Compression of Laser Radiation in Plasmas Using Electromagnetic Cascading

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Compressing high-power laser beams in plasmas via generation of a coherent cascade of electromagnetic sidebands is described. The technique requires two copropagating beams detuned by a near-resonant frequency $\Omega \approx \omega_p$. The ponderomotive force of the laser beat wave drives an electron plasma wave which modifies the refractive index of plasma so as to produce a periodic phase modulation of the laser field with the beat period $\tau_b = 2\pi/\Omega$. A train of chirped laser beat notes (each of duration $\tau_b$) is thus created. The group velocity dispersion of radiation in plasma can then compress each beat note to a few-laser-cycle duration. As a result, a train of sharp electromagnetic spikes separated in time by $\tau_b$ is formed. Depending on the plasma and laser parameters, chirping and compression can be implemented either concurrently in the same plasma or sequentially in different plasmas.

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For nearly two decades the chirped-pulse amplification (CPA) has been the dominant technique of generating ultrashort high-power laser pulses [1]. The basic premise of the CPA is to avoid optical nonlinearities of the amplified pulse by stretching it in time, thereby preserving its spectral characteristics. Recently, the exact opposite approach has been proposed by several groups: an advantage is taken of the nonlinearities in order to increase the frequency bandwidth of the pulse and reduce its duration. For example, nonlinear processes in gaseous media such as Raman cascading [2] and generation of high harmonics [3] have been utilized for making ultrahigh-bandwidth pulses. These pulses can be compressed to an ultrashort duration in another gaseous medium (further referred to as a compressor) with a high group velocity dispersion (GVD). These techniques cannot be extended to ultrashort laser intensities exceeding the ionization threshold. Fortunately, nonlinear properties of a fully ionized plasma can be used for increasing the laser bandwidth via phase modulation due to a near-resonant beat wave excitation of electron plasma waves (EPWs). The natural GVD of plasma can compress the resulting broad-bandwidth radiation into spikes of femtosecond duration.

In this Letter we demonstrate that nonlinear coupling occurring in plasma between two incident copropagating laser beams with a near-resonant detuning, $\Omega = \omega_{p(M)} = \sqrt{4\pi e^2 n_{0(M)}/m_e}$, results in the generation of a coherent electromagnetic (EM) cascade that consists of multiple Stokes–anti-Stokes components (here, $n_{0(M)}$ is the electron plasma density, and $m_e$ and $-|e|$ are the electron rest mass and charge). These EM sidebands, having frequencies $\omega_n = \omega_0 + n\Omega$ and wave numbers $k_n = k_0 + nk_\Omega$, are arranged in time and space so as to produce a periodic frequency modulation with the period of laser beat wave $\tau_b = 2\pi/\Omega$ [here, $k_0 = \Omega/v_{s(M)}$, $-M \leq n \leq M$ is an integer, $v_{s(M)} = c\sqrt{1-d_M}$ is the group velocity, $d_M = n_{0(M)}/n_e$ is the normalized electron density, and $n_e = m_e\omega_0^2/(4\pi e^2)$ is the critical plasma density for the fundamental laser frequency $\omega_0$]. If the number of sidebands $2M$ is of the order of $\omega_0/\Omega$, and $\Omega < \omega_{p(M)}$, the GVD of plasma can transform the broadband frequency-modulated laser pulse into a train of ultrashort (few-laser-cycles) radiation spikes separated in time by the beat period $\tau_b$. This can be done in a separate higher-density plasma where the laser beat wave is detuned far from the plasma resonance, and the EM sidebands are redistributed in space and time by the plasma GVD with the resulting compression effect. The envisioned setup (Fig. 1) thus consists of two plasma stages: (i) a low-density plasma (further called the modulator) with $\Omega < \omega_{p(M)}$ in which the incident two-frequency laser becomes frequency modulated (chirped), and (ii) a higher-density compressor with $\omega_{p(C)} \gg \Omega$ where the linear amplitude modulation (compression) of the laser beat notes occurs.

![Diagram](https://via.placeholder.com/150)

**FIG. 1.** Schematic of a two-stage EM cascade compressor. The frequency modulation (FM) occurs in a rarefied plasma. Denser plasma is used for the compression.

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The physical mechanism of the EM cascading is the following. The ponderomotive laser beat wave drives an electron density grating \( \delta n(\xi) \), comoving with the laser beams (where \( \xi/\nu_{p(M)} = t - z/\nu_{p(M)} \) is the retarded time in the modulator), thereby creating a comoving index grating \( \delta N = -(\delta n/n_{0(M)})d_M/2 \). A time-dependent phase shift, \( \Phi(z, \xi) \approx \delta No_0z/c \), can result in the nonlinear phase modulation, frequency shifting, and chirping [4]. Our approach is based on preserving a nearly sinusoidal temporal variation of \( \delta n \). Hence, the resulting laser beam consists of the equally spaced in time and space sidebands. Those are arranged in time and space so as to give the laser pulse a periodic in time chirp with the period \( \tau_\nu \). The number of sidebands \( 2M \) proportional to the plasma length \( z \) in the idealized case of negligible GVD can be large, and considerable broadening of the laser bandwidth can occur. When the GVD of radiation in the modulator is non-negligible, the laser undergoes an amplitude modulation as well. We show that proper adjustment of the plasma length and density can reduce the GVD effect while preserving the desirable frequency bandwidth.

In a realistic plasma, the EM cascading is a complicated interplay between the GVD of radiation and the sideband coupling through the driven electron density perturbations, the nonlinearities due to the relativistic increase of an electron mass, and the forward stimulated Raman scattering (FSRS). Our nonlinear analysis accounts for all these effects and describes the cascade development in time and one spatial dimension (1D). Because the longest time scale of the problem is only a few ion plasma periods, parametric decay of the EPW and consequent plasma heating [5] are ignored, and plasma ions are assumed to be a positive neutralizing background. Relativistic nonlinearities and FSRS are found to be relatively unimportant for the chosen parameters, and the EM cascading and compression proceed in agreement with the above qualitative scenario. Numerical modeling also reveals the parameter regime for which frequency and amplitude modulation occur concurrently, resulting in a single-stage compression of laser beat notes.

We assume that the circularly polarized planar laser beam incident on the modulator entrance, \( z = 0 \), consists of two spectral components with the frequencies \( \omega_0 \) and \( \omega_1 = \omega_0 + \Omega \) and amplitudes \( E_0(\omega_0, k_0) \) and \( E_1(\omega_0 + \Omega, k_0 + k_\Omega) \),

\[
a(0, \xi) = \text{Re} [e_0 e^{-i\omega_0 \xi/\nu_{p(M)}} a_0(0, \xi) + a_1(0, \xi) e^{-ika_\xi}],
\]

where \( |e_0| = 1 \) and \( a_n = eE_n/(m_n \omega_0 c) \). The EM cascade in the modulator,

\[
a(z > 0, \xi) = \text{Re} \left\{ e_0 \sum_{n=-\infty}^{+\infty} a_n(z, \xi) e^{-i\omega_n \xi/\nu_{p(M)}} \right\},
\]

is induced by the electron density perturbation \( \delta n/n_e \) of two spectral components with the frequencies and amplitudes \( \omega_n/\nu_{p(M)} = \omega_n - \omega_0, a_n(0, \xi) \). The nonlinear terms due to the relativistic increase of an electron mass are combined into the nonlinear frequency shifts

\[
\frac{\partial}{\partial z} - n_M \frac{\partial}{\partial z} - \frac{n_M}{\omega_0} \cdot \frac{\partial}{\partial \omega_n} \left( \frac{\omega_n}{\omega_0} \right) \frac{a_n}{a_n} = \frac{\omega_0}{\omega_n} \frac{n_M}{\omega_0} \left( \frac{\omega_n}{\omega_0} \right) \frac{a_n}{a_n}.
\]

The nonresonant components of the electron density perturbation as well as the relativistic corrections to the electron mass are combined into the nonlinear frequency shifts proportional to \( \Delta_n = n_M \rho_1 f^{(M)} + n_M \rho_2 f^{(M)} - f^{(M)} - \Omega^2 \), where \( \rho_1 = \rho_2 \), and \( f^{(M)} = \frac{\omega_n^2}{\omega_0^2} - f^{(M)} \). The term proportional to the normalized electron density \( n_M \) on the left-hand side (LHS) of Eq. (3) describes the GVD of EM sidebands. The EPW amplitude satisfies the weakly nonlinear equation

\[
(i \kappa_\Omega^{-1} \partial/\partial \xi + \delta \omega_1/\Omega) N_e + (3/16)|N_e/d_M|^2 N_e + (\rho_0 N_e + \rho_{-2} N_e^2)/8 = d_M (\rho_1/4),
\]

where the second and third terms on the LHS are the nonlinear frequency shifts coming from the relativistic mass corrections due to the longitudinal and transverse electron motion, respectively. The initial condition is \( N_e(z, -\infty) = 0 \) (unperturbed plasma ahead of the laser pulse); \( \rho_{-1}(z, \xi) \) is the normalized amplitude of the near-resonant ponderomotive force, and \( \delta \omega_1 = (\Omega^2 - \omega_n^2)/(2\Omega) \) is the beat wave detuning from the plasma resonance.

Equations analogous to Eq. (3) (with \( \omega_{p(M)}, \nu_{p(M)}, d_M, \xi \), and \( \Delta_n^{(M)} \) replaced by \( \omega_{p(C)}, \nu_{p(C)} = c \sqrt{1 - \beta^2}, d_e = n_{0(C)} / n_e, \xi = \nu_{p(C)} t - z, \text{and } \Delta_n^{(C)} \), respectively) model the nonlinear evolution of the cascade in the dense compressor plasma. The compressor density is such that \( \omega_{p(C)}^2 \neq n_e \Omega^2 \) for an integer \( n \); hence, the plasma response is nonresonant. The amplitude \( N_e \) does not explicitly appear in the compressor equations: it is absorbed into \( \Delta_n^{(C)} \) as \( f_{\pm 1}^{(C)} = \omega_{p(C)}^2/\omega_{p(C)}^2 - \Omega^2 \). Solution of Eq. (3) at the exit of the modulator (\( z = z_M \)) is the initial condition for the cascade equations in the compressor.

In the nonrelativistic case with GVD neglected, scaling laws for the EM cascading are particularly simple. From Eq. (3) with \( d_M = 0 \) and \( \omega_n = \omega_0 \) follow the conservation laws:

\[
\partial \rho_1/\partial z = 0.
\]

Hence, \( \rho_{-1}(z, \xi) \equiv |a_0(0, \xi)|^2 + |a_1(0, \xi)|^2, \rho_1 = a_1(0, \xi) a_0(0, \xi), \rho_{-2} = \text{for } l = 0, -1, \text{and, in the comoving frame, } N_e \text{ is independent of } z \text{ despite the laser evolution. Thus simpli-}

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fied Eq. (3) has the analytic [7] solution
\[ a_n(z, \xi) = \sum_{\nu=0}(-e^{-z/\omega_p})J_{\nu}(z\omega_p/\nu^2) \]
where \( J_{\nu}(z\omega_p/\nu^2) \) are the Bessel functions, and
\( a(z, \xi) \) and \( W(z, \xi) \) are the phase and absolute value of
the generating function
\[ w(z, \xi) = W e^{i\phi} = i(\xi/4)N_e(\xi) \].
Substituting \( a_n(z, \xi) \) into Eq. (2) yields the expression for a
train of phase-modulated beat notes:
\[ a(z, \xi) = \sum_{\nu=0}\sum_{m=0}(-e^{-z/\omega_p})\sin(k_m\xi + \nu/2) \]
where \( \nu/2 = (k_m/2)N_e(\xi)\cos(k_m\xi) \). The physical meaning
of this result is that, without GVD, the laser pulse undergoes
only frequency modulation. The magnitude of the plasma
wave depends only on the laser amplitude which remains
unchanged. This is valid for any pair of \( a_0(0, \xi), a_1(0, \xi) \),
and the corresponding \( N_e(\xi) \).

The frequency modulation is periodic in time with the
beat period \( \tau_b \) when \( N_e(\xi) \) is almost constant (this is the
cases when the inequality \( |\partial \rho_{-1}/\partial \xi| << |(\delta \omega)/\nu_p(M)\rho_{-1}| \)
holds). For \( |\delta \omega| >> (\Omega/4)^2 |\rho_{-1}|^2 |\Omega/\delta \omega| \), Eq. (4) yields
\[ a = \sum_{n=0}(-1)^n \cos(k_n\xi + (k_0/2)N_e(\xi)\cos(k_m\xi)) \].

Hence, a modulator plasma slab of thickness \( z_M = 2M/(N_c k_0) \) produces \( M \) sidebands on each side of
the fundamental, and a frequency bandwidth \( \Delta \omega \sim \sqrt{2\Delta_M \Omega} \).
Conversely, \( \Delta \omega \sim r_c z_c \bar{\alpha}_0 |\rho_{-1} - \rho_0| \), where
\( \bar{\alpha}_0 = 2\pi \omega_p/\omega_0 \) is the fundamental laser wavelength,
and \( r_c = c^2/(\omega_p k_0) \) is a classical electron radius.

From Eq. (5) follows that only for \( \delta \omega_1 < 0 \) the laser
wavelength decreases with time near the amplitude
maximum of each beat note. We define this chirp as positive.
The anomalous GVD in the dense compressor plasma
compresses thus chirped beat notes; the shorter-wavelength
(blueshifted) sidebands catch up with the longer-wavelength
(redshifted) ones, eventually building up the field amplitude near the beat note center. With the compressor nonlinearities neglected (\( \Delta p_{\rho(C)} = 0 \)), and plasma density, laser frequency, and the number \( 2M \) of the Stokes–anti-Stokes satellites held fixed, the peak compression occurs at a distance
\[ z_c = \pi(3k_0 M)^{-1}(\omega_p/\omega_p)^2(\omega_c/\Omega)^2. \]

This estimate assumes that the outer sidebands were initially
separated in time by roughly \( \tau_b/2 \) within one beat
note. To catch up with the red sidebands at the beat note
center, the blue sidebands need the propagation time
\( z_c/c = (c/\Delta v_g)(\tau_b/2) \), where the group velocity mismatch is
\( \Delta v_g = 2M \Omega(\partial v_g/\partial \omega)_{\omega_0} = (3M \Omega/k_0) \times \)
\( (\omega_p/\omega_0)^2 \).

We model the EM cascading by numerically solving the
set of coupled nonlinear equations (3) and (4) with the boundary condition \( a_0(0, \xi) = a_1(0, \xi) = A \exp(-\xi^2/\xi\tau_b^2) \] for the EM fields, and \( N_e(\xi, \xi) = -\infty \approx 0 \) for the EPW. The beat note compression in the
second stage is modeled by numerically solving the compressor
equations described in the paragraph following
Eq. (4). The nonlinear effects of relativistic mass correction,
nonresonant electron density perturbations, and FSRS
are included in the model. In all our simulations, \( \lambda_0 = 0.8 \mu \mbox{m} \). Spectra and amplitudes of the chirped and compressed single beat note (selected near \( \xi = 0 \) where \( N_c \) has the maximum, and the phase modulation is the strongest)
are shown in Fig. 2. The chosen spectral width of the laser
at the exit of the modulator (\( M = 8 \) sidebands on each side)
and the maximum density perturbation \( |N_e(\zeta = 0)|_{\max} = 0.5 \times 10^{-4} \) determine the modulator length \( z_s \) = 4.1 cm. Such interaction length could be implemented in a
plasma guiding structure such as a plasma channel.

Alternatively, using a loosely focused laser with a focal
spot of \( r_0 = 70 \mu \mbox{m} \) would match the modulator length \( z_s \)
with the twice the Rayleigh length, \( 2r_0^2/\lambda_0 \). The initial laser amplitude \( A = 0.2 \), the modulator density \( n_0(M) = 8.75 \times 10^{17} \ \mbox{cm}^{-3} \), and \( \delta \omega_1 = -0.1 \omega_2(M) \) were used for
this simulation. The laser pulse duration is taken as \( \tau_1 \) =
\( 4.5 \times 10^{-12} \) s (about one ion plasma period for a fully
ionized Helium).

Figure 2(a) shows the beat note compression by a factor of
7.2 in intensity and shrinkage from the initial duration
of \( \tau_{b(n)} = 1.2 \times 10^{-13} \) s to roughly five laser cycles. This is
achieved in the compressor plasma of density \( n_0(C) = 25n_0(M) \)
and length \( z_c = 0.0275z_s = 1.1 \) mm (such a short
dense plasma can be created by ablation of a microcapillary [9]). Note that the linear formula (6) overestimates \( z_c \) by a factor of 3 since it ignores both precompression of the
pulse in modulator [dash-dotted line in Fig. 2(a)] and
additional bandwidth increase due to the nonlinear fre-
quency shifts in the compressor.

By comparing the dash-dotted (with GVD) and dashed
(without GVD) lines, we observe a surprisingly small
effect of GVD on the laser pulse propagating in the
modulator. For the plasma density \( n_0 = n_0(M) \) the estimate (6)

\[ z_c = \pi(3k_0 M)^{-1}(\omega_p/\omega_p)^2(\omega_c/\Omega)^2. \]
Recalling that \( j \) later and have less time to catch up with the fundamental.

One beat note. (b) Laser spectrum near the pulse maximum, the analytic formula of its maximum value, the half-growth length \( z \) case under the parameters of Fig. 2, where simulate it for the same plasma length the practically convenient single-stage compression. We the GVD is important in the modulator plasma.

\[ z_{C} = 1.65z_{b} \]
yields \( z_{C} \), which suggests that a beat note compression at the exit of modulator could virtually be large. A qualitative reason for the lack of compression is that the higher-order Stokes–anti-Stokes sidebands are generated later and have less time to catch up with the fundamental. Recalling that \( |a_{M}| \approx |J_{M}(z)| \) in the modulator, we define the half-growth length \( z_{M,1/2} \) at which \( |a_{M}| \) reaches half of its maximum value, \( |J_{M}(z_{M,1/2})| = |J_{M}(z_{M})|/2 \).

Thereby, compression effectively takes place over the shorter distance \( \Delta z = z_{M} - z_{M,1/2} < z_{M} \). We find that the analytic formula \( \Delta z = M^{2/3}z_{M} \) accurately fits \( \Delta z \) for \( M \approx 5 \). Therefore, \( \Delta z \ll z_{M} \) for \( M \gg 1 \). Hence, for \( z_{M} < M^{2/3}z_{C} \) the effect of the GVD is negligible in the modulator section because the distance \( \Delta z \) actually available for the compression is less than \( z_{C} \). This is indeed the case under the parameters of Fig. 2, where \( z_{C}/\Delta z = 6.6 \). Otherwise, if \( z_{M} > M^{2/3}z_{C} \) or \( z_{M} > (\lambda_{0}/6)M^{-1/3}d_{g}^{2} \), the GVD is important in the modulator plasma.

We find this GVD-dominated regime advantageous for the practically convenient single-stage compression. We simulate it for the same plasma length \( z_{b} \) and the EPW initial amplitude \( N_{e} \) as in Fig. 2; the electron density is now doubled to \( n_{0}(M) = 1.75 \times 10^{18} \text{ cm}^{-3} \). The laser amplitude is \( A = 0.071 \), and the beat wave detuning is \( \delta \omega_{i} = -0.025\omega_{p}(M) \). Figure 3(b) shows the generation of primarily redshifted cascade. The linear estimate gives \( z_{C}/\Delta z = 1.7 \) under the simulation parameters. In effect, the compression accompanies phase modulation, and the compression rate, as follows from Fig. 3(a), is about an order of magnitude in intensity.

In conclusion, we have demonstrated an approach for generating trains of high-intensity femtosecond radiation spikes via coherent EM cascading. Laser bandwidth broadening comparable to the fundamental laser frequency can be achieved for realistic experimental parameters. The GVD of radiation in plasma can compress positively chirped laser beat notes and thus create a train of few-laser-cycle EM spikes with a fixed time separation. Our method shows robustness against plasma nonlinearities and unwanted parametric instabilities. We conjecture that this technique could be used for compressing radiation beams to petawatt power and for electron acceleration [10].

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