Compression of Laser Radiation in Plasmas Using Electromagnetic Cascading

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Compressing high-power laser beams in plasmas via generation of a coherent cascade of electromagnetic sidebands is described. The technique requires two copropagating beams detuned by a near-resonant frequency $\Omega \lesssim \omega_p$. The ponderomotive force of the laser beat wave drives an electron plasma wave which modifies the refractive index of plasma so as to produce a periodic phase modulation of the laser field with the beat period $\tau_b = 2\pi/\Omega$. A train of chirped laser beat notes (each of duration τ_b) is thus created. The group velocity dispersion of radiation in plasma can then compress each beat note to a few-laser-cycle duration. As a result, a train of sharp electromagnetic spikes separated in time by τ_b is formed. Depending on the plasma and laser parameters, chirping and compression can be implemented either concurrently in the same plasma or sequentially in different plasmas.

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For nearly two decades the chirped-pulse amplification (CPA) has been the dominant technique of generating ultrashort high-power laser pulses [1]. The basic premise of the CPA is to avoid optical nonlinearities of the amplified pulse by stretching it in time, thereby preserving its spectral characteristics. Recently, the exact opposite approach has been proposed by several groups: an advantage is taken of the nonlinearities in order to increase the frequency bandwidth of the pulse and reduce its duration. For example, nonlinear processes in gaseous media such as Raman cascading [2] and generation of high harmonics [3] have been utilized for making ultrahigh-bandwidth pulses. These pulses can be compressed to an ultrashort duration in another gaseous medium (further referred to as a compressor) with a high group velocity dispersion (GVD). These techniques cannot be extended to ultrahigh laser intensities exceeding the ionization threshold. Fortunately, nonlinear properties of a fully ionized plasma can be used for increasing the laser bandwidth via phase modulation due to a near-resonant beat wave excitation of electron plasma waves (EPWs). The natural GVD of plasma can compress the resulting broad-bandwidth radiation into spikes of femtosecond duration.

In this Letter we demonstrate that nonlinear coupling occurring in plasma between two incident copropagating laser beams with a near-resonant detuning, $\Omega \approx \omega_{p(M)} = \sqrt{4\pi e^2 n_{0(M)}/m_e}$, results in the generation of a coherent electromagnetic (EM) cascade that consists of multiple Stokes–anti-Stokes components (here, $n_{0(M)}$ is the electron plasma density, and m_e and $-|e|$ are the electron rest mass and charge). These EM sidebands, having frequencies $\omega_n = \omega_0 + n\Omega$ and wave numbers $k_n = k_0 + nk_\Omega$, are arranged in time and space so as to produce a periodic frequency modulation with the period of laser beat wave $\tau_b = 2\pi/\Omega$ [here, $k_\Omega = \Omega/v_{g(M)}$, $-\mathcal{M} \lesssim n \lesssim \mathcal{M}$ is an integer, $v_{g(M)} = c\sqrt{1-d_M}$ is the group velocity, $d_M = n_{0(M)}/n_c$ is the normalized electron density, and $n_c = m_e\omega_0^2/(4\pi e^2)$ is the critical plasma density for the fundamental laser frequency ω_0]. If the number of sidebands $2\mathcal{M}$ is of the order of ω_0/Ω, and $\Omega < \omega_{p(M)}$, the GVD of plasma can transform the broadband frequency-modulated laser pulse into a train of ultrashort (few-laser-cycles) radiation spikes separated in time by the beat period τ_b. This can be done in a separate higher-density plasma where the laser beat wave is detuned far from the plasma resonance, and the EM sidebands are redistributed in space and time by the plasma GVD with the resulting compression effect. The envisioned setup (Fig. 1) thus consists of two plasma stages: (i) a low-density plasma (further called the modulator) with $\Omega < \omega_{p(M)}$ in which the incident two-frequency laser becomes frequency modulated (chirped), and (ii) a higher-density compressor with $\omega_{p(C)} \gg \Omega$ where the linear amplitude modulation (compression) of the laser beat notes occurs.

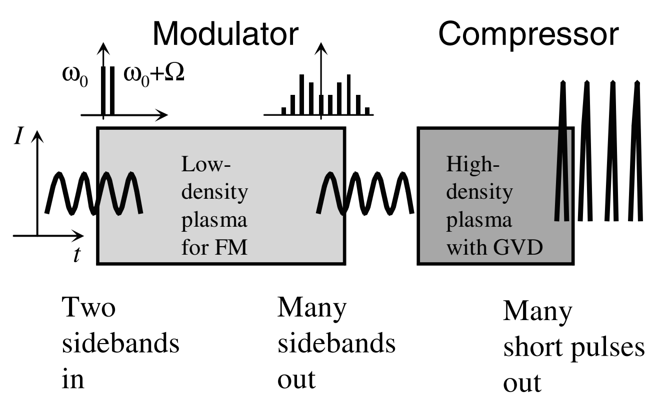

FIG. 1. Schematic of a two-stage EM cascade compressor. The frequency modulation (FM) occurs in a rarefied plasma. Denser plasma is used for the compression.

The physical mechanism of the EM cascading is the following. The ponderomotive laser beat wave drives an electron density grating $\delta n(\xi)$ comoving with the laser beams (where $\xi/v_{g(M)} = t - z/v_{g(M)}$ is the retarded time in the modulator), thereby creating a comoving index grating $\delta N = -(\delta n/n_{0(M)})d_M/2$. A time-dependent phase shift, $\Phi(z, \xi) \approx \delta N \omega_0 z/c$, can result in the nonlinear phase modulation, frequency shifting, and chirping [4]. Our approach is based on preserving a nearly sinusoidal temporal variation of δn. Hence, the resulting laser beam consists of the equally spaced in frequency sidebands. Those are arranged in time and space so as to give the laser pulse a periodic in time chirp with the period τ_b. The number of sidebands $2\mathcal{M}$ proportional to the plasma length z in the idealized case of negligible GVD can be large, and considerable broadening of the laser bandwidth can occur. When the GVD of radiation in the modulator is non-negligible, the laser undergoes an amplitude modulation as well. We show that proper adjustment of the plasma length and density can reduce the GVD effect while preserving the desirable frequency bandwidth.

In a realistic plasma, the EM cascading is a complicated interplay between the GVD of radiation and the sideband coupling through the driven electron density perturbations, the nonlinearities due to the relativistic increase of an electron mass, and the forward stimulated Raman scattering (FSRS). Our nonlinear analysis accounts for all these effects and describes the cascade development in time and one spatial dimension (1D). Because the longest time scale of the problem is only a few ion plasma periods, parametric decay of the EPW and consequent plasma heating [5] are ignored, and plasma ions are assumed to be a positive neutralizing background. Relativistic nonlinearities and FSRS are found to be relatively unimportant for the chosen parameters, and the EM cascading and compression proceed in agreement with the above qualitative scenario. Numerical modeling also reveals the parameter regime for which frequency and amplitude modulation proceed concurrently, resulting in a single-stage compression of laser beat notes.

We assume that the circularly polarized planar laser beam incident on the modulator entrance, $z = 0$, consists of two spectral components with the frequencies ω_0 and $\omega_1 = \omega_0 + \Omega$ and amplitudes $E_0(\omega_0, k_0)$ and $E_1(\omega_0 + \Omega, k_0 + k_\Omega)$,

$$\mathbf{a}\,(0, \xi) = \mathrm{Re}\{\mathbf{e}_0 e^{-i\omega_0\xi/v_{g(M)}}[a_0(0, \xi) + a_1(0, \xi)e^{-ik_\Omega\xi}]\}, \quad (1)$$

where $|\mathbf{e}_0| = 1$ and $a_n = eE_n/(m_e\omega_0 c)$. The EM cascade in the modulator,

$$\mathbf{a}\,(z > 0, \xi) = \mathrm{Re}\left\{\mathbf{e}_0 \sum_{n=-\infty}^{+\infty} a_n(z, \xi)e^{-i\omega_n\xi/v_{g(M)}}\right\}, \quad (2)$$

is induced by the electron density perturbation $\delta n/n_c \equiv \mathrm{Re}[N_e(z, \xi)e^{-ik_\Omega\xi}]$ driven near resonantly by the laser beat wave. Both a_n and N_e vary slowly on the time and space scales Ω^{-1} and k_Ω^{-1}. The cascade experiences a nonlinear evolution in the modulator and then in the compressor plasmas. In the weakly relativistic limit of $|a|^2 < 1$, we find from 1D Maxwell's equations and hydrodynamic equations of electron fluid that in the modulator plasma the amplitudes of sidebands evolve as

$$\left[\frac{2i}{k_0}\frac{\partial}{\partial z} - d_M\left(\frac{\omega_n - \omega_0}{\omega_0}\right)^2\frac{\omega_0}{\omega_n}\right]a_n \approx \frac{\omega_0}{\omega_n}\left(-\frac{d_M}{4}\Delta_n^{(M)} + \frac{N_e a_{n-1} + N_e^* a_{n+1}}{2}\right). \quad (3)$$

The nonresonant components of the electron density perturbation as well as the relativistic corrections to the electron mass are combined into the nonlinear frequency shifts proportional to $\Delta_n^{(M)} = \Sigma_l a_{n+l}\rho_l f_l^{(M)}$, where $\rho_l \equiv \Sigma_m a_m a_{m+l}^*$, and $f_l^{(M)} \approx \omega_{p(M)}^2/(\omega_{p(M)}^2 - l^2\Omega^2)$ for $l \neq \pm 1$, and $f_{\pm 1}^{(M)} = 1$. The term proportional to the normalized electron density d_M on the left-hand side (LHS) of Eq. (3) describes the GVD of EM sidebands. The EPW amplitude N_e satisfies the weakly nonlinear equation

$$(ik_\Omega^{-1}\partial/\partial\xi + \delta\omega_l/\Omega)N_e + (3/16)|N_e/d_M|^2 N_e + (\rho_0 N_e + \rho_{-2}N_e^*)/8 = d_M(\rho_{-1}/4), \quad (4)$$

where the second and third terms on the LHS are the nonlinear frequency shifts coming from the relativistic mass corrections due to the longitudinal and transverse electron motion, respectively [6]. The initial condition is $N_e(z, -\infty) \equiv 0$ (unperturbed plasma ahead of the laser pulse); $\rho_{-1}(z, \xi)$ is the normalized amplitude of the near-resonant ponderomotive force, and $\delta\omega_l = (\Omega^2 - \omega_{p(M)}^2)/(2\Omega)$ is the beat wave detuning from the plasma resonance.

Equations analogous to Eq. (3) (with $\omega_{p(M)}$, $v_{g(M)}$, d_M, ξ, and $\Delta_n^{(M)}$ replaced by $\omega_{p(C)}$, $v_{g(C)} = c\sqrt{1 - d_C}$, $d_C = n_{0(C)}/n_c$, $\zeta = v_{g(C)}t - z$, and $\Delta_n^{(C)}$, respectively) model the nonlinear evolution of the cascade in the dense compressor plasma. The compressor density is such that $\omega_{p(C)}^2 \neq n^2\Omega^2$ for an integer n; hence, the plasma response is nonresonant. The amplitude N_e does not explicitly appear in the compressor equations: it is absorbed into $\Delta_n^{(C)}$ as $f_{\pm 1}^{(C)} \approx \omega_{p(C)}^2/(\omega_{p(C)}^2 - \Omega^2)$. Solution of Eq. (3) at the exit of the modulator ($z = z_M$) is the initial condition for the cascade equations in the compressor.

In the nonrelativistic case with GVD neglected, scaling laws for the EM cascading are particularly simple. From Eq. (3) with $d_M = 0$ and $\omega_n \approx \omega_0$ follow the conservation laws: $\partial\rho_l/\partial z = 0$. Hence, $\rho_0(z, \xi) \equiv |a_0(0, \xi)|^2 + |a_1(0, \xi)|^2$, $\rho_{-1}(z, \xi) \equiv a_1(0, \xi)a_0^*(0, \xi)$, $\rho_l \equiv 0$ for $l \neq 0, -1$, and, in the comoving frame, N_e is independent of z despite the laser evolution. Thus simplified Eq. (3) has the analytic [7] solution $a_n(z,\xi)=\sum_{\sigma=0,1}a_\sigma(0,\xi)e^{i(n-\sigma)(\psi+\pi)}J_{n-\sigma}(2W)$ satisfying the initial condition (1) [here, $J_n(x)$ are the Bessel functions, and $\psi(z,\xi)$ and $W(z,\xi)$ are the phase and absolute value of the generating function $w(z,\xi)\equiv We^{i\psi}=i(k_0z/4)N_e(\xi)$]. Substituting $a_n(z,\xi)$ into Eq. (2) yields the expression for a train of phase-modulated beat notes: $a(z,\xi)=\sum_{\sigma=0,1}a_\sigma(0,\xi)\cos[k_\sigma\xi+\varphi(z,\xi)]$, where $\varphi(z,\xi)=(k_0z/2)|N_e(\xi)|\sin(\psi-k_\Omega\xi)$. The physical meaning of this result is that, without GVD, the laser pulse undergoes only frequency modulation. The magnitude of the plasma wave depends only on the laser amplitude which remains unchanged. This is valid for any pair of $a_0(0,\xi)$, $a_1(0,\xi)$, and the corresponding $N_e(\xi)$.

The frequency modulation is periodic in time with the beat period τ_b when $N_e(\xi)$ is almost constant (this is the cases when the inequality $|\partial\rho_{-1}/\partial\xi|\ll|(\delta\omega_l/\nu_{g(M)})\rho_{-1}|$ holds). For $|\delta\omega_l|\gg(\Omega/4)\sqrt[3]{3|\rho_{-1}|^2/2}$ [8], Eq. (4) yields $N_e(\xi)\approx d_M[\rho_{-1}(\xi)/4](\Omega/\delta\omega_l)$. For real ρ_{-1},

$$a=\sum_{n=0,1}a_n(0,\xi)\cos[k_n\xi+(k_0z/2)N_e(\xi)\cos(k_\Omega\xi)]. \quad (5)$$

Hence, a modulator plasma slab of thickness $z_{\mathcal{M}}\approx 2\mathcal{M}/(N_ek_0)$ produces $\mathcal{M}$ sidebands on each side of the fundamental, and a frequency bandwidth $\Delta\omega\sim 2\sqrt{d_M}\mathcal{M}\omega_0$. Conversely, $\mathcal{M}\sim r_ez_{\mathcal{M}}\lambda_0|n_e-n_0|$, where $\lambda_0=2\pi c/\omega_0$ is the fundamental laser wavelength, and $r_e=e^2/(m_ec^2)$ is a classical electron radius.

From Eq. (5) follows that only for $\delta\omega_l<0$ the laser wavelength decreases with time near the amplitude maximum of each beat note. We define this chirp as positive. The anomalous GVD in the dense compressor plasma compresses thus chirped beat notes: the shorter-wavelength (blueshifted) sidebands catch up with the longer-wavelength (redshifted) ones, eventually building up the field amplitude near the beat note center. With the compressor nonlinearities neglected ($\Delta_n^{(C)}\approx 0$), and plasma density, laser frequency, and the number $2\mathcal{M}$ of the Stokes–anti-Stokes satellites held fixed, the peak compression occurs at a distance

$$z_C\approx\pi(3k_0\mathcal{M})^{-1}(\omega_0/\omega_{p(C)})^2(\omega_0/\Omega)^2. \quad (6)$$

This estimate assumes that the outer sidebands were initially separated in time by roughly $\tau_b/2$ within one beat note. To catch up with the red sidebands at the beat note center, the blue sidebands need the propagation time $z_C/c\approx(c/\Delta v_g)(\tau_b/2)$, where the group velocity mismatch is $\Delta v_g\approx 2\mathcal{M}\Omega(\partial v_g/\partial\omega)_{\omega_0}\approx(3\mathcal{M}\Omega/k_0)\times(\omega_p/\omega_0)^2$.

We model the EM cascading by numerically solving the set of coupled nonlinear equations (3) and (4) with the boundary condition $a_0(0,\xi)=a_1(0,\xi)=A\exp[-\xi^2/(c\tau_L)^2]$ for the EM fields, and $N_e(z,\xi=-\infty)\equiv 0$ for the EPW. The beat note compression in the second stage is modeled by numerically solving the compressor equations described in the paragraph following Eq. (4). The nonlinear effects of relativistic mass correction, nonresonant electron density perturbations, and FSRS are included in the model. In all our simulations, $\lambda_0=0.8\ \mu$m. Spectra and amplitudes of the chirped and compressed single beat note (selected near $\xi=0$ where N_e has the maximum, and the phase modulation is the strongest) are shown in Fig. 2. The chosen spectral width of the laser at the exit of the modulator ($\mathcal{M}\approx 8$ sidebands on each side) and the maximum density perturbation $|N_e(z=0)|_{\max}\approx 0.5\times 10^{-4}$ determine the modulator length $z_8\approx 4.1$ cm. Such interaction length could be implemented in a plasma guiding structure such as a plasma channel. Alternatively, using a loosely focused laser with a focal spot of $r_0\approx 70\ \mu$m would match the modulator length z_8 with the twice the Rayleigh length, $2\pi r_0^2/\lambda_0$. The initial laser amplitude $A=0.2$, the modulator density $n_{0(M)}=8.75\times 10^{17}$ cm^{-3}, and $\delta\omega_l=-0.1\omega_{p(M)}$ were used for this simulation. The laser pulse duration is taken as $\tau_L=4.5\times 10^{-12}$ s (about one ion plasma period for a fully ionized Helium).

Figure 2(a) shows the beat note compression by a factor of 7.2 in intensity and shrinkage from the initial duration of $\tau_{b(\mathrm{in})}\approx 1.2\times 10^{-13}$ s to roughly five laser cycles. This is achieved in the compressor plasma of density $n_{0(C)}=25n_{0(M)}$ and length $z_C\approx 0.0275z_8\approx 1.1$ mm (such a short dense plasma can be created by ablation of a microcapillary [9]). Note that the linear formula (6) overestimates z_C by a factor of 3 since it ignores both precompression of the pulse in modulator [dash-dotted line in Fig. 2(a)] and additional bandwidth increase due to the nonlinear frequency shifts in the compressor.

By comparing the dash-dotted (with GVD) and dashed (without GVD) lines, we observe a surprisingly small effect of GVD on the laser pulse propagating in the modulator. For the plasma density $n_0=n_{0(M)}$ the estimate (6)

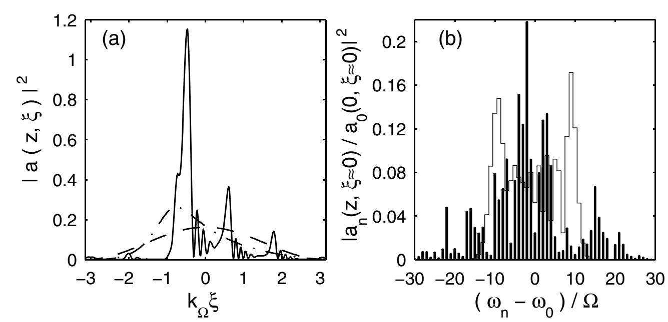

FIG. 2. The two-stage compression of a chirped beat note. (a) The beat note intensity at the entrance ($z=0$, dashed line) and exit of the modulator ($z=z_8$, dash-dotted line) and after the compressor ($z=z_8+z_C$, solid line). (b) The laser spectra near $\xi=0$ shown after the modulator ($z=z_8$, stairs) and after the compressor ($z=z_8+z_C$, bars). The nonlinearities and GVD in both plasmas are included.

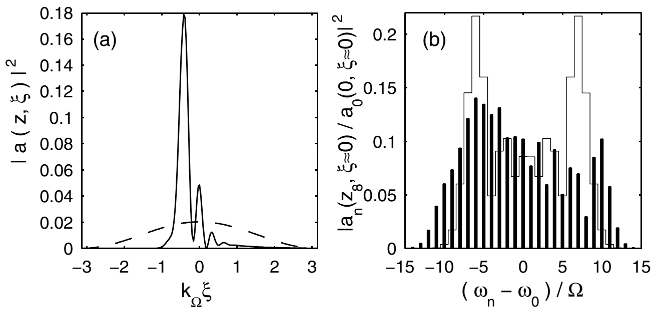

FIG. 3. The single-stage compressor with concurrent phase modulation (chirping) and compression. (a) Initial ($z = 0$, dashed line) and final ($z = z_8$, solid line) intensity profiles of one beat note. (b) Laser spectrum near the pulse maximum, $\xi \approx 0$, at the plasma border $z = z_8$, with (bars) and without (stairs) GVD and all the nonlinear frequency shifts.

yields $z_C \approx 1.65 z_8$, which suggests that a beat note compression at the exit of modulator could virtually be large. A qualitative reason for the lack of compression is that the higher-order Stokes–anti-Stokes sidebands are generated later and have less time to catch up with the fundamental. Recalling that $|a_{\mathcal{M}}| \propto |J_{\mathcal{M}}(z)|$ in the modulator, we define the half-growth length $z_{\mathcal{M},1/2}$ at which $|a_{\mathcal{M}}|$ reaches half of its maximum value, $|J_{\mathcal{M}}(z_{\mathcal{M},1/2})| = |J_{\mathcal{M}}(z_{\mathcal{M}})|/2$. Thereby, compression effectively takes place over the shorter distance $\Delta z \approx z_{\mathcal{M}} - z_{\mathcal{M},1/2} < z_{\mathcal{M}}$. We find that the analytic formula $\Delta z \approx \mathcal{M}^{-2/3} z_{\mathcal{M}}$ accurately fits Δz for $\mathcal{M} \gtrsim 5$. Therefore, $\Delta z \ll z_{\mathcal{M}}$ for $\mathcal{M} \gg 1$. Hence, for $z_{\mathcal{M}} < \mathcal{M}^{2/3} z_C$ the effect of the GVD is negligible in the modulator section because the distance Δz actually available for the compression is less than z_C. This is indeed the case under the parameters of Fig. 2, where $z_C/\Delta z \approx 6.6$. Otherwise, if $z_{\mathcal{M}} > \mathcal{M}^{2/3} z_C$ [or $z_{\mathcal{M}} > (\lambda_0/6)\mathcal{M}^{-1/3} d_M^{-2}$], the GVD is important in the modulator plasma.

We find this GVD-dominated regime advantageous for the practically convenient single-stage compression. We simulate it for the same plasma length z_8 and the EPW initial amplitude N_e as in Fig. 2; the electron density is now doubled to $n_{0(M)} = 1.75 \times 10^{18}\ \mathrm{cm}^{-3}$. The laser amplitude is $A \approx 0.071$, and the beat wave detuning is $\delta\omega_l = -0.025\omega_{p(M)}$. Figure 3(b) shows the generation of primarily redshifted cascade. The linear estimate gives $z_C/\Delta z \approx 1.7$ under the simulation parameters. In effect, the compression accompanies phase modulation, and the compression rate, as follows from Fig. 3(a), is about an order of magnitude in intensity.

In conclusion, we have demonstrated an approach for generating trains of high-intensity femtosecond radiation spikes via coherent EM cascading. Laser bandwidth broadening comparable to the fundamental laser frequency can be achieved for realistic experimental parameters. The GVD of radiation in plasma can compress positively chirped laser beat notes and thus create a train of few-laser-cycle EM spikes with a fixed time separation. Our method shows robustness against plasma nonlinearities and unwanted parametric instabilities. We conjecture that this technique could be used for compressing radiation beams to petawatt power and for electron acceleration [10].

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