed poloidal field $B_y(x) = \partial\psi/\partial x$ vanishes at $x=0$ is held fixed at $B_y = \pm B_y^0$ at $x = \pm L$. The electric and magnetic fields in the plasma are

$$\underline{\tilde{E}} = -\nabla\phi(x,y,t) + \frac{1}{c} \frac{\partial\psi}{\partial t} \hat{z} \quad (1)$$

$$\underline{\tilde{B}} = \hat{z} \times \nabla\psi + B\hat{z}, \quad (2)$$

and thus the component of the electric field parallel to $\underline{\tilde{B}}$ is

$$E_{\parallel} = \frac{\underline{\tilde{B}} \cdot \underline{\tilde{E}}}{B} = \frac{1}{c} \left(\frac{\partial\psi}{\partial t} + \underline{v} \cdot \nabla\psi \right) \quad (3)$$

with

$$\underline{v} = \frac{c\hat{z} \times \nabla\phi}{B} \quad (4)$$

and the current density is

$$\underline{j} = \frac{c}{4\pi} \nabla \times \underline{\tilde{B}} = \frac{c}{4\pi} \nabla^2 \psi \hat{z}. \quad (5)$$

In the turbulent motion the fields are split into their mean and fluctuating components by

$$\langle F \rangle = \int \frac{dy}{L_y} \int \frac{dt}{T} F \quad \text{and} \quad \delta F = F - \langle F \rangle$$

where L_y is the period of the system in the y direction and $T\Delta\omega \gg 1$ and yet T is short compared with the time scale of the nonlinear evolution of the background.

The space-time scales of the fluctuations are determined by the width of the current sheet x_0 and the poloidal Alfvén speed $v_{Ay} = B_y^0 / (4\pi n m_i)^{1/2}$ with

$$\Delta k_{\perp} \leq \frac{1}{\Delta x} \sim \frac{1}{x_0(t)} \quad \text{and} \quad \Delta k_{\parallel} = \Delta k_y B_y(x) / B \quad (6)$$

$$\Delta\omega \lesssim \max\left(\frac{v_e B_y^0}{x_0 B}, \frac{v_{Ay}}{x_0}\right)$$

in the laboratory frame. The first characteristic frequency is the inverse of electron diffusion time in turbulent magnetic fields.¹ The second is the inverse of the Alfvén transit time. The thermal electrons see a correlation time $\tau_c^e = 1/\Delta k_{\parallel} v_e$ and the ions a correlation time $\tau_c^i = \min(\gamma_k^{-1}, 1/\Delta k_{\perp} \tilde{v})$ where $\tilde{v} = \langle v^2 \rangle^{1/2}$.

For the mean profile $\langle \psi \rangle$ and $\langle \phi \rangle$ we take $\langle \phi \rangle = 0$ and

$$\begin{aligned}\langle \psi \rangle &= x_0 B_y^0 \ln \left[\cosh \left(\frac{x}{x_0} \right) \right] \\ \langle B_y \rangle &= B_y^0 \tanh \left(\frac{x}{x_0} \right) \\ \langle j_z \rangle &= \frac{B_y^0}{x_0} \operatorname{sech}^2 \left(\frac{x}{x_0} \right) .\end{aligned}\tag{7}$$

The analysis of the linear stability of the system at large magnetic Reynolds numbers $S = x_0 v_{Ay} / \eta$ is given by Drake and Lee.⁶ The linear outside solution of $\underline{B} \cdot \nabla j_{\parallel} = \hat{z} \cdot \nabla \psi \times \nabla \nabla^2 \psi = 0$ gives

$$\psi_{k_y}(x) = \psi_{k_y}(0) e^{\mp k_y x} \left[1 \pm \frac{\tanh \left(\frac{x}{x_0} \right)}{k_y x_0} \right] \cos(k_y y)\tag{8}$$

with

$$\Delta'_{k_y} = \left. \frac{2 \partial_x \psi_{k_y}}{\psi_{k_y}} \right|_{x=0} = \frac{2}{x_0} \left[\frac{1}{k_y x_0} - k_y x_0 \right] .\tag{9}$$

The collisionless growth rate is

$$\gamma_{k_y} = \frac{B_y^0 k_y v_e c^2 x_0 \Delta'_{k_y}}{2 \pi^{1/2} B_x^0 \omega_{pe}^2}\tag{10}$$

and the collisional growth rate is

$$\gamma_{k_y} = \left(\frac{v_{Ay}}{x_0}\right) (Cx_0 \Delta'_{k_y})^{4/5} (k_y x_0)^{2/5} s^{-3/5} \quad (11)$$

with $C = \Gamma(1/4)/\Gamma(3/4)$.

In the tearing layer the magnetic fluctuations $\psi_{k_y}(x)$ drive an electrostatic potential $\phi_{k_y}(x)$ through Poisson's equation or in the quasi-neutral limit valid for $k_{\perp} \lambda_{De} \ll 1$ through $\nabla \cdot \underline{J} = -\partial_t \rho_{\phi} = 0$.

II. Nonlinear Electron Dynamics and Ohm's Law

Even at a low amplitude of the fields $\psi_{\underline{k}}, \phi_{\underline{k}}$ the electron motion becomes nonlinear. The distribution $f(\underline{x}, y, t)$ of magnetized or adiabatic electrons evolves by

$$\frac{\partial f}{\partial t} + v_{\parallel} \frac{\underline{B}}{B} \cdot \frac{\partial f}{\partial \underline{x}} + \frac{c \underline{E} \times \underline{B}}{B^2} \cdot \frac{\partial f}{\partial \underline{x}} - \frac{e}{m_e} E_{\parallel} \frac{\partial f}{\partial v_{\parallel}} = \hat{C} f \quad (12)$$

where we include a weak electron-ion collision operator \hat{C} to define the classical resistivity η_0 .

With the 2D spatial dependence of the simulations and the electromagnetic fields in Eqs. (1)-(3) the kinetic equation (12) becomes

$$\frac{\partial f}{\partial t} + \frac{\hat{z}}{B} \cdot \nabla (v_{\parallel} \psi + c \phi) \times \nabla f - \frac{e}{m_e c} \frac{d\psi}{dt} \frac{\partial f}{\partial v_{\parallel}} = \hat{C} f \quad (13)$$

with

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \frac{c}{B} \hat{z} \cdot \nabla\phi \times \nabla\psi \equiv \frac{\partial\psi}{\partial t} + \frac{c}{B} [\phi, \psi] . \quad (14)$$

In Eqs. (13) and (14) it is useful for the nonlinear analysis to introduce the Poisson brackets $[f, g]$ defined by

$$[f, g] = \hat{z} \cdot \nabla f \times \nabla g = \frac{\partial f}{\partial x} \frac{\partial g}{\partial y} - \frac{\partial f}{\partial y} \frac{\partial g}{\partial x}$$

because of their obvious conservation and symmetry properties. We note that $\int dx f[f, g] = \int dx g[f, g] = 0$ and

$$\langle [f, g] \rangle_y = -\partial_x \langle g \partial f / \partial y \rangle = \partial_x \langle f \partial g / \partial y \rangle .$$

One of the most important aspects of the electron motion during field line reconnection is the form of Ohm's law parallel to the fluctuating magnetic field. Thus, we take the parallel current moment $j_{\parallel} = -e \int v_{\parallel} f dy$ of Eq. (14) and then average over the fluctuations to obtain the generalized Ohm's law

$$-\frac{1}{e} \frac{\partial \langle j_{\parallel} \rangle}{\partial t} + \frac{1}{m_e B} \frac{\partial}{\partial x} \left\langle -\frac{\partial \psi}{\partial y} \delta p_{\parallel} \right\rangle + \frac{c}{B} \frac{\partial}{\partial x} \left\langle \frac{\partial \phi}{\partial y} \delta j_{\parallel} \right\rangle \quad (15)$$

$$+ \frac{e \langle n \rangle}{m_e} \langle E_{\parallel} \rangle + \frac{e}{m_e} \langle \delta n \delta E_{\parallel} \rangle = \frac{v_e}{e} \langle j_{\parallel} \rangle .$$

There are three important quasilinear transport processes described by Eq. (15).

1. Radial Transport of $\langle j \rangle$

The fluctuations in the electron pressure δp_{\parallel} combine with the cross-field tilting of the magnetic field $\delta B_x = -\partial\psi/\partial y$ to produce a net flow of current across the neutral sheet. In addition, the current is convected by the $\underline{E} \times \underline{B}$ flow across the current sheet. The net current flux F_j is given by (second and third terms of Eq. (15))

$$F_{j_{\parallel}} = \frac{c}{B} \left\langle -\frac{e}{m_{ec}} \frac{\partial\psi}{\partial y} \delta p_{\parallel} + \frac{\partial\phi}{\partial y} \delta j_{\parallel} \right\rangle \quad (16)$$

$$= -\frac{c}{B} \sum_{\underline{k}} ik_y \left[\langle \phi_{\underline{k}}^* \delta j_{\parallel \underline{k}} \rangle - \frac{e}{m_{ec}} \langle \psi_{\underline{k}}^* \delta p_{\parallel \underline{k}} \rangle \right].$$

A lengthy calculation of δp_{\parallel} and δj_{\parallel} using renormalized turbulence theory⁷ shows that the principal contribution to $F_{j_{\parallel}}$ reduces to an anomalous electron viscosity

$$F_{j_{\parallel}} = -\mu_A \frac{d\langle j_{\parallel} \rangle}{dx} \quad (17)$$

with

$$\mu_A = -\frac{c^2}{B^2} \sum_{\underline{k}} \int d\underline{y} \langle f_c \rangle \frac{k_y^2 v_{\parallel}^2}{v_e^2} \left| \phi_{\underline{k}} + \frac{v_{\parallel}}{c} \psi_{\underline{k}} \right|^2 \text{Im } G_{\underline{k}}(v_{\parallel}) \quad (18)$$

where $G_{\underline{k}}(v_{\parallel}) = (\omega - k_{\parallel}(x)v_{\parallel} + i\nu_{\underline{k}})^{-1}$ and

$$\nu_{\underline{k}} = i\nu_0 \sum_{\underline{k}_1} (\underline{k} \times \underline{k}_1 \cdot \hat{z}/B)^2 |v_{\parallel} \psi_{\underline{k}_1} + c \phi_{\underline{k}_1}|^2 G_{\underline{k}-\underline{k}_1}(v_{\parallel}).$$

For the regime $v_0 < v_k \leq \Delta k_{\perp} v_e \langle \delta B^2 \rangle^{1/2} / B < (v_e / x_0) (B_y^0 / B)$ we estimate that the maximum anomalous viscosity is given by

$$\mu_A \equiv \sum_k \frac{v_e^2 |\delta B_k|^2}{v_k B^2} \lesssim \frac{v_e |B_y^0|}{x_0 B} \quad (19)$$

where the last formula applies for large $\langle \delta B_{\perp}^2 \rangle^{1/2}$ and $v_k \sim kv_e \langle \delta B_{\perp}^2 \rangle^{1/2} / B$.

2. Mean Parallel Electric Field $\langle E_{\parallel} \rangle$

The convection of the magnetic flux $\tilde{y} \cdot \nabla \psi$ produces an average inductive change in the electric field parallel to the neutral sheet current. The total mean field is given by

$$\langle E_{\parallel} \rangle = \frac{\partial \langle \psi \rangle}{\partial t} + \frac{\partial F_{\psi}}{\partial x} \quad (20)$$

where F_{ψ} is the flux of B_y -magnetic flux into the turbulent layer given by

$$F_{\psi} = \langle v_x \psi \rangle = \sum_{\tilde{k}} \frac{ick_y}{B} [\langle \phi_{\tilde{k}}^* \psi_{\tilde{k}} \rangle - \langle \phi_k \psi_k^* \rangle] . \quad (21)$$

This flux F_{ψ} depends on the cross-correlation function of the magnetic and kinetic fluctuations. In Sec. III we calculate this flux in detail for a simple dynamical model. In general $\Delta E_z = \partial F_{\psi} / \partial x$ can have either sign -- ($\Delta E_z > 0$) reinforcing the ambient resistive E_z or ($\Delta E_z < 0$) reacting against it.

3. Parallel Friction from Density Fluctuations

The correlation function

$$\langle \delta n \delta E_{\parallel} \rangle = \sum_{\tilde{k}} \langle \delta n_{\tilde{k}}^* E_{\parallel \tilde{k}} \rangle \quad (22)$$

produces a parallel friction to the passage of the electron current. The most common mechanisms giving rise to this type of anomalous resistivity are the lower hybrid and ion-acoustic waves. These mechanisms for producing anomalous resistivity are reviewed by Papadopoulos.⁸

III. Evolution of the Mean Flux and the Effective Resistivity

In this section we neglect processes (1) anomalous electron viscosity and (3) parallel friction from density fluctuations in Sec. II and calculate process (2) the turbulent transport of magnetic flux.

For $\partial_t \langle j_{\parallel} \rangle \ll (e^2 \langle n \rangle / m_e) \langle E_{\parallel} \rangle$ the generalized Ohm's law (15) and Eq. (20) yield

$$\frac{\partial \langle \psi \rangle}{\partial t} + \frac{\partial F_{\psi}}{\partial x} = \frac{c^2 \eta_0}{4\pi} \frac{\partial^2}{\partial x^2} \langle \psi \rangle \quad (23)$$

where η_0 is the collisional resistivity and F_{ψ} is given by Eq. (21). The value of F_{ψ} requires knowledge of the ψ - ϕ cross-correlation function.

The electrostatic potential is determined by Poisson's equation in the quasi-neutral limit

$$\nabla \cdot \mathbf{J}_{\perp}^i = -\mathbf{B} \cdot \nabla (J_{\parallel}^e / B) = -\frac{1}{B} [\psi, \nabla^2 \psi]$$

where in the cross-field ion current we include the effect of a weak ion viscous force $\mu_0 \nabla^2 \mathbf{v}$. The magnetic flux ψ is determined by Ohm's law parallel to magnetic field $E_{\parallel} = \eta_0 j_{\parallel}$ using Eqs. (3) and (5). The model nonlinear equations for ψ and ϕ are

$$\frac{\partial \psi}{\partial t} + \frac{c}{B} [\phi, \psi] = \frac{\eta_0 c^2}{4\pi} \nabla^2 \psi \quad (24)$$

$$\frac{1}{v_A^2} \nabla^2 \frac{\partial \psi}{\partial t} = \frac{c}{B} [\psi, \nabla^2 \psi] + \frac{\mu_0}{v_A^2} \nabla^4 \phi \quad (25)$$

Although the system (24) and (25) is an overly simplified description of the reconnection dynamics, it possess the correct conservation laws and the tearing mode instability, and thus is a useful model for calculating the ψ - ϕ correlation function.

Small amplitude fluctuations $\delta\psi_{k_y}(x)$, $\delta\phi_{k_y}(x)\exp(ik_y y + \gamma t)$ in Eqs.(24)-(25) satisfy the linear system

$$\begin{bmatrix} \gamma + \eta_0 (k_y^2 - \partial_x^2) & -ik_y \frac{\partial \psi}{\partial x} \\ -ik_y \frac{\partial \psi}{\partial x} (k_y^2 - \partial_x^2) - ik_y \frac{\partial^3 \psi}{\partial x^3} & \gamma (k_y^2 - \partial_x^2) + \mu_0 (k_y^2 - \partial_x^2)^2 \end{bmatrix} \begin{bmatrix} \delta\psi_{k_y}(x) \\ \delta\phi_{k_y}(x) \end{bmatrix} = 0 \quad (26)$$

The sixth order system (26) with suitable boundary conditions is an eigenvalue problem for $\gamma = \gamma(S, M, k_y x_0)$ with $S = x_0 v_{Ay} / \eta_0$ and $M = x_0 v_{Ay} / \mu_0$. The same equation is solved by Dalhburg et.al.⁹ for the problem of 2D incompressible resistive-viscous magnetohydrodynamics. For $SM \gtrsim 2$ the system is unstable for all modes with $k_y x_0 \lesssim 1$ and the growth rate is given by Eq. (11) for large $S = x_0 v_{Ay} / \eta_0$.

The linear solutions of Eq. (26) determine the phase relation between $\psi_{k_y}(x)$ and $\phi_{k_y}(x)$ or $\psi(k_x, k_y)$ and $\phi(k_x, k_y)$ by a Fourier transform in x . [In general the relationship between $\psi(\underline{k})$ and $\phi(\underline{k})$ is nonlocal due to coupling from $B_y(x/x_0)$. We neglect this coupling here assuming $k_y x_0 > 1$ and consider the effect in a future study.] From Eq. (26) we write the quasilinear phase relations as

$$\delta \psi_{\underline{k}} = \frac{ik_y \psi'(x)}{\gamma + k_{\perp}^2 \eta_0} \delta \phi_{\underline{k}} \quad (27)$$

$$\delta \phi_{\underline{k}} = \frac{ik_y (k_{\perp}^2 \psi' + \psi''')}{k_{\perp}^2 (\gamma + k_{\perp}^2 \mu_0)} \delta \psi_{\underline{k}0} \quad (28)$$

Substituting relationships (27) and (28) into Eq. (21) we obtain the flux formula

$$F_{\psi} = - \sum_{\underline{k}} \left[\frac{k_y^2 \langle |\delta \phi_{\underline{k}}|^2 \rangle}{\gamma_{\underline{k}} + k_{\perp}^2 \eta_0} - \frac{k_y^2 (k_{\perp}^2 + \psi''' / \psi')}{k_{\perp}^2 (\gamma_{\underline{k}} + k_{\perp}^2 \mu_0)} \langle |\delta \psi_{\underline{k}}|^2 \rangle \right] \frac{\partial \psi}{\partial x} \quad (29)$$

The first term in the square bracket in Eq. (24) comes from diffusion by ExB drift; the second term originates from the electron meandering

in the turbulent magnetic fields. Formula (29) is instructive for several purposes.

First we consider the short wavelength, dissipationless limit, $k_{\perp}^2 \gg \psi''''/\psi' \approx x_0^{-2}$ and $\eta_0 = \mu_0 = 0$. Formula (29) then reduces to the Biskamp formula¹⁰

$$F_{\psi} = - \eta_A \frac{\partial \psi}{\partial x} \quad (30)$$

with

$$\begin{aligned} \eta_A &= \sum_{\tilde{k}} \gamma_{\tilde{k}}^{-1} k_y^2 [\langle |\delta\phi_{\tilde{k}}|^2 \rangle - \langle |\delta\psi_{\tilde{k}}|^2 \rangle] \\ &= \tau_c [\langle v_x^2 \rangle - \langle \delta B_x^2 \rangle] \end{aligned} \quad (31)$$

with τ_c the average correlation time in the laboratory frame. Biskamp uses formula (31) along with the observation that the 3D simulations of tokamaks show the buildup of a short scale magnetic turbulence with $\langle \delta B^2 \rangle \gg \langle \delta v^2 \rangle$ to propose an explanation of the major disruption based on the negative value of η_A .

Formula (31) shows that a spectrum of Alfvén wave turbulence does not produce a transport of magnetic flux since $|\delta\phi_{\tilde{k}}|^2 = |\delta\psi_{\tilde{k}}|^2$ for Alfvén waves and thus $F_{\psi} = 0$.

Finally, we observe that for tearing mode turbulence, where formula (29) must be used, the magnetic and kinetic contributions in Eq. (29) are related through the local dispersion relation

$$k_{\perp}^2 (\gamma + k_{\perp}^2 \mu_0) (\gamma + k_{\perp}^2 \eta_0) + k_y^2 (k_{\perp}^2 \psi'^2 + \psi' \psi'''') = 0$$

from Eq. (26). Along this dispersion relation (\underline{k}, γ) the magnetic and kinetic contributions are equal and combine to yield

$$F_{\psi} = - \sum_{\underline{k}} \frac{2k_y^2 \langle |\delta\phi_{\underline{k}}|^2 \rangle}{\gamma_{\underline{k}} + k_{\perp}^2 \eta_0} \frac{\partial \psi}{\partial x} \quad (32)$$

giving a positive definite anomalous η_A from tearing mode turbulence.

IV. Summary and Discussion

To complete the quasilinear calculation we carry out the integration of

$$\frac{\partial \psi}{\partial t}(\underline{x}, t) = \frac{\partial}{\partial x} \left(\eta(\underline{x}, t) \frac{\partial \psi}{\partial x} \right)$$

with

$$\eta(\underline{x}, t) = \eta_0 + \sum_{k_y} \frac{2k_y^2 \langle |\delta\phi_{k_y}(\underline{x}, t)|^2 \rangle}{\gamma_{k_y} + k_{\perp}^2 \eta}$$

(34)

with $\delta\phi_{k_y}(\underline{x}, t)$ given by the inner solution of the tearing mode equations and $\gamma_{\underline{k}} = \gamma_{\underline{k}}(\Delta'_{\underline{k}})$ calculated from the solution of the outer equation for $\delta\psi_{\underline{k}}(\underline{x})$ using $\psi''''(\underline{x}, t)/\psi'(\underline{x}, t)$ for the local potential.

The results are shown schematically in Fig. 3 and will be reported in detail in a later work. The principal features are that there is a strong inward connection of the magnetic flux to the turbulent reconnection layer. This flux induces an electric field $E_0 + \Delta E_z$ with $\Delta E_z = \partial F_{\psi} / \partial x$ which has $E_0 \Delta E_z < 0$, inhibiting the current flow in the

center of the current sheet, and $E_0 \Delta E_z > 0$, strengthening the current flow, in the outer regions of reconnection. As the current layer broadens the spectrum shifts to longer wavelengths. If the turbulence remains weak then we expect the turbulent broadening to continue until the last mode $k_y = 2\pi/L_y$ is stabilized.

Acknowledgments

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Figure Captions

Fig. 1(a) and (b) Contour of constant poloidal magnetic flux ψ at times t_1 and $t_2 > t_1$ showing the growth of the turbulent layer $x_0(t)$. (c) The approximately linear increase of the flux ψ trapped in the turbulent layer.

Fig. 2

Schematic of the evolution described by the quasilinear system of equations.

Fig. 3

Evolution of the quasilinear effective anomalous resistivity, the inward flux F_ψ of the poloidal magnetic flux and the change ΔE_z in the parallel electric field produced by the inward turbulent convection of magnetic flux.

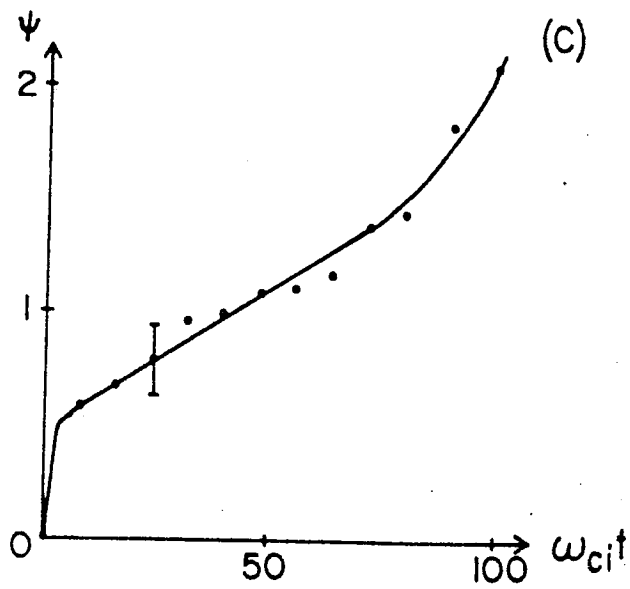
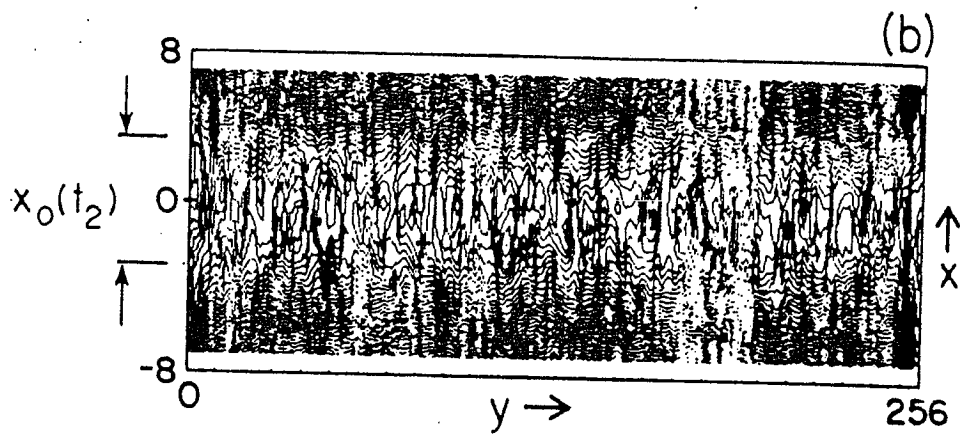
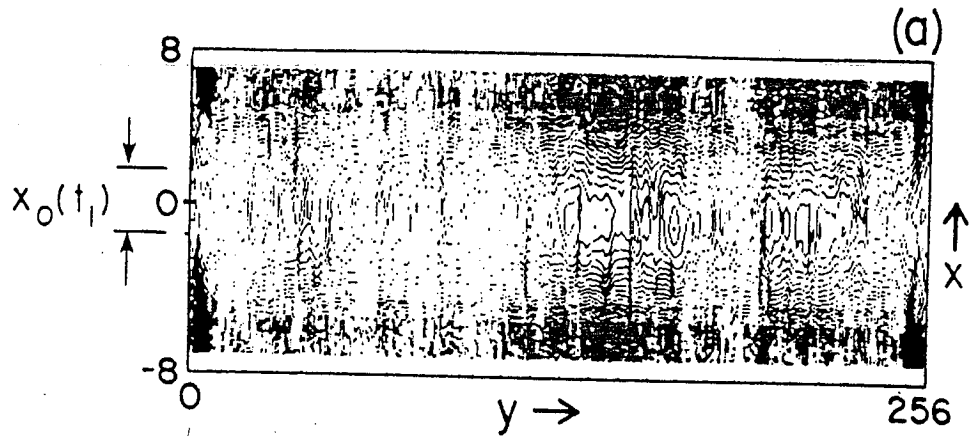


FIG. 1

$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial x} \left((\eta_0 + \eta_A) \frac{\partial \psi}{\partial x} \right) + \text{B.C.}$$

$$\delta \phi_k(x, t) = \delta \phi_k(x) e^{\gamma_k t}$$

$$\text{New } \eta = \eta_0 + \sum_k \frac{2k_y^2 |\delta \phi_k(x, t)|^2}{\gamma_k + k_{\perp}^2 \eta}$$

$$\left(\partial_x^2 - k_y^2 - \frac{\psi''(x, t)}{\psi'(x, t)} \right) \delta \psi_k(x) = 0$$

New $\Delta'(k, t)$ and $\gamma_k(t)$

FIG. 2