

# Parametric mechanism of the rotation energy pumping by a relativistic plasma

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An investigation of the kinematics of a plasma stream rotating in the pulsar magnetosphere is presented. On the basis of an exact set of equations describing the behavior of the plasma stream, the increment of the instability is obtained, and the possible relevance of this approach for the understanding of the pulsar rotation energy pumping mechanism is discussed. © 2005 American Institute of Physics. [DOI: 10.1063/1.1924315]

## I. INTRODUCTION

One of the important stages in the development of pulsar radiation theory was the discovery that the rotation energy could transform into energy in the electrostatic field; this possibility stimulated the modeling of pulsar magnetospheres (see Refs. 1 and 2). In a typical model, the electrostatic field  $\mathbf{E}_0$ , generated over the pulsar surface, has a finite projection along the dipole magnetic field  $\mathbf{B}_0$ . The resulting electric force,  $eE_0$  exceeds the gravitational force near the neutron star surface, and ends up uprooting charged particles from the surface layer. These particles are accelerated in the electric field and radiate  $\gamma$  photons because of the curvature radiation in the dipolar field. For photon energies greater than twice the electron rest mass ( $\epsilon_\gamma \geq 2m_e c^2$ ), the electron-positron pair production ( $\gamma + \mathbf{B}_0 \rightarrow e^+ + e^- + \gamma'$ ) in the ambience of the magnetic field becomes possible. The particles, so produced, repeat the cycle; they are accelerated, generating  $\gamma$  radiation which creates more electron-positron pairs. This cascading leads to the build-up of a large flux of relativistic  $e^+e^-$  pairs; the process continues till the pair plasma screens the electric field  $\mathbf{E}_0$  as shown in Refs. 3 and 4.

The  $e^+$ ,  $e^-$  population in the pulsar atmosphere may be conveniently divided into three principal components: (1) the basic plasma mass (Bulk) with concentration  $n_{pl}$  and a Lorentz factor  $\gamma_{pl}$ , (2) a tail with concentration  $n_t$  and Lorentz factor  $\gamma_t$ , and (3) the remnant of the primary electron or ion beam (most likely electrons). Note that cascading due to pair production is possible only in a highly relativistic scenario, because for nonrelativistic or even mildly relativistic velocities, the pair annihilation dominates pair production (see, for example, Refs. 5 and 6). The reader is referred to Refs. 7 and 8 for a guide to processes that define and control the behavior of radiation in the pulsar magnetosphere.

The essence of this “standard model” is that the pulsar magnetosphere is permeated with a multicomponent plasma. It is generally assumed that the energy contents of the three major components are of the same order:  $n_{pl}\gamma_{pl} \approx n_t\gamma_t \approx n_b\gamma_b$ . Theoretical estimates for the density and the energy of the primary component are  $n_b \approx (10^{11} - 10^{12}) \text{ cm}^{-3}$  (also called the Goldreich-Julian concentration) and  $\gamma_b \approx 10^6 - 10^7$ . The origin of the pulsar radiation is supposed to be in the magnetosphere, and hence the energy contained in the  $e^+e^-$  plasma must be enough to account for the observable radiation.

In these pulsar models, the measure of the region over which the electric field is nonzero is of great significance. The distance between the star surface and the region where it is screened out is called the vacuum gap, and may be estimated to be  $(10^3 - 10^4) \text{ cm}$  for  $\mathbf{E}_0 \approx 10^7 \text{ G}$  (see, for example, Ref. 9). Unfortunately the particle energy inventory accumulated within the gap is not sufficient to explain the observed visible radiation. Several mechanisms have been invoked to increase the gap size. The intermediate formation of positronium (electron-positron bound state) which would hold back the decay and lead to an increase in the gap was proposed in Ref. 10. According to Ref. 11, the pulsar magnetic field lines, supposedly curved towards rotation, must be screwed even more; the twisted magnetic field lines will be rectified, and as a result will enlarge the gap. A different mode, the pair formation front mechanism was introduced instead of the gap. Taking the heating of the stellar surface and the electron thermoemission into account leads to nonstationarity, and explains nonstationary behavior of pulsars (see Ref. 12).

In a series of papers, Muslimov and Tsigan have attempted to solve the gap problem via the general relativity route. Realizing that in the vicinity of the rotating neutron star, the space time is slightly curved, they worked out the creation of the electric field in Kerr metric (see Ref. 13). As a result, the gap size is somewhat enlarged but not enough to make a difference.

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It seems that some additional source of energy or some new mechanism will be necessary to surmount the problem arising out of the “insufficiency” of the energy content in the vacuum gap. For instance, it seems perfectly plausible and possible that one could draw upon the pulsar rotation energy to augment the energy content of the magnetospheric  $e^+e^-$  plasma; the rotation energy, for example, could be transformed into energy associated with oscillations in the  $e^+e^-$  plasma far from the pulsar surface. In this region the complicating effects of unremovable gravitation (having to use the Kerr metric, for example) will not exist. This paper is an attempt to formulate and investigate the problem associated with the parametric pumping of plasma oscillations in the pulsar magnetosphere taking into consideration the complicated nature of the multicomponent electron-positron substrate.

Let us review a standard system to examine the plausibility of the proposed mechanism. For the Crab Nebula, the nebula radiation can be surely sustained by the rotational energy of the pulsar PSR 0531 located in the vicinity of the nebular center. The power of the nebula radiation exceeds the pulsar radiation power by two orders of magnitude (it is  $\approx 2 \times 10^{38}$  erg/s). The only source capable of providing such a prodigious power is the slowdown of the pulsar rotation:  $\dot{W} \approx I\dot{\Omega}$ , where  $I$  is pulsar’s moment of inertia with mass of order  $(1.5-2.5)M_\odot$  ( $M_\odot$  is solar mass),  $\Omega$  and  $\dot{\Omega}$  are star’s angular velocity and angular acceleration, respectively. This power is equal to  $5 \times 10^{38}$  erg/s. The rotation energy loss rate is described by the ratio  $\dot{W}/W \approx 2\dot{\Omega}/\Omega = 2\dot{P}/P$ , where  $P = 2\pi/\Omega$  is the neutron star’s rotation period, and  $P$  and  $\dot{P}$  are measurable parameters for pulsars. The ratio  $\dot{P}/P$  for different pulsars ranges from  $10^{-11} \text{ s}^{-1}$  (PSR 0531) to  $10^{-18} \text{ s}^{-1}$  (PSR 1952+29). Thus, for ordinary plasma oscillations to be pumped by the pulsar rotation, their growth rate must not exceed  $\dot{P}/P$ , the measurable rate of rotation energy loss.

A hydrodynamic approximation will be used to study the problem of rotation induced wave generation in plasmas. For relative simplicity one will assume that the  $e^+e^-$  plasma has two distinct energy ranges: the lower range (still highly relativistic) with  $n_{\text{pl}}$  and  $\gamma_{\text{pl}}$ , and the beam with  $n_b$  and  $\gamma_b$ . These flows propagate along the rotating monopolelike magnetic field lines.

## II. THE MAIN CONSIDERATION

It is supposed that for distances less than the radius of curvature of the field line, the magnetic field is monopolelike (in this approximation the magnetic field lines may be supposed to be rectilinear). The problem of the motion of charged particles in the pulsar magnetosphere can be considered in the local inertial frame of the observers, who measure the physical quantities in their immediate vicinity. They are called the zero angular momentum observers (ZAMOs). Naturally, the proper time of the observer riding the particle is different from the proper time of ZAMOs.

Transformation from the inertial frame to the frame con-

nected with the pulsar magnetosphere can be done in the following way:

$$t = t', \quad \varphi = \varphi', \quad r = r', \quad z = 0, \quad (1)$$

then the interval in the corotating frame will have the form

$$ds^2 = -(1 - \Omega^2 r^2) dt^2 - dr^2, \quad (2)$$

where  $\Omega$  is angular velocity of rotation.

Note that the lapse function  $\alpha = \sqrt{1 - \Omega^2 r^2}$  ( $c=1$ ) not only connects the proper time of ZAMOs with the universal time  $d\tau = \alpha dt$ , but also gives a gravitational potential:

$$\mathbf{g} = -\frac{\nabla \alpha}{\alpha}. \quad (3)$$

The equation of motion of the particle in the ZAMOs frame is expressed as

$$\frac{d\mathbf{p}}{d\tau} = \gamma \mathbf{g} + \frac{e}{m}(\mathbf{E} + [\mathbf{V}\mathbf{B}]), \quad (4)$$

where  $\gamma = (1 - \mathbf{V}^2)^{-1/2}$  is the Lorentz factor and  $\mathbf{V} = d\mathbf{r}/d\tau$  is the velocity of the particle determined in the so-called 1+1 formalism, and  $\mathbf{p} \rightarrow \mathbf{p}/m$  is the dimensionless momentum.

As is shown in Ref. 14 the transition from the particle equation of motion to the Euler equation for fluid dynamics in the 1+1 formalism may be easily fulfilled if one changes  $d/d\tau$  in Eq. (4) by  $1/(\alpha \partial t) + (\mathbf{V}\nabla)$ . The resulting equation describing the stream motion (neglecting the stream pressure) takes the following form:

$$\frac{1}{\alpha} \frac{\partial \mathbf{p}}{\partial t} + (\mathbf{V}\nabla)\mathbf{p} = -\gamma \frac{\nabla \alpha}{\alpha} + \frac{e}{m}(\mathbf{E} + [\mathbf{V}\mathbf{B}]), \quad (5)$$

where  $\mathbf{V}$  and  $\mathbf{p}$  are now hydrodynamic velocity and momentum, respectively. In order to rewrite this equation in the inertial frame, let us note that the ZAMOs momentum coincides with the momentum in the inertial frame. In fact, from the definitions  $\mathbf{p} = \gamma \mathbf{V}$ ,  $\gamma = \alpha \gamma'$ , and  $\mathbf{V}' = d\mathbf{r}/d\tau$  (prime refers to quantities in the inertial frame), one can easily find that  $\mathbf{p} = \mathbf{p}'$ . In the inertial frame, Eq. (5) converts to (omitting primes for all quantities)

$$\frac{\partial \mathbf{p}_i}{\partial t} + (\mathbf{v}_i \nabla)\mathbf{p}_i = -\gamma \alpha \nabla \alpha + \frac{e_i}{m}(\mathbf{E} + [\mathbf{v}_i \mathbf{B}]), \quad i = b, e, p, \quad (6)$$

where  $b$ ,  $e$ , and  $p$  denote the beam, electron, and positron components, respectively. In Eq. (6), the force  $\mathbf{F} = -\gamma \alpha \nabla \alpha$  is the analog of the centrifugal force.

Adding the continuity and the Poisson equations,

$$\frac{\partial n_i}{\partial t} + \nabla(n_i \mathbf{v}_i) = 0, \quad (7)$$

$$\nabla \mathbf{E} = 4\pi e(n_e - n_p + n_b), \quad (8)$$

completes the dynamical system.

As we are interested in the evolution of fluctuations, one can look for solutions of Eqs. (6)–(8) in the framework of a

perturbation theory—an expansion in which terms such as  $E^1/mn\gamma$  (the small parameter in the approximation of weak turbulence for the plasma) are small:

$$\mathbf{E} = \mathbf{E}^0 + \mathbf{E}^1 + \dots, \quad (9a)$$

$$\mathbf{B} = \mathbf{B}^0 + \mathbf{B}^1 + \dots, \quad (9b)$$

$$\mathbf{p}_i = \mathbf{p}_i^0 + \mathbf{p}_i^1 + \dots, \quad (9c)$$

where  $\mathbf{E}^0$ ,  $\mathbf{B}^0$ , and  $\mathbf{p}_i^0$  are the leading terms, and  $\mathbf{E}^1$ ,  $\mathbf{B}^1$ , and  $\mathbf{p}_i^1$  constitute the perturbations. In the zeroth approximation (taking into consideration the fact that the ejected particles not only move along the radius, but also corotate with the pulsar magnetosphere because of the *frozen-in* condition,  $\mathbf{E}_0 + [\mathbf{v}_0 \mathbf{B}_0] = \mathbf{0}$ ), Eq. (6) will be reduced to the form (see, for example, Refs. 15 and 16)

$$\frac{d^2 r}{dt^2} = \frac{\Omega^2 r}{1 - \Omega^2 r^2} \left[ 1 - \Omega^2 r^2 - 2 \left( \frac{dr}{dt} \right)^2 \right], \quad (10)$$

where we have neglected the term  $(\mathbf{v}_0 \nabla) \mathbf{p}_0$  (taking this term into account is a separate problem, and is not examined within the framework of this paper). As is shown in Ref. 17, Eq. (10) allows an exact solution for particular initial conditions,  $r(t_0=0)=0$ ,  $V(t_0=0)=V_0$ :

$$r(t) = \frac{V_0 \operatorname{Sn}(\Omega t | \tilde{m})}{\Omega \operatorname{dn}(\Omega t | \tilde{m})}, \quad (11)$$

where Sn and dn are Jacobian elliptical functions, the sine and the modulus, respectively, and  $\tilde{m} = 1 - V_0^2$ .

Using the following properties of the mentioned Jacobian elliptical functions  $\operatorname{Sn}(x|0) = \sin(x)$  and  $\operatorname{dn}(x|0) = 1$  (see Ref. 17), one may easily reduce Eq. (11) for the ultrarelativistic regime ( $V_0 \rightarrow 1$ ) relevant to this paper:

$$r(t) = \frac{V_0}{\Omega} \sin \Omega t. \quad (12)$$

With this known asymptotic solution, the first-order Equation (6) reads

$$\frac{\partial \mathbf{p}_i^1}{\partial t} + (\mathbf{v}_i^0 \nabla) \mathbf{p}_i^1 = \mathbf{F}_i^1 + \frac{e_i}{m} \mathbf{E}^1, \quad (13a)$$

$$\mathbf{F}_i^1 = \Omega^2 r v_i^0 \mathbf{p}_i^1, \quad (13b)$$

$$v_i^0 = V_{i0} \cos \Omega t, \quad (13c)$$

where, in addition to Eq. (12), we have used the Lorentz-factor expansion in the small parameter  $p_i^1/p_i^0$ :  $\gamma \approx \sqrt{1 + \mathbf{p}_i^0{}^2} [1 + \mathbf{p}_i^0 \mathbf{p}_i^1 / (1 + \mathbf{p}_i^0{}^2)]$ .

In a similar fashion, the linearized Equation (7) becomes

$$\frac{\partial n_i^1}{\partial t} + \operatorname{div}(n_i^0 \mathbf{v}_i^1) + \operatorname{div}(n_i^1 \mathbf{v}_i^0) = 0. \quad (14)$$

The electron-positron continuity equations may be combined to obtain the evolution equation for the effective charge density  $n_{\text{pl}} = n_e - n_p$ :

$$\frac{\partial n_{\text{pl}}^1}{\partial t} + \operatorname{div}(n_e^0 \mathbf{v}_{\text{pl}}^1) + \operatorname{div}(n_{\text{pl}}^1 \mathbf{v}_{\text{pl}}^0) = 0, \quad (15)$$

where  $\mathbf{v}_{\text{pl}}^1 = \mathbf{v}_e^1 - \mathbf{v}_p^1$  and it has been assumed that  $\mathbf{v}_e^0 = \mathbf{v}_p^0 \equiv \mathbf{v}_{\text{pl}}^0$  and  $n_e^0 = n_p^0$ .

We seek here a solution in which the density perturbations have no spatial dependence so that the last terms of Eqs. (14) and (15) are identically zero. Let us choose  $n_i^1$  (now  $i = \text{pl}, b$ ) to have the form

$$n_i^1 = N_i e^{-(ikV_{0i}/\Omega) \sin \Omega t}. \quad (16)$$

Then, if the rest of the perturbed quantities are allowed the spatial dependence  $\mathbf{p}_i^1(\mathbf{r}; t) = \mathbf{p}_i^1(\mathbf{k}; t) e^{i\mathbf{k}\mathbf{r}}$ , one obtains from Eqs. (14) and (15)

$$ik p_i^1 = - \frac{\gamma_{0i}^3}{n_i^0} \frac{\partial N_i}{\partial t} e^{-ikR_i}, \quad (17)$$

where  $\gamma_{0i}$  is the initial Lorentz factor and  $R_i = (V_{0i}/\Omega) \sin \Omega t$ , and the relation  $v_i^1 = p_i^1 / \gamma_{0i}^3$  ( $p_{\text{pl}}^1 = p_e^1 - p_p^1$ ) that is satisfied for  $\Omega t \sim \Omega / \omega \ll 1$  has been used.

Let us now go back to Eqs. (13) and write them separately for the two components—the plasma and the beam:

$$\frac{\partial \mathbf{p}_b^1}{\partial t} + (\mathbf{v}_b^0 \nabla) \mathbf{p}_b^1 = \mathbf{F}_b^1 + \frac{e}{m} \mathbf{E}^1, \quad (18a)$$

$$\frac{\partial \mathbf{p}_{\text{pl}}^1}{\partial t} + (\mathbf{v}_{\text{pl}}^0 \nabla) \mathbf{p}_{\text{pl}}^1 = \mathbf{F}_{\text{pl}}^1 + 2 \frac{e}{m} \mathbf{E}^1. \quad (18b)$$

Repeating the procedure applied to Eqs. (18) and taking Eqs. (12) and (17) into consideration, one can find that the beam and the plasma components evolve as

$$\frac{\partial^2 N_b}{\partial t^2} = -i \frac{e n_b^0}{m \gamma_{b0}^3} e^{ikR_b} k E_1, \quad (19a)$$

$$\frac{\partial^2 N_{\text{pl}}}{\partial t^2} = -2i \frac{e n_{\text{pl}}^0}{m \gamma_{p0}^3} e^{ikR_{\text{pl}}} k E_1. \quad (19b)$$

Combining Eqs. (19a) and (19b) eliminates the electric field to yield

$$\frac{\partial^2 N_b}{\partial t^2} = \frac{n_b^0 \gamma_{p0}^3}{2 n_{\text{pl}}^0 \gamma_{b0}^3} e^{ik(R_{\text{pl}} - R_b)} \frac{\partial^2 N_{\text{pl}}}{\partial t^2}. \quad (20)$$

Equation (20) is a rather complicated nonautonomous equation in time. To solve it, one can take its Fourier time transform (restoring the speed of light) along with that of the Poisson Equation (8) [cf. Appendixes A and B, Eqs. (A2) and (B2)] with the electric field eliminated to arrive at the coupled system

$$\omega^2 N_b(\omega) = \frac{n_b^0 \gamma_{0\text{pl}}^3}{2 n_{\text{pl}}^0 \gamma_{0b}^3} \sum_s (\omega + s\Omega)^2 J_s(a) N_{\text{pl}}(\omega + s\Omega), \quad (21)$$

$$\left(\omega^2 - \frac{\omega_{\text{pl}}^2}{\gamma_{0\text{pl}}^3}\right) N_p(\omega) = \frac{\omega_{\text{pl}}^2}{\gamma_{0\text{pl}}^3} \sum_s J_s(a) N_b(\omega - s\Omega), \quad (22)$$

where  $a = kc/2\Omega\gamma_{0\text{pl}}^2$  and  $\omega_{\text{pl}} = \sqrt{8\pi n_{\text{pl}}^0 e^2/m}$ . Naturally one can see the appearance of convolution sums on the right-hand sides of both the equations.

Substituting  $N_b$  from Eq. (21) into Eq. (22), one can find that

$$\left(\omega^2 - \frac{\omega_{\text{pl}}^2}{\gamma_{0\text{pl}}^3}\right) N_{\text{pl}}(\Omega) = \frac{\omega_b^2}{\gamma_{0b}^3} \sum_{s,l} J_s(a) J_l(a) \left(\frac{\omega - (s-l)\Omega}{\omega - s\Omega}\right)^2 \times N_{\text{pl}}[\omega - (s-l)\Omega], \quad (23)$$

where  $\omega_b = \sqrt{8\pi n_b^0 e^2/m}$ . One could try to find the nature of the time evolution from Eq. (23), but it seems to be a little better to go back to  $n_{\text{pl}}$  given by  $N_{\text{pl}} = n_{\text{pl}}^1 e^{ikR_{\text{pl}}}$ . Carrying out the algebra given in Appendix C [Eq. (C2)], one may derive

$$N_{\text{pl}} = \sum_{s=-\infty}^{+\infty} J_s(b) n_{\text{pl}}(\omega - s\Omega), \quad (24)$$

where  $b = kc/\Omega$ . Substituting Eq. (24) into Eq. (23), one finally obtains the rather complicated dispersion:

$$\left(\omega^2 - \frac{\omega_{\text{pl}}^2}{\gamma_{0\text{pl}}^3}\right) \sum_s J_s(b) n_{\text{pl}}(\omega - s\Omega) = \frac{\omega_b^2}{\gamma_{0b}^3} \sum_{j,l,m} J_l(a) J_{j+l+m}(a) J_m(b) \left(\frac{\omega + (j+m)\Omega}{\omega - l\Omega}\right)^2 \times n_{\text{pl}}(\omega + j\Omega). \quad (25)$$

To extract some sense out of the above result, one can explore the dispersion relation near the resonant condition,  $\omega^2 \approx \omega_{\text{pl}}^2/\gamma_{0\text{pl}}^3$ . Near the resonance, the basic contribution to the sum  $\sum_s J_s(b) n_{\text{pl}}(\omega - s\Omega)$  comes from  $\omega \approx s_0\Omega$ . Similarly the right-hand side of Eq. (25) is reduced to a single term corresponding to  $\omega \approx -j_0\Omega$ ,  $\omega \approx l_0\Omega$  and  $m = m_0 = -j_0 = l_0 = s_0$ . Rewriting  $\omega^2 - \omega_{\text{pl}}^2/\gamma_{0\text{pl}}^3$  as  $2\omega_{\text{pl}}\Delta/\gamma_{0\text{pl}}^{3/2}$  (where  $\Delta = \omega - \omega_{\text{pl}}/\gamma_{0\text{pl}}^{3/2}$ ), then, reduces Eq. (25) to the simple cubic equation

$$\Delta^3 \approx \frac{\omega_b^2 \omega_{\text{pl}}}{2\gamma_{0b}^3 \gamma_{0\text{pl}}^{3/2}} J_{s_0}^2(a), \quad (26)$$

where  $s_0 = [\omega_{\text{pl}}/\Omega\gamma_{0\text{pl}}^{3/2}]$ . In addition to the real root, the dispersion relation of Eq. (26) allows the complex conjugate pair

$$\Delta_{1,2} \approx -\frac{1}{2}M \pm \frac{i\sqrt{3}}{2}M, \quad (27a)$$

$$M = \left[ \frac{\omega_b^2 \omega_{\text{pl}}}{2\gamma_{0b}^3 \gamma_{0\text{pl}}^{3/2}} J_{s_0}^2(a) \right]^{1/3}, \quad (27b)$$

with comparable real and imaginary parts. The root with the positive imaginary part implies the instability that we were seeking.

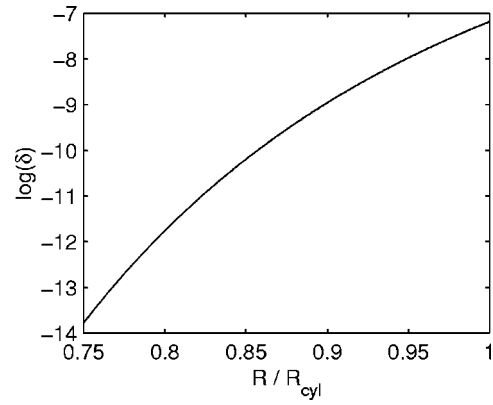


FIG. 1. Graph of  $\ln[\delta(R/R_{\text{cyl}})]$ . Set of parameters is the following:  $n_{0b} \approx 10^7 \text{ cm}^{-3}$ ,  $\gamma_{0b} \approx 10^6$ ,  $\gamma_{0\text{pl}} \approx 10$ ,  $\Omega \approx 190 \text{ Hz}$ ,  $r \approx 10^6 \text{ cm}$ .

### III. ESTIMATES

The instability turns out to be rather strong. For typical pulsar parameters (that of the Crab Nebula)  $n_{0b} \approx 10^7 \text{ cm}^{-3}$ ,  $\gamma_{0b} \approx 10^6$ ,  $\gamma_{0\text{pl}} \approx 10$ ,  $\Omega \approx 190 \text{ Hz}$ , considering the longitudinal waves, one can plot the graph of the growth rate  $\delta$  ( $\delta = \sqrt{3}M/2$ , see Eqs. (27)) as a function of the radial distance (normalized by the light cylinder radius, thus the radius, where the rotation velocity equals to the speed of light). For simplicity let us examine longitudinal waves in a coupling point. In this case all three modes of a cold plasma ( $o, x$ , and Alfvén modes) are indistinguishable, then one can estimate the wave number according to the following approximate formula  $kc \approx \omega_{\text{pl}}/\gamma_{0\text{pl}}^{3/2}$  (see Ref. 18). Using mentioned pulsar parameters, from Eq. (27a) the growth rate is estimated. In Fig. 1 one may see that  $\delta$  is sensitive to the radial distance, in the range  $0.75 \leq R/R_{\text{cyl}} \leq 1$  it increases from  $\sim 10^{-14} \text{ s}^{-1}$  to  $\sim 10^{-7} \text{ s}^{-1}$ . (Here  $R_{\text{cyl}}$  is the light cylinder radius). It is clear that  $\delta$  becomes unreasonably large for  $R/R_{\text{cyl}} \geq 0.85$ , which means that the linear assumption will be grossly violated long before these distances are reached. The instability will have a linear growth over certain distances and then nonlinear saturation will set in after which other nonlinear mechanisms may take place in the energy pumping phenomenon. In estimating the growth, one has invoked  $n_{0\text{pl}}\gamma_{0\text{pl}} \approx n_{0b}\gamma_{0b}$ , and used the fact that the density goes as  $n \approx n_0(r/R)^3$  (where  $r \approx 10^6 \text{ cm}$  is a radius of the neutron star) because the magnetic field is monopolelike.

### IV. CONCLUSIONS

The purpose of this paper was to explore the possibility of pumping the rotational energy of the pulsar into the plasma. Considering a highly idealized system, the linear instability caused by rotation in a two component relativistic plasma embedded in a uniform magnetic field was examined. By using the hydrodynamic equations of motion, the continuity and the Poisson equation, it has been shown that the plasma waves can grow on the rotation energy with rather high growth rates. In fact the perturbation growth rate for distances  $R/R_{\text{cyl}} \geq 0.85$  is quite high in comparison with the supposed instability rates we were seeking. If this mechanism is, indeed, operational, then the only consistent scenario

is that the linear stage with this growth rate is very short, and nonlinearities are turned in soon enough to considerably reduce the growth. Thus the need for a nonlinear theory is immediately and strongly indicated. Sooner or later we should consider this particular nonlinear effect, which will comprise one more step closer to the real scenario. There are two other shortcomings of this effort: (a) the straight magnetic field lines have been examined, whereas real profiles are curved, and (b) only electrostatic waves have been considered. The preliminary results, however, unambiguously show that the energy content of the magnetospheric plasma can grow at the expense of the stellar rotational energy.

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## APPENDIX A: DERIVATION OF EQ. (21)

Note that

$$N_f(t) = \int d\omega' e^{-i\omega't} N_f(\omega'), \quad (\text{A1a})$$

where

$$f = b, \text{pl},$$

then

$$\begin{aligned} & \int dt e^{i\omega't} \frac{\partial^2}{\partial t^2} \int d\omega' e^{-i\omega't} N_f(\omega') \\ &= \int \int dt d\omega' e^{i\omega't} (-i\omega')^2 e^{-i\omega't} N_f(\omega') \\ &= - \int \int dt d\omega' e^{it(\omega-\omega')} (\omega')^2 N_f(\omega') \\ &= -2\pi \int d\omega' (\omega')^2 \delta(\omega-\omega') N_f(\omega') = -2\pi\omega^2 N_f(\omega), \end{aligned} \quad (\text{A1b})$$

where the following representation of the delta function has been examined:

$$\delta(x) = \frac{1}{2\pi} \int dk e^{ikx}. \quad (\text{A1c})$$

Also if

$$\begin{aligned} R_{\text{pl}} - R_b &= [(V_{0\text{pl}} - V_{0b})/\Omega] \sin \Omega t \\ &\approx \frac{c}{\Omega} \left( \frac{1}{2\gamma_{0b}^2} - \frac{1}{2\gamma_{0\text{pl}}^2} \right) \sin \Omega t \\ &\approx -\frac{c}{2\gamma_{0\text{pl}}^2 \Omega} \sin \Omega t, \end{aligned} \quad (\text{A1d})$$

then

$$e^{-ik(R_{\text{pl}}-R_b)} \approx e^{ia \sin \Omega t}, \quad (\text{A1e})$$

$$a = \frac{kc}{2\gamma_{0\text{pl}}^2 \Omega}. \quad (\text{A1f})$$

We have considered the approximate expression  $V_{0i} \approx 1 - (1/2\gamma_{0i}^2)$ , and the following observable fact:  $\gamma_{0b} \gg \gamma_{0\text{pl}}$ . As one can see, the speed of light has been restored again.

By using the following identity:

$$e^{\pm ix \sin \Omega t} = \sum_s J_s(x) e^{\pm is\Omega t}, \quad (\text{A1g})$$

one may easily transform  $e^{-ik(R_p-R_b)}$  into its Fourier mode,

$$\begin{aligned} & \int dt e^{i\omega t} \sum_s J_s(a) e^{is\Omega t} \frac{\partial^2}{\partial t^2} \int d\omega' e^{-i\omega't} N_p(\omega') \\ &= - \sum_s J_s(a) \int \int dt d\omega' (\omega')^2 N_p(\omega') e^{it(\omega+s\Omega-\omega')} \\ &= -2\pi \sum_s J_s(a) \int d\omega' (\omega')^2 N_p(\omega') \delta(\omega+s\Omega-\omega') \\ &= -2\pi \sum_s J_s(a) (\omega+s\Omega)^2 N_p(\omega+s\Omega). \end{aligned} \quad (\text{A1h})$$

Combining Eqs. (A1b) and (A1h) and taking into consideration Eq. (20), one will finally obtain

$$\omega^2 N_b(\omega) = \frac{n_b^0 \gamma_{0\text{pl}}^3}{2n_{\text{pl}}^0 \gamma_{0b}^3} \sum_s (\omega+s\Omega)^2 J_s(a) N_{\text{pl}}(\omega+s\Omega). \quad (\text{A2})$$

## APPENDIX B: DERIVATION OF EQ. (22)

One can easily transform Eq. (8) into the following form:

$$\mathbf{kE}_1 = 4\pi e [N_b(t) e^{-ikR_b} + N_{\text{pl}}(t) e^{-ikR_{\text{pl}}}], \quad (\text{B1a})$$

Taking Eq. (19b) into account, one can have

$$\frac{\partial^2 N_{\text{pl}}(t)}{\partial t^2} = -\frac{8\pi e n_{\text{pl}}^0}{m \gamma_{0\text{pl}}^3} [N_{\text{pl}}(t) + N_b(t) e^{ik(R_{\text{pl}}-R_b)}]. \quad (\text{B1b})$$

For Fourier expansion of the left-hand side of Eq. (B1b) one can analogously go to Eq. (A1b) and write it down as

$$\int dt e^{i\omega't} \frac{\partial^2}{\partial t^2} \int d\omega' e^{-i\omega't} N_{\text{pl}}(\omega') = -2\pi\omega^2 N_{\text{pl}}(\omega) \quad (\text{B1c})$$

while, for the second term in a bracket, one finds

$$\begin{aligned} & \sum_s J_s(a) \int \int dt d\omega' e^{it(\omega-s\Omega-\omega')} N_b(\omega') \\ &= -2\pi \sum_s J_s(a) \int d\omega' N_b(\omega') \delta(\omega-s\Omega-\omega') \\ &= -2\pi \sum_s J_s(a) N_b(\omega-s\Omega). \end{aligned} \quad (\text{B1d})$$

Combining Eqs. (B1c) and (B1d) we will obtain Eq. (22):

$$\left(\omega^2 - \frac{\omega_{\text{pl}}^2}{\gamma_{0\text{pl}}^3}\right) N_{\text{pl}}(\omega) = \frac{\omega_{\text{pl}}^2}{\gamma_{0\text{pl}}^3} \sum_s J_s(a) N_b(\omega - s\Omega), \quad (\text{B2a})$$

where

$$\omega_{\text{pl}} = \sqrt{\frac{8\pi n_{\text{pl}}^0 e^2}{m}}. \quad (\text{B2b})$$

### APPENDIX C: DERIVATION OF EQ. (24)

From Eq. (16) we have

$$N_f(t) = n_f^1(t) e^{ikR_f}. \quad (\text{C1a})$$

Taking into consideration Eq. (A1g) and a condition  $\gamma_a \gg 1$ , one finds

$$N_f(t) = \sum_s J_s(A_f) e^{is\Omega t} n_f^1(t), \quad (\text{C1b})$$

where

$$A_b = \frac{kc}{\Omega}, \quad (\text{C1c})$$

$$A_{\text{pl}} = \frac{kc}{2\Omega \gamma_{0\text{pl}}^2}. \quad (\text{C1d})$$

Fourier analysis of the right-hand side of the Eq. (C1b) gives the following:

$$\begin{aligned} & \sum_s J_s(A_f) \int \int dt d\omega' e^{i(\omega+s\Omega-\omega')t} n_f(\omega') \\ &= -2\pi \sum_s J_s(A_f) \int d\omega' n_f(\omega') \delta(\omega + s\Omega - \omega') \\ &= -2\pi \sum_s J_s(A_f) n_f^1(\omega - s\Omega). \end{aligned} \quad (\text{C1e})$$

Hence we will have

$$N_f(\omega) = \sum_s J_s(A_f) n_f^1(\omega - s\Omega). \quad (\text{C2})$$

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