Yang-Mills Magnetofluid Unification

Bindu A. Bambah
School of Physics, University of Hyderabad, Hyderabad, Andhra Pradesh, 500 046, India

Swadesh M. Mahajan
Institute of Fusion Studies, University of Texas, Austin, Texas, 78712, USA

Chandrasekher Mukku
International Institute of Information Technology, Hyderabad, Andhra Pradesh, 500 032, India
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We generalize the hybrid magnetofluid model of a charged fluid interacting with an electromagnetic field to the dynamics of a relativistic hot fluid interacting with a non-Abelian field. The fluid itself is endowed with a non-Abelian charge and the consequences of this generalization are worked out. Applications of this formalism to the quark gluon plasma are suggested.

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Recent experiments at the BNL Relativistic Heavy Ion Collider (RHIC) have shed light on the behavior of hot dense nuclear matter [1]. The conjecture, that quarks and gluons deconfine and become a plasma in extreme conditions [2], is close to being experimentally proved. However, it has been realized that the deconfined quark gluon matter that has been revealed at RHIC, far from being the weakly interacting collisionless plasma, is, in fact, behaving more like a quark gluon liquid [3]. The quark gluon plasma (QGP) liquid is dense, but seems to flow with very little viscosity approximating an ideal fluid, governed by the standard laws of hydrodynamics. Thus, most of the phenomenological input for the explanation of the data at RHIC comes from adopting a hydrodynamic approach to the plasma for deducing its properties. For a proper fluid dynamical description of the quark gluon fluid, the non-Abelian charges of the quarks and gluons have to be taken into consideration. Many theories focusing on various aspects of an ideal fluid in interaction with Yang-Mills (YM) fields have been proposed [4–8]. The RHIC data coupled with recent studies by Jackiw and his collaborators on the "Clebsch representation" of YM fluid dynamics have lead to a resurgence of interest in these theories [1,9]. In tune with this revival, we investigate the dynamics of a relativistic hot fluid with a non-Abelian charge in terms of a model which unifies the YM field with the flow field [10]. Apart from its possible phenomenological applications, the motivation for the following work is based on the aesthetic criterion of unifying the fluid field and the YM field into a YM "magnetofluid" by a "gauge principle". Using a gauge theoretic analogy, it has been shown that a fully antisymmetric flow tensor, resembling the electromagnetic field, can be constructed and the unification is achieved by defining an effective field strength tensor that combines appropriately weighted electromagnetic and flow fields [10]. Is a consistent and useful non-Abelian generalization of this genre of flow-field unification possible? This investigation constitutes the theme of this Letter.

First, let us recapitulate the salient features of Abelian (Maxwell) Magnetofluid unification [10]. Although Maxwell's electrodynamics provides equations of motion for the electric and magnetic fields, for describing their interaction with matter fields (charged particles), the Lorentz force law has to be independently postulated. In contrast, in a gravity coupled plasma, a natural consequence of general covariance allows for the Lorentz force law for charged particles moving in a gravitational field [11] to be derived from the field equations. In the limit of a weak gravitational field (the flat space-time limit, $\nabla_\mu \to \partial_\mu$), the component form of the Lorentz force law reads

$$U^\mu \partial_\mu U^\nu = \frac{q}{m} U^\mu F^\nu{}_\mu. \tag{1}$$

With no loss of generality, the coefficient of U^μ, on the left hand side of the equation, can be antisymmetrized. Since $U^\mu U_\mu = -1$ and $U^\mu \partial^\nu U_\mu = 0$, Eq. (1) becomes

$$U^\mu \partial_\mu U^\nu - U^\mu \partial^\nu U_\mu = \frac{q}{m} U^\mu F^\nu{}_\mu. \tag{2}$$

Defining $P_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu$, we can write Eq. (2) as

$$U^\mu \left(\frac{m}{q} P_{\mu\nu} + F_{\mu\nu} \right) = 0. \tag{3}$$

For deriving equations of motion for point particles, usually a limiting procedure is invoked [11]. For the motion of fluids, however, a small volume element of the fluid is the limiting element and the statistical properties of the fluid come into play. Recently it has been suggested [10] that the $S_{\mu\nu} = \partial_\mu f U_\nu - \partial_\nu f U_\mu$ must replace the particle $P_{\mu\nu}$ for a new and natural "minimal coupling" to describe a fluid interacting with the Maxwell fields, where f represents a temperature dependent statistical attribute of the fluid, and is related to the enthalpy h, number density n, and mass m

of the fluid by the relation $h = mnf(T)$. In terms of $S_{\mu\nu}$, the "fluid Lorentz equation" derived in Ref. [10] is $T\partial^\nu\sigma = q(F^{\mu\nu} + \frac{m}{q}S^{\mu\nu})U_\mu$, where σ is an entropy density. The limiting procedure to the point particle case, then, is simply equivalent to $fU^\mu \rightarrow U^\mu$, $S_{\mu\nu} \rightarrow P_{\mu\nu}$, as $f \rightarrow 1$ ($T \rightarrow 0$).

The curvature $F_{\mu\nu}$ corresponding to the connection A_μ is obtained from the commutator of two covariant derivatives $D_\mu = \partial_\mu - iqA_\mu$. In a similar vein, we can define a "unified" connection $Q^\mu = A^\mu + \frac{m}{q}fU^\mu$, which corresponds to a minimally coupled hot magnetofluid. The new Abelian covariant derivative for the unified field $D_\mu = \partial_\mu - iqA_\mu - \mathrm{im}fU_\mu$ leads to the unified curvature

$$[D_\mu, D_\nu] = -iqF_{\mu\nu} - \mathrm{im}S_{\mu\nu}, \tag{4}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $S_{\mu\nu} = \partial_\mu fU_\nu - \partial_\nu fU_\mu$

Now we return to the main theme of this work—the dynamics of a non-Abelian fluid. The non-Abelian gauge field is represented by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a{}_\mu - ig[A_\mu, A_\nu]^a, \tag{5}$$

where, $[A_\mu, A_\nu]^a = iC^a_{bc}A^b{}_\mu A^c{}_\nu$, $C^a{}_{bc}$ are the structure constants of the gauge group and g is the gauge "charge".

Since F now has a gauge index, the right hand side of the Abelian equations of motion suggests a generalization of the fluid flow vector to include a gauge, or a non-Abelian index. The right hand side of Eq. (2) can then be written as: $(q/m)U^\mu{}_aF^{a\nu}{}_\mu$. Correspondingly, the left hand side, $U^\mu\partial_\mu U^\nu$, requires a non-Abelian generalization. This mandates giving the flow field U_μ a non-Abelian index, and we are led to a generalization of the Abelian flow tensor $S_{\mu\nu}$ to $S^a{}_{\mu\nu}$.

Following the Abelian route, an explicit form of $S^a{}_{\mu\nu}$ is given by evaluating the curvature for the generalized non-Abelian covariant derivative, $D_\mu = \partial_\mu - ig[A_\mu,] - \mathrm{im}[fU_\mu,]$:

$$[D_\mu, D_\nu]^a = -igF^a_{\mu\nu} - \mathrm{im}S^a_{\mu\nu} \tag{6}$$

and defines the YM generalization of $S_{\mu\nu}$ written, succinctly, as

$$S^{\mu\nu}_a = \mathcal{D}^\mu(fU^\nu_a) - \mathcal{D}^\nu(fU^\mu_a) - \mathrm{im}f^2[U^\mu, U^\nu]_a, \tag{7}$$

where $\mathcal{D}_\mu = \partial_\mu - ig[A_\mu,]$ is the ordinary non-Abelian gauge covariant derivative. Notice that $S^{\mu\nu}_a$ encompasses the pure flow field as well as the interaction. In the presence of a matter gauge current, the YM field evolves as $\mathcal{D}_\mu F^{\mu\nu}_a = -J^\nu_a$. The left hand side this equation is easily related to the energy momentum tensor $\Theta_{\mu\nu}$ through the Bianchi identity for $F^a{}_{\mu\nu}$ and gives

$$\partial^\nu\Theta_{\nu\mu} = -F^a_{\nu\mu}\mathcal{D}_\rho F^{\rho\nu}_a = F^a{}_{\mu\nu}J^\nu_a. \tag{8}$$

The evolution of YM potential $A_\mu = A^a_\mu T_a$ is dictated by the matter current J_μ in addition to the inherent nonlinearities in field equations. For strongly interacting matter such as a QGP, the current J_μ, is constructed from a collective many-body wave function. This procedure is very cumbersome and sometimes not very illuminating. Thus, a fluid description, in terms of flow and thermodynamic variables, is useful, because it captures a complicated dynamics in terms of a few collective variables. We are "viewing" a strongly interacting many particle system through a set of representative "flow" fields. We consider not just one but several flow fields (labeled by a species index s) denoting the particles (quarks, antiquarks) interacting with the non-Abelian gauge field.

For the fluid energy momentum tensor, there are several forms which have been postulated [8]. Ideally, one should include the fluid terms for the massive quarks and the radiation reaction terms of the gluon fields which impart energy to the fluid. These terms for the gluon fluids are of the order of g^2T^2. We believe that the non-Abelian generalization of this simpler problem captures the essence of the strongly interacting QGP. Thus the gluons of relevance are fully represented through $F^a{}_{\mu\nu}$ and we retain only the massive fluid species in the energy momentum tensor given below.

Each species can in principle have different charges, densities, temperatures, etc. Each species is taken to be a perfect fluid with an energy momentum tensor of the form $T^{\mu\nu}_s = p_s\eta^{\mu\nu} + h_sU^\mu_{a,s}U^{\nu a,s}$, where p_s is the pressure, and the enthalpy density h_s is given by $h_s = m_sn_{R,s}f_s(T)$, and $n_{R,s}$ measures the density in the rest frame for the given species. Each of the non-Abelian species contributes a flux $\Gamma^\mu{}_{a,s} = n_{R,s}U^\mu_{a,s}$, toward the total non-Abelian current $J^\mu_a = \sum_s g_s\Gamma^\mu{}_{a,s}$, where g_s is the gauge coupling for the species "s." The matter fields contributing to J^μ_a evolve covariantly under the action of the non-Abelian covariant derivative. The multispecies formulation allows the possibility $\sum_s g_sn_{R,s} = 0$, that is, the charge density can be zero in the rest frame of the fluid. We have already identified the charge weighted flux sum (over matter species) with the current J^μ_a that drives the gauge field equation. This identification is perfectly sensible and is consistent with the choice of the fluid equation of motion

$$\partial_\mu T^{\mu\nu}_a = g_sn_{R,s}U^\nu{}_{a,s}. \tag{9}$$

Summing Eq. (9) over species, we obtain

$$\sum_s\partial_\mu T^{\mu\nu}_s = \partial_\mu\mathcal{T}^{\mu\nu} = \sum_s g_sF^{\nu\sigma}_a\Gamma^{a,s}_\sigma = F^{\nu\sigma}_aJ^a_\sigma, \tag{10}$$

where $\mathcal{T}^{\mu\nu}$ is the total fluid tensor. Combining it with (8), we arrive at the expected conservation law for the total energy momentum tensor (matter plus field)

$$\partial^\mu(\Theta_{\mu\nu} + \mathcal{T}_{\mu\nu}) = 0, \tag{11}$$

justifying the expression for the current. For the rest of the Letter, we shall drop the species index unless it is essential for clarity.

From the continuity equation, generalized to the non-Abelian case,

$$\mathcal{D}^a_\mu \Gamma^\mu{}_a = 0 \Rightarrow \partial_\mu(n_R U^{\mu a}) = -g n_R C^{abc} A^b{}_\mu U^{c\mu}. \tag{12}$$

and from the definition of $T_{\mu\nu}$ (for a perfect fluid), we have

$$\partial_\mu T^{\mu\nu} = \partial^\nu p + m\partial_\mu(f n_R U_a{}^\mu U_a{}^\nu) = \partial^\nu p + m n_R U_{a\mu}[\mathcal{D}^\mu f U^\nu]_a. \tag{13}$$

The second term on the right hand side of Eq. (13) is related to $S_a{}^{\mu\nu}$ given by Eq. (7):

$$U_{a\mu} S_a{}^{\mu\nu} = N\partial^\nu f + U_{a\mu}[\mathcal{D}^\mu f U^\nu]_a. \tag{14}$$

Combining Eqs. (13) and (14),

$$\partial_\mu T^{\mu\nu} = \partial^\nu p + m n_R (U_{a\mu} S_a{}^{\mu\nu} - N\partial^\nu f), \tag{15}$$

where N stands for the dimension of the gauge group and we have applied $U^{a\mu}U_{a\mu} = -N$ as a natural generalization of $U^\mu U_\mu = -1$ in the Abelian case. Substituting appropriately from Eq. (10), we find

$$\partial^\nu p - N m n_R \partial^\nu f = g n_R \left[F_a{}^{\mu\nu} + \frac{m}{g} S_a{}^{\mu\nu} \right] U_{a\mu}. \tag{16}$$

In analogy with the Abelian case [10], we define the new unified non-Abelian tensor

$$M_a^{\mu\nu} = F_a^{\mu\nu} + \frac{m}{g} S_a^{\mu\nu}, \tag{17}$$

representing the matter and gauge field (including their interaction). Then, Eq. (16) becomes

$$\partial^\nu p - N m n_R \partial^\nu f = g n_R M_a^{\mu\nu} U_{a\mu}. \tag{18}$$

Defining the entropy σ, in analogy with Ref. [10], $\sigma = \ln[(p/K_2)(\frac{m}{T})^2 \exp(-\frac{mK_3}{TK_2})]$, Eq. (18) may be reduced to

$$T\partial^\nu \sigma = g M_a^{\mu\nu} U_{a\mu}. \tag{19}$$

For a homentropic fluid (a relevant limit for the QGP), the equation of motion becomes even simpler:

$$g M_a^{\mu\nu} U_{\nu a} = \left[F_a{}^{\mu\nu} + \frac{m}{g} S_a{}^{\mu\nu} \right] U_{a\mu} = 0. \tag{20}$$

We have just shown the existence of a unified minimally coupled potential for hot non-Abelian fluids $Q_a^\mu = A_a^\mu + \frac{m}{g} f U^\mu{}_a$, with its corresponding field tensor $M_a^{\mu\nu} = \partial^\mu Q_a^\nu - \partial^\nu Q_a{}^\mu + g C_a{}^{bc} Q_b^\mu Q_c^\nu$. Through $M_a^{\mu\nu}$ and Q_a^μ, we have put the non-Abelian flow field and the non-Abelian gauge field on the same footing. The unification opens up an opportunity to apply the powerful machinery of gauge theories to the unified gauge-flow field; this formulation complements the previous work on the subject [6,9].

The equation of motion given by (20) is the analogue of the two equations for the continuum version of Wong's equations [9]. However, unlike the generalization given in Ref. [9], which seems to couple YM fields with a fluid of non-Abelian charges, our proposal provides a natural nonlinearity within the coupled system since $S^a{}_{\mu\nu}$ contains an interaction with the YM fields through the connection $A^a{}_\mu$. This natural nonlinearity provides a mechanism to bypass the theorem forbidding the existence of solitonic configurations in a plasma [12].

To explore interesting consequences of this formalism for YM fluids, let us return to the Abelian results on helicity conservation. For the isentropic case, the spatial components of the equation of motion in the Abelian case are

$$U_0\left(\frac{m}{q} S^{0i} + F^{0i}\right) + U_j\left(\frac{m}{q} S^{ji} + F^{ji}\right) = 0. \tag{21}$$

Since F^{0i} is just the electric field, let us call the combined factor $(\frac{m}{q} S^{0i} + F^{0i})$, the fluid-generalized electric field $\hat{E}^i$. Let us make the same prescription for the magnetic components. Then, since U_0 is just the relativistic factor, γ, and $U_i = \gamma\vec{U}$, Eq. (21) corresponds to

$$\gamma\hat{\vec{E}} + \gamma\vec{U} \times \hat{\vec{B}} = 0. \tag{22}$$

An immediate consequence is the condition $\hat{\vec{E}} \cdot \hat{\vec{B}} = 0$. Since the product of $M_{\mu\nu} = \frac{m}{q} S_{\mu\nu} + F_{\mu\nu}$ with its dual $\mathcal{M}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho} M^{\lambda\rho}$, is proportional to $\hat{\vec{E}} \cdot \hat{\vec{B}}$, this demands the boundary term $\frac{1}{2} M^{\mu\nu}\mathcal{M}_{\mu\nu} = 0$ and implies the existence of a topological current K^μ, which for electrodynamics, is conserved: $\partial_\mu K^\mu = 0$. This K^μ is identified with the fluid-field equivalent of the Abelian Chern-Simons (CS) vector $A_\nu \mathcal{F}^{\nu\mu}$ (see Ref. [10]). The quantity $\frac{1}{2} M^{\mu\nu}\mathcal{M}_{\mu\nu}$ represents the topological winding number (charge) of solutions to the fluid-field equations of motion and the fact that it is zero supports the widely held view that an ordinary electron positron plasma, for example, does not support stable, self confining knot like solutions. This is upheld by a virial theorem due to Shafranov which states that a static configuration of a plasma in isolation is dissipative. Recently in Ref. [13] it has been proposed that this no go theorem is circumvented by introducing nonlinear interactions. We shall now show that the generalization to the YM plasma overrides this limitation and can support stable knot like solutions.

For the isentropic non-Abelian case, the spatial part of Eq. (20) is somewhat more complicated:

$$U_0{}^a\left(\frac{m}{q} S_a{}^{0i} + F_a{}^{0i}\right) + U^a{}_j\left(\frac{m}{q} S_a{}^{ji} + F_a{}^{ji}\right) = 0. \tag{23}$$

To manipulate Eq. (23), we have to face the question of factoring the non-Abelian four velocity $U_\mu{}^a$. At this stage there is no immediate compulsion for a factoring out of the generators (charges) of the gauge group. We take $U^a{}_\mu U_a{}^\mu = -N$ implying the existence of a full non-Abelian flow and that the velocity four vector is normalized in each flow. In terms of the fluid-generalized electric and magnetic fields, Eq. (23) becomes

$$\sum_a \gamma \hat{\vec{E}}_a + \gamma \vec{U}^a \times \hat{\vec{B}}_a = 0. \quad (24)$$

Because of the trace over the group indices, Eq. (24) implies that unlike in the Abelian case, the product $M^a_{\mu\nu}\mathcal{M}_{a\mu\nu} \neq 0$. Consequently, the non-Abelian generalization of the topological charge is not necessarily zero, opening up the possibility of nontrivial topological structures being supported by the non-Abelian fluid plasma. We define a generalized CS vector, C^μ, as

$$\partial_\mu C^\mu = \frac{1}{2} M^a_{\lambda\rho}\mathcal{M}_{a\lambda\rho} = Q^a{}_\nu\left[\mathcal{M}^{a\mu\nu} - \frac{g}{6}\epsilon^{\mu\nu\lambda\rho}C^{abc}Q_{b\lambda}Q_{c\rho}\right]. \quad (25)$$

To analyze this topological term, we look at the gauge properties of $Q^a{}_\mu$. Under a gauge transformation Ω_g, since we know that $\mathbf{A}_\mu$ transforms as $\mathbf{A}'_\mu = \Omega_g \mathbf{A}_\mu \Omega_g^{-1} - \frac{i}{g} \times (\partial_\mu \Omega_g)\Omega_g^{-1}$, and the non-Abelian fluid velocity vector $U^a{}_\mu$ transforms covariantly $\mathbf{U}'_\mu = \Omega_g \mathbf{U}_\mu \Omega_g^{-1}$, it follows that $\mathbf{Q}'_\mu = \Omega_g \mathbf{Q}_\mu \Omega_g^{-1} - \frac{i}{g}(\partial_\mu \Omega_g)\Omega_g^{-1}$. Hence, $Q^a{}_\mu$ can be strictly identified with a non-Abelian gauge connection. An immediate consequence of this is that C^0 is indeed a generalized CS invariant associated with the generalized connection Q^a. In addition, the YM connection $A^a{}_\mu$ will provide us with the standard CS invariant. To understand the topological implications of our result, consider the transformation of the quantity $\frac{1}{8\pi}\int C_0 d^3x$, under gauge transformations. For this we use the wedge product notation $A \wedge F = \epsilon^{ijk} A_i F_{jk} d^3x$ to write

$$I = \frac{1}{8\pi^2}\int_{\partial M} C_0 d^3x = \frac{1}{8\pi^2}\int \mathrm{Tr}\left(Q \wedge M + \frac{1}{3} Q \wedge Q \wedge Q\right). \quad (26)$$

Using the transformation properties established for Q we get the gauge transformed I_g as $I_g = \frac{1}{8\pi^2}\int \mathrm{Tr}(Q \wedge M_g + \frac{1}{3} Q' \wedge Q' \wedge Q')$ where $M_g = \Omega_g M \Omega_g^{-1}$. Thus we have

$$I_g - I = \frac{1}{24\pi^2}\int \mathrm{Tr}[d\Omega_g\Omega_g^{-1} \wedge d\Omega_g\Omega_g^{-1} \wedge d\Omega_g\Omega_g^{-1}], \quad (27)$$

which is the second Chern class describing the winding number of the space-time manifold. The integral in Eq. (27) is over the boundary of the space-time manifold and Stoke's theorem to relate it to TOP $= \int \mathrm{Tr} M \wedge M$, which classifies the topological solutions. Thus, unlike the Abelian case (where the divergence of the generalized helicity four vector for the combined system was forced to be zero), we have two topological quantities in the non-Abelian case: one coming from the combined fluid + YM case and one from the YM case with the fluid velocity vector tying the two together. To see this explicitly we write

$$\mathrm{TOP} = \int_M \mathrm{Tr}(F \wedge F) + \frac{2m}{g}\int_M \mathrm{Tr}(S \wedge F) + \frac{m^2}{g^2} \times \int_M \mathrm{Tr}(S \wedge S), \quad (28)$$

where the integral is over space time. In the non-Abelian magnetofluid we may associate this nonzero Pontryangin index with a non-Abelian magnetofluid helicity implying the existence of stable self confining nondissipative solutions. In fact, the nontriviality of this invariant ensures that flux lines can be knotted and solitonic configurations are inevitable. These can have a number of consequences, such as the existence of knotted solitons having the non-Abelian helicity as a topological quantum number, which may survive in the QGP in the interior of a heavy ion collision or in the early universe. We have given the foundations of a consistent theory of non-Abelian fluid-field system, in which the flow field and the gauge field are unified in a single minimally coupled gauge flow field. We have shown that this gives rise to a quantity which is the fluid-field generalization of the non-Abelian CS term, and shown that knotted fluid-field non-Abelian solitons may exist. We can, by using standard techniques in pure YM theories, find explicit forms of these and look for phenomenological signatures in the context of QGP. The formalism is simple and unified and should lead to new and interesting phenomenon such as non-Abelian Alfven waves and other non-Abelian counterparts of magnetohydrodynamics which may lead to new signals for collective flow in the QGP. Such studies are under investigation.

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